Book Review, "Mathematics in Ancient Egypt: A Contextual History, by Annette Imhausen"

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Published in:
Ganita Bharati

Publication date:
2018

Document Version
Early version, also known as pre-print

Citation for published version (APA):
Høyrup, J. (2018). Book Review, "Mathematics in Ancient Egypt: A Contextual History, by Annette Imhausen". *Ganita Bharati*, *39*(2), 163-183.

***Mathematics in Ancient Egypt: A Contextual History* by Annette Imhausen.**

Princeton and Oxford: Princeton University Press. xi+234 pp. Indexes of subjects, Egyptian words and phrases, and of mathematical texts. ISBN 978-0-691-11713-3. Cloth.

All major sources – and almost all sources, major or minor – for ancient Egyptian mathematics "proper" have been known and described since decades, the most important of all for more than a century (what is meant by "proper" will be made clear below). A new volume describing its history can therefore only be justified if it makes a new approach, raises new question or provides new answers to old questions.

That is precisely what Annette Imhausen does in her "contextual history".

Comparison with an article by Walter Reineke from [1978] containing "thoughts about the probable age of mathematical capabilities in ancient Egypt" shows that the very term "history" implies novelty. Reineke was at the time (as Imhausen today) the Egyptologist who by far knew most about ancient Egyptian mathematics, and his article can be taken to represent the best that would be done at the moment. Written by a competent Egyptologist, it was deliberately contextual, arguing from technical and social needs to the plausible existence of corresponding mathematical knowledge – but it is hardly history. From the technological feats of the early third millennium[1] it is concluded that the most advanced mathematics of the Middle Kingdom was created during the first three dynasties, while the basics – including the solution of simpler distribution problems and "simple equations"[2] – was developed already before the unification of Egypt, in the later fourth millennium. Reineke does not raise the question whether, for example, the aliquot parts[3] so characteristic of second-millennium mathematics were really

[1] All ancient Egyptian dates are evidently BCE.

[2] My translation (here from Reineke's German), as all translations in the following where no translator is indicated. Meant are problems which, translated with approximation into modern symbols, become $ax = b$.

[3] Given the question arising too easily from the use of the term "unit fraction",

the best tool for practice, or perhaps sub-units (also in ample use during the second millennium) would do better. Admittedly, at the time there was little material at hand from which the character of third-millennium mathematics could be derived – Sethe's still essential monograph on "numbers and number words" from [1916] was too limited in scope – and the adage that "the absence of evidence is not evidence of absence" can evidently be used both ways; in any case, however, Reineke's default assumption can be seen to have been that ancient Egyptian mathematics *had* no history, it *was there*, basically unchanged as long as Pharaonic Egypt itself existed. This, by contrast, was the reason that my essay review [Høyrup 1999] of volumes I–II of Marshall Clagett's *Source Book* [1989; 1995] carries the title "A Historian's History of Ancient Egyptian Science"; Clagett, indeed, has his eyes wide open to historical change, while at the same time being quite aware, for example, that the claim about a Middle Kingdom water clock that "never was made the like of it since the beginning of time" repeats a commonplace and is no proof of actual innovation (Imhausen quotes a very similar formulation from a Sixth Dynasty dignitary on p. 37).

Clagett's volumes were source books, albeit with ample commentary, and not meant as a general history. Gillings' *Mathematics in the Time of the Pharaos* from [1972], on the other hand, was almost exclusively dedicated to Middle Kingdom material. To a limited extent it was contextual (beyond the mathematical texts "proper" it presents the reader with excerpts from the volume calculations of the Reisner Papyri and a temple account (from Papyrus Berlin 10005, see below); but on the whole its historiography was that of a time capsule, informative but static. Sylvie Couchoud's *Mathématiques égyptiennes* from [1993] was even more of a Middle Kingdom time capsule, and less contextual (the passage from Papyrus Anastasi I on pp. 183*f*

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namely, "why didn't the Egyptians use general fractions, so much more efficacious than their unit fractions", I shall follow Eric Peet [1923] throughout and speak of "aliquot parts" (in French it would be *quantièmes*, in German *Stammbrüche*). Imhausen does much to dismiss the question about general fractions as irrelevant and misleading, but speaks in her book about "fractions", evidently hoping the reader will remember the objections.

is too short to count – on this papyrus much more below); yet Couchoud is explicit in her claim that everything in the capsule represents perennial ancient Egyptian mathematics, from before the pyramids until the Papyrus Akhmîm from the 6th century CE (pp. 11, 189*f*).[4]

In contrast, Imhausen's fairly slim volume is genuine history, and genuinely contextual. After an introduction discussing "Past Historiography" and presenting the "Aims of This Study" follow five main parts,

[1] "Prehistoric and Early Dynastic Period" (Chapters 1–4, pp. 11–29),

[2] "Old Kingdom" (Chapters 5-8, pp. 31–56),

[3] "Middle Kingdom" (Chapters 9–11, pp. 57–126),

[4] "New Kingdom" (Chapters 12–16, pp. 127–178),

[5] "Greco-Roman Periods" (Chapter 17, pp. 179–204).

