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A crowd size estimation task in the context of protests in Chile

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From a notion of critical statistical literacy, one of the purposes of learning statistics is for citizens to be aware of how statistical information, methods, and arguments shape society. Moreover, they should be able to critique and transform their society through statistics. In this paper, I tell the story of a 12th-grade statistical estimation task, anchored in the context of students' protests in Chile. Students' reflections suggest it is possible to design and engage in a teaching practice that lives up to the ideals of reading and writing the world with statistics. I display the teacher's voice throughout the paper in the form of excerpts from the post-intervention interview. It means to reflect our collaboration in the process of design, implementation and analysis.

Keywords: Confidence intervals, critical mathematics education, sample distribution, statistical literacy, statistical estimation.

Introduction

One of the shared quests among the Nordic countries, regarding mathematics education, is their push to connect it to democratic life and values. In the pre ICME 10 document, Dahl and Stedøy (2004) tell a shared story that goes from the enlightenment inspiration for equal access to education, to educational objectives involving the development of pupils' critical awareness of the role of mathematics in society. However, as they admit, "it is one thing to have a curriculum; classroom practice might be very different" (Dahl & Stedøy, 2004, p. 8). Concrete links between democratic ideals and actual mathematics educational practices are challenges to be tackled.

A similar drive is taking place in Latin America, since its process of post-dictatorship democratisation in the 1990s. In this context, one possible response arises to the challenge aforementioned, namely the notion of deliberative mathematics education (Valero, 1999). Among other practices, it involves a critical reflection on the shaping of society by mathematical models, by acknowledging how decision-making draws upon mathematical arguments and encouraging students to take part in such type of discussions in the mathematics classroom.

Statistics plays a significant role in how modern societies are read and written. As school mathematics subjects, probability and statistics have grown in relevance and the educational research community agrees on the critical stance all citizens are entitled to engage towards statistics. The role of educational practices in this regard is an ongoing agenda. This paper is embedded in a PhD research project exploring the *problématique* of coherence between the critical justification for the inclusion of statistics in high school, and a teaching practice living up to it.

The case of estimating crowd sizes

One exemplary case of mathematics in action is the use of estimates of attendees to demonstrations in order to validate – or invalidate – their cause. Organisers and authorities often provide different estimates reported in the media based on non-transparent methods and frame the results according to their respective agendas. This phenomenon provides an opportunity for students to engage in

critical reflections about the use of mathematics in society. Recent contingency in Chile provides a rich and up-to-date Latin American context for the exercise of the democratic right to gather and demonstrate. In late 2019, massive protests have led to starting a new constitutional process in 2020.

The timeline of this study may be relevant to the reader. This study draws the attention to a students' march occurred in April 2018 in Santiago, Chile, as a warning for recently elected government's educational policies. The task was implemented in October 2018, based on that demonstration. One year later, in October 2019, an unprecedented explosion of demonstrations and protests were triggered by high school students. Later, in December 2019, I interviewed the teacher about the task and its potential, considering the new national circumstances. I include excerpts from the post-intervention interview with the teacher about the whole process, positioning the teacher as a 'research subject' as opposed to 'research object' (Skovsmose & Borba, 2004).

The core of the case is a statistical estimation task, based on aerial pictures of a demonstration, implemented and discussed with a high school teacher in Chile. I describe the design process, outcomes of its implementation and its potential to produce critical reflections. The general research question to be addressed is: How and to what extent is it possible to design and implement a teaching practice coherent with critical purposes of high school statistics?

Theoretical framework: A dual-purpose

Statistics as being relevant to all citizens and justified to be taught for that reason is a common theme in the literature on statistical literacy. Most – if not all – frameworks for statistical literacy include some notion of *critical*, in the form of critical questions as knowledge elements, or a critical stance as a dispositional element (Gal, 2002). Moreover, hierarchical models such as Watson and Callingham's (2003) characterise higher levels involving some critical engagement with context. As a problematisation of the connection between statistical and critical literacies, Weiland (2017) addresses three problems when joining these literacies: the meaning of critical, the distinction between reader and writer of statistical messages, and the role of context. Briefly put, a critically statistically literate citizen can understand how social structures are shaped by statistics in real and eventually divisive contexts and contribute to change them.

