

A focus group study of Danish and English high-achieving high school pupils of mathematics

What can we learn from their verbalised explanations of how they learn mathematics

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A focus group study of Danish and English high-achieving high school pupils of mathematics:

**What can we learn from their verbalised explanations
of how they learn mathematics**

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Thesis submitted for the degree of Ph.D.

Centre for Research in Learning Mathematics:
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In the loving memory of my maternal grandfather
Einar Peter Dahl, 1910-1995

ABSTRACT

How do the high-achieving pupils say they come to understand a mathematical concept that is new to them? How can these reports be informed by various psychological learning theories of mathematics? These are the research questions of this study. The focus is on the metacognitive awareness of ten high-achieving high school pupils of mathematics from Denmark and England. Through qualitative un-structured interviews in smaller groups, I investigate how the pupils talk about their learning of a mathematical concept that is new to them. I focus on the cognitive learning process.

To answer the research questions I develop a model for analysis to get an understanding of what the pupils tell. I call this model the “CULTIS model for analysis” where ‘CULTIS’ stands for “Consciousness, Unconsciousness, Language, Tacit, Individual, and Social”. I interpret that these are six themes in which the various learning theories express themselves. I organise the six themes in three “pairs” of what seems to be opposite themes and I use the model as a systematic way to handle the many theories and it is a way to sort the pupils’ statements into areas. Subsequently I discuss the pupils’ explanations in each theme and relate them to the other themes. The choice of having qualitative interviews with open-end questions is mainly owing to a necessity of avoiding a self-fulfilling process if both the method of interview and the method of analysis are strongly influenced by theories.

Building on work by others, the study assumes that the pupils are able to talk about their learning. The study confirms this assumption, as it is very clear that the pupils are able to speak of their learning process in a way that makes sense to them and to the pupils they are interviewed with. They explain their learning in their own words but most of the time it is quite easy to identify some theoretical notions which reflects what the pupils say. A result of the study is that it seems that the pupils each have their own way of learning. However, there are also similarities, particularly if we look at each of the three pairs of themes. Seemingly contradictory theories are furthermore often seen in function within one single pupil. Sometimes it also depends on the branch of mathematics they learn. I do therefore also discuss the concept of complementarity between theories as well as the theoretical possibility of a synthesis. Towards the end of the thesis I also discuss the effect these results may have for teacher education and education policy.

SAMMENDRAG (Abstract in Danish)

Hvordan fortæller højtpræsterede gymnasieelever at de lærer et begreb som er nyt for dem? Hvordan kan disse forklaringer blive analyseret ud fra forskellige psykologiske læringsteorier om matematik? Det er forskningsspørgsmålene i dette studium. Dette projekt arbejder med den meta-kognitive erkendelse hos ti højtpræsterende gymnasieelever fra Danmark og England set i forhold læringen af et matematisk begreb. Gennem kvalitative ustrukturerede interviews i mindre grupper undersøges elevernes viden om deres egne kognitive læringsprocesser. Disse forklaringer bliver sammenlignet med et antal psykologiske læringsteorier om matematik. Fokus i projektet er således på den kognitive side af læring.

Jeg udvikler en analysemodel, som døbes: "CULTIS model for analysis" hvor 'CULTIS' står for "Consciousness, Unconsciousness, Language, Tacit, Individual og Social". På dansk: bevidsthed, ubevidsthed, sprog, tavst, individ og social. Dette er seks temaer, som jeg tolker, at de forskellige teorier udtrykker sig om. Disse seks temaer er organiseret i tre "par" i forhold til, hvad der umiddelbart ser ud til at være modsætningspar. Analysemodellen bruges som et systematisk værktøj til at arbejde med de mange teorier på én gang, og det er yderligere en måde, hvorpå man kan sortere elevernes forklaringer. Elevernes forklaringer inden for hvert tema bliver ydermere diskuteret og relateret til de andre temaer. Valget af kvalitative interview med åbne spørgsmål skyldes primært, at det er nødvendigt at undgå en situation, hvor både interviewsituationen og analyseapparatet er meget styret af teoretiske antagelser, hvorved hele processen kan antages at være selvopfyldende.

Ved at bygge på andres arbejder antager projektet, at eleverne er i stand til at snakke om deres læringsprocesser. Denne antagelse bliver bekræftet, da det er tydeligt, at eleverne er i stand til at tale om deres læringsproces på en måde, der giver mening for, ikke kun dem selv, men også for dem(n), de bliver interviewet med. Deres forklaringer om deres læringsproces bliver udtrykt med deres egne ord, men det er som oftest forholdsvis nemt at finde teoretiske begreber, der vurderes til at være dækkende for, det eleverne siger. Et andet resultat er, at det ser ud til at eleverne, hver har deres egen måde at lære på. Der er dog også visse fællestræk, specielt hvis man fokuserer på hvert af de tre par af temaer. De læringstyper, det var muligt at genkende gik endvidere på tværs af landegrænser. Samme elev kan også referere til modstridende teorier. Somme tider afhænger det af hvilken type matematik, der er tale om. Jeg diskuterer derfor også komplementaritetsbegrebet i forhold til forskellige teorier, lige som

jeg også diskuterer den teoretiske mulighed for en sammenfatning af forskellige teorier. Sluttelig fremkommer jeg med forslag til, hvordan resultaterne i studiet kan informere undervisningspolitikken og uddannelsen af lærere.

*A is for Algebra, thoroughgoing bore.
To pass it is asked you, no less and no more.
For though algebra's dreary, complex, and abstruse,
Thank God, out of school, it's of no further use.
("An Alphabet for Schoolboys", in: O'Rourke, 1987, p. 156)*

PREFACE AND ACKNOWLEDGEMENTS

This thesis is the end product of a process that began almost ten years ago. At that time I studied mathematics as a major subject at Aalborg University, Denmark, and I had chosen to write my Master thesis (*speciale*) within the didactics and theory of science of mathematics. To help me decide on the particular topic for my thesis I read the drafted Master thesis of Vithal (1992), and here I came across the issue of the role of language in learning mathematics. At once I knew what my topic should be and the curiosity that was born in this instant drove me throughout the process. Briefly described, my Master thesis was about mathematics as a language-game and learning mathematics seen as language-games transition mainly using the later Wittgenstein. I developed the so-called SIMUR-model to describe the important factor in such a process (in Danish it is called the FROMY-model). For a revised version, see Dahl (1995). I really enjoyed all the reading, writing, and thinking connected with working on a thesis. My model was my "child" and of course I conceived it to be the eighth wonder of the world! However, at the same time there began to grow a sense of dissatisfaction. What was the use of this? Could this rather philosophical work actually be used to anything but giving me my exam? The world was going crazy and here I was with all my books; really not doing anything that seemed useful. Well, I finished my thesis and in the summer of 1994 I got my major subject (*hovedfag*) in mathematics.

In Denmark it was not possible to get a Master of Science (*cand.scient.*) degree only in mathematics, one needed to study a second subjects to get a complete degree. I therefore decided to study social science as my minor subject (*sidefag*, 1½ years). Typically students in Denmark begin studying their other subject right after they end studying the first, but I needed a brake. I needed to *do* something. In the autumn of 1994 I therefore went to Rumania with *Danish Balkan Mission* to visit poor (mainly old) people and distribute food, clothes,

evangelical material and work in an orphanage, visit kindergartens, and distribute toys. I could write a book about my experience in Rumania but I would particularly like to draw the reader's attention to two observations, which to some extent determined the topic of this Ph.D. thesis. (1) One of the families I visited was an unemployed single-mom living in a tiny very old and ramshackle house with no front door or electricity. The front door was a carpet and the glass in the windows broken. Yet the nine-year old daughter was in the top of her class! (2) In the orphanage there was an eight-year old boy who very often sat by himself, doing mathematics, and he was extremely good at it. What was particularly surprising about this was that the children at this orphanage were rather under-stimulated. Through getting contact with an American adoption agency as well as through my own organisation, I learnt that the staff preferred the children to be rather passive and non-demanding. The staff had actually complaint about the Americans playing with the children as that had developed the children so much that "light came to their eyes" and they began to become more (normal) active children. Some of them also complaint about Americans adopting "their" children or helping the children's own families to take the children back, as some of the staff were afraid of loosing their jobs if all the children disappeared. At a neighbouring baby-orphanage I saw that in one room there were rows of cribs with altogether 30 children aged 0-3, sitting "rocking" and two (2!) women taking care of them as well as the cleaning and firewood.

The question that came to my mind was how was it possible that such environments could "produce" such high-achieving children? These two children might not be representative of all the children in the world and naturally this was not the whole story of these two children. But regardless of various methodological issues, these children nevertheless existed, and they made me wonder and think about how high-achievers "made it".

When I came back home to Denmark, I worked as a student teacher in mathematics at the university, studied my minor subject, and then worked for a school year, 1997-1998, as a high school teacher in mathematics and social science. I saw this as my chance to apply my SIMUR-model to a real teaching situation. However, this was far from easy. So many other things seemed to go on in the classroom and the learning process. But it was to some extent useful to consider the force of language to either convey a point or confuse a meaning, but it seemed to be far from all that described a learning process or which would make a learning situation successful. Issues like will for learning, self-confidence, practice, discussions,

ability, hard work and discipline, classroom culture, the book, home support etc. etc. “blurred” the picture.

At the end of this school year, a Ph.D. scholarship was advertised at Roskilde University and I applied for it. At the same time, I also applied for a research assistant position at the Danish National Institute for Educational Research (DPI), and I got this job from 1st of August 1998. DPI was an institute for research within the public sector under the Ministry of Education, but since July 2000 it forms part of the Danish University of Education. After working some months at DPI, I got the 3-year full time Ph.D. scholarship and began 1st of November 1998 at Roskilde University, Centre for Research in Learning Mathematics. This centre was a cross-university centre between Aalborg University, the Danish University of Education, and Roskilde University.

The idea behind my Ph.D.-work was initially to find out more about how pupils learn mathematics. At some point during the thinking I got the idea to ask the high-achieving pupils how they had actually learnt mathematics. Asking the high-achievers could perhaps result in “getting their secrets”. A lot of methodological issues were involved with this topic and I hope the reader would find them answered satisfying in the thesis.

As part of a Danish Ph.D. programme the Ph.D. student by law must work 840 hours for the university, mainly teach, the student must go abroad for a certain amount of time to be “international”, and the students must furthermore take courses corresponding to ½ year full-time study. To fulfil the last two requirements, I decided together with my supervisor Morten Blomhøj to try to come to University of Oxford, Department of Educational Studies to study for the M.Sc. in Educational Research Methodology. The courses here were what the Probationer Research Students at this department were required to follow at the beginning of their D.Phil. study. In this connection Anne Watson was my supervisor and I would like to thank her for introducing me to the psychology of learning mathematics and for all the interesting discussions. I would also like to direct a general thank to various people at the department, or connected with it, for interesting and valuable discussions. I would also like to thank my Ph.D. supervisor Morten Blomhøj for support, valuable comments, ideas throughout the whole period, and for being very flexible. I would also like to thank the teachers and pupils that took part in this research.

When I was in Oxford I also got the opportunity to work as a Research Officer for a European Union (EU) funded network. The work consisted of studying the impact of the EU

education and training policy and EU programmes on national level and was supervised by David Phillips. As such it did not have anything to do with the present thesis, but it was extremely interesting to work with education at another level. As I all together worked part-time for the network from April 2000 until August 2002, it extended the period of which I was paid from my scholarship since I took several breaks from being a Ph.D. student and thus took “months out” of the scholarship and placed these at the end. This way the length of the scholarship was extended 11 months. I would here like to thank the Institute of Mathematics and Physics (IMFUFA) at Roskilde University for being very flexible and allowing me, on several occasions, to do so and go to Oxford and work, precisely as I liked. At IMFUFA I would also like to thank Birthe Saltoft for her kindness and Tomas Højgaard Jensen and Charlotte Krog Skott for many joyful times and interesting discussions. Another organisation to thank is the Centre for Educational Development in University Science (CEDUS/DCN) at Aalborg University where I physically spend the second half of the Ph.D. study. I would like to address a general thanks to the people at the centre for a very cheerful time, which also including interesting discussions of topics in education research. I would also like to thank Ole Skovsmose, course lectures, and the group of Ph.D. students connected with DCN for all the interesting discussions we had at the half-year Ph.D. courses. In this connection I would also like to thank the board behind the Centre for Research in Learning Mathematics for at various occasions having commented on my work. Particularly Tine Wedege and Ole Skovsmose. Finally I would like to thank friends and family for encouragement and support, particularly my mother, Mona Dahl. Last, but not least, I would like to thank my Everlasting Father for everything He has given me.

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*To Fitzgerald's remark
that the rich are different from us,
Hemingway's riposte was:
'Yes, they have more money than us'.
(Bryant & Bradley, 1985, p. 6)*

1. INTRODUCTION

This study investigates how four Danish high school pupils and six English mathematics A-level¹ pupils say they learn a mathematical concept that is new to them. All the pupils are just about to graduate and have been taught mathematics at the highest level possible in their respective education system. They are also all high-achieving. The study uses various psychological learning theories to get a greater understanding of what the pupils say. I focus on the cognitive level of the development of the pupil's knowledge and understanding. The study is not centred on the existing school systems but on the high-achieving pupils' understanding of how they learn school mathematics, which one might assume is influenced by the school systems in which they were taught mathematics. I choose to use psychological learning theories, but one could argue that other areas within research of learning mathematics might also inform or explain the pupils' learning experiences. However, through focusing on the cognitive level of learning mathematics and the psychology of learning mathematics one gets an understanding of what *this* angle explains and how an understanding of the pupils' narratives can be informed by psychology. Furthermore Skemp (1993, p. 14) states that to fully understand how learning mathematics takes place requires knowledge from psychology:

Problems of learning and teaching are psychological problems, and we can expect little improvement in the teaching of mathematics until we know more about how it is learnt.