In the end comes a "Conclusion: Egyptian Mathematics in Historical Perspective", a bibliography, and indexes.

With the partial exception of [5] (which is said on p. 183 to be "mainly an overview of the extant sources and some questions that can be raised"), all of the main parts discuss the socio-cultural context within which mathematics lived and served. No straitjacket is imposed on the material, for two good reasons: the source situation differs from one period to the other, and the relationship between mathematics and its socio-cultural context also cannot be reduced to an interaction between a fixed set of "factors". In later periods, for example, scribal culture and pride as inoculated in school are of importance; not so, evidently, before schools came into existence, which seems only to have happened after the disintegration of the Old Kingdom.[5]

So, [I] starts by a brief general sketch of the ample millennium preceding the Old Kingdom, 3900–2700 – where the process leading in its second half

[4] Couchoud's book is also marred by errors of all kinds, cf. [Høyrup 1996] and [Caveing 1995].

[5] Thus [Brunner 1957: 13]. Imhausen, who cites Brunner in general terms, is even more cautious on p. 60.

to the formation of the unified state now seems more open to questioning than it appeared a few decades ago (the earliest written sources, known since long, indicate conquest and submission, while recent archaeology suggests a gradual process). Next the invention of writing and the number notation is addressed, both of them first visible in connection with documents related to the unification but apparently results of a preceding development process. Since the notation for integers remained the same throughout the ensuing three millennia (apart from the use of cursive scripts and the introduction of multiplicative writings of the highest numerals), this notation is presented in detail here. In the end, the evidence for Pre- and early Dynastic *use* of numerals and numeracy is presented. The most technically precise information comes from the "Palermo Stone", annals written on stone during the Fifth Dynasty but indicating from the third king of the First dynasty onward the yearly level of the Nile (no doubt measured in order to determine the level of possible taxation for the year) in units of cubits, palms and fingers (almost certainly already standard units indicated on the nilometer).

Imhausen points out that even though archaeological conditions have favoured the survival of documents in funerary contexts (that is, in the extremely dry desert area), the copious appearance there of numbers and measures must reflect their use in the administrative daily life of scribes. This selective survival is taken up again later, in connection with the preservation of papyrus (probably an invention made in very early dynastic times); papyrus only has a chance to survive in the desert, not is the more humid living areas closer to the Nile. As Imhausen points out on p. 157, this "imbalance (and a general modern fascination with mythical Egypt) has led to the presumably wrong impression that the Egyptians were constantly focused on death and afterlife".

The closing summary of [I] points out that scribal literacy and scribal numeracy were intimately linked already during the earliest Dynastic phase, as also later.

[2] deals with the Old Kingdom, Third to Sixth Dynasties (c. 2667–2160), which produced the great pyramids, indubitable evidence of great technical as well as geometrical and administrative skills. This is also the epoch where

the solar civil calendar with a year of 365 days was introduced for administrative purposes – Imhausen does not enter into the technical discussions of precisely how it was done, but some kind of arithmetic was certainly involved; instead, true to her contextual aim, she explains the agricultural conditions for the calendar in greater detail than often done. Scribal autobiographies (funerary once again) demonstrate the general importance of numerate administration of temples and royal property. In spite of this somewhat richer documentation, the technical aspect is still mostly in the dark – we have evidence for metrologies with standardized sub- and sub-subunits, but nothing sufficiently complete to allow us to follow calculations. In any case (this particular adds to Imhausen and is taken from the translation in [Clagett 1989 İ, 80–87]), from the Third Dynasty onward indication of Nile heights on the Palermo Stone might include what Neugebauer [1926: 10; 1927: 5; 1969: 26] called "natural fractions" ($^1/_2$, $^2/_3$ and $^3/_4$ of a finger – the same fractions of cubits and palms also occur). It is clear from the few surviving papyri, moreover, that tabular formats were in general use, as well as systems of units with standardized sub-units.

In [2] Imhausen therefore presents the Egyptian metrologies for length, area, capacity and weight, in as far as they were in use during the Old Kingdom, together with their social use and the changes they underwent in later epochs. Also in [2], beyond the natural fractions Imhausen introduces the later use of aliquot parts or "unit fractions" and the way to transcribe them, without claiming that they were in use during the Old Kingdom (which indeed they were not according to the evidence we have).

