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Experimental Evidence for a State-Point-Dependent Density-Scaling Exponent of Liquid Dynamics

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A large class of liquids obey density scaling characterized by an exponent, which quantifies the relative roles of temperature and density for the dynamics. We present experimental evidence that the density-scaling exponent γ is state-point dependent for the glass formers tetramethyl-tetraphenyl-trisiloxane (DC704) and 5-polyphenyl ether (5PPE). A method is proposed that from dynamic and thermodynamic properties at equilibrium estimates the value of γ. The method applies at any state point of the pressure-temperature plane, both in the supercooled and the normal liquid regimes. We find that γ is generally state-point dependent, which is confirmed by reanalyzing data for 20 metallic liquids and two model liquids.

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Decreasing the temperature (T) or increasing the density (ρ) by applying pressure to a liquid leads to a slowing down of the molecular dynamics and eventually to a glass transition if crystallization is avoided [1,2]. It has been demonstrated that for many low-molecular weight liquids and polymers, the relaxation time and other dynamic quantities can be superimposed onto master curves within the experimental uncertainty when plotted as a function of ρ/T [3]. The density-scaling exponent γ is often regarded as a material constant [4–6]. Indeed, recent experiments have shown that in certain cases γ is constant over a huge pressure range [7,8]. In this Letter we show, however, that γ for other systems is not constant but state-point dependent; this is done by using a novel formalism that allows γ to be determined from properties exclusively referring to the state point in question.

In the following we use subscripts to distinguish between different γ’s. Let τ be the structural relaxation time determined, e.g., from the dielectric loss-peak frequency. We quote τ in reduced units (i.e., in units of \( \sqrt{mp^2/3k_BT} \)) where m is the atomic, molecular, or polymer-segment mass and define the density-scaling exponent of τ as [9]

\[
γ_s(\rho, T) = \left( \frac{\partial \log T}{\partial \log \rho} \right)_s.
\]  

We use base 10 logarithms. In this definition the exponent generally depends on the state point in question and the quantity kept fixed, which is τ in the above case. The physical interpretation of \( γ_s \) is that it quantifies the relative contributions of volume and temperature to the variation of τ in the thermodynamic phase diagram [10].

In analogy to the structural relaxation time, the configurational adiabats can also be associated with a density-scaling exponent:

\[
γ_s(\rho, T) = \left( \frac{\partial \log T}{\partial \log \rho} \right)_{S_{ex}}.
\]

Here \( S_{ex} \) is the entropy in excess of the ideal gas contribution \( S_{id} \). \( S_{ex} \equiv S - S_{id} \). In general, two density-scaling exponents of different observables (structural, dynamical, or thermodynamic) differ, e.g., \( γ_τ \neq γ_s \). In Ref. [11] it was shown experimentally that \( γ_τ = γ_s \) at a state point of the silicone oil DC704, confirming a prediction of the isomorph theory [9] believed to apply to systems where van der Waals interactions dominate (but not to systems where hydrogen bonds dominate the Hamiltonian [12–14]).

The isomorph theory [9,12–15] is a framework for describing systems where the potential energy function \( U(R) \) possesses hidden scale invariance (here, \( R \) is the collective coordinate of the system). Hidden scale invariance can be formulated as the following criterion for two configurations \( a \) and \( b \): if \( U(R_a) < U(R_b) \) then \( U(\lambda R_a) < U(\lambda R_b) \) to a good approximation where \( \lambda > 0 \) is a scaling parameter [15]. For these systems, density-scaling exponents of many properties have the same state-point dependence because they are all controlled by the excess entropy. Thus, there is just one density-scaling exponent, namely, the one of Eq. (2) that is given by the slope of the so-called isomorphs [9] along which dynamical, structural, and some thermodynamic properties are
constant in reduced units. As an application, the isomorph theory explains Rosenfeld’s excess entropy scaling law [16–19], i.e., the fact that the relaxation time in many systems is a function of the excess entropy: \( \tau(S_{\text{ex}}) \) [20].