In this chapter I discuss the background, goal, and assumptions behind this study. I furthermore see it in relation to previous works of other researchers as well as discuss its relevance. This leads to a formulation of the research question.

¹ The English "A-level" is equivalent to the Danish high school. When I write "level-A" it refers to the high-level mathematics in the Danish high school.

1.1 BACKGROUND

A possible aim of this piece of work is well formulated by Niss:

if we understood the possible paths of learning mathematics, and the obstacles that may block these paths, for ordinary students, we would get a better understanding of what mathematical knowledge, insight, and ability are (and are not), of how they are generated, stored, and activated, and hence how they may be promoted (and impeded) for other categories of students, including those with severe learning difficulties, as well as those with a remarkable talent.
(Niss, 1999, p. 4)

In this quote Niss discusses how “ordinary” pupils learn mathematics and how this might be useful for other categories of students. Instead I focus on the pupils with a “remarkable talent” but one might still assume that an understanding of how one group of pupils learns mathematics could benefit other groups of pupils. This however rests on an assumption that the difficulties and ways of learning for various groups of pupils are *qualitatively* the same and only *quantitatively* different. Being qualitatively the same means that the pupils encounter the same *type* of problems, but they learn the same ways using the same methods. The main difference between the various groups of pupils is that the low-achievers experience these problems on a *larger scale*. Skemp (1993) seems to argue in favour of a view that there is a certain way to learn mathematics. He states, for instance, that the first principle of learning mathematics is: “Concepts of a higher order than those which people already have *cannot* be communicated to them by a definition, but *only* by arranging for them to encounter a suitable collection of examples” (my italicisation) (Skemp, 1993, p. 30). Sfard (1991, p. 2) discusses what she calls the peculiarity of mathematical thinking through reflecting on the epistemological and ontological status of mathematical constructs. She argues that mathematical concepts “can be defined - thus conceived - both structurally and operationally” (Sfard, 1991, p. 5). The processes of learning consist of an interplay between operational and structural conceptions of the same notions (Sfard, 1991, 1). Hence, she seems to argue that owing to the nature of mathematics there are some general features of how one learns mathematics. Some argue the contrary; that there seem to be qualitative differences between various sorts of pupils, since “the less able do not simply learn the same techniques more slowly. They develop different techniques” (Gray & Tall, 1994, p. 129). Furthermore “the possibility of formal definition is one factor which distinguishes advanced mathematical thinking” (Tall, 1991, p. 3). In favour of this view is also Krutetskii (1976), who describes

what distinguishes able pupils in mathematics from average pupils and incapable pupils. Also Brown (1997) investigates how undergraduate students perceive group theory, and he shows that there are different ways. However, one may argue that differences on a “single-concept-level” of mathematical learning does not necessarily rule out the possibility of greater similarity on a higher level.

The distinction between qualitative and quantitative differences may not always be clear-cut. There may also be borderline cases where one could argue that two strategies are so quantitative different, that it makes more sense to call them qualitative different. Nevertheless the distinction is useful to distinguish between learning problems/strategies as a matter of difference in *style* or difference in *number*. Furthermore this distinction is also used in educational research of reading where Bryant and Bradley writes, referring to the top-quote of this chapter: “The first possibility is that the difficulties which backward readers suffer are exactly the same in kind as those encountered by any other child. The only difference is that those of the backward reader are considerable greater. It is Hemingway’s alternative: no difference in kind but a large difference in amount. The second possibility - Scott Fitzgerald’s - is that backward readers do not just have more difficulties: they have an altogether different kind of difficulty” (Bryant & Bradley, 1985, p. 6). The question is therefore which of the positions (qualitative difference or quantitative difference) that is correct. The answer to this question is crucial for determining if, and how, weaker pupils may benefit from the result of this investigation. I see the options as follows:

Similarity Thesis: If the difference between weaker and stronger pupils is only quantitatively, I see two ways that the weaker pupils may benefit from this study:

(1) *Import Strategy:* If it turns out that among the ten pupils in this study, it is possible to identify a moderate (well below ten) number of general learning styles or types of strong pupils, the teachers could “show” the weaker pupils the stronger pupils’ “tricks”. A variation of this strategy is to find a personality type among the strong pupils, which is similar to a given weak pupil, and then introduce this specific strong pupil’s “learning style” to the weak pupil.

(2) *Individualism Strategy:* If it turns out that there is not a moderate number of learning styles or types of strong pupils, this may indicate that there are several, perhaps infinite, ways of learning mathematics. The reason why some pupils are weak could therefore

be that they have been “made weak” by being taught as if there is one, or only a very few, way(s) of learning mathematics and the ways the pupils have been taught just happened not to be “their ways”. On the other hand, even though there may be infinite ways of learning mathematics, it does not imply that everybody is able² to learn mathematics at the highest high school level. Here I will also stress that learning styles are not necessarily equivalent to abilities.

Diversity Thesis: If the difference between weaker and stronger pupils is qualitatively, other type of research is needed to improve the level for weaker pupils. Examples of research with the prime aim of improving the learning for pupils with difficulties learning mathematics is for instance seen in Magne et al. (1976). The Import Strategy is therefore not helpful here, but the Individualism Strategy may still be relevant.

I do not want to choose between the Similarity Thesis and the Diversity Thesis, but instead discuss the consequences of each of the thesis. In case of the Similarity Thesis, it seems to be quite straightforward to use the result of the investigation. The Diversity Thesis, on the other hand, needs some clarification: *what* might be the differences between the weaker and the stronger pupils? This is discussed below as well as the relevance of a study that may “only” help pupils who are already good at mathematics.

1.1.1 SOME CHARACTERISTICS OF DIFFERENCES BETWEEN VARIOUS GROUPS OF PUPILS

This section reveals that following the Diversity Thesis, one might assume that the high-achieving pupils differ from lower-achieving pupils in areas such as ways of analysing a mathematical problem, generalisation, procedures and emotional factors, and memory (Hadamard, 1945; Krutetskii, 1976; Tall, 1991). When Krutetskii writes about able, average,

² I will not go deeper into a discussion of the concept of ableness for learning mathematics as this is not the scope of this study. The views differ, and do mainly have to do with to what extent the ability of learning mathematics is “nature” or “nurture”. See for instance Gillis, DeFries, and Fulker (1992) who did an investigation on reading disabilities and mathematics disabilities using twin study. The investigation has as a goal to assess the genetic and environmental etiologies of both reading and mathematics disabilities.

and incapable pupils, I interpret this as saying something about high-achieving, medium-achieving, and low-achieving pupils.

Analysing

When high-achieving pupils work on a mathematical problem they perceive the mathematics of it *analytically*, which means that they isolate and assess the different elements in its structure, systematise them, and determine their ‘hierarchy’. At the same time they perceive the mathematical material *synthetically*, and here combine the elements into complexes and investigate the mathematical relationships (Krutetskii, 1976, pp. 227-228; Tall, 1991, p. 15). High-achieving pupils perceive problems as a composite whole, while medium-achieving pupils see a problem in its separate mathematical elements. It is only through analysing the problem that the medium-achievers are able to find the connections of the mathematical elements. Low-achieving pupils have great difficulties in establishing these connections, even when they achieved help (Krutetskii, 1976, p. 228). The speed of the analytical-synthetic process in the high-achievers is so fast that they see its ‘skeleton’ at once, and it is often impossible to trace the analytic-synthetic process. The fast grasping of a problem’s structure has been observed to be the result of exercises. However, the same exercises produce different results in pupils of different abilities, but high-achieving pupils need only a minimal number of exercises to make the analytical-synthetic perception arise ‘on the spot’ (Krutetskii, 1976, pp. 231-232). For low-achieving pupils “generalization came only after many exercises and after special training, which in our experience was never broad enough” (Krutetskii, 1976, p. 253). Thus, the analytical-synthetic process is therefore very slow, or non-existent, for the low-achieving pupils.

High-achievers therefore perceive the mathematics of a problem in a fast analytical-synthetic process. They furthermore perceive problems as a composite whole. This fast grasping is the result of exercises. Following this, low-achieving pupils need another kind of teaching as they seem not to benefit from exercises the same way the high-achievers do.

Generalising

The ability to ‘grasp’ structural relationships in a generalised form is a central feature for the productive thinking (Krutetskii, 1976, p. 234). Again the high-achieving pupils do this on the spot whereas low-achieving pupils have to have a lot of practice and exercises covering all possible cases and levels before an elementary level of generalization is possible (Krutetskii, 1976, pp. 240-242). High-achieving pupils can analyse one phenomenon and generalise from this by separating the essential features from inessential. Their method, explained by Krutetskii, is to infer “the features’ generality from their essentiality. ... to be essential means to be necessary and, consequently, it *should be* common to a number of phenomena of this type, that is, it should inevitably be repeated” (Krutetskii, 1976, p. 259). Lower-achieving pupils perceive the generality of features by contrast. Krutetskii therefore *inter alia* concludes that there is more than one method to generalize.

One might therefore expect the high-achieving pupils in this study to be qualitatively different from other pupils in how they generalise (using contrast or inferring the essentiality from one example) as well as the speed of it. What the high-achieving pupils report might therefore not be immediately applicable to lower-achieving pupils.

Procedures for problems-solving and emotional factors

The trials for problem-solving for low-achieving pupils are blind, unmotivated, and unsystematic. On the contrary high-achieving pupils have an organised plan of searching (Krutetskii, 1976, p. 292). High-achieving pupils switch easily from one mental operation and method to another, they have great flexibility and mobility in their mental processes in solving mathematical problems, and it is therefore easy for them to reconstruct established thought patterns. For medium-achieving pupils it is much harder to switch to a new method of problem-solving. Low-achieving pupils experience even greater difficulties in that (Krutetskii, 1976, pp. 278-282). For the high-achieving pupils the trials are a way to thoroughly investigate the problem through extracting information from each trial. Without having finished the trial, high-achieving pupils seem to know if they are on the right track. This is owing to the existence of an acceptor, which is a psychological control-appraisal

mechanism, where ‘line-of-communication’ is received from each mathematical operation. Under this acceptor lies a generalised and concentrated system of past mathematical experience (Krutetskii, 1976, p. 293). The high-achieving pupils thoroughly investigate the problem, which may suggest that they enjoy working with mathematics. This emotional factor is also seen in that these pupils often try to solve the problem in a more simple way or improve the solution and they show satisfaction when the solution was economical, rational, and elegant (Krutetskii, 1976, p. 285).

Here one might therefore assume that what the high-achieving pupils tell is unique for them, and therefore that lower-achieving pupils may not find the high-achievers’ procedures helpful. It would, at least, require that the lower-achieving pupils got a similar emotional attachment to mathematics. On the other hand the low-achievers may anyway benefit from learning a more organised way of problem-solving.

Memory

High-achieving pupils do not have a “better” memory than lower-achievers, but high-achievers usually remember the general character of a problem-solving operation and not the problem’s specific data. On the contrary, low-achieving pupils usually only remember the problem’s specific facts (Krutetskii, 1976, p. 299). The mathematical memory of high-achieving pupils is selective and only keeps the mathematical information that represents generalised and curtailed structures. This means that the brain is not loaded with extra information which makes it possible to retain the information longer and use it more easy (Krutetskii, 1976, p. 300).

This might influence how the high-achieving pupils describe how they learn. Their descriptions might be general as they remember the general character of problem-solving operations.

Differences between high-achievers

There are different mathematical minds (Hadamard, 1945, p. 5), and “even among men who are born mathematicians, important mental differences may exist” (Hadamard, 1945, p. 11). One can, for instance, distinguish between logical and intuitive minds (Hadamard, 1945, p. 106). In relation to intuition Tall writes that “Intuition is the product of the concept images of the individual. The more educated the individual in logical thinking, the more unlikely the individual’s concept imagery will resonate with a logical response” (Tall, 1991, p. 14). Thus, primary intuition is developed by itself before and independently of teaching, while secondary intuition is a result of systematic intellectual training (Tall, 1991, p. 14).

One may therefore not expect the high-achieving pupils’ descriptions to be completely alike. There may also be a difference in how they learn and how they say they learn and it therefore requires an analysis to see what there may be of similarities. Following the discussion about the Import Strategy, the above mentioned possible differences between weaker and stronger pupils’ learning style makes it clear why the Import Strategy is not in general useful if the Diversity Thesis is correct. The weaker pupils will not benefit from knowing for instance that other pupils analyse without using contrast, as it is not within the power of weaker pupils to use this tool. On the other hand, as also stated above, the lower-achieving pupils might benefit from learning a more organised way of learning.

1.1.2 RELEVANCE OF IMPROVEMENTS FOR PUPILS WHO ARE ALREADY HIGH-ACHIEVING

When one discusses the relevance of a study I will argue that one needs to clarify if one talks about research being *immediately* useful, or basic research being useful *in the longer run* when the research contributes to the general body of knowledge, and thus becomes useful indirectly. In this connection Niss describes that the “over-arching, ultimate end of the whole enterprise [the educational studies of mathematics] is to promote/improve students’ learning of mathematics and acquisition of mathematical competencies” (Niss, 1999, p. 5). Given his ideas are accepted, a guideline for didactical work is whether it improves the learning of mathematics, directly or indirectly. Regarding being useful indirectly, this study is relevant as

it poses a research question that has not been approached before in Denmark. The result of the study may be that it provides a clarification, which together with other information may be used in future planning of teaching or teacher education. Another possible outcome is that if it, for instance, becomes possible to suggest some types of high-achieving pupils, and this may make it possible to discuss things that were not possible to discuss before. On the other hand, even though one talks about research as something contributing the body of knowledge or being useful on a long term basis one does, in my view, not get away from the question if *any* new research is relevant, just because it is new. In other words: Is any “adding” to the body of knowledge relevant? Should the fact that something is “new” be a kind of “carte blanche” for researchers to do anything? Furthermore, from a more practical perspective, since research resources are limited, would it not be *more relevant* with a study that focussed on helping pupils having problems learning mathematics? Taxpayers’ money (or private funds’ money) can only be spent once and any government (or fund) must therefore prioritise. For instance: more money to research in cancer treatment or more money to (perhaps) irrelevant educational research? Is it not the duty of governments and funds to make sure that money are being spent where they are needed most? This does however not mean that research should only follow trends, and further, it is sometimes experienced that doing research in one area gives side-developments in others, or one might find something one never thought about looking for. Even still, my opinion is that when discussing the relevance of a study one must argue more precisely for the usefulness of the study.