Weiland (2017) further proposes a framework for characterising critical statistical literacy in eight aspects, four regarding the ability to read and four to write the world with statistics, respectively. Given the limitations of this paper, I define only four of them, exemplary of reading and writing, which I use later for the analysis. Hereby, an individual is able to read the world through statistics by (R1) "making sense of language and statistical symbols systems and critiquing statistical information and data-based arguments encountered in diverse contexts to gain an awareness of the systemic structures at play in society" (p. 41). He or she can also read the world by (R2) "evaluating the source, collection and reporting of statistical information and how they are influenced by the author's social position and socio-political and historical lens" (p. 41). As for writing the world, critical statistical literacy enables an individual to (W1) "negotiate societal dialectical tensions when formulating statistical questions, data collection and analysis methods, and highlighting such tensions in the results of a statistical investigation" (p. 41). Additionally, one should (W2)

“communicate one’s social location, subjectivity, and political context to others, and how it shapes one’s meaning-making of the world when reporting results of a statistical investigation” (p. 41).

Inquiry-based task design

Because of its anchoring in real-life modelling activities, inquiry-based mathematics education is the framework of choice for action. It can be defined roughly as “a way of teaching in which students are invited to work in ways similar to how mathematicians and scientists work” (Artigue & Blomhøj, 2013, p. 797). However, “researching the inner change process that learners’ undergo as they develop the ‘critical lens’ that is part of statistical literacy requires different and less direct approaches” (Petocz, Reid, & Gal, 2018, p. 81), such as open-ended investigations:

Teacher: I like structure very much because it allows any person to follow the class. Now, that structure does not have to be so rigid, and one can make changes, because, evidently, excellent ideas will emerge, different from what one had prepared.

Previous studies echo this idea. For example, by asking openly to interpret a chart relating school recess time with race, Brantlinger’s (2014) students engage spontaneously in discussions about systemic racism, on the grounds of statistical information. Kuntze, Aizikovitsh-Udi and Clarke (2017) show it is possible to design hybrid tasks that provoke both statistical or critical thinking, employing a “thinking-aloud” task consisting of the evaluation of a claim based on a diagram of births and deaths in Germany since 1945. These studies make use of real data from sensible contexts with problematic representations and a variety of possible interpretations. As a key improvement to the tasks, the authors propose a reflection-oriented framing, i.e. to have the learners ask themselves critical questions after solving the task.

Methodology: Collaborative classroom intervention

As methodological guidance for critical mathematics educational research, Skovsmose and Borba (2004) propose a general model consisting of a triad: *current*, *imagined* and *arranged situations*; (CS, IS and AS, respectively). Critical research possibilities are facilitated by “researching *with*, and not *on* teachers” (p. 220). Therefore, the definition of the CS, IS and AS are done in dialogue and collaboration with the teacher. The CS consists of a 12th-grade class in a school where:

Teacher: We work with class participation. However, usually, mathematics is a– among 45 characters you have in the class, only ten really enjoy it. The rest –a bit– suffer it.

The statistics curriculum for that grade includes sampling distributions and confidence intervals and the goal to evaluate information gathered from media outlets critically. Nevertheless, teachers felt that, so far, critical reflections are not possible:

Teacher: Because the amount of content we have to cover in all disciplines is excessive. It does not allow for reflection or having like this paused learning, respecting the students, respecting their time.

The IS is that students’ own ideas to approach the inquiries lead to statistical notions. As a researcher, I looked for opportunities to allow and provoke such reflections by proposing a set of real-life politically relevant inquiries connected to high school ideas of probability and statistics.

Production of data: The arranged situation

Teacher: It is useful too because they were fun activities. So you showed us like four activities, and we said: “all right, this is the best”.

That chosen inquiry is inspired by the “Counting People” task by Triantafillou, Psycharis, Bakogianni, and Potari (2018). The teacher pointed out that many students had attended the march in April 2018. It also fits the need to cover sampling distributions and confidence intervals.

As for the structure of the sequence, Blomhøj (2016) describes three main stages in inquiry-based teaching: setting the scene, independent work, and joint reflection. Accordingly, the AS starts by students reading two news articles from the same outlet; one where the organisers of the march predicted 60,000 attendees and one after the march including aerial photos, where authorities provide aftermath of 30,000 participants. The general inquiry is given: “How many people attended the students’ march?” Students’ independent work, guided by a working sheet given by the teacher and researcher, consisted of students making their estimates based on aerial pictures and a map. In Figure 1, students divide a picture in a mesh, take random samples, count and provide the mean to understand how the sampling distribution varies. Later on, they construct a confidence interval for the mean amount of people in a frame. They use Figure 2 to scale up their estimates to the full demonstration. Eventually, they challenge the use of this picture, since many of them attended and knew that the march went beyond what is showed in Figure 2. In the last part of the sequence, representatives of each group of students provide their results. Finally, they are encouraged to share their reflection about the methods used (as suggested by Kuntze et al., 2017), variety of results, the information given in the news articles, and form of work throughout the experience.