In the isomorph theory, \( \gamma \) generally depends on both density and temperature [15]. Previously, Alba-Simionesco, Kivelson and Tarjus (AKT) investigated the validity of a scaling law for activated dynamics where the density-scaling exponent is a function of only the density [2,21,22]. Regarding \( \gamma \) as a material constant is an even more restrictive assumption, which only applies rigorously if the potential part of the Hamiltonian can be approximated by a sum of inverse power-law (IPL) pair interactions \( r^{-n} \) plus an arbitrary constant (the “IPL hypothesis”; \( n \) is the same for all terms but prefactors may differ) [4,23–27]. In that case, the density-scaling exponent is state-point independent and the relaxation time is simply a function of \( \rho^2/T \) with \( \gamma = n/3 \).

In this Letter we investigate the state-point dependence of \( \gamma \) without any model assumptions in a treatment that emphasizes error estimates. In general, the following four scenarios are possible: (A) \( \gamma \) is a material constant (the IPL hypothesis), (B) \( \gamma \) is a function only of density (the AKT hypothesis), (C) \( \gamma \) varies throughout the two-dimensional thermodynamic phase diagram in matter controlled by the excess entropy (the isomorph-theory prediction [15]), or (D) the completely general case in which there are different density-scaling exponents for different quantities, each of which generally varies throughout the phase diagram.

We first give an expression for \( \gamma_T \) in terms of quantities that can be measured at isobaric or isothermal conditions that most experiments are performed. Inspired by Angell [28] we define the following generalized fragility [6,10,22,28–30]:

\[
m^A_T = \left( \frac{\partial \log \tau}{\partial A} \right)_B,
\]

where \( A \) and \( B \) are thermodynamic variables such as \( T, \rho \), or \( p \). Note that the generalized fragility is not defined as a dimensionless Angell-type index. Moreover, the temperature fragility \( m^T_T \) (which is negative) is related to the apparent activation enthalpy [6] \( H_p \equiv k_B(\partial \log \tau/\partial T)|_p \) by \( H_p = -k_B\tau^2 m^T_T \). When the temperature fragility \( m^T_T \) is evaluated at the glass transition temperature \( T_g \) at ambient pressure (0.1 MPa), the relation to Angell’s dimensionless fragility index [31] is given by \( m^A_T \equiv [\partial \log \tau/\partial (T_g/T)]_{p,T_g} = -T_g m^T_T |_{p=0.1 \text{ MPa}} \). The isochoric activation energy \( E_V \equiv k_B(\partial \log \tau/\partial (1/T))|_p \) [6] can also be expressed via a generalized fragility as \( E_V = -k_B T^2 m^V_T \).

We next consider the pressure fragility \( m^p_T \) and temperature fragility \( m^T_T \) because these two quantities are directly experimentally accessible. We rewrite \( \gamma_T \), Eq. (1), in terms of the ratio between two generalized fragilities by using the identity \( (\partial \log T/\partial \log \rho)_p = -(\partial \log \tau/\partial \log \rho)_T (\partial \log T/\partial \log \rho)_\rho \), resulting in

\[
\gamma_T = -\frac{\rho m^p_T}{T m^T_T}.
\]

Typically the pressure and not the density is controlled in an experiment. Thus, we aim for an expression involving \( m^p_T \) and \( m^T_T \). From the chain rule one has \( m^p_T = (\partial \log \tau/\partial p)_T (\partial p/\partial \rho)_\rho \). It follows that

\[
m^p_T = K_T m^T_T/\rho,
\]

in which \( K_T \equiv (\partial p/\partial \log \rho)_T \) is the isothermal bulk modulus.

Using the identity \( (\partial \log \tau/\partial T)_p = (\partial \log \tau/\partial \rho)_T + (\partial \log \tau/\partial p)_T (\partial \rho/\partial \rho)_T \), and inserting the thermal-expansion coefficient at constant pressure \( \alpha_p = -(\partial \log \rho/\partial \rho)_T \), one obtains

\[
m^p_T = m^T_T + \alpha_p K_T m^T_T.
\]

Finally, by combining Eqs. (4)–(6) we arrive at an expression that relates \( \gamma_T \) to directly measurable equilibrium properties

\[
\gamma_T = -\frac{K_T m^p_T}{T m^T_T + \alpha_p K_T m^T_T}.
\]

Thus, the state-point dependence of the density-scaling exponent \( \gamma_T \) can be obtained from the thermal-expansion coefficient \( \alpha_p \), the pressure and temperature fragilities \( m^p_T \) and \( m^T_T \), and the isothermal bulk modulus \( K_T \). The idea of computing \( \gamma \) from two fragilities has been discussed previously in studies of supercooled liquids [6,10,21,30]. Here we have generalized this approach to the entire liquid state, i.e., also in the normal equilibrium liquid phase far from the glass transition.