Thus, the question is that if the Diversity Thesis mentioned above is correct, the information of the study might only be useful to improve the learning of pupils who already do well in the education system. Some may raise objections to the relevance of such a study. This might be the case particularly in Denmark owing to our culturally feature characterized by the *Jantelov* (‘the law of Jante’, where Jante is the name of a fictional small town; Sandemose, 1955). The *Jantelov* is basically a cultural effort to keep everyone in the society down in mediocrity and phrases and ways of thinking such like “do not think you are anyone special” or “do not think that you are better than the rest of us” is common. Below I therefore argue for the relevance of the study, even when the Diversity Thesis may be correct. I will firstly refer to the latest year’s discussion about a decline of the level of knowledge in mathematics. This has included a discussion of problems of correspondence between various steps in the educational system as well as the expectations from working life. Secondly I will

refer to the rather poor results for the Danish pupils in international comparisons. Thirdly I will argue from a perspective of equal opportunity, and fourthly refer to research showing that the high-achieving cannot just take care of themselves.

The expectations from other educational institutions and working life

A pamphlet from the non-governmental organisation (NGO) ‘The Confederation of Danish Industries’ (Dansk Industri, 2000a) states that globalisation makes it necessary with more co-operations between the industry and the universities, and that there is too much width and too little elite at the Danish universities. One can pose the hypothesis that knowledge of how high-achieving last-year high-school pupils learn mathematics might prove to be useful to create even better last-year pupils and hereby create a better basis for an elite at the university. In another pamphlet from the same NGO there is a call for a raise of the level of the whole education area (Dansk Industri, 2000b). The then Minister of Education, from the centre-left government, on the whole praised this latter initiative, even though there was a disagreement in the objective of creating an elite (Internetavisen Jyllands-Posten, 25 September 2000).

On European Union (EU) level there is a similar concern, for instance seen in a press release, 2 March 2001, about education and youth affairs titled: “How can education and research meet the demographic challenge?” The press release was from an informal meeting among the ministers of education and research held in Uppsala on March 1-3, 2001 as a preparatory meeting prior to the European Council in Stockholm on March 23-24, 2001. An important topic of discussion at these meetings is the goal of making the EU the most competitive and dynamic knowledge-based economy in the world. The press release states that the EU ministers of education and research have discussed how the interest among young people for natural sciences and technology can be stimulated and how the EU Member States can increase the recruitment to education and research in these areas. The demand for competence and skills at all levels is expected to continue to grow. Owing to the very rapid technological and economic development there is also a real and growing deficit in trained specialists, in information, and in other leading technologies. It is furthermore stressed that measures to stimulate recruitment must comprise a general renewal of pedagogy and good

links to working life and industry through the whole educational system.³ On the EU agenda one can see that having a good and competitive economy is linked to having for instance ‘trained specialists’ particularly in the area of mathematics and natural sciences.

Finally, mathematics professors at the universities have in later years complained about the decline of the level of knowledge of the pupils from high school. High school teachers also complain about the “material” they get from the elementary school. In an evaluation report evaluating the higher education in mathematics, physics, and chemistry from 1998 it is written that the previous very problematic transition from high school to university has become smaller but the report also states that this conclusion is only based on discussions with graduate students and that for instance students who dropped out are not present in the study (Evalueringssentret, 1998, p. 32). Furthermore, the evaluation report states that the level of education is high, also seen in a Nordic as well as an international perspective, but there is still basis for improvements in both the number and the quality of graduates (Evalueringssentret, 1998, p. 31).

Another argument for the demand for a study like the present is that this study has partly been undertaken at a Danish general academic high school, and research in this area is not very developed. Szomlajski (1997, p. 4) writes for instance that research in education in this field in Denmark until now has rested on a few “fiery souls”.

In summary, both positive and negative things can be said about the education system, but regarding the expectations from education institutions and the working life, there seem to be two problems. First, it seems that the various steps in the education system to some extent have failed to educate the pupils according to their present curriculum, at least there is some disagreement of what to expect of pupils who graduates from the elementary and the high school system. Second, there are too few pupils at the highest level and the highest level is too low. One could here argue that the definition of “levels” is problematic and that the purpose of education is more than serving working life. I would agree with these views, but nevertheless I would state that education is *also* for working life.

³ http://www.eu2001.se/eu2001/news/news_read.asp?iInformationID=12545

Danish high-school pupils in international comparisons

Danish pupils at elementary school and high school have not performed that well in international comparison in the 1990s and early 2000s. The TIMSS (The Third International Mathematics and Science Study) for mathematics high level at high school placed Denmark rather high (5 of 16) (Allerup et al., 1998, p. 84) but in this study the very high scoring Asian countries from a previous international study for elementary school-level did not participate (Allerup et al., 1998, p. 183). The Danish mean was rather high, but the dispersion very low, which means that Denmark had very few pupils among the 5% highest scoring pupils internationally. This means that Denmark did not have an elite group compared with the other countries (Allerup et al., 1998, p. 85). Also, the ranking of countries would furthermore not have been different had the participating countries been tested with “Danish” problems (Allerup et al., 1998, p. 186). A rather similar result of 15-year old is seen in the first results from the OECD programme for international student assessment (PISA) from 2001. Denmark perform significantly above the OECD mean and seems to be in the middle of all the OECD countries, but she is also one of the countries with the smallest difference between the 75th and 25th percentiles (OECD, 2001, pp. 80-81).

The results of the international assessments are many, but to round of the present discussion, then taking into account that the EU aims to raise the education level and the fact that other countries have an elite compared to Denmark, one might argue that this would be an element leading to Denmark lacking behind. One could furthermore assume that this would damage the economy and the whole welfare system in the longer run.

Equal opportunity

I will argue that it is a question of equality of opportunity to provide for the needs of the high-achieving. This is connected with a view of equality as not meaning equal results but equal rights and (perhaps) different outcome. One could here argue against what Held (1987, p. 295) labels a tyranny of sameness. Instead the focus of the education system could be on

meeting every pupil where he⁴ is and help him to reach his full potential. Special emphasis on the needs of the high-achieving is for instance seen in the United Kingdom where Ofsted (Office for Standards in Education) considers the needs of able pupils as part of equality of opportunity. The DfEE (Department for Education and Employment) has furthermore in two Circulars (14/94 & 15/94) in 1994 recommended that in primary and secondary schools all School Prospectus should include details of arrangements to identify and provide for exceptionally able pupils (Eyre, 1995, pp. 16-17). One can argue as follows:

If we accept that it is the duty of society, through the public school system, to provide educational opportunities for all children appropriate to their individual abilities and aptitudes, and if one further accepts that some children are exceptional ... then the issue is settled. ... For children to receive specialized educational treatment in such circumstances is not for them to get more than their fair share; they are simply receiving what, in their individual circumstances, is appropriate. (McLeod & Cropley, 1989, p. 4)

Therefore, it is not “un-just” to help high-achieving pupils to become even better; they too have a right to receive what fits them. The case is unfortunately not that easy. A problem for special education for the able pupils is the fear of elitism. Some argues that this was also on the agenda of Nazi Germany. However, the Nazi government actually abolished special measures for promoting academic excellence in schools in a decree of 1935 (McLeod & Cropley, 1989, p. 2). There is also a fear of capitalism. For instance McLeod and Cropley (1989, p. 8) quote Luria, a major figures in Soviet psychology, for saying that the selection of more able students for an advanced form of education is a capitalist ploy to keep the proletariat in its place. However, Communist China has a steep pathway to the best education, as well as a school system with competitive exams and where progress is measured solely in marks. Furthermore every republic in the Soviet Union had at least one special school for the gifted and talented (McLeod & Cropley, 1989, pp. 8-9). What happened historically was that in 1957 the Soviet Union launched *Sputnik 1*, and after a lot of self-criticism in the USA a reaction was that the President vowed:

that the United States would be the first on the moon and the massive NASA (National Aeronautics and Space Administration) program was launched. Congress passes the National Defense Education Act ... no doubt spurred on by reports such as which appeared in Newsweek on 29 October 1956,

⁴ When I do not refer to any particular person I will for the sake of convenience always only write ‘he’, ‘him’, or ‘his’ instead of the very space filling ‘he and/or she’, ‘him and/or her’, or ‘his and/or her’ which I find is also very annoying to read. Furthermore the masculine tense has traditionally been used to refer to a person, male or female, whose gender is not stated or known (Oxford Advanced Learner’s Dictionary, 1995). This will be the policy throughout this thesis.

that the USSR could expect twice as many graduates in science, engineering and technology between 1957 and 1961 as the USA. Thus, in the late 1950s and early 1960s the United States launched massive programs designed to foster excellence and talent. (McLeod & Cropley, 1989, pp. 16-17)

The USA therefore wanted to make sure that she had enough talent owing to the Cold War and the fight against totalitarianism. Young and Tyre furthermore call the question of elitism or egalitarianism for a false dichotomy and irrelevant. Instead they state that “all children are born as unique individuals, each different from the other, and in developing them we need to make them more equal by overcoming whatever inabilities they may have and more different from one another by developing their abilities and propensities” (Young & Tyre, 1992, p. 31). In that sense, special education (for both weak and strong pupils) is, in my view, justifiable as it both improves inabilities as well as develops the individual person’s talents.

High-achievers cannot take care of themselves

To help high-achieving pupils could be seen as taking resources from disadvantaged children. But on the other hand no pupil can progress towards the limit of his capacity unless he has an opportunity to learn. “Mozart might have had an extraordinary aptitude for music, but this could hardly have been realized unless his parents possessed a piano. It is at best inefficient to rely on nature or chance to develop talents, while for potentially gifted children in homes with limited cultural horizons it borders on neglect” (McLeod & Cropley, 1989, p. 5). Studies have also shown that some gifted pupils are underachieving and sometimes suffer psychological disturbances including poor concentration, exaggerated conformity, excessively inhibited behaviour, anxiety, social isolation and aggressiveness, or the opposite such as extreme passivity (McLeod & Cropley, 1989, p. 6). Another argument for special education for the gifted is that studies have shown that if gifted pupils are held back or bored in school, some of them will be ‘turned off’ by school, achieve far below the level of which they are capable, drop out, fail, or even become delinquent (McLeod & Cropley, 1989, p. 14). It is further argued that some talented pupils deliberately hold themselves back:

Some able students receive a shock when they move on to university. The leisurely study habits which had ensured reasonable grades in the mixed ability classes in secondary schools prove to be inadequate for the more intellectually demanding environment of the university. ... there are too many students of high ability who wastefully drop out. ... it is very probable that many gifted

*children 'learn to be average' or deliberately hold themselves back in order to have a quiet life in school: this is the phenomena of 'faking bad'.
(McLeod & Cropley, 1989, pp. 14-15)*

Another reason is that gifted pupils need adequate stimulation. There is evidence that suggests that association with other pupils of high ability raises a pupil's level of performance. One study showed that the "overall intellectual level within a group had an effect on the development of the level of individuals within the group - contact with clever people tended to raise the level of ability of the less clever" (McLeod & Cropley, 1989, p. 13). Another study showed that "down to an IQ of about 65, mentally retarded students taught with normal peers achieved better than those who were taught in self-contained classes" (McLeod & Cropley, 1989, p. 13). And further "that students of *high* ability were penalized academically by being taught with students of lesser ability" (McLeod & Cropley, 1989, pp. 13-14). McLeod & Cropley (1989, p. 14) argues that

Refusing to make special provision for the unusually able, on the grounds that they are necessary for the optimal development of the other children, means that adults shrug off the task of promoting the development of less gifted youngsters onto the shoulders of clever children. Naturally, educators should be looking at the needs of the less gifted, but not at the expense of the gifted and talented.

Thus, it might seem to be an unsolvable problem that when each pupil seem to do better when they are taught with more clever pupil, and suffer from being with less clever pupils, there will always be a "looser" in the "game". However, this does not have to be a problem if it is the teachers' duty to stimulate the pupils according to their abilities.

1.1.3 THIS STUDY SEEN IN RELATION TO PREVIOUS WORKS

This study has got inspiration from previous works by others but it also distinguishes itself from these studies.

Some studies of mathematicians and successful graduate students

Burton (1999) has done research on how mathematicians say they learn mathematics and her investigation is based on interviews. The questions were about “how they came to know mathematics, what their feelings were about coming to know mathematics and what career experiences they had had that might have influenced their thinking about knowing mathematics” (Burton, 1999, p. 121). One of her findings is that very few of her participants do individual work, instead it is common practice to collaborate. She reports that many of the mathematician explains that there has been a change in the culture of mathematics from mainly being a field dominated by individualism to a field which highly values team work (Burton, 1999, p. 131). About how they understand knowing, she writes that what she is told is how certainty feels, not how the mathematics is learnt (Burton, 1999, p. 133). Burton also writes that learning is represented by feelings, particularly the aha experience (Burton, 1999, p. 135). She concludes that “the world of knowing described by my participants, a world of uncertainties and explorations, and the feelings of excitement, frustration and satisfaction ... but above all a world of connections, relationships and linkages” (Burton, 1999, p. 138). She also states that: “Learning is *neither* wholly individual *nor* wholly social” (Burton, 1999, p. 139).

Burton’s investigation is based on interviews and is centred on what it is mathematicians do to come to know mathematics and how they work with it. This investigation thus touches the surface of what I want to do, as she does not concentrate on cognitive processes. However, it might be interesting to compare the statement that learning is neither wholly social or wholly individual as well as the statements about the importance of emotions in learning mathematics with what the ten pupils in this study says.