Figure 1: Source: Ahora Noticias

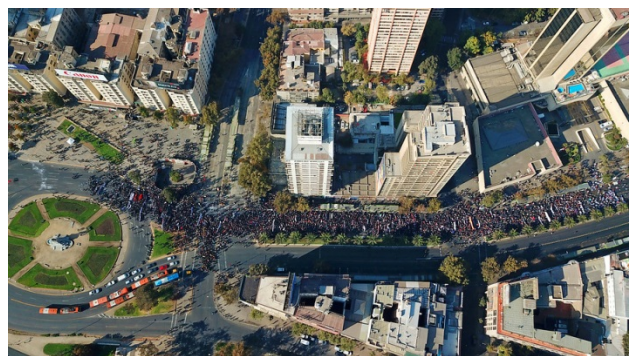


Figure 2: Source: Ahora Noticias

The overall *problématique* this study is embedded in is concerned about possibilities and challenges in actual teaching practice, so the implementation of the task was intended to be done in a real teaching context, as opposed to laboratory conditions. The classroom experience took place in three 12th grade mathematics sessions, during a week where the teacher had planned to teach the statistics part of the programme. In total, 35 12th-grade students (around 17 years old) participated forming six groups, led by both teacher and researcher. Data were collected in the form of audio recordings

on each group of students, a video recording as reference for transcription, and scanned written work by the students. Audio transcripts from the final joint reflection are used for the analysis.

Analysis: Reading and writing the world through statistics

For me to address the research question, aspects of critical statistical literacy must be evoked by the students. By selecting and coding students' interventions into categories by a single researcher, there is a risk for the method's reliability. To address this issue, in the spirit of the study, I let the voice of the teacher confirm the code indirectly. Her comments during the post-intervention semi-structured interview contribute to a triangulation of the coding process. In some cases, I showed excerpts from the intervention to the teacher to prompt a personal interpretation of the episodes.

Exemplary results for four of the critical statistical literacy aspects are summarised in Table 1. The first column indicates which reading (R1, R2) and writing (W1, W2) abilities are found. The second column shows a brief excerpt of the classroom intervention exemplifying this finding, where R stands for the researcher, T represents the teacher, and GXSY is the Y-th student from group X to intervene in the session. On the third column, there are teacher's comments from the post-intervention interview that provide insight and reliability to the findings.

Discussion

The task allows a student to make sense of statistical variation in the context of the task (R1). Acknowledging that everyone in the class had different estimates by using the same methods, he refuses to judge the media so lightly and gives them the benefit of the doubt (Table 1, see the first row). The teacher recognises this feature of the task, by explicitly saying that it makes sense to use statistical estimation, compared to the standardised test, they are soon to take finishing high school.

Students also read beyond the data and methods, evaluating the source, collection and its consequences (R2). They make use of their context knowledge (transportation patterns and shadow features) to realise that the pictures are taken too early, and therefore providing an underestimate on any case (Table 1, second row). The teacher was shown a partial transcript of this discussion where one student simply says it was early. Nevertheless, she remembered the observation about the shadows even after a year, making it clear how relevant this argument was.

As for writing the world, students from different groups negotiate dialectical tensions (W1) regarding the scaling of their first estimate to the full demonstration. The key issue, as the teacher also observes (Table 1, third row), is that the march is moving, and the pictures (data) are not. Students tackle this issue by providing results computed under different assumptions of what corresponds to the full picture.

On Table 1, the fourth row, I intended to go deeper and challenge the very importance of crowd sizes when it comes to support or reject a cause, and it is not engaged. This absence shows how a direct question will not necessarily provoke critical utterances if it is not correctly framed within the task. Instead, a student shares a simple reflection on the possibility of being agents and communicating one's social location when reporting their statistical investigation (W2). As can be seen, the teacher was sceptical of her students to achieve this level of reflectivity until I showed this transcript.

