Designing for students’ decision-making and critical reflections
A tax paying game
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As a first step towards collaborative work with upper secondary school teachers, this paper proposes a didactic sequence that involves students making decisions under uncertainty and risk, building strategies using basic probabilistic contents. Within the framework of critical mathematics education, individual and collective reflections are addressed in the context of a taxation system game. The sequence is displayed into three units that set the scene: a game theoretical setting with a possibility of a bonus, a setting with risk of penalty, and a collective reflection on outcomes of taxation systems in a fantasy (but realistic) social context. Six entry points for assessing reflective knowing are presented, and connected to the intended learning outcomes.

Keywords: Critical mathematics education, reflective knowing, decision-making, probability.

Introduction

In this paper we present and discuss a didactical design for students’ personal and collective decision-making and related reflections in relation to an issue of great societal importance, namely tax paying and taxation systems.

The research reported is part of an ongoing Ph.D. project by the first author, in which he addresses the relationship between probabilistic reasoning and decision-making, within the framework of critical mathematics education. This relationship is meant to be investigated both theoretically and empirically.

In the empirical part of the project the main focus will be on the discussions in the mathematics classroom concerning the students’ mathematical based decision-making and their related reflections. Didactic sequences related to the tax paying game—and to other contexts—will be designed, tested and analyzed with respect to the students learning of curricula mathematics within areas of probability and statistics at upper secondary level. Particular focus in the designs will be placed on challenging the students’ decision-making and related individual and collective reflections. It is the research ambition to collect and analyze empirical data so as to develop further the possible meaning of critical mathematics education in relation to the teaching of probability and statistics at upper secondary level.

In particular, the aim of this paper is to give two main contributions. The first one is a proposal for a research-based didactical design. The design intends to engage decision-making under risk scenarios, within a context of societal importance.

The second contribution appeals to critical mathematics education research. The proposed sequence will help to address the following research question: how do students handle decision-making when uncertainty and risk are involved, under a fantasy (but realistic) tax paying scenario? There are many foci to describe and explain how students engage the activity. From a critical mathematics
education framework, we focus in mathematics education as empowerment for critical reflection about mathematical structures in society.

**Theoretical framework**

Since the German sociologist Ulrich Beck developed his theory on the risk society in late eighties, it has been generally accepted that the handling of risk phenomena and political decision-making under uncertainty are important major characteristics of technological societies. Beck (2006) summarized the meaning of the concept of (world) risk (society) in the following eight points: It means (1) neither destruction nor trust/security but real virtuality; (2) a threatening future, (still) contrary to fact, becomes the parameter of influence for current action; (3) both a factual and a value statement, it combines in mathematicised morality; (4) control and lack of control as expressed in manufactured uncertainty; (5) knowledge or unawareness realized in conflicts of (re)cognition; (6) simultaneously the global and local are reconstituted as the ‘glocality’ of risks; (7) the distinction between knowledge, latent impact and symptomatic consequences; (8) a man-made hybrid world which lost its dualism between nature and culture. (p. 222)

This characterization of postmodern societies places mathematics in the center of general education. Mathematical modelling and models are essential in establishing and dealing with risk phenomena on the societal level. And mathematical competences are indispensable for developing, understanding and criticizing such models and their applications in relation to both personal life and societal issues. However, in order for mathematics teaching to really contribute to a general education, which is relevant in the risk society and in order for the students to experience this relevance, in the practice of mathematics teaching, the students should sufficiently often be working with modelling phenomena or systems of societal importance and be challenged to handle risk situations and decision-making under uncertainties.

**Designing for students’ decision-making**

Games can support the promotion of probabilistic reasoning by relating it to decision-making. For example, Hernández, Yumi and Silva (2010) develop a sequence of game proposals, where students have to decide if they play or not. Batanero, Fernandes and Contreras (2009) show a full semiotic analysis of the Monty Hall problem and their didactical consequences. From a more societal context, de Souza, de Lima, Campos and Gerardini (2015) propose a didactical sequence involving bank loan systems for adults.

From a behavioral research point of view, individual decision-making processes are subjected to heuristics and biases (Kahneman, 2011; Kahneman & Tversky, 2007). School mathematics education may help to control these misconceptions, but it has been reported that in some cases it does not or even makes it worse (Fischbein & Schnarch, 1997; Serrano, Batanero, Ortíz, & Cañizares, 1998). The complexity of these decision processes has also been addressed from other perspectives. For example, Nilsson (2007) proposes a dice game and analyzes it from a contextualization framework, locating the attention in what is the problem students interpret from the given task.
Critical mathematics education

The research framework takes into account critical mathematical education, placing mathematics education in an uncertain relationship with democracy. There is no neutrality, neither an intrinsic resonance nor dissonance between these two notions (Skovsmose & Valero, 2012). According to Valero, Andrade-Molina & Montecino (2015), critical mathematics education’s preoccupations can be summarized in three: (1) the notion of mathematics in action; (2) the relationship between mathematics education, democracy and social justice; and (3) the imagination of new educational practices.