Carlson publishes in 1999 an article about the mathematical behaviour of six successful mathematics graduate students where she investigates the influences leading to mathematical success. She observes their behaviour while they complete complex mathematical tasks and she assesses their beliefs by a written survey. Her study explores the non-cognitive factors that play a major role in the high-achieving students’ success and further mathematical study. She therefore investigates their mathematical beliefs, behaviours, and backgrounds (Carlson, 1999, p. 238). The students tell that “they enjoy the challenge of attempting complex mathematical tasks and believe that they possess abilities and strategies that facilitate their

problem solving success” (Carlson, 1999, p. 242). They also report that their interest in mathematics is facilitated by a mentor, which they describe as an individual who “required regular work and provided incentives for working ‘lots of problems’. They created a non-intimidating environment where students were encouraged to pose questions until they acquired understanding” (Carlson, 1999, p. 244).

Both studies look at successful students or mathematician, but not in the same age group as the present study and furthermore they do not focus on the cognitive factors involved in learning mathematics. It would also be relevant to see if the pupils in this study mention non-cognitive factors when asked about their cognitive learning.

Educational studies in different areas of mathematics

I stated above that I wanted to know how the pupils say they learn *mathematics*. But which type of mathematics do I focus on? Krutetskii (1976) investigated how pupils solved some mathematical problems (Krutetskii, 1976, p. 97). His problems came from arithmetic, algebra, and geometry (Krutetskii, 1976, p. 89). Some problems did only require knowledge that was available to all the pupils, some problems were given in recently learnt material, and some problems went ahead of the curriculum, or were not covered by the curriculum. The problems had elements of mathematical creativity and were non-standard (Krutetskii, 1976, p. 95). I chose to look at a smaller part of these mathematical activities, namely the exploration/learning of mathematics. I focus on how the high-achieving pupils approach and explore a mathematical *concept* that is new to them, not a *problem*, which is therefore a bit different from Krutetskii’s (1976) works. Furthermore, Krutetskii’s study of mathematical ability took place over a period of twelve years (1955-1966) and involved a total of 201 persons. The pupils in his study were school children in various ages from primary to tenth grades (Krutetskii, 1976, pp. 81-83). This is therefore in relation to another age group than the sixth form pupils and also pupils from another (school) system. My study is more similar to that of Schoenfeld (1985, 1992) who analysed mathematical thinking and behaviour of university students in problem-solving, and by ‘problem’ he meant: “if one has ready access to a solution schema for a mathematical task, that task is an exercise and not a problem” (Schoenfeld, 1985, p. 74). In his study, problem-solving does not mean doing exercises, but

through a problem exploring the mathematics. As his investigations focus on the learning of *mathematics* through *problem-solving*, my study is both extension of his as well as different from his, as I focus on how *they say* they learn *mathematics* when meeting a new *mathematical concept*. I will not go into a deeper discussion of whether mathematical concepts are *the* core of mathematics, only state that mathematical concepts are *a* central thing in learning mathematics (see for instance Dahl, 1995; 1996a&b, Dowling, 1998; Mellin-Olsen, 1987; Pimm, 1987). Furthermore the study builds on previous work by the author, (Dahl, 2000), where some of the methodological issues were discussed.

1.1.4 CHOICE OF PSYCHOLOGICAL THEORIES

In this section I will explain my choice of theories. There are a great number of different theories that focus on the cognitive side of learning mathematics. Which theory is “in” seems to change over time and in waves. Hansen (2002, pp. 60-61) gives an example of this from the Danish school history. He writes that there is a surprising modernity in the debate about teaching of arithmetic in the first decade of the 1900s. To Hansen, this sounds like old wine on new bottles. The keywords at that time were, as today, that the child’s motivation, experience, and suggestions should be at the centre; there was an emphasis on actions, the principle of induction etc. My aim is to attempt to a more timeless use of theories. Some theories seem contradictory to other theories but I do not want to “choose side” but instead be curious and see how the pupils can be understood from different theories *to understand the pupils better*. The theories are tools. As general criterion I want to use “classics”, which means rather well known cognitive psychological theories and educational studies of mathematics that have as focus mathematical thinking. Briefly described, the choices are the following. They are discussed further in particularly Chapter 3:

- Piaget (1962, 1969, 1970, 1971) and Vygotsky (1962, 1978) are two of the main figures within cognitive psychology. Piaget was a Swiss psychologist who worked with the cognitive development and he is seen as a founding father of constructivism. According to Mellin-Olsen (1987, p. 30), activity theory has its roots in Soviet psychology, of which Vygotsky is recognised as one of the founders.
- Glaserfeld (1995) is a key representative for radical constructivism.
- Ernest (1991) is a central author within the area of social constructivism.
- Mason (1985) and Polya (1971). The study is related to that of Mason and Polya who both worked with mathematical thinking and problem-solving procedures. Schoenfeld states that “Polya had identified something significant” (Schoenfeld, 1985, p. xi). Tall criticises Mason for only describing elementary

and not advanced mathematical thinking (Tall, 1991, p. 20), but this is not important for the choice of theories since I have decided not to choose among the Diversity Thesis and the Similarity Thesis.

- Hadamard (1945). The study is furthermore related to Hadamard's work on invention, which is part of exploring a piece of mathematics, which is new to the learner. One could assume that there are similarities between learning an existing concept that is new to the learner, and discovering a completely new concept, as the concept in both cases is new to the pupil. On the other hand, there might be a difference between the procedures leading to (re)creating a given concept and the discovering of a concept for the first time.
- Krutetskii (1976) works on the psychology of mathematical abilities in schoolchildren and tries to describe different aspects of mathematical thinking.
- Skemp (1993) works inter alia with how the mathematical concepts are created and is therefore essential in understanding which cognitive factors the pupils might express.
- Sfard (1991) and Dubinsky in Asiala et al. (1996) discuss concept formation in mathematics. I have chosen to mainly refer to Sfard's works, as they are quite similar.

This list is potentially never-ending but I have chosen to settle with these authors. At various places a few other authors might briefly be used to illustrate a certain point; examples of these are Ausubel (1978), Mellin-Olsen (1987, 1989), Russell (1948), and Tall (1991).

1.1.5 ETHICAL ISSUES WITH SOME OF THE SELECTED THEORIES

My awareness of the need to consider when one for ethical reasons, perhaps, ought not to use a theory came while I was reading Krutetskii (1976). The work was made in the Soviet Union and one meets Soviet "fanfares" in a number of statements like the following: "A goodly number of foreign works are devoted to the problem of abilities and giftedness. Most of them, however, are not in the spirit of true science. They are permeated by tendencies profoundly alien to the ideas of dialectical materialist psychology" (Krutetskii, 1976, p. 8). Another example: "Antiscientific bourgeois pedology, proceeding from class interests, 'proved' the special giftedness of representatives of the exploiting classes and the 'higher' races as well as the spiritual poverty and foredoom of representatives of the working classes and the 'lower' races. Of course, to adopt and transfer even a few ideas of bourgeois pedology uncritically into our psychology and pedagogy, for all the reservations and 'corrections,' was to do great harm to Soviet science and education" (Krutetskii, 1976, p. 49). At page 83 he furthermore thanks what he calls his "comrades" for their comments on a certain issue. Two questions then came to me. The first was how, or why, to use research where one must expect the academic freedom to have been limited. The second, if it was responsible, morally, to use

research conducted as part of a totalitarian regime in order to support it and/or to use research that more directly may have been harmful for the participants. I will discuss this below.

Limitation of the academic freedom

In the “Ethical Guidelines for Educational Research”, adopted by the British Educational Research Association (BERA) in 1992, the following is stated:

*The British Educational Research Association has been aware for some time of a concern amongst the educational research community about increasing restrictions being imposed by government agencies on the conduct and dissemination of the educational research and evaluation which they sponsor. ... Such a concern must be seen in a context ... where central government now controls access to large amount of funding for research in a field which it increasingly views as its policy domain. In this context there is a great temptation for educational researchers and their institutions to accept sub-optimal conditions which compromise the canons of intellectual inquiry in a free society. These conditions tend to impose restrictions on the freedom of researchers to publish and disseminate their findings. But there is also increasing evidence of a tendency to impose restrictions on the conduct of the inquiry itself, e.g. on the questions to be addressed, and on methods of data collection and analysis.
(BERA, 1992, p. 1)*

The view is thus that funding agencies such as governments can pose a risk for the academic freedom in terms of research questions, methods, and publishing. BERA therefore adopted some ethical guidelines. About the relationship with funding agencies it is specifically stated that censorship and research which is in conflict with academic freedom is not acceptable:

*Educational researchers should remain free to interpret and publish their findings without censorship or approval from individuals or organizations, including sponsors, funding agencies, participants, colleagues, supervisors, or administrators. ... Educational researchers should not agree to conduct research that conflicts with academic freedom, nor should they agree to undue or questionable influence by government or other funding agencies.
(BERA, 1992, pp. 2-3, point 15 & 16)*

A rather similar emphasis is seen in the American Sociological Association (ASA) “Code of Ethics”, where it is written about conflicts of interest that: “Sociologists maintain the highest degree of integrity in their professional work and avoid conflicts of interest and the appearance of conflict. Conflicts of interests arise when sociologists’ personal or financial interests prevent them from performing their professional work in an unbiased manner” (ASA, 1997, p. 6). The work of Krutetskii is part of the Soviet project and is clearly written

within a frame of thought where certain conclusions and ideas are excluded and forbidden. In the Soviet Union, psychologists “were expected to produce a ‘Marxist psychology’ in sync with Marxism-Leninism’s materialist, environmentalist, egalitarian ideology. ... A 1936 meeting of the Communist Party of the Soviet Union (CPSU) went so far as to actually ban ‘testy’ (psychological tests) and much of Western scientific psychology as anti-Marxist” (Nalchajian et al., 1997, p. 22). The work has therefore the character of being “commissioned work” with not much possibility for academic freedom.

This, I feel, is a general question that is always relevant when dealing with research that is commissioned and controlled. Another example of such potential problems for the academic freedom is seen in the so-called V6-Case in November 1999 in Denmark. The chewing gum company Dandy sponsored researchers at Aarhus University in Denmark to investigate the tooth hygienic chewing gum V6 and its effect to prevent holes in the teeth. The result of the research was that the gum did not have any effect in preventing holes and it seems that the researchers, by Dandy, were forced to redraw the investigation.⁵ I will not go into more details pro and con these accusations of Dandy, but the case shows the potential problems for the academic freedom with for instance an economically dependence between the researcher and the sponsor. Reading research articles or reports one should always be critical, but *all other things being equal*, one ought to be extra careful when it comes to research where there is a risk of loss of academic freedom. On the other hand, an advantage with sponsored research is that it may be more relevant research, at least more immediately relevant, and there are many examples of sound research that has been sponsored from various sources.

I feel that the question of academic freedom is an even more relevant question to pose in relation to the Soviet regime, as researchers (anybody actually) who did not have the “right” opinions risked imprisonment, being sent to a working camp, exile, or execution. The psychologist Bekhterev was for instance poisoned (Nalchajian, 1997, p. 22). I assume that this must have made them even more inclined to “follow order”, or exercise self-censorship, and not be as critical as researchers in a democratic country who in worst case risk social exclusion in their research field or being fired. This is very serious in itself, but is not a matter of life and death. Furthermore the researcher in a democratic country has the option of going to the media, or to the ombudsman, as was for instance seen in the V6-Case.

⁵ Tandlægernes Nye Landsforening http://www.tnl.dk/tidsskrift/arkiv/v6_sagen.html.

Moral responsibility

Another thing that came to my mind was that if I used Soviet research, would that not be a “support”, or excuse, for a totalitarian regime? Leninism and Stalinism are responsible for the death of about 20 million people in the Soviet Union (Courtois, 1999, pp. 4ff.). Censorship and suppression was the rule during the whole existence of the Soviet Union. If I can use Krutetskii, why not Mengele?⁶ Where do we stop? Do we ever stop in the name of progress of science?

Decision on the ethical issues

After these considerations, I chose after all to use the work of Krutetskii (1976). About the limitations to academic freedom: All type of research is based on some values and an advantage with Krutetskii is that these values are easy to see and therefore to take extra precautions against. Furthermore it might be interesting to evaluate if some of the pupils’ explanations reflect this particular “lens”. Furthermore, Krutetskii’s work/experiments did not harm his participants; they were asked to solve some mathematical problems and were interviewed by some researchers. Mengele’s work most certainly harmed the children, which is where I think we need to draw the line. Furthermore, as Kilpatrick writes in the introduction to Krutetskii’s book:

The reader should be aware that Soviet research reports do not meet Western standards of thoroughness and objectivity. ... What then is the value of Krutetskii’s report? It is no exaggeration to say that this work could have the same sort of impact on mathematics education that Piaget’s work has had. ... The work reported in this book may help educators and researchers break free from their reliance on test scores as indicators of ability and may stimulate the search for more productive ways of measuring the processes of mathematical thought.
(Krutetskii, 1976, pp. xv-xvi)

The focus of this research has not to do with test scores, instead it has to do with the processes of mathematical thought and it is thus useful.

⁶ Dr. Josef Mengele experimented with about 3,000 twins in Auschwitz in World War II. Only about 200 survived <http://www.candles-museum.com/mengele.htm>. CANDLES stands for “Children of Auschwitz Nazi Deadly Laboratory Experiments Survivors”.

This discussion about academic freedom could in some sense seem “contradictory” to what I argued above about the need to justify the relevance of one’s research. From what might be termed an *ideally* perspective, academic freedom should be really “free”, but from what one might term a *pragmatic* perspective the society and/or private funds have limited resources and prioritisation is necessary. Perhaps a *realistic* perspective would be to reconcile the two; balance might be a keyword.