| No. | Students' exchange (October 2018) | Teacher's comment (December 2019) |
|-----|---|---|
| R1 | <p>G1S1: Beyond casting doubt about the media, and – as it is an own estimation, it cannot be known how many people exactly there are in the march, so I would not stay with that doubt. Since it is an estimate, it can be by itself or not a mistake, so I would not stay with it.</p> | <p>T: I believe everyone is engaged with the activity. I do not know if with learning the confidence interval or with applying it. What happens is that there it (...) made sense using it. Compared to the test, where they are asked for the formula directly.</p> |
| R2 | <p>G2S2: Because at that time, after an hour, they block the transit and no buses pass by. So, in this photo there are buses, it means it was too early.</p> <p>G3S1: (...) G2S2 must be right. That about the [photo of the] march being early, because it is like, observing more, yeah, but if it is known that the north is down [in the picture] and the shadows are from there [points left/East], because we are super early, before 12.</p> | <p>T: OK, there are like fun things emerging.</p> <p>R: Yes.</p> <p>T: But there is no mention to that about the shadows.</p> <p>R: No. No, that emerges – the one thing is that here it is said that is early.</p> <p>T: That is early.</p> |
| W1 | <p>G4S2: Yes, that the march was supposed to reach [to Los Héroes], but with the same mass, I mean, “relatively” with the same mass. It is like they were moving all together with the same people.</p> <p>G2S1: But we also computed until Torre Entel. More or less we suppose anyway that all this section [is] full of people. So, we computed [as if it were] complete anyway. How much did it give?</p> | <p>T: I mean, I see two things here, like... the thing about them managing to perceive that it is not something constant, that area is not entirely full. Another thing is that they manage to perceive that if it was earlier, there are fewer people, and then more people accumulate.</p> |
| W2 | <p>R: Now that you have these numbers, what – do they produce something in you? Do they change any of your opinions about the march, its causes...?</p> <p>G2S2: That oneself can verify whether it is true, and take one's own conclusions (pause) and believe oneself.</p> | <p>T: Now, nowadays, I mean, with all that has happened, I believe that now I could feel that reflection: “yes, it is really important to know a bit of mathematics to be able to be – not to believe so much in what the press tells me”. Still, I do not know if that reflection could have been reached. (...) Perhaps I am wrong.</p> |

Table 1: Excerpts from the classroom intervention and post-intervention interview

From the last comment on Table 1, fourth row, the teacher highlights that circumstances have changed, and current social unrest may represent an opportunity to let students reflect on the formatting power of statistics in society. Moreover, group work was innovative for her, but the tides are also changing at a local school level:

Teacher: We just had collaborative work training, and then that disposition [in groups] is quite normal. However, at that time, we had not had such training.

The confluence of evolving circumstances opens new possibilities for transformation. In the language of the methodological approach, a new current situation is reached (CS2). The teacher highlights that Chile's most massive demonstration in history just occurred on October 25th, 2019.

Conclusion

This paper shows the potential of a statistical estimation task to provoke students' reflections to read and write the world with statistics, addressing the research question of *how* and *to what extent* it is possible to design and implement a teaching practice coherent with a critical statistical literacy.

In principle, this is possible in close collaboration with the teacher throughout the design, implementation and the subsequent reflection process, enabling her to cover mandated mathematical content, namely sampling distributions and confidence intervals. We chose an inquiry-based approach, asking students openly to reflect on their investigation, and a real, contingent and relatable context; a demonstration to which some students, in fact, attended.

The didactical design allowed students to read the world through statistics. They made sense of statistical estimation, and the variation among different social actors' estimates found on the media. Being personally engaged with context enriched the possibilities to evaluate pictures as valid sources of data for a dynamic event such as a march. These tensions were the basis for writing the world with statistics, to the extent of negotiating assumptions to produce different calculations and results. Questioning broader structures – e.g. the use of numbers to stir the adherence to a cause – is a level yet to be achieved and requires more research. Being sensitive to national and local circumstances defines a new concrete situation to investigate, making the experience not only possible to replicate as a new iteration of an ever-transforming process.

Latin America is geographically far from the Nordic countries, but the journey to connect mathematics education and democracy is shared. The critical approach to probability and statistics shaping society is one path that teachers are willing to take:

Teacher: So, with all the revolution currently taking place, (...) we were thinking about which current topics, where there are problems, we could approach from mathematics. So, I do not know, we reached as a conclusion that we could teach about pension fund managers (...) and that is a very mathematical topic (...)

Researcher: And, in particular, related to uncertainty.

Teacher: Yes. And life expectancies!

In Chile, the current shake of the status quo can also respond to the conference theme of projecting the Nordic model into the future.

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