Because of the measuring methods favored in our lab [32], we determine the isothermal bulk modulus from the adiabatic modulus \( K_S \) measured from the speed of sound [32] via \( K_T = K_S p C_p / [p C_p + \tau C_S K_{S}] \), in which \( C_p \) is the isobaric heat capacity (which in the data discussed below was measured with differential scanning calorimetry).

We focus the investigation on two well-studied van der Waals liquids, tetramethyl-tetraphenyl-trisiloxane (DC704) and 5-polyphenyl ether (5PPE), with values of \( \Delta H_{f,0} \) and \( \Delta S_{f,0} \) at ambient pressure of 211 and 245 K, respectively [33]. As an example, Fig. 1 collects the quantities needed to calculate \( \gamma_T \) for DC704 from Eq. (7) where \( K_T \) is computed from \( K_s \) and \( C_p \) as mentioned above. Figure 2(a) shows the dielectric relaxation times at atmospheric pressure of 211 and 245 K, respectively. Figure 2(b) displays \( \gamma_T \) at four temperatures along the 0.1 MPa isobar. Figure 2(b) shows an increase of \( \gamma_T \) as temperature increases or density decreases (inset) at ambient pressure.
In order to evaluate to which extent the measured quantities contribute to the uncertainty of $\gamma_T$ and, in this way, to predict where one should give particular attention to reduce as much as possible the uncertainty, we use statistical tools. A large population of values for each variable ($N = 10^5$) are sampled by a Monte Carlo approach that assumes a Gaussian distribution centered about the mean with variance determined from the estimated error. The error is calculated by sampling a random collection of different scenarios for the variables in Eq. (7). The $P_{\gamma,i}$ values in the panels of Fig. 1 represent the pairwise Pearson correlation coefficients between $\gamma_T$ and the variables involved in the computation of Eq. (7). If $P_{\gamma,i}$ equals 100%, there is a total positive correlation while zero indicates an absolute lack of correlation. We find that the properties with the strongest correlation with $\gamma_T$ are the pressure fragility $m_p^T$, followed by the temperature fragility $m_T^p$. It is therefore recommended to measure these two generalized fragilities with high accuracy in order to reduce the uncertainty in $\gamma_T$. On the other hand, slight variations in the density and in the thermal-expansion coefficient have only a minor effect on the resulting values of $\gamma_T$. The final error estimates on $\gamma_T$ are shown as error bars in Fig. 2(b).
TABLE I. Density-scaling exponent of the structural relaxation time $\gamma_t$ at $p = 0.1$ MPa for different substances obtained from Eq. (7).

<table>
<thead>
<tr>
<th>Substance ($T_0$)</th>
<th>$T$</th>
<th>$\gamma_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>DC704 (211 K)</td>
<td>218 K</td>
<td>4.5 ± 0.8</td>
</tr>
<tr>
<td></td>
<td>222 K</td>
<td>6.1 ± 1.1</td>
</tr>
<tr>
<td></td>
<td>235 K</td>
<td>7.1 ± 1.5</td>
</tr>
<tr>
<td></td>
<td>242 K</td>
<td>7.2 ± 1.3</td>
</tr>
<tr>
<td>5PPE (245 K)</td>
<td>268 K</td>
<td>5.4 ± 0.2</td>
</tr>
<tr>
<td></td>
<td>284 K</td>
<td>7.8 ± 0.5</td>
</tr>
<tr>
<td>Glycerol (185 K)</td>
<td>230 K</td>
<td>0.88 ± 0.09</td>
</tr>
<tr>
<td></td>
<td>258 K</td>
<td>0.43 ± 0.05</td>
</tr>
<tr>
<td>DPG (195 K)</td>
<td>240 K</td>
<td>1.3 ± 0.2</td>
</tr>
</tbody>
</table>

Results are based on measurements presented in this study and data in Refs. [3,6,11,35,37–42].