Critical mathematics education plays three roles in this study, in line with the three concerns mentioned. First, the notion of mathematics in action gives the basis to a design that involves real life mathematical structures in society. Social and private economic policies are often influenced by prescriptive mathematical models which, for example, define socially disadvantaged areas or ghettos (Sánchez & Blomhøj, 2016), or help to manipulate the audience for fundraising campaigns (Sánchez, 2014). In political processes, mathematical models are used to legitimize de facto decisions, sometimes on partly rational grounds and sometimes through direct manipulations with the models and their concrete application (Skovsmose, 2005, p. 94).

Secondly, it provides an opportunity to discuss about these structures. From a mathematics education point of view it is important to realize that the use of mathematics in form of models change the condition for democratic decision-making processes. This is analyzed in detail in Blomhøj & Kjeldsen (2010, pp. 558-560).

Finally, it provides guidance for imagining a new educational practice to work with teachers. The overall project will involve close collaboration with mathematics teachers on the development, testing out, evaluation and redesign of didactic sequences. This part of the project will use the methodology for interplay between the development of teaching practice and research within critical mathematics education as developed by Skovsmose and Borba (2004).

The didactic sequence

One example to be inspired was developed by Ole Skovsmose and Henning Bødtkjer, called “Family Support in a Micro-Society” (Skovsmose, 1994, pp. 125–140). It is not related to stochastics reasoning, but it helps to set the scene. The aim was to build a fictitious (but realistic) society and to distribute a District family support budget. The class was divided in groups and each of them represented a single District authority. The main goal was to illustrate how reflective knowing takes place.

In our context, we intend to involve explicitly basic notions of probability. The intended learning outcomes of the activity are that students should be able to:

ILO1. Distinguish between a situation under uncertainty and a situation under risk.
ILO2. Build strategies for decision-making under risk, with given probabilities of success with and without replacement.
ILO3. Reflect upon their own and others’ decision-making strategies in a social context.
ILO4. Engage in discussions about the role and function of mathematics in taxation systems.

As described in the theoretical framework, many policies are based upon mathematical models. We have chosen to engage students in the taxation system, since it is one illustrative example that concerns every citizen, helping the teacher, researcher and students to set the scene.

**Unit 1: Game theoretical uncertainty**

The first unit of the activity is intended to have students make decisions under uncertainty. The uncertainty will be about the other players’ – teams of students’ – behavior. The teams will not have the means for estimating the probabilities of the different possible choices of the other players’ scenarios.

Each team will represent a company or a family. Each of them will have a different monthly income and tax rate. They must calculate and decide individually if they will pay their tax or not. The teacher will collect their payment and if every team pays their corresponding taxes, every team will receive an equal amount of money as a bonus as a benefit of living in a society with tax financed services. This bonus must be attractive for lower income groups and not significant for the higher income groups.

Each team will only know their own income and tax rate. They will not have access to others’ information and decisions. The teacher will receive the payment as calculated and decided by each group in a closed envelope. Each simulated month they will either receive the bonus or not, and they can modify their strategies if they consider this information relevant.

**Unit 2: Penalty risk**

The second unit has the purpose of engage students into a situation under risk. The difference is that in a situation under risk, probabilities of different outcomes are known *a priori* or can be estimated based on data (Leitgeb & Hartmann, 2014).

This time, there will be no bonus, but a possible penalty. Each month, the State will audit a number of teams picked randomly with an equal probability, as an urn problem with replacement. After one year of simulations, one relevant part of the discussion will be how did they “learn” from each monthly outcome. It is very likely that students will fall into the gambler’s fallacy (Taleb, 2001; Serrano, Batanero, Ortiz, & Cañizares, 1998), acting as if each month’s outcome is dependent on the previous results.

Afterwards, the rules change in order to take the form of an urn problem without replacement. This is, once a team is audited, it cannot be audited again until everyone has been audited. Under these conditions, teams will decide under different probabilities of being audited for each month.

**Unit 3: Social context reflection**

So far, students have not been engaged with the purpose of tax collection. It is expected that in the previous units, decision-making processes would involve personal interest. Groups would build their strategies in terms of maximizing their expected revenue or at least betting to what they intuitively see as being most favorable. The teacher should now guide a discussion where the teams
share their strategies with the rest of the class. Some guiding questions for the groups to consider might be: How did we come up with such decisions? What was our goal in each case? Were we trying to achieve or avoid something in particular? Did our strategies work and how do we know this? Does the change from the first and second scenario influence our ways for making decisions?

The reflection should move towards a collection now (Skovsmose, 2005, pp. 176–179), as a collective discussion about the mathematical structures in context. The teacher will now show the tax collection results on each unit, and their respective evolution throughout time. To set the scene, the teacher must now engage the discussion about why this society has a taxation system. Students should propose possible needs for a common fund and where can this money come from.