1.2 RESEARCH QUESTION

How do the high-achieving pupils say they come to understand a mathematical concept that is new to them? How can these reports be informed by various psychological learning theories of mathematics?

Do the high-achieving pupils give similar or different accounts, or does there seem to be certain “types” of pupils? If there are “types”, are these “cross-national”? How can the answers to my research question be used to improve the learning/teaching of other (high-achieving) high school pupils of mathematics?

The research does not “test” if the pupils are able to say anything sensible about their learning process, or in other words: do the pupils have a meta-cognitive awareness which they can verbalise? I *assume* that they can and I argue for this in Chapter 2. Instead the research question is about *what* is it the pupils actually say. In other words: what I really investigate is how high-achieving pupils verbally express how they reach understanding of a new mathematical concept. This means that the responses I get say as much about the pupils’ ability to (a) reflect on own learning process (their metacognition), (b) verbalise this; as it tells about (c) how this learning process really is. The main target of this study is not to answer (a) or (b). In some sense the underlying research question is to get wiser on what the “real” learning process “really” is. But as investigating this is impossible since ‘learning’ is not ‘visible’ to the eye, one has to “sneak” into the learning process, which is the problem of any learning theory or research in learning. The assumption is that the discrepancy between what the pupils say and what the learning process really is, is not too big. Therefore (a) and (b) are not part of the research question, but very interesting and important issues to be discussed

again in the discussion and conclusion. The existence of types of pupils is not a hypothesis or a “success criterion”, but more an open question - whether the answer is yes, no, or ‘not sure’, would be equally interesting. The thesis is about learning mathematics and it is implied that, unless otherwise stated, ‘learning’ also denotes learning mathematics. Through being theoretical wise, I might understand the pupils (better than they have understood themselves) and the result might be something that could inform policy makers and teachers.

1.3 GUIDANCE FOR THE READER

The study is a focus group study with semi-structured group interviews that, for the English pupils, were combined with participant observation of how the pupils work with a piece of mathematics that was new to them, in the style of a psychology experiment. Reasons for anticipating that the pupils can say anything about their mental processes, possible difference between how they say they learn and how they actually learn as well as other methodological issues will be discussed in Chapter 2. This chapter will also discuss how I have collected the data. In Chapter 3 I will discuss the theories of the above-mentioned authors within the psychology of learning mathematics and here extract a model for analysis. Chapter 4 will discuss the possibility of a synthesis of various theories of learning. Chapter 5 will argue for how I have processed the data, and Chapter 6 to 8 will be the analysis of the data. Then follows Chapter 9, which is a discussion and conclusion of the existence of types of pupils as well as a discussion of general issues. The bibliography is in Chapter 10. I have placed the full transcriptions of the interviews in the appendices for anyone to use with reference. As the data, as well as the analysis of them, to a very high extent form the centre of my study, I will also include methodological discussions throughout the thesis. This is for instance seen in Chapter 5 where I discuss in-depth why and how I chose to transcribe the interviews.

*U is for the Unemployment rates,
Still rather grim in most cities and states.
There may be no jobs no matter your knowledge,
By the time that you matriculate from college.
So work and study and practice night and day
At something to give you social entrée.
There may be no jobs, not for doctor nor dentist,
But you'll marry an heiress if you're real good at tennis.
(O'Rourke, 1987, p. 161)*

2. METHODOLOGICAL ISSUES

The core of the study is the high-achieving pupils' understanding of their cognitive learning process. This raises several questions about methodology and methods. I use the concept of 'methodology' to denote an outline of how to go about studying the research question, defined broadly (Silverman, 2000, p. 79). This is in line with Robson's (1993, p. 37) definition of a 'strategy' as the style and general orientation in approaching a particular research question. I understand 'method' as the more specific ways of investigation and data gathering (Robson, 1993, pp. 37 & 187; Silverman, 2000, p. 79). In this chapter I will concentrate on both issues.

2.1 WHAT CAN THE PUPILS KNOW AS WELL AS SAY

The thesis depends on that the high-achieving pupils (1) know something about their learning process and (2) they are able to verbalise this knowledge.

2.1.1 THE PUPILS' KNOWLEDGE OF OWN LEARNING PROCESS

Schoenfeld (1985, 1992) discusses the concept of metacognition, which can be understood either as knowledge about or regulation of cognition (Schoenfeld, 1992, p. 334). Knowledge about cognition means to have relatively stable information about one's own cognitive

processes. This knowledge develops with age and “performance on many tasks is positively correlated with the degree of one’s metaknowledge” (Schoenfeld, 1985, p. 138). Metacognition, understood as regulation of cognition, includes the planning before beginning to solve a problem and the monitoring and assessing “on-line” during problem-solving and learning (Schoenfeld, 1992, p. 355). The presence of this has a positive impact on intellectual performance and the absence a strong negative effect. The pupils’ conceptual model for the problem influences their problem-solving behaviour. “Expert behavior, in which the appropriate resources are routinely accessed, is a result of the experts’ possession of stable conceptual models. Conversely, many students’ difficulties are due to the fact that their conceptual models are unstable” (Schoenfeld, 1985, p. 139).

Furthermore, according to Schoenfeld, “one of the hallmarks of good problem solvers’ control behavior is that, while they are in the midst of working problems, such individuals seem to maintain an internal dialogue regarding the way that their solutions evolve. Plans are not simply made, they are evaluated and contrasted with other possible plans. ... one might say that part of competent problem solvers’ behavior is that they argue with themselves as they work” (Schoenfeld, 1985, p. 141). It would be relevant to get an idea of this “internal dialogue”. One could therefore assume that high-achieving pupils have knowledge of how they learn mathematics.

2.1.2 THE PUPILS’ ABILITIES TO EXPRESS THEIR KNOWLEDGE OF OWN LEARNING PROCESS

According to Vygotsky (1978, p. 61), psychological processes can be difficult to observe but they can be artificially provoked in a laboratory and Hadamard furthermore discusses the so-called subjective (‘introspective’) method about ‘observing from the inside’: “information about the ways of thought is directly obtained by the thinker himself who, looking inwards, reports on his own mental process” (Hadamard, 1945, pp. 1-2). A problem with this method is however that the pupil might disturb the phenomenon he is investigating, as he has to think and observe his thinking at the same time, but this according to Hadamard (1945, p. 2) this is a minor problem in an inventive process. I would argue that the learning process has similarities with an inventive process as, to the pupils, what they learn is new to them. Mental

processes take place inside a pupil's mind but the subconsciousness must be distinguished from unconsciousness and it is the subconsciousness, which is valuable for psychology as it is accessible to introspection, which generally is not possible for the more remote unconscious processes (Hadamard, 1945, p. 25). Hadamard then explains that it is difficult to distinguish between consciousness and fringe-consciousness (subconsciousness), but "the difficulty happens to be much less in the case of invention, which interests us. The reason for that is that invention work itself implies that thought be inflexible directed toward the solution of the problem: when obtaining the latter, and only then, the mind can perceive what takes place in the 'fringe-consciousness'" (Hadamard, 1945, p. 26). According to Hadamard invention or discovery takes place by combining ideas and that "invention is discernment, choice" (Hadamard, 1945, p. 30).

Other psychologists find that introspection cannot provide a correct or accurate description, for instance Vygotsky (1978, p. 67). Also Schoenfeld is critically of the use of introspection, and instead uses another verbal method inspired by Piaget and Krutetskii (Schoenfeld, 1985, pp. 273-274). A basic idea is here that observing people solving problems and then interview them might give information about their problem-solving processes (Schoenfeld, 1985, p. 277). Schoenfeld's students worked in pairs, were taped-recorded, and the researcher did not interrupt this process as this might have altered their decision or caused them to learn from him. His work consisted of four phases:

(1) The students generate noninterventionist protocols. (2) The students provide retrospective reports of their work, with minimal interventions from the experimenter. (3) The students and the experimenter watch the videotapes of their work together, and the students are asked detailed questions about what happened during the problem session. (4) There is a clinical interview that explores any issues of interest that surfaced in the first three phases. (Schoenfeld, 1985, pp. 282-283)

Krutetskii's works also consisted of a mix of observation and interviews, again mainly using interviews. Four things supported his analysis of the pupils' process of thinking during solving experimental problems: (1) An objective record of the solution and diagrams, sketches etc. (2) A record of the verbally process of reflection. (3) The nature of the answers to questions. (4) Material from discussion about the solution after its completion (Krutetskii, 1976, p. 94).

What these two designs have in common is that some phase of working with the mathematics is followed by interviews about what they did.

Whether the pupils really know something about how they learn mathematics, or they just think they do, is impossible to answer completely. The adequately answer to this question would require that one had access to knowledge about how the real learning process was and therefore could compare the real learning process with what the pupils think it is. But we do not have access to this kind of knowledge and if we had, there would be no need to consider asking the pupils about it; unless the asking had other purposes than finding out how one learns a new mathematical concept.

For Vygotsky the relationship between what people say and the cognitive processes behind is that speech completes thinking rather than reflects it (Vygotsky, 1962; see Section 3.3.1). An advantage of asking high school pupils, as in this study, is that they are quite young and they have therefore not been taught about any theories of learning in school and are thus “unspoiled” and they would therefore not merely repeat what they might have learnt in a pedagogical course. A disadvantage with this is that they might not always have the words to express what it is that they mean. Instead I will presume that they will use their daily language and metaphors, which I may be able to unveil by using theory. However, one could state that even though the pupils are unspoiled, their teachers are not, and they might have, directly or indirectly expressed their views of learning mathematics. On the other hand, as will be clear below, the data shows that the pupils say very different things, even from within the same class.

2.2 METHODS OF DATA COLLECTION

This study involves four Danish pupils interviewed as one group and six English pupils interviewed in pairs. Altogether four groups and ten pupils. The design is inspired by the studies of Schoenfeld and Krutetskii mentioned above, which means that I created a “mix” of interviews and observation; also called hybrid strategies (Oppenheim, 1999, p. 12; Drever, 1995, p. 8; Robson, 1993, p. 41; Hammersley, 1992, pp. 196-197). This does not mean that the method was determined before the research question; but rather that I do not want to imply that there has never before been done research which has similarities with this study, and therefore I want to draw on the experiences of these studies and then mould it into something which is particularly useful in the context of the present study.

Each session with the English pupils consisted of three phases: 1. Interview. 2. Observation of the pupils learning a piece of new mathematics. 3. Interview. The Danish pupil did not have the second phase. This chapter will include a discussion about how the Danish and the English study were designed and carried out which will be illustrated with examples from the transcriptions and, for the English study, also the observations. To ease references to a certain place in a particular interview I will use following notation: “IE3, 879” to refer to something that was said in the third English interview, transcribed in line 879. “ID, 1657” refers to something said in the Danish interview in line 1657.

2.2.1 THE MATHEMATICS THE PUPILS WERE PRESENTED TO

I chose to use a piece of knot theory in an observation phase of the English interviews. One reason was that since my knowledge of the English system, for natural causes, was not as deep as that of the Danish, I needed something extra on which to prompt a discussion and to hang and exemplify the pupils’ explanations. The purpose of the knot theory was not primarily to see how they learnt the knot theory but that sitting and working with this mathematics might prompt them to say how they usually learn mathematics. In other words, this intervention was made to, in Vygotsky’s (1962) word, provoke artificially a psychological process of learning mathematics. As the pupils had to work out some new mathematics, it might give rise to invention, which according to Hadamard (1945) makes introspection possible. The knot theory would also give me as an interviewer from another context an extra thing to refer to in the questioning. Thus, the English pupils were given a copy of two pages about basic concepts of knot theory leading up to Reidemeister’s Theorem (Nelson & Wilson, 1990; Appendix B). I did also not give them exercises, as this study was about how they approach a mathematical concept that was new to them. The topic was in an area they (according to the Head of Mathematics Department) had not worked with before, as this would give all the pupils equal opportunities and it would also prevent them from using strategies they had been taught by their teachers. There are pupils of both sex, but gender issues were not a topic in this study. What I study is what these pupils have in common, namely that they are high-achieving and from the same level in their education.

They were also given the book where the copy was taken from, another book about knots (McLeay, 1994), paper, pencils (in different colours so I could later see who wrote what), and a sheet with the questions I was going to ask them afterwards (Appendix C). They were also invited to write notes on this sheet, if they wanted. Only the third English group did that. I also gave them the opportunity to go into another room, sit each by himself, and if they desired they could ask me. I told them that they should do whatever they found was necessary to get some understanding of this piece of mathematics.

Schoenfeld states that “having subjects explain the reasons for their actions as they solve problems will disrupt the subjects’ problem-solving processes” (Schoenfeld, 1985, p. 281), and perhaps alter it. I therefore let the pupils be on their own, unless *they* involved me. I told them they had about 15 minutes, which I estimated was time enough for them to get some understanding of the mathematics, but not get a complete understanding. I assumed that if their process of learning was interrupted, as they were still becoming familiar with the new mathematics they would be more able to say what they would do next.

In this study, I chose to focus on how pupils explore and get to know a mathematical concept that is new to them as distinct from doing exercises and applying mathematics. It seemed that the pupils were all very able to distinguish between doing exercises and doing problems.

I used a piece of mathematics, in the English interviews, which was very different from what they were used to. One could discuss if it would have been better to give them some mathematics that was less strange to them, and which was explained a way that they were used to, and in a familiar language. On the other hand, following Schutz (1964) the stranger is good at discerning the particularities of a strange situation. Therefore I did not try to create a learning situation that was completely similar to the one they were used to as when the pupils experience something different this may make them aware of what is typical. For instance is it unusual for the pupils to only get a piece of paper and having to sit and read this to get an understanding of a new piece of mathematics. The language of the piece of knot theory was also different from what the pupils were used to and thus I anticipated that this would probe a discussion about the role of language. This might seem manipulating, but still less than if I directly asked them. Another reason why knot theory was chosen was that it involved features of both graphical and algebraic representation which I anticipated could span the range of different kind of mathematics taught at high school level. Had everything been like it usually

was it might not have given rise to the discussions about the role of language, discussions, time, visualisation, and difference between doing exercises and explore mathematics etc. These themes will be discussed in Chapter 3. In that sense the piece of knot theory was “leading”, as I hoped that this strange piece of mathematics would probe a discussion about these issues. However, the balance is sharp. I could not have given this mathematics to other than pupils like them. Less able pupils in mathematics might have got a shock, and there is an ethical issue in this. Another issue is that too difficult mathematics would destroy any discussion about mathematics.