[3] covers the Middle Kingdom, Eleventh–Thirteenth Dynasties (2055–1650), which is the first period from which we have "mathematical texts proper", that is, texts whose purpose it is to *present* or *teach* mathematics (be it mathematical tables, be it problem statements accompanied or not by a description of the way the problems are solved), not just to use mathematical techniques for instance for administrative purposes. Imhausen, as many others, simply speaks of them as "mathematical texts", which I shall also do in the following. She suspects that the genre of such texts is a *creation* of the Middle Kingdom, since they seem to belong in an educational context,

in itself an innovation.[6] They are dealt with in Chapter 9 under a heading "mathematical Texts (I): The Mathematical Training of Scribes"[7].

At first Imhausen offers a complete survey of the corpus of extant Middle Kingdom texts: the Rhind Mathematical Papyrus alias RMP (strictly speaking dating from c. 1550, in the Second Intermediate or Hyksos period, but claiming to be a copy of a Middle Kingdom original); the Lahun Mathematical fragments; the Papyrus Berlin 6619 (also fragments); the Cairo wooden Boards; the mathematical Leather Roll; and the Moscow mathematical Papyrus alias MMP. By far the most important of these is the RMP; it contains on one hand several tables, among which in particular the tabulation of 2 divided by odd numbers N from 3 to 101 (expressing 2 as a sum of aliquot parts of N),[8] as well as a large number of problems with detailed calculations; on the other it is systematically ordered, presenting so to speak the *elements* of Middle Kingdom mathematic. An important supplement is the MMP, also containing problems (but with fewer details of the calculations and not in systematic order), some of them going beyond the RMP (the volume of a truncated pyramid and the surface of an "basket", the meaning of which is disputed and left open by Imhausen). The Lahun and Berlin fragments principally confirm what is found in the RMP and the MMP, while the wooden boards and the leather roll corroborate the use of metrological reference tables and the way aliquot parts are handled.

After this brief survey of the corpus (5 pages in total) follows a mathematical analysis of select aspects of the contents of the texts. It begins, however, by a presentation of a tool for precise analysis of the single

[6] Since Imhausen refers earlier on to [Ritter 1992], which dates the unfolding of the calculation with general aliquot parts to the early second millennium, texts listing such calculations in table form can also hardly be imagined to have existed before the Middle kingdom, which may be part of her underlying argument.

[7] "Mathematical Texts (II)" belongs to [5], the Greco-Roman period being the other epoch from which mathematical texts are known. See below.

[8] The normal reference to a $2\div N$-"table" is actually somewhat misleading – a table would only state the results, but the papyrus also sets out the calculations. Rather than a table, we have an ordered sequence of solved problems.

problems, in the shape of a rewriting as a symbolic algorithm. This tool, inspired by Jim Ritter ([1998; 2004] – but also personal contact between Ritter and Imhausen), was used extensively in [Imhausen 2003], where it allowed a more subtle classification of problems than previously made. On the present occasion, however, Imhausen points out (p. 73) that

> the method is not without limitations, and a modern reader should be aware of them. The rewriting is straightforward as long as a step of the procedure involves a simple arithmetic operation [...]. Establishing the first step, however, is not trivial. [...] Hence the rewriting of the first step depends on the interpretation of the modern reader. [... It] includes a certain amount of interpretation, and different readers may arrive at different algorithms (where they "guess" missing instructions).
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> Certain parts of the problem text itself are not included in the symbolic algorithm, for example, [...] written calculations found after the instructions. [...]. If instructions are missing and not indicated by written calculations, the reader can only guess which steps were taken to fill the blank. However, in analyzing the texts, this might be considered a strength of the method, because it alerts the researcher to the fact that certain steps were not recorded. Thus, none of the previously mentioned limitations is a flaw in the methodology itself – but one should be aware of them.

As Ritter, but in contrast to many others who have recently begun speaking loosely about "algorithms" in early mathematical texts, Imhausen thus uses the symbolic algorithms "as an analytical, nearly linguistic tool that helps to foreground particular aspects of old texts",as expressed by Maarten Bullynck [2015: 5], and makes no claim that the Egyptian texts themselves *consist of* or *are* algorithms. Accordingly, she goes on to look closer at the principles governing the construction of the texts themselves, looking at the "formal elements" of procedure texts, that is (p. 76), "a title, a presentation of data, and a procedure (consisting of a sequence of instructions) to solve the given problem", and in many cases also "drawings, calculations performed in writing, and a verification"; then Imhausen takes up the global formal organization of the RMP, the MMP, and a short "handbook" containing two problems which belongs to the Lahun corpus.