Table I summarizes our findings for DC704 and 5PPE. The average $\gamma_t$ of DC704 is $6 \pm 1$ for the investigated state points. This is consistent with the density-scaling exponent 6.2 reported in Refs. [11,37] assuming a constant gamma (scenario A), suggesting that the exponent reported in Refs. [11,37] is the average of the actual, state-point dependent $\gamma_t$'s. In Ref. [11] it was shown that $\gamma_t = \gamma_S$ for DC704. Thus, of the possible scenarios outlined above one concludes that DC704 belongs to either B or C. To demonstrate the general applicability of our approach, we include in Table I the values of $\gamma_t$ for the two hydrogen-bonded liquids glycerol and dipropylene glycol (DPG) using own data, as well as data from the literature. Unlike the van der Waals bonded liquids DC704 and 5PPE, the hydrogen-bonded liquids have been shown to belong to scenario (D) with density-scaling exponents that depend on the observable [12]. The $\gamma_t$ values of the hydrogen-bonded liquids are much lower than those of the van der Waals bonded liquids. From the definition in Eq. (1) it follows that the dynamics of hydrogen-bonded liquids is less affected by a (relative) change of density than by a (relative) change of temperature.

Figure 3 shows that the density-scaling exponents are also state-point dependent for model liquids and metals. Figure 3(a) shows $\gamma_{S_{ex}}$ computed from molecular dynamics simulations [43] of the Lennard-Jones (LJ) liquid [44] and of the Lennard-Jones trimer suggested by Lewis and Wahnström [45] as a coarse-grained model for ortho-terphenyl (LW-oTP). In line with the experimental findings, the LW-oTP model shows an increase of the exponent with temperature, though less dramatic than in experiments (we note that this is an inaccurate model of real oTP, for instance, it predicts a wrong crystal structure [46]). In Fig. 3(b) we reanalyze experimental data for 20 metallic liquids [47], including metals where ab initio density functional theory calculations demonstrate hidden-scale invariance [48]. For the monatomic metallic liquids, the density-scaling exponent of excess entropy is estimated using $\gamma_{S_{ex}} = [\gamma_G - k_B/c_v]/[1 - 3 k_B/c_v]$, where $\gamma_G = \alpha_p K_F/\rho c_v$ is the thermodynamic Grüneisen parameter [49]. Figure 3 shows that the $\gamma_{S_{ex}}$ for liquid metals computed by reanalyzing experimental data compiled in Ref. [47].

FIG. 3. (a) Comparison of experimentally determined density-scaling exponents (open symbols) with values of two model liquids (filled symbols). (b) The density-scaling exponent of the configurational adiabat $\gamma_{S_{ex}}$ for liquid metals computed by reanalyzing experimental data compiled in Ref. [47].
density-scaling equation of state \[51\]. In this way, it is possible to determine the evolution of \(\gamma\) with density from PVT measurements, as was recently reported for several organic liquids \[52\].

Finally, we note that Eq. (7) for \(\gamma\) can be evaluated for any liquid regardless of its vitrification ability and that the formalism does not assume activated dynamics \[21\]. Thus, the suggested procedure can be used at any given point in the temperature-pressure plane. It has been shown that the isomorph theory allows one to make predictions for the variation of properties along the freezing line \[53\] and out-of-equilibrium relaxation \[54–56\], assuming that a liquid belongs to scenarios A–C. For these applications it is pertinent to know \(\gamma\) at the state points of interest.

In summary, we have shown that it is possible to determine the density-scaling exponent \(\gamma\) from dynamic and thermodynamic properties of liquids. The analysis shows that \(\gamma\) in general is state-point dependent. The expression presented in Eq. (7) connects \(\gamma\) to measurable quantities that can be estimated over a wide range of thermodynamic conditions, from state points near the glass transition to well above the melting point (including elevated pressures). The new route is free of model assumptions.

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