One could also discuss if what they said after the intersection was only relevant for this particular piece of mathematics. I experienced that the rather different mathematics often became a good starting point for a more general discussion (IE2, 853-866):

E: I think the the English is worse than the maths in this (D: Yea), I'm not, I'm

I: Does this, I mean, is that a a an obstruction for for the learning, the language?

E: Yea.

I: Is it always like this?

[2 sec silence]

D: It's always an obstruction if you don't understand it (I: mmm) (E: a) it's frustrating cause you think that it's probably quite simple, and you would understand this in, and this is very simple [laughs] knowing what a knot is, well, you know, when we discuss it with bits of our own words then it was fine, but this “mutually disjoint simple closed curves” [laughs]

One could therefore conclude that the extent to which unfamiliar mathematics can provide information about more general ways of approaching mathematics has more to do with how the pupils are probed than the particular piece of new mathematics.

2.2.2 INTERVIEW AND OBSERVATION

The structure of each English session was inspired by Schoenfeld's (1985) and Krutetskii's (1976) studied mixed with some ethnographical principles. The structure of the Danish study is the same except for the observation part and the questions related to that:

Semi-structured group interview. I asked them to describe a normal mathematics lesson, what they *do* (learning strategies) when they meet some new mathematics they do not

understand at first, and how they know if they have learnt some mathematics (their own criteria). I also asked them to describe their learning process. This phase is not directly part of the structure of Schoenfeld (1985) and Krutetskii (1976), but is included as a way of “warming up” and develop trust (see Appendix D & E). This phase is planned to last about 10 minutes.

Participant observation of how they worked with the new mathematics. They got some mathematics, knot theory (Nelson & Wilson, 1990; see Appendix B), on a piece of paper. It was not a problem to solve but some basic knot theory that they were supposed to get some understanding of. I observed them while they worked on it and here gave them the opportunity to involve me in the learning process. They also got a sheet with some of the questions I would ask afterwards to make them think about what they did while working with the mathematics (see Appendix C). This phase was planned to last about 15 minutes.

Unstructured qualitative group interview with open-ended questions. I asked them what they *did* to try to understand, why they did the various things I observed, how and why it helped, what they would *do next* to fully understand this mathematics, if (how) this learning situation is different from what they normally experience, and how they would present this to the class. This phase was planned to last about 30 minutes. As there was not any intersection in the Danish interview the pupils here choose examples from their daily life, as well as the English pupils sometimes did.

There are a number of ways to conduct an interview and a deeper explanation of how it is done in this study is found in Section 2.3.1. Generally, there are two different kinds of observation: ‘participant observation’ and ‘structured observation’. For the present study, participant observation is mostly relevant as one here tries to become a member of the group to get a deep understanding of the cultural context of behaviour, communication, meaning and how things operate in their natural setting (Yin, 1994, pp. 87-89; Robson, 1993, pp. 194-196). The focus is on what the pupils “naturally” do, when they meet a new mathematical concept. A more elaborate discussion of the observations in this study will take place in Section 2.3.2.

2.3 DESIGN AND DATA COLLECTION

I will in the following sections explain the interviews and observations more detailed. I have let myself be inspired by general methodological standards and “kit” such as Morgan’s (1998a&b) which talks about focus group investigation in general and not just in educational.

2.3.1 THE GROUP INTERVIEWS

The Danish interview was a group interview and in the English study, the first and the third part of each session consisted of group interviews as “the group discussion creates a process of sharing and comparing among the participants. In a lively group discussion, the participants will do the work of exploration and discovery for you” (Morgan, 1998a, p. 12).

Size of groups

In the English interview the teachers “paired” the pupils according to how they thought they would fit. There were seven pupils (called A, B, C, D, E, F, G), and the first pair was supposed to consist of three boys, A, B, and C. Pupil B forgot to turn up but came instead at the third session where G could not make it. The composition of the six pupils was therefore: A & C, D & E, and F & B. The Danish study consisted of one group of 4 pupils. I call the Danish pupils: Z, Æ, Ø, and Å, using mainly Danish letters to distinguish them from the English pupils.⁷ The group size was therefore two in each of the English groups and four in the Danish study.

In general the number of participants in a group interview is around 6-9/10 (Jacobsen, 1982, pp. 61-62 & Morgan, 1998b, p. 71). But for this particular study I wanted smaller groups which is also what one mainly uses when the participants have a lot to say, have a high involvement, are “experts”, the researcher wants details, depth, and personal accounts (Morgan, 1998b, p. 73), which is the case in this study. Usually one-to-one interviews

⁷ The Danish alphabet consists of three extra letters (vowels) placed at the end of the Latin alphabet, Æ (æ), Ø (ø), and Å (å). The Danish alphabet does therefore consist of 28 letters as we do not have W (w).

generate an extensive amount of data about single individuals which group interviews can lose (Morgan, 1998a, pp. 32-33). This was another reason for the choice of small groups as I anticipated that this loss would not be as great when the groups were all small.

The opinions of groups are usually more radical than individuals (Jacobsen, 1982, p. 60), and in larger groups there might be interpersonal conflicts (Fontana & Frey, 1993, p. 34). Furthermore Albrecht et al. writes that the group affects the opinion articulation (Albrecht et al., 1993, pp. 55-56). The small groups were therefore an advantage here.

Schoenfeld also states that recording pupils in pairs “helps to alleviate the kind of environmental pressures that weights so heavily on students as they solve problems individually. ... [and] two-person protocols will often provide better information about individual students’ decision-making processes than do single-person protocols” (Schoenfeld, 1985, p. 281). The latter has to do with that when two work together, there is more verbalisation and the reasons for decisions are open. Also Drever (1995, p. 16) points to the fact that children may feel more comfortable meeting an adult if they are interviewed in groups.

A criticism of group interviews is that it might produce conformity (Morgan, 1998a, pp. 50-51; Carey, 1994, p. 236; Frey & Fontana, 1993, p. 34). However, the discussions in this study were not aimed at creating decisions about right or wrong ways to learn mathematics. I specifically told the pupils that there were no right or wrong opinions, and that they should speak freely and not omit things they found obvious (Jacobsen, 1982, pp. 51-57).

The pupils were not paired at random, which might have been a kind of compromise between purposive sampling and random sampling. Firstly, I did not need random sampling as I did not aim at generalisation in the quantitative sense, secondly a random mix of the pupils might have led to groups that were not working (Morgan, 1998b, pp. 56-57).

Interview style

As I wanted to uncover, explore, and describe the diversity of experience and perceptions, and the range of possible approaches, I decided on unstructured interviews for the main part of the interview sessions. The reason was that I wanted to explore the pupils’ wider ranging experiences, to which unstructured qualitative interview with open-ended questions are very

suitable (Kvale, 1996, p. 97). In my style of leadership of the group, I therefore wanted to ask open questions from a prepared list and then see where it took us. I also aimed at listening and delaying the next question. I probed to “open-up” in-talk, and also used something someone has said previously and for instance reformulate their answers or referred to what they had said to get a more elaborate explanation (Jacobsen, 1982, pp. 66-74). When I discovered contradictions, I aimed at being gentle and show that I wanted to understand everything as correctly as possible (Jacobsen, 1982, p. 85). I attempted not to ask leading questions, stay neutral, and keep an open mind to what they might say that was not covered by the theoretical framework for analysis. On occasions, I also challenged what they said. I also tried to ignore information about a pupil, which did not come from the pupil himself. I was careful about not interrupting them and allowed periods of silence, sometimes for several seconds, to make sure that I did not interrupt anyone’s train of thought. An example of this is in IE2, 1126-1141, towards the end of the interview, where the dialogue below led to a longer discussion about the role of the unconsciousness, which had not come up previously. The style of transcription is discussed in Chapter 5. “I” is the interviewer:

I: OK, I have a last question, well, it's not really a question it's about, is there anything we haven't talked about tonight which you think is important to mention in relation to learning maths or handling problems learning maths?

[3 sec silence]

E: Er

[7 sec silence]

D: [inaudible]

[3 sec silence]

D: I don't I don't think so, cause a lot of this is hiding it's subconscious.

Being “patient” and “quiet” may therefore lead to that the interviewee discloses further information. For the first part of the English interviews as well as the Danish interview I had a more structured interview as I wanted some specific information and it was also intended as a warming-up phase.

Strangers or acquaintances

The pupils were not strangers to one another. A group of strangers “is useful when you want the participants to think about and talk about their taken-for-granted assumptions” (Morgan, 1998a, p. 49). To make the pupils talk about these possible assumptions, I used instead that I as a stranger could ask “silly” questions (Morgan, 1998b, p. 69). I probed them, rephrased their statements, and asked why-questions, to make them elaborate on what they said, as the interpretation otherwise could have been much more influenced by my bias. Also, if they expected the interviewer to know what they referred to, they would perhaps feel awkward about explaining more than “you know”. These things, I assume, would balance the effect of the group consisting of non-strangers. It is an advantage that people know each other if “we want to re-create some of the context that you are trying to understand” (Morgan, 1998a, p. 49). Also Albrecht et al. writes about focus groups as a way to “simulate a microcosm” (Albrecht et al., 1993, p. 59). If the group is homogeneous, it “increases the participant’s comfort in talking with similar others” (Morgan, 1998a, p. 59). For these reason it was an advantage that the pupils knew each other, as I wanted them to tell how they (usually) learn a mathematical concept that is new to them. It was the teachers who paired the English pupils, which could also raise the pupils’ comfort, which is essential in voicing their views (Morgan, 1998a, p. 61). This, however, depends on how well the teachers know the pupils. Putting together people who knows one another might result in that friends “pair up” or some may “break off into private conversation” (Morgan, 1998b, pp. 67-68) and it is therefore an advantage that the groups only consisted of two or four pupils and in the four-person Danish group interview it is seen that the opinions are diverse and criss-crossing.

2.3.2 PARTICIPANT OBSERVATION OF THE ENGLISH PUPILS

The second part of each English session consisted of participant observation. There are several ways to conduct participant observations (Robson, 1993, pp. 194-198; Lincoln & Guba, 1985, pp. 273-274). I decided on a role between the ‘participant-as-observer’ and ‘observer-as-participant’ (Robson, 1993, pp. 197-198). In participant-as-observer, the researcher enters an existing role and tries to establish a close relationship with the group,

participate in activities, and through this asks the members about what is going on. In observer-as-participant the researcher constructs a role within the organisation, but does not take part in the activity but from his role asks questions. What I did was somewhere in between of these two roles. I did not enter as a teacher, but on the other hand I gave them the opportunity to involve me in their learning process.

In IE3 the pair worked for about around 25 minutes in the intersection as they were not as talkative as the first two pairs. An illustration of this is that before the intersection a pupil's narrative could in IE1 last 18 lines (IE1, 315-332), in IE2 10 lines (IE2, 424-433), while in IE3 not above 6 lines (IE3, 84-89). The order of the questions was the same. I therefore decided to give them more time as their knowledge of how to come to learn mathematics could be tacit (Polanyi, 1967) and that they could instead "show it" to me. Another reason why they were less talkative could be that the teachers did not originally put this pair together.

Recording

The observation phase was audio recorded to have an account of how the pupils discussed the mathematics and their process of discussing (Silverman, 2000, p. 149). I used 'descriptive observations' (Robson, 1993, p. 200) and therefore took notes describing the whole setting and some of my initial reaction to what they said they did. In the intersection I noted down actions and used that in the following continuation of the interview, for example seen in IE2, 793-820.

2.4 SAMPLING AND ACCESS TO THE PUPILS

In this section I discuss the procedure behind sampling the schools and pupils, how I got entry, and how this may influence the research findings.

2.4.1 THE DANISH AND ENGLISH EDUCATION SYSTEM

I will now briefly describe the Danish and English education system up until university level. The purpose is to give the reader who may not know these systems a view of the background of the pupils in this study.

Denmark has nine years of compulsory schooling beginning the year the child is seven years old. These nine years take place in the *folkeskole* (The People's School). Some pupils choose a 10th grade. After the *folkeskole* young people have five options: (1) The *gymnasium* (General Academic High Schools) is a three-year upper secondary education. (2) The *Højere Forberedelseksamen (HF)* (Higher Preparatory Examination Course) is a two-year course that is meant for adults who wanted a high school exam, or for pupils who have completed the 10th year of the *folkeskole*. These two are preparatory for higher education. (3) The three-year Commercial School (*Handelsskole/Handelsgymnasiet*). (4) The three-year Technical School (*Teknisk skole/Teknisk Gymnasium*). Both of these give access to higher education as well as prepare for professional activities in the private sector. (5) Vocational education and training courses (*erhvervsuddannelserne*) with theoretical training (1/3) at technical schools and practical training as an apprentice (*lærling*) (2/3) at an enterprise.

I am interested in the pupils in the gymnasium and it is these pupils that I refer to when I write "Danish high school pupils". This is divided into a linguistic and a mathematical line. I am interested in the pupils on the mathematical line. The subjects are divided into three levels: A, B, and C. A is the highest level. Pupils on the mathematical line are obliged to have level B in mathematics. At the time of the study, this level ended by the end of the second year and the pupils could then freely choose level A for the third year. Later it has become the law that pupils choose a level A+ after the end of the first year.