Chapter 9 closes (p. 81) with a discussion of the *types* of mathematical problems encountered in the mathematical texts. Firstly, those without a (direct) practical background (or from which the practical background has been abstracted, leaving a technique with multiple possible applications) – the $2 \div N$ "table" belongs to this category. Secondly, "practical problems", most of which "can be interpreted as 'real´ practical problems, that is, those that are likely to have occurred in the work life of a scribe". Others, however, though dealing with entities that the scribe might regularly encounter in his work, would never present themselves to him *as problems*; in these cases, "a pseudo-practical background is used to phrase a mathematical problem" (on pp. 193 and 208 Imhausen speaks of such problems as "suprautilitarian"). Most of the practical problems (true or pretended) refer to administrative practice, a few are linked to architecture (slopes of pyramids, the volume of the truncated pyramid).

Chapter 10, "Foundation of Mathematics", draws on what can be extracted from the mathematical texts: the terminology for arithmetical operations, the techniques for the multiplication and division of integers (the latter easily involving difficult work with aliquot parts), tables for work with aliquot parts (primarily the RMP, but also the Mathematical Leather Roll) and tables for converting capacity measures.

Chapter 11, the last chapter in [3], turns to the relation between what we find in the mathematical texts and what was done in the mathematical practice of working scribes as we know it from a variety of sources: reliefs and models in tombs as well as administrative documents and letters – none of which informs us about the details of the mathematical work as do the mathematical texts but which can still be seen to be related to their general themes. 22 pages deal with distribution of rations as treated in the mathematical texts and in administrative life, with architectural calculations, and with land measurement.

No mathematical texts beyond two fragments on ostraca have come down to us from the New Kingdom, the Eighteenth to Twentieth Dynasties (1550–1069), dealt with in [4]. The period is rich, on the other hand, in material illuminating the social setting of mathematics.

Chapter 13 presents two lengthy administrative papyri with extracts – one dealing with endowments to temples meant to provide for the royal cult, the other with land administration. Chapter 14, “Mathematics in Literature”, first gives extracts from papyri serving the moral upbringing of future scribes and referring among other things also to the learning of calculation and to their prospective numerate-administrative tasks, and a model letter in which a scribe informs his master about how he has executed ration distribution in agreement with instructions. Eight pages summarize and bring extracts from the famous satirical letter Papyrus Anastasi I, in which one scribe chides another for his incompetence: he cannot determine the volume of a lake to be dug, nor calculate the rations for the workers performing the work; when it comes to finding the bricks needed for the construction of a ramp he fares no better, nor when he has to ascertain the number of workers needed to transport an obelisk or to erect a colossal monument. His handling of the rations for a military expedition is so confused that the responsible for the granary refuses him his seal. Chapter 15, “Further Aspects of Mathematics from New Kingdom Sources”, quotes wisdom literature for the importance of mathematical justice on the part of an overseer of the tax and land register and the “vizier” (the highest administrative official) – as pointed out, much more precise in this respect than comparable Old Kingdom texts, where closeness and faithfulness to Pharaoh overshadow other themes. Next, Chapter 15 deals with the importance of certain mathematical instruments in the effort to secure afterlife – the pair of scales on which the heart is weighed, and cubit rods whose symbolic value is not quite as clear. The chapter closes with a discussion of architectural plans, which however were not made to scale and are for this and other reasons difficult to interpret; the use of square grids in pictorial art is also mentioned briefly but the appurtenant system of “canonical proportions” only indirectly, by references to publications where it is dealt with.

During the seventh century Egypt was conquered for a while by Assyria, and from 525 to 404 and again from 343 to 332 by the Achaemenid Persians.[9] Alexander’s conquest followed in 332, and after his death Ptolemy

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[9] Dates according to [Shaw 2000: 364]. Imhausen only dates the first, long Persian period.

established his own dynasty – ruling until the Romans took over in the later first century. This is the "Demotic" period – thus named after the new cursive – partially stenographic – script that came into use.

As observed in the introduction to [5], it is "probable that during this time some exchange of Egyptian and Mesopotamian knowledge (including that of mathematics) took place" (p. 179). This was already pointed out by Richard Parker [1972: 6], according to whom "such influence is only too likely and can in certain cases be documented", and – in reverse perspective – by Reineke [1980: 1238], who saw no evidence for and little likelihood of inspiration from West Asia in Pharaonic mathematics – "such connections seem to have existed only after the Persian epoch". The introduction goes on with a description of the continued uses of mathematics for administrative purposes, documented in the land registers of temples (which were by now responsible for keeping them), in land lease contracts and in tax documents – rarely, as usual, allowing us to ascertain *how* calculations were made.

Here, however, the second batch of mathematical texts can assist. Almost all of those that are known were published in [Parker 1972]. Imhausen herself has worked for long on the material and intends to provide a new edition "in near future" (p. 183 n. 1), but she has reasonably chosen to base the discussion on the available edition.