**Research method**

The sequence is meant to both contribute as a teaching activity and a research instrument. In the following, we describe the plan for its implementation and analysis.

**Execution**

By the date of submission of this paper, the researchers have made a verbal agreement with one Danish upper secondary school teacher to apply the sequence. It will be implemented in a second year gymnasium class (15 to 16 years old), who are studying a curriculum with special focus in mathematics and social sciences. In principle, they will be around 25 students, who will be divided in five groups for the activity.

As described in the sequence, the teacher will be the executor, collector of students’ tax payment and expositor of final results of tax collection. One researcher will participate explaining the main ideas behind the study, and record students’ discussions in video format to be fully transcribed.

**Plan for analysis**

In order to address the research question, our findings must be informed by a coding of the transcriptions. The activity units are meant to engage and observe collective reflection among students about how probabilities and statistics mean action in their society. This reflective knowing can be summed up in the following set of questions or entry points, ordered in a level of reflectivity (Skovsmose, 1994, pp. 97–124):

S1. Have we done the calculation right?
S2. Have we done the right calculation?
S3. Are the results reliable for the purpose we have in mind?
S4. Do we in fact need mathematics to answer this question?
S5. How does this application affect our conception of a part of the world?
S6. How are we reflecting upon the use of mathematics?

Alternatively, within the framework of statistical literacy, Gal (2002) proposes ten sample “worry questions” to be asked by a statistically literate adult. There is an overlap between some of these questions and Skovsmose’s entry points. But there is an essential focus distinction: statistical literacy is a concept regarding consumers of statistical messages, not producers. So these questions are not meant for decision-making, but for evaluating others’ statistics-based claims.
The qualitative strategy of the analysis consists in detecting if students are asking themselves or the teacher these questions, and what leads to these reflections. Researcher and teacher will define the criteria for coding the appearance of the entry points (S1 to S6) and a level of spontaneity (say from 0 to 2). The role of the teacher as researcher is relevant, in the sense that, “if (…) the teacher becomes ‘objectified’, then, for instance, the quality of pedagogical imagination may be reduced” (Skovsmose & Borba, 2004, p. 221). Once obtained an inter-coder reliability of at least 80% in one minute samples, the researcher will finish the coding.

**Discussion**

The following discussion will be about how the units address the given intended learning outcomes, and on how the analyses of the students’ activities can contribute to answering the research question.

**Unit 1**

Our main expected outcome is that everything will depend on the first month outcome. If they get the bonus after the first month, it is most likely they will continue to do so. Otherwise, they will not count on others paying their taxes, and so stop paying as well. Of course, the conditions can be varied and additional information made available for the teams – for example the distribution of income and the tax rates.

This unit engages part of ILO1, in terms of experiencing a situation under uncertainty. Three entry points are expected to be engaged. S1 is engaged within each team via revision of calculation of tax payment involving percentage, addition of bonus, and result of subtracting the payment and adding the bonus. S2 arises as they will choose to pick which calculation is the right one according to their strategy. Some will not pay more than what they would receive as a bonus. S4 becomes important if students do not count on others paying, they might as well realize that they do not need to calculate anything, just avoid paying their tax.

**Unit 2**

In order to finally achieve ILO1, it is important that the teacher asks questions related to the difference between both settings, and guides the discussion towards the actual distinction. Under a risk situation, knowing the probabilities of different outcomes should bring more security and feeling of empowerment to students.

ILO2 and part of ILO3 come together as a constant building and self-assessing of strategies. After the setting involving replacement, the teacher should ask the students about how they have built their strategies over time, in order to check if the gambler’s fallacy has come to play. Then the no-replacement setting is presented, making clear that strategies are dependent on the previous outcomes.

As in the first unit, entry points S1 and S2 are addressed according to revision and choice of calculations. Percentage of income, subtraction of penalty, probabilities based on Laplace’s law, and expected values are meant to be discussed. S3 takes place as a reflection about the meaning of probability and expected value. Is the choice with a maximum expected value the best one? Is it more reliable just to pick the one with the best possible outcome, regardless of its probability?
S4 comes straightforward when a team realizes they are the only one left for auditing. In this case, the only calculation needed is the tax payment.

**Unit 3**

Regarding entry point S4, the discussion should be about the necessity of creating individual strategies, given the importance of collecting taxes.

Further, entry points S5 and S6 might be addressed with questions such as: Does our way of deciding have an impact in the total collection of taxes? Is it fair that some of you have higher tax rates than others? What would happen when people avoid paying their taxes? Why are we not just auditing everyone each period?

In general, we expect debate between individual profit and collective welfare emphasis. We expect a discussion regarding the fairness of auditing at random and the way it is performed. Regarding empowerment, we see two levels: mathematics as a tool to defend oneself, and mathematics as a resource to engage a collective discussion, questioning the mathematical structure behind.

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**References**