The education system in England and Wales are not the same as that in Northern Ireland and Scotland, which also differ from each other. The 1988 Education Reform Act (ERA) changed the school system in England and Wales considerably (Convey & Merritt, 2000, p. 377). Usually primary education begins at the age of five (Eurydice, 1991, p. 186). One change was a compulsory national curriculum for all pupils in state schools from the ages of 5 to 16; national standardised tests of pupils' achievement at ages 7 (Key Stage 1), 11 (Key Stage 2), 14 (Key Stage 3), and 16 (Key Stage 4) (Convey & Merritt, 2000, pp. 378-80). At 16 the pupils sit for the General Certificate of Secondary Education (GCSE) examination. In

England and Wales, primary education takes place either in a primary school or first schools followed by a middle school. Secondary education takes place either in a comprehensive school, a grammar school, secondary modern school, or a technical school or pupils from middle schools go to a high school. Upper secondary education can also take place in a separate Sixth Form College or a two-year Youth Training in co-operation with private employers (Eurydice, 1991 & 1997). What is equivalent to the Danish high school examination is the “General Certificate of Education at Advanced Level (GCE A-level)” or the “Advanced Supplementary Examination (AS)” (Eurydice, 1997, p. 46). The latter is for the pupils who want to gain greater knowledge of mathematics. A module system allows for further study.

2.4.2 SAMPLING OF SCHOOL, ENTRY, AND ACCESS

The Danish school was chosen through purposive sampling (Bernard, 1994, p. 95 & Robson, 1993, pp. 141-142) and fulfilled following criteria: (1) For practical reasons I needed a high school in the town in which I lived, which is Aalborg. (2) I needed a school that would be willing to co-operate. I am a former pupil at the school and in the school year 1997-98 I had a one-year vacancy at the school in mathematics and social science. Therefore entry and access was very easy. The pupils did not know me, not did I know them. I asked the four teachers who teaches mathematics at Level A to each find me about three pupils who were good at mathematics in their classes. Two of the teachers could not find any pupils who wanted to participate. The week after the interview there was mid-term test at the school, and the next week Easter holiday (from Friday before Palm Sunday until Tuesday after Easter). The teachers found it best to interview the pupils before Easter as the pupils after Easter would only have the forthcoming examination in their heads. I therefore interviewed four pupils in March 1999. I wanted to interview the pupils towards the end of the school year as I wanted the pupils who had been high-achieving and to know who had, generally, been high achieving, one has to wait until the end of the school year. A problem that arose was that the teachers had been on strike in the 3rd and 4th lesson owing to dissatisfaction about the collective bargaining about the agreement on wages and working conditions. Pupil Ø was the only pupil from her class.

Before the interview begins there is an introductory chat. I tell about the project and that I would like to hear about their experience learning mathematics. I told them that they are the experts who I shall learn from. They should say whatever comes to their minds, also things that may to them seem obvious, it may not be obvious for me. They do not need to raise their hands to be allowed to talk; they should just talk. I also tell them that they will remain anonymous in the transcriptions from the tape.

I also chose the English school through purposive sampling. The criteria were: (1) For practical reasons I needed a school that taught A-level Mathematics in the town I lived and studied in, which was Oxford. (2) I needed a school that would be willing to co-operate. (3) I wanted to interview the pupils at approximately the same time of year as the Danish pupils. I decided on an independent boarding school.

I wrote a letter to the Head of the school and got a positive reply through my supervisor the same day the letter was received. Getting entry was therefore quite easy (Hammersley, 1995, p. 55). I had a meeting with the Head of the Mathematics Department and I experienced her as very interested and helpful. At the school, there were four A-level Mathematics classes. One of these was a class doing AS Level Mathematics consisting of seven pupils. Originally, there were 14 in the class. The class consisted now of six boys and one girl. It was these pupils that I interviewed.

The Danish interview took place in the pupils' common room, which was also the smoking area and a place where pupils sometimes sat and worked in groups during class. We sat in soft chairs. The interview began Thursday 18 March 1999 at 2.50pm, just after the seventh lesson. The interview lasted approximately 1½ hour and was taped. In relation to the location of the English interviews I asked for the existence of a common room for the pupils, but the Head of Mathematics Department suggested the pupils' classroom, which was easy for the pupils to reach. We sat round two tables put together in a square so I did not look like a teacher. I did not want to be assigned the role of an evaluator/examiner but the role of a professional friendly novice researcher who wanted to learn from them and someone they could have faith in (Hammersley & Atkinson, 1995, p. 103).

I assumed that the pupils would take an interest in my research topic, as they had chosen to study either the advanced Level-A in Denmark or the English AS Level Mathematics. The Danish pupils' teachers and the Head of Mathematics Department at the English school told me that all the pupils had agreed to participate. Partly to "pay" them for

their trouble, and partly for making ‘hesitant’ participators (Morgan, 1998a, p. 67) more motivated towards participation, I bought them each fizzy drinks and chocolate-chip cookies and “bonded” by drinking the same drink as them (Morgan, 1998b, pp. 110 & 128). To me, they seemed grateful, surprised, and engaged. The English interviews were on a Thursday, Monday, and Tuesday the 17th, 21st, and 22nd of February 2000 at 7pm. The interviews all lasted 65-70 minutes.

2.5 METHODS OF ANALYSIS

In this section I will discuss how I used the various psychological learning theories.

Theory is a systematic inspiration to get a greater understanding of how the pupils say they learn. One could state that theory is basically other people’s experiences being put into a system. The theory cast light on the data, and the different theories do it in a different way. I have predetermined themes in this study as I do not want to pretend that nobody has done theoretical thinking in this area before. I use theories to put concepts on the pupils’ explanations. I will use the framework for analysis that will be developed in Chapter 3 to identify some of the things the pupils mention and to identify what they mention that are not supported by these theories. This analysis takes place mainly from transcripts of the interviews and observations. A discussion of the transcripts will take place in Chapter 5.

Kvale discusses various contexts of interpretation (Kvale, 1996, pp. 214-217). The first context is the ‘self-understanding’ where the researcher in a condensed form rephrases what the interviewee said. Another context is the ‘theoretical understanding’, where a theoretical framework for analysis is applied to interpret the meaning of the statements. These two levels fit with the aim of this study as the first level aims at understanding the pupils’ understandings while the other is an application of a theoretical framework for analysis in order to describe and analyse more deeply what the pupils expressed about their own learning. Another advantage of this separation is to avoid confusion about what the pupils say themselves and what they say through the eyes of the theoretical framework for analysis. To exhibit the narratives, which reflect the pupils’ self-understanding’, I use a so-called ‘checklist matrix’ (Miles & Huberman, 1984, pp. 95-100). These are seen in Appendix A. This is also the first

step of the analysis as it groups the pupils' statements and it is also to help systematise the analysis and create overview.

In practice, the analysis will take place in three steps: 1. Categorise the utterances according to which theme it belongs to. 2. Understand the pupils' self-understanding. 3. Get a theoretical understanding which means to see which theory under the appropriate theme seems to be the "best fit" and discuss the extent but of which there are holes. An example: A pupil talks about the first and the third step of a theory, but not the second. Can we then assume that the theory still explains how the pupil actually learns mathematics? Does it mean that the theory is not (always) true or does a second step exist, but the pupil has either forgotten to mention it or the pupil is not aware of it. The answer to this question will depend on individual judgement of the whole of what the pupil say, phrases etc. The result might either be (suggestions to) a new theory or a decision to regard it as irrelevant.

Fundamentally, this is a general discussion of the relationship between theory and data, briefly stated: does theory enlighten data or does data check theory. I find it is problematic to just say that it is a two-way process as if I look at the data through the eye of theory, the theory will influence what it is that I see, and then when I later try to compare this knowledge of the pupils with the theories then it would not be surprising that there are some similarities. And vice versa. On the other hand, we have to make it a kind of two-way process for the following reasons: (1) Basically the theories are per definition never finite. One issue here is that the theories have arising on the background of various experiments and investigations etc. There is a variety of for instance the age of the participants in these studies. For instance, as discussed in Chapter 1, Krutetskii's work is based on investigations on school children, Schoenfeld's work is on university students, and Piaget and Vygotsky's work is with children. This is one of the reasons why we cannot assume that the theories will fit perfectly to what any pupil say. Therefore theory needs "input" from data to develop further. But (2) on the other hand, the pupils might not know *everything* about their learning process either, and even if they do, they might not have expressed everything they know, despite that the methods of the data collection tried to take everything into account. Furthermore, the researcher is different from the pupils in age and background and for some of the pupils also different in gender, nationality, and language. According to Schutz (1964), a stranger discerns peculiarities with great clear-sightedness, and Hammersley and Atkinson (1995, p. 9) write about the necessity to remain 'anthropologically strange'. But, even so, the social world does

only make sense from within (Winch, 1990, pp. 88-89) and “to notice something is to identify relevant characteristics, which means that the noticer must have some *concepts* of such characteristics” (Winch, 1990, p. 85). Thus, to understand the pupils, one also needs the theories.

In this study I will begin by putting the main emphasis on what the pupils say. This is owing to that according to various theoreticians mentioned in the beginning of this Chapter, the high-achieving pupils can be assumed to have something important to say. Some theoretical assumptions have determined the creation of the themes for analysis, but, at the same time, the interview sessions were very “un-controlled”. This means that one avoid that if theoretical assumptions both controls the interviews (creates utterances) and the frame for analysis (analyse the utterances), then the process is very controlled and one could fear that it goes in circles. On the other hand, one needs theories to be able to see something more. As described above, the starting point of the analysis is therefore at the pupils’ explanations and then afterwards I apply the theories to discuss these utterances. Therefore, for this study, one must be careful in giving suggestions and feedback to the original theories.

2.6 ETHICAL CONSIDERATIONS

This study involves participants, and, following Fog, “we cannot hope to put a priority on research and still hope this to be justifiable, if at the same time this is bad for the people researched” (Fog, 1993, p. 158). Foster recommends that a researcher considers the morality of the research itself, the conduct of it, and the reporting (Foster, 1996, p. 116). I therefore aimed at conforming to the BERA (British Educational Research Association, 1992) guidelines for ethical research. I therefore made it clear for the teachers that I was not evaluating their teaching, and I made it clear to the pupils that I was not “examining” their mathematical abilities, and that they (the English pupils) were not supposed to be able to get a full picture of the mathematics that I gave them. The pupils in the interviews knew each other in advance and were also going to meet afterwards. There was therefore a danger that self-disclosure might cause harm afterwards (Morgan, 1998a, p. 91). However, I do not think that there were any ethical problems here because the topics they discussed were not sensitive

(Morgan, 1998a, p. 50). They were interviewed about something they were good at and they were chosen because they were good, which they knew.

Results and tapes were treated confidentially, and I asked for permission to tape-record before each interview. The pupils and teachers are identified with pseudonyms to protect their anonymity and confidentiality. Another reason for the anonymity was that as the transcriptions were verbatim in style (see Chapter 5), where people's talk looks incoherent, this might cause distress for the pupils if they are being recognised. The names of the schools are also not mentioned to protect the identity of the participants.

2.7 VALIDITY AND RELIABILITY

In this section I will generally discuss the notions of validity and reliability. How to secure validity and reliability in the data collection and analysis will be discussed and considered again as they later come up.

The term 'validity' has been defined in many ways, but it generally means the extent to which an account accurately represents the phenomena it refers to. Validity is another word for truth, correctness, or 'trustworthiness' (Silverman, 2000, p. 175; Hammersley, 1998, p. 62; Kvale, 1996, p. 236; Lincoln & Guba, 1985, p. 290).

2.7.1 INTERNAL VALIDITY

In relation to this study one can discuss if the methods chosen give a valid account. Kvale's rejection of absolute truth leads him to stating that: "The quest for absolute, certain knowledge is replaced by a conception of defensible knowledge claims. Validation becomes the issue of choosing among competing and falsifiable interpretations" (Kvale, 1996, p. 240). Kvale's discussion about choosing among interpretations becomes important, even if one does not share his postmodern position. In this connection, Hammersley (1998, pp. 64-69) argues for accepting something as true when it is beyond reasonable doubt, not beyond all possible doubt. This is connected with the view of Evans (1983, p. 183) where 'internal validity' in qualitative research is about to what extent other explanations have been ruled out.

One could here discuss that it becomes crucial for the validity that the teachers choose the best pupils. However, since I only want pupils who have taken the Level-A/AS-level option in mathematics, *any* pupil in these classes would be good at mathematics.

To Kvale “validity is ascertained by examining the sources of invalidity” (Kvale, 1996, p. 241). In this connection, “to say that ... our data, are constructed does not automatically imply that they do not or cannot represent social phenomena” (Hammersley & Atkinson, 1995, p. 18). In relation to this is the problem of reactivity. The issue is not if the researcher affected the behaviour, but “whether they have affected it in respects that are relevant to the claims made (and to a significant degree)” (Hammersley, 1998, p. 86). It therefore becomes important how I probed. An example: after asking the third English pair about how they would describe their learning process in IE3, 222-231, they say:

B: Learning process (F: We, nah) I think that for maths it is certainly different than subjects cause when you learn, it is learning how to use a technique rather than, like lists of meanings or whatever, and er so I think the learning process is just practice, have it explained to you, and then practice it to sort of get it, get it over in your head so you can do it easily at a time.

F: It is er we get taught through the principles, been given one example and just go practice it, that's the basic learning process I think.

I: But why does this, I mean, what kind of practice?

Here the researcher changed her mind halfway through the question and decided not to probe more about why practice helps. Therefore the pupils did not get the opportunity to elaborate on this, which they may or may not have been able to. The change of direction happened as this pair discussed learning as something externally, practice, not as cognitive processes, while for instance the first pair talked about that new knowledge requires a change of the cognitive structure. The researcher estimated that this, together with their relatively short narratives, meant that they were not as able to articulate the information about their learning process, and the researcher did not want to make them feel uncomfortable by asking them questions they could not answer. Being uncomfortable could have affected the rest of the interview negatively, which the researcher estimated would have been worse than not getting an “answer” to this single question.