Chapter 17 consists of three parts. The first of these gives a complete survey of the published Demotic mathematical texts, referring also to a few fragments and ostraca not known to Parker. The second presents Demotic arithmetical techniques, which are somewhat different from what we know from earlier times. For instance, a multiplication 13×17 is found as the sum 10×10+3×10+7×10+3×7, not by doublings and decuplings. In this connection Imhausen wonders whether multiplication tables were used, and points to a list in one of the Demotic mathematical texts which *could* at least look like one but could as well be, for example, an arithmetical exercise: a list of numbers which are not identified as products but are indeed products n×64, n going from 1 to 16. From multiplications Imhausen passes on to "Division (and a Note on Types of Fractions)" (p. 189). The parenthesis refers to a calculation where the outcome of 100÷47 is given as $2\frac{6}{47}$. This is obviously a transgression of the Middle Kingdom canon, according to which fractional values were to be stated as a sum of *different* aliquot parts. But the break is only partial: this is indeed a partial calculation, and the number $2\frac{6}{47}$ is thus an intermediate result – an unfinished division, which however is calculated with *as if it were a number,* so to speak.[10]

The third part of Chapter 7 presents a selection of problems from the mathematical papyri “focusing on those examples that occur as clusters of problems, thus enabling a more substantial analysis” (namely because damages in one problem may correspond to preserved text in one of the others). The first cluster deals with the transformation of the dimensions of rectangular pieces of cloth which conserve their area. In later times such problems would appeal to the inverse rule of three.[11] The Demotic way of thinking is much more concrete: a strip is imagined to be cut alongside one side of the piece, its area is determined, and finally this area is divided by the other side, which tells how much must be added. As Imhausen points out, this would certainly give rise to practical difficulties if done with real textiles – the problems, as argued, are suprautilitarian.

The other cluster consists of (equally suprautilitarian) “pole-against-the-

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[10] $2\frac{6}{47}$ is obviously not written with this notation – the fraction line was only introduced in the twelfth century CE. About this and similar expressions in the Demotic papyri Parker explains [1972: 9] that the “numerator is written first, and the denominator follows on the same line. In problems 2, 3 [Imhausen’s example], 10, and 13 (the Cairo papyrus) the numerator is underlined. In problems 51 and 72 the denominator is underlined”. One may perhaps see such fluctuation in notation as evidence of a conceptualization still in corresponding flux.

Kurt Vogel [1929: 43] observes two similar slips in RMP #81. First, the scribe writes $\dot{5}$ instead of $\dot{2}$ $\dot{8}$, betraying that he has something like 5 times $\dot{8}$ on his mind; in the next line 3 takes the place of $\dot{4}$ $\dot{8}$, 3 times $\dot{8}$ ($\dot{n}$ stands for the aliquot part $^1/_n$). However, these *are* slips, so to speak betrayals of forbidden knowledge; allowing the use of similar expressions within calculations is certainly an innovation.

[11] For example, in Mahāvīra in the 9th century CE [ed., trans. Raṅgācārya 1912: 88], and in the *Bamberger Rechenbuch* from 1483 CE [ed. Schröder 1988: 99 (facsimile), 216 (transcription)].

wall problems", a general type also known from Mesopotamia (and the chief evidence that transmission had taken place): a pole of length ℓ first stands vertically along a wall; its foot then moves out a distance d from the wall, and at the same time the top slides down a distance a. This dress is found in the Old Babylonian text BM 85196 (probably late 17th century according to the "Middle chronology"), with given ℓ and a. d is then found by simple application of the "Pythagorean rule". The dress is also found in the Seleucid text BM 34568; here, however, a and d are given, which from the mathematical point of views is a much more intricate problem.

In the Demotic Papyrus Cairo JE 89127-30, 89137-43, probably from the third century (the same as contains 10 variants of the cloth problem), the dress is used thrice for the Old Babylonian problem type, thrice for the equally easy type where ℓ and a are given, and twice in problems of the Seleucid type. Imhausen uses the algorithmic schemes to compare the Mesopotamian and the Demotic procedures, and concludes that they are similar but cannot be identical because of different ways to perform division (this concerns the comparison Seleucid-Demotic), and because there is no specific term for square root in the Demotic text (which touches the comparison Old Babylonian-Demotic). The Demotic solution further doubles a divisor, while the Seleucid procedure halves the quotient. Already on p. 198, before making the comparisons, Imhausen states (p. 198) that

> this is the only very distinct case, in which a problem existed in Old Babylonian times, still exists in Seleucid sources, and makes its first appearance in the Egyptian material of the Greco-Roman Period. The lack of evidence between the two periods, however, for Egypt as well as for Mesopotamia, makes it impossible to definitely prove a specific transmission, let alone when and how it happened.