As part of the examinations of the sources of invalidity I will also state that another option here is to make the whole process of data-collection and analysis transparent whereby

the reader becomes able to check for himself how the work is being done. This is one of the reasons why I have chosen to be rather detailed in the methodological discussions.

In groups, one can furthermore observe the group validate each other's statements (Morgan, 1998a, p. 52). The non-verbal actions add to this (Frey & Fontana, 1993, p. 32). In the end of the first English interview (IE1, 718-736) there is an example that shows both some pupils validating each other about an issue but also that this does not necessarily is the same as complete agreement:

A: Yea, cause [C giggles] some people want the sort of, just be told, like I say just to be told to do examples, find it helps them learn.

[1-2 sec silence]

I: You don't think it learn, er, you learn anything by just doing example?

A: Yea, you do, you you learn how to apply the method. (C: You don't understand it) [some words are lost due to the interruption] helpful in terms of [C sighs] say what you gonna have to do in the exams (C: mmm) (I: mmm). It's being able to, you know, it helps you to work (C: mmm [very silent]) around slightly different forms of the problems being able to apply the method quickly without making any stupid mistakes, and so it is definitely useful in the form of exams, but

C: That's what I DO like, for revision (A: Yea) cause I understand (A: Yea yea, once you) if I if you know that basic integration for example, there is no point going through, for revision, you've got 2 weeks to the exam (I: mmm), there is no point in going through your notes on integration (I: mmm), cause you know how to integrate. What you need to do is have loads of loads of practice

Triangulation is “the checking of inferences drawn from one set of data sources by collecting data from others” (Hammersley, 1998, p. 230). A triangulation during the interviews was that I asked them to think about how they would present this mathematics to the class, as this was another way to talk about learning new mathematics. An example was in the third English interview (IE3, 670-676) shortly after the pupils stopped working on the mathematics:

I: I notices when when you talked about presenting it to the class you wanted to give them examples and you also mention while you were talking about, that it would be nice, it would be nice with examples. Why, I mean?

B: That's because we always, the way that we've always been taught is, because this using examples thoroughly to explain, so that's the way we think the people in our class will understand it easiest, how explain through examples

If an issue is complex, it can be difficult to know what questions to ask, but the group discussions can allow one to hear the questions the pupils want to ask each other. “This provides an excellent opportunity to uncover things that you never knew existed. At the same

time, you do not surrender to your own ability to ask questions” (Morgan, 1998a, p. 58). Another advantage is that group members respond to each other, which could stimulate recall and opinion elaboration, and then diffuse the interviewer’s influence on the interview (Fonatana & Frey, 1993, p. 26). In the interviews the participants sometimes asked questions to each other, which made the interviewer’s influence less than had the interviews been one-to-one.

Another issue here is how the pupils interpret the things I ask them about and if they can have a discussion between themselves about how they learn mathematics. Do they seem to recognise something in what the other pupil talks about and does it seem to make sense to them to have such a conversation. If this is a case, this would suggest that the pupils do have a metacognitive awareness and are able to articulate it.

2.7.2 EXTERNAL VALIDITY

Generalisability is in the sense of producing laws that apply universally not a useful standard or goal for qualitative research (Guba & Lincoln, 1989, p. 61). Therefore, the notion of generalisability, ‘external validity’, is replaced by ‘fittingness’, “the degree to which the situation matches other situations in which we are interested” (Schofield, 1990, p. 207). Goetz and LeCompte (1984) have a similar emphasis on the importance of clear and detailed description to determine the applicability of one study to other situations. They use the notion ‘translatability’ to denote if the theoretical frames and research techniques are understood by other researchers in the same field, and the notion of ‘comparability’ to mean if a situation has been “sufficiently well described and defined that other researchers can use the results of the study as a basis for comparison with other studies addressing related issues” (Goetz & LeCompte, 1984, p. 228). ‘Thick descriptions’ are therefore vital for others to be able to judge if the attributes compared are relevant (Kvale, 1996, p. 233). This is in line with my discussion about making the process transparent.

As this study aims at exploring and describing a smaller number of pupils that are not randomly sampled, one cannot hope to make generalisations in the quantitative sense. However, one can through thick descriptions of the pupils’ explanations and the methods hope that future investigations within this topic to some extent could build on the findings.

2.7.3 RELIABILITY

The term reliability has (like validity) been defined in a variety of ways. To some it means the extent to which a study can be replicated (LeCompte & Goetz, 1982, p. 35), to others the consistency, predictability, or stability of findings (Lincoln & Guba, 1985, p. 292).

According to Hammersley (1987, p. 78) the usefulness of these definitions depends upon 'reliability' being understood in a realist or nominalist way. A realist focuses on the results of the measurements in relation to the property being measured, while a nominalist focuses on the relationship between the results of the measurement and the tools that produce these measurements (Hammersley, 1987, p. 75). A realist focus is therefore not only about measuring the same, but also measuring in a consistent and precise way what it is intended to measure (Hammersley, 1987, p. 76). Contrary, a nominalist focus discusses things such as intra-observer and inter-observer reliability (Robson, 1993, p. 221), or if two different interviewers, following the same guidelines, do the same, or if the same interviewer would conduct the same kind of interview if he had to repeat it (Kvale, 1996, p. 208). Nominalist reliability has only a value "if what it measures has a high validity" (Hammersley, 1987, p. 75). Turning this argument around, one could argue that *if* what is measured *is* valid, *then* it is desirable that the method can be replicated so other researchers may be able to use it. In this connection Eisenhart (1988, p. 108) writes that ethnographic research can be made replicable if it in details describe the choices made, the settings, conditions, the researcher's role, the methods of data collection and analysis.

According to Kvale (1996, pp. 157-159 & 235), interviewer reliability has mainly to do with leading questions, unless they are used on purpose to "check repeatedly the reliability of the interviewee's answers, as well as to verify the interviewer's interpretations. Thus ... leading questions do not always reduce the reliability of interviews, but may enhance it" (Kvale, 1996, p. 158). I used leading questions a bit to challenge them, and tried to not show emotions if they said something very interesting. Instead I quickly noted down my reaction. It may be leading that the pupils observed me taking notes hinting they said something "good", but I estimated that it was "less" leading.

*Personally I am always willing to learn,
although I do not always like being taught.
(Churchill, 1998, p. 13)*

3. THEMES FROM THEORIES OF LEARNING MATHEMATICS

This chapter will present and discuss different themes from a variety of psychological theories of learning mathematics. I will try to find out which “themes” or aspects the various theories find are important for the cognitive side of learning mathematics. The way I will do this is to investigate the various theories mentioned in Chapter 1 and, as seen below, here notice that for instance ‘language’, ‘the social’, and ‘the unconsciousness’ are some of the themes mentioned by some of these authors. The themes, so to speak, go “across” the theories. How I find these themes will be explained in more detail below. These themes should not be mistaken for the types or styles of learning that I write about in Chapter 1. Instead it could be the case that one type of learning has special characteristics within several themes. This chapter will show that I found six themes, which I divide into three binary opposite-pair. The degree to which these pairs actually are “opposite” or if they rather support each other will be discussed in Section 3.5 and at various places throughout the thesis. Included here is also a discussion of to what extent the internal relationship between each of the pairs are the same or if some pair’s are different from others. I will use the six themes to label what the pupils say in various places in the interview. The fact that I, for instance, have a themes named ‘social’ only means that when I read through the transcribed interviews then, whenever a pupil mentions something about for instance the importance of a social element for the learning, I will label this remark ‘social’ without, in the first instance, going any deeper into what the pupil means. The names of the themes are thus “thesis-empty” overall boxes, shelves, to sort out various areas and topics the pupils mention. This means that even when various and different theories are mentioned under a label such as ‘social’, I will use the label ‘social’ as an “empty” theme to group and sort the pupils’ explanations to create an overview of what they say instead of having one huge amount of data and a bunch of theories talking about everything.

One could perhaps argue that this splitting-up of the theories would destroy the theories, as a theory should be seen as a whole. But this is exactly why I chose to look at *all* six themes in the analysis, as this would ensure that I get “around” the theories. There might be theoretical concepts that could be put in more than one theme. In which theme it is being put, means less for the final analysis of what the pupils say. It is merely a question of in which section it is being written. The themes, so to speak, “cut” the theories into “modules”. This will therefore have an impact on the lenses through which the pupils’ narratives are understood. The disadvantages of this dissection of the theories is minor since the pupils are being looked upon from all six themes, and therefore the various pieces of a specific theories are being put together again in the end. The six themes are thus my construction.

To summarize, I will first investigated a number of theories, then create some overall themes, which are seen in Section 3.2-3.4. These themes, and the areas and ideas that are included in them, thus provide me with a framework for analysis. In Chapter 4 I will go “behind” some of the theories and discuss some of their underlying theses. I will identify two main opposite trends, the Piaget and the Vygotskian views and then discuss these in relation to each other, particularly the possibility of a synthesis and if they are really that different.

3.1 ESSENTIAL THEMES IN LEARNING MATHEMATICS

The six themes are: (1) Consciousness, (2) Unconsciousness, (3) Language, (4) Tacit, (5) Individual, and (6) Social. I will choose to label this the “CULTIS model for analysis” using the first letter from each of the themes’ names. This framework for analysis is developed to thematize how the pupils say they learn a concept from school mathematics at a cognitive level. The CULTIS model for analysis consist of the themes which various theories talk about in, sometimes, different ways. When I analyse the reports of the pupils (Chapter 6 onwards), I will thus in the transcriptions code what each pupil says according to which of the six themes it belongs to and notice if some narratives does not belong anywhere. After this I will analyse what it is the pupils say more deeply. What the pupils say could be seen as “islands” revealing parts of their cognitive learning process. To theorise on the islands, I need psychological theories. I use theories to be able to say something more about how they work than they are able to themselves. For instance, if they begin to talk about language, do they, in their own

word, say something that can be explained from the Vygotskian positive understanding of language in the learning process, or do they have a slightly more reserved view of the role of language like Piaget (1970).

Below I will develop and describe the six various themes for learning mathematics mentioned above. The way I developed these themes was firstly to read all these authors, write resumes of essential parts of their work, put all these resumes in one big document and then use word processing and comparison to find the general themes. I noticed for instance that several mentioned the topic of language and therefore I created a subsection titled “language” and put everything said about the (dis)effect of language under this headline. In the remaining document I then noticed that the unconsciousness was also mentioned a lot and subsequently created a new subsection with this title. And so forth. The CULTIS model for analysis was therefore developed gradually. At one point there were four themes (Dahl, 2000), but later I split up two of them. Previously the two themes tacit-language was one theme, as well as the social-individual themes were one. One could call this an iterative approach. Furthermore I decided that the depth of description seen below is suitable for the purpose of this study. For this frame for analysis I aimed at a balance between details and overview, and furthermore that the purpose of writing about the theories is not a goal in itself, but a tool. The level of detail in the description is also seen in relation to that even though the pupils are quite detailed in their, there are also limits to how detailed they are.

3.2 FIRST PAIR OF THEMES: CONSCIOUSNESS - UNCONSCIOUSNESS

In this section I will firstly discuss which role the conscious features such as practice, reflection, and planning has for the learning of mathematics from the point of view of various authors. Secondly I will discuss that various authors argue that part of the learning takes place in the unconsciousness. This will show how they discuss the experience that “a problem, after prolonged absence, may return into consciousness essentially clarified, much nearer to its solution than when it was dropped out of consciousness” (Polya, 1971, p. 198).

3.2.1 FIRST THEME: THE CONSCIOUSNESS

This subsection will consist of a discussion of mainly Polya's (1971) and Mason's (1985) ideas of different stages of working with mathematics. I will therefore discuss the role of planning, motivation, practice, and reflection. Mason (1985, pp. 27-28) describes three phases of work: Entry - Attack - Review, where one goes back and forth among the stages. I will use these phases as a way of structuring this subsection.

1. Before starting to work with the mathematics: understanding, motivation, and planning

Polya distinguishes between four phases of the work: 1. Understand the problem, 2. Devise a plan, 3. Carry out the plan, 4. Look back at solution, review and discuss. According to Polya "The worst may happen if the student embarks upon computations or constructions without having *understood* the problem" (Polya, 1971, p. 6). Furthermore the problem should be interesting and neither too easy or too difficult. For the second phase, Polya states that a good idea of a plan is "based on past experience and formerly acquired knowledge. Mere remembering is not enough for a good idea, but we cannot have any good idea without recollecting some pertinent facts" (Polya, 1971, p. 9). The planning is also according to Krutetskii (1976, p. 292) (see Chapter 1) an essential part of the high-achieving pupils' style.

In the first phase, the pupil must furthermore understand the problem before starting to work on it, and the pupil should "desire its solution" (Polya, 1971, p. 6), or in other words be motivated. However, for Vygotsky and activity theory, motivation is not just one "point on a list" but something very essential. The usual view of motivation is that motives steer actions, while for activity theory, the motive is in the object for learning. The cognitive drive does, so to speak, move into the object for learning. For activity theory, the object for learning is different, if the learner has different motives. Mellin-Olsen (1989, p. 16-17) argues that it is a central point in Vygotsky's work that when the child is learning, he is in an activity that is goal-oriented, and it is also a central point that the child, so to speak, owns the goal. As a contrast to this is the use of Piaget, where the children has been working with certain topic, to give them experience, but where they did not know the purpose of what they were doing. There is, however, no agreement about the exact role of motivation. Some argue that motivation "by no means [is] an indispensable condition" (Ausubel et al., 1978, p. 400) and

that the cognitive drive (the desire for knowledge as an end in itself) is “the most important kind of motivation in meaningful learning” (Ausubel et al., 1978, p. 403). Others state that “an affective element is an essential part in every discovery or invention ... it is clear that no significant discovery or invention can take place without the will of finding” (Hadamard, 1945, p. 31). Mellin-Olsen (1987, p. 157-158) identifies what he calls two major drives for school