As already quoted, [V] is said on p. 183 to be "mainly an overview of the extant sources and some questions that can be raised". Imhausen goes on as follows:

> our knowledge of the Egyptian culture during that period is still growing rapidly at the moment. It should also be noted that the material that is available from this period is immensely rich and would necessitate writing a second book on mathematics in Egypt during the Greco-Roman Periods,

> which would then include the mathematical texts (and the contemporary Greek and Seleucid material), the relevant administrative material, as well as other related texts (e.g., the inscription about fields on the walls of the Edfu temple).

In consequence, nothing more is said about the late period here. The closing Chapter 18 is a summary, which also looks at the parallel between Mesopotamia and Egypt. In both places, in spite of different starting points for the state formation process, mathematics is seen to have become "a state-directed activity" (p. 206) – one might even, in view of everything that has been said before in the book, call it a state-carrying and almost state-defining activity even in Egypt (as it certainly was in early Mesopotamia). Many of the tasks performed in the service of the state retained the same global character throughout, even though, in the Greco-Roman period, they were delegated to the temples. Global continuity, however, did not prevent change on other levels – for example in the metrologies in use. For this reason, and for others depending on transformations of scribal culture, mathematics itself developed over the millennia.

The final paragraph of the conclusion runs:

> Evidence presented in this book was selected to show individual aspects of Egyptian mathematics over several thousand years. A different selection may highlight further aspects of the same picture. May many more of these be painted in the future.

It may therefore be legitimate to have a general look at the aspects which were selected, and those which have been more or less left out of the picture.

The book is primarily a contextual, one might almost say a socio-cultural history. It does not attempt to make what we might call "cognitive history", that is, a hermeneutic tracing of ways of thought – and doing both satisfactorily within less than 250 pages would hardly be feasible, not least because cognitive history is the key battle-ground in the historiography of Egyptian mathematics; I shall go through one example summarily in order to show how far such discussions can go (yet without repeating the arguments advanced, which would take up many pages).[12] The discussion began when

[12] In a different perspective – namely as part of a discussion of "an outmoded

Léon Rodet objected to August Eisenlohr's explanation of the addition of difficult fractions in the RMP in terms of a "common denominator". Eisenlohr had not claimed explicitly that this was how the Egyptian scribe had thought, he appears to have had no pretention to make cognitive history (nor to have imagined it as a possibility). Rodet, on the other hand, saw precisely this as the crux and reproached Eisenlohr to let "a writer from the eighteenth century BCE [...] think and act too much as we think and act today" [Rodet 1881: 187*f*]. Instead of a common denominator he suggested the use of a *bloc extractif* – we may call it a "reference magnitude". We may illustrate the disagreement with an example from RMP 31: 1, $\ddot{3}$, $\dot{2}$ and $\dot{7}$ are to be added. The papyrus has this scheme:

1	42
$\ddot{3}$	28
$\dot{2}$	21
$\dot{7}$	6

The sum is given by error as 99 instead of 97. According to Eisenlohr, this can be understood (by us, presumably) as an addition $\frac{42}{42}+\frac{28}{42}+\frac{21}{42}+\frac{6}{42}$. Rodet. pointing out that common denominators are a much later Indian invention and using a wealth of parallels, suggests instead that the fractions are taken of the reference magnitude 42. The whole of it is 42, its $\ddot{3}$ is 28, etc. The sum is 97, which has to be measured by 42.

Peet [1923: 18*f*], loath to ascribe any cognitive particularism to the ancient Egyptian except deep-rooted conservatism, saw no difference except one of notations. Already in 1881–1882, Eisenlohr and Moritz Cantor had answered much in the same way, without understanding the cognitive point, and in [1881: 287–303] Eugène Revillout[13] made an extensive attack carried by arrogant zeal and ignorance. Raymond Archibald's bibliography of Egyptian

historiography of (ancient) mathematics" – Imhausen does much the same in [2009: 793–798].

13 [Revillout 1881]. The whole section "Revue bibliographique" carries the author names Eugène et Victor Revillout, but the pages in question speak in the first person singular.

mathematics quotes all three [1927: 143*f*] – from Revillout only the *ad hominem* assault in the last paragraph); it seems Archibald had never seen Rodet's article himself. Since then nobody except Kurt Vogel [1929: 30, 32 and *passim*] appears to have looked at Rodet.[14]

Imhausen does not enter this discussion. When going through the calculations of RMP #31 she merely observes (p. 91*f*) that an auxiliary number 42 is used,[15] and refers to [Neugebauer 1934: 137–147] for more examples; she does not consider whether this means that an underlying common denominator is involved (which Neugebauer argues against).[16]

Similarly, Imhausen observes in just so many words (p. 123) that "there is no evidence for the use of the rule of Pythagoras" without polemicizing against those writings which claim without support in sources that it was known. Nor does she waste her forces on pyramid speculations involving π or the Golden section.[17] The closest she comes (stated in connection with the area calculations of the RMP) is that the

> procedure given in the Egyptian sources indicates that the relation of diameter and circumference did not play any role in establishing the area of a circle. The extant types of problems with circles further corroborate this. Contrary

[14] Vogel, however, explicitly endorses Peet's view on p. 32, in spite of his general tendency to look for thinking and not just calculations. I myself had the accidental good luck four decades ago to read Rodet before I got hold of Archibald's bibliography. Sometimes indeed, in Orwell's phrase, "ignorance is strength".

[15] This is not what other writers on the topic since Neugebauer [1926: 24] have meant by the term. In the above example from RMP 31, they would speak of 42, 28, 21 and 6 (written in red in the papyrus) as *Hilfszahlen*/"auxiliary numbers".

[16] Imhausen's main analytical tool, the algorithmic transcription, is not fit for hermeneutics – it is by necessity neutral with respect to the thought patterns behind the numerical operations which it describes. But Imhausen is aware of its delimitations, as quoted above, and in the actual passage she does not even make use of it. She has simply chosen not to get lost in questions where – as the actual discussion shows – no answer can be firmly established.

[17] For those who are interested (in particular in critical analysis) I can point to [Borchardt 1922] (the classic) and [Herz-Fischler 2000] (much more thorough yet much less well known).

> to the pyramid problems, where individual problems calculate one of three parameters that are linked (base of a pyramid, height of a pyramid, inclination of a pyramid), the Egyptian procedure calculates the area of a circle only from a given diameter. The circumference is not even mentioned in any of the hieratic mathematical problems,

and her reference, quoted above, to the "general modern fascination with mythical Egypt". This latter remark is, I believe, the only somewhat polemical observation in the whole book.

All of this corresponds well to the program that is set out at p. 7:

> by presenting a variety of available sources and pointing out the limitations of the available material, one aim of this book is to encourage its readers to judge speculations about Egyptian mathematics with a critical and informed eye.

The critical eye, as we see, is to be that of the reader, while the author restricts herself to supporting it by information, leaving the many speculations unmentioned.

Others would probably have weighted the single matters differently – we all do. However, those whose main interest is the esoteric wisdom of the Egyptians, its refutation, or the agreement/disagreement between the Egyptian way of thinking mathematics and that of other cultures (not least our own) should shelve their own preferences for a while and learn from Imhausen that there is much more to be said about the history of Egyptian mathematics.

My own objections and disagreements are few. Firstly – quite technically – I would have liked to learn more about the use of rounding, which is indubitably a topic pertaining to the relation between mathematics and its social use, and which might illuminate for which purpose general aliquot parts were adopted. My personal guess (which *is* a guess, which is why I would like to know more) is that the adoption had more to do with the dynamics of the Middle Kingdom school institution than with needs generated by administrative practice. The precision of scribal calculations with aliquot parts often exceeded what could be measured in practice, so rounding was the natural practical choice. For instance, an accounting calculation from

Papyrus Berlin 10005 (one of the Lahun papyri), quoted by Imhausen on p. 109, gives to the temple worker 3̈ 4̇ 180̇ *h̲phw* jug of beer.[18] 180̇ of a jug is obviously below the limit of measurability. It is thus for good reasons that the scribe rounds to easier numbers: in Borchardt's reading [1902: 116] at one moment he divides by 42 and not by 41 3̈, as he should, and then makes a further error; Imhausen, p. 110, describes the rounding differently (and seemingly with reason), stating that 1 3̈ 75̇ is replaced by 1 3̈ and 2 3̈ 10̇ 250̇ 750̇ by 2 3̈ 10̇. From scattered reading I know there are other, more glaring examples; Egyptologist working on economic papyri must know much about them but may not have thought the information interesting ("I do not remember having seen it – but I am sure that if I had seen it I would not have thought about it", as an illustrious Egyptologist once answered a question I asked about a particular kind of fraction).

Secondly, I am afraid Imhausen sometimes takes ideology or self-indulgence for reality. So, on p. 58, we read that the "individual success of a nomarch [the efficient ruler of a *nome*, namely during the First Intermediate Period, after the collapse of the Old Kingdom], as expressed in the auto-biographies, was consequently no longer measured through his relation with a superior entity but through his ability to ensure social and economic stability within his own region and through his conduct toward the weak members of its society". That, at least, is what he has taken care to have inscribed on his funerary stela and therefore what he wanted to tell some-body – perhaps the judges of afterlife. But as Imhausen points out on p. 168, the weighing of the heart is meant to unmask possible fraud in such declarations, which should therefore not be taken for more than they are.

At least at one point Imhausen forgets that her readers are less competent than she is herself (in general she is a very good pedagogue). P. 70 n. 28 states that

[18] Actually, Imhausen writes 3̈ 41̇ 80̇, but this must be a printing or an overlooked scanning error – she tells to follow [Borchardt 1902: 180] with corrections from [Gardiner 1956] – but Borchardt has 3̈ 4̇ 180̇, and Gardiner only points out that the account must state daily, not monthly rations.

> Clagett, *Egyptian Mathematics* [...], useful as it is, should be used with care. The Rhind papyrus is given in the facsimile of Chace, Bull, Manning, and Archibald, *Rhind Mathematical Papyrus*, rather than in the form of accurate photographs, which can be found in Robins and Shute, *Rhind Mathematical Papyrus*.

Clagett's book shall certainly be used with care – which book should not? But photographs of the hieratic writing of the RMP is hardly very useful for anybody except trained Egyptologists, if not for judging the precision of geometric diagrams (which are anyhow meant as suggestions and not made to scale, as Imhausen points out on pp. 42*f*, 76, and 171). As Gillings [1972: 6] explains,

> with Egyptian scribes as with present-day handwriters, no two people write the same hand [...]. So standard practice among Egyptologists is first of all to transliterate the "cursive" hieratic into "printed" hieroglyphics, and then to translate the hieroglyphics into a modern language.

[Robins & Shute 1987] is explicitly made for "the not-too numerate ancient historian, the educated layman and the student of mathematics" (Preface, p. 7), and the photographs are certainly an attractive feature of a book to be sold in the British Museum Shop. However, if the booklet should succeed in the stated aim to arouse the interest of members this group, these would probably be better served by Clagett's reproductions (apart from the deplorable fact that their use presupposes a strong magnifying glass). Here they would find not only the old British Museum facsimile of the Hieratic text (which, quite likely, suffers from some imprecisions – I am unable to judge that) but also a Hieroglyphic transcription with interlinear transcription in phonetic Latin letters, which would allow them to identify the technical terms. Still better, of course, would be the original in [Chace et al 1929], which confronts this with another interlinear system, a literal translation written under the alphabet transcription. This volume, unfortunately, has never been republished, and it can only be found in the public and private libraries that acquired it when it was first published by the Mathematical Association of America; nor has anything similar appeared since then. So, I believe Imhausen to be mistaken when she states (p. 66) that while Egyptologists "tend to use the excellent edition by Peet", historians of mathematics simply prefer "the edition that was made by people they knew". They have other reasons.

Similarly, the statement (p. 94) is misleading that [Neugebauer 1927] and [Vogel 1929] (both dealing principally with the "table" 2÷*N*) "analyze the representations we find in the 2÷*n* table through modern mathematical formulas". Both use formulas to analyze conditions and possibilities – Neugebauer, for instance, says this on his p. 21:

> However, let us first leave the Egyptian way to calculate aside and ask when at all such a method can lead to a result.

Vogel, on his part, offers a Chapter I, "Theory of the partition of a fraction $\frac{2}{n}$ into aliquot parts" (p. 61), which similarly begins by a "Section A. (Not taking the Egyptian method into account". Section B (p. 81) is said to be "Using the main fraction used in the papyrus", which still uses modern writings and tools to analyze what is possible. But this is clearly distinct from Chapter II, "The 2÷*N* able in the papyrus itself (without the anomalous numbers)" (p. 103) and Chapter III (p. 157), "The anomalous numbers and the method of auxiliary numbers", both of which only use formulas (when they do so) to describe in short form what is done – similarly, one may say, to Imhausen's algorithmic transcriptions, and no more invasive of the analysis.

Formulas expressing elementary mathematics leave little space for disagreement – hermeneutics leaves much. One need only look at Neugebauer's review of Vogel's book [1931] to discover disagreement and thus hermeneutics.

My final objection is a bibliographic trifle. The claim that the "first two researchers to publish monographs on aspects of Egyptian fraction reckoning were Otto Neugebauer and Kurt Vogel" (p. 4) seems unjust to Friedrich Hultsch, who published "The Elements of Egyptian Fraction Calculation" in [1897] – a monograph of 192 pages.

Of these objections and disagreements, only the one concerning rounding touches Egyptian mathematics itself; the others have to do with historiography, not history. All are peripheral. In conclusion, Annette Imhausen's volume is a most welcome addition to the modest corpus of works describing the history of ancient Egyptiam mathematics in some depth – an addition moreover not only to the corpus but to the perspectives applied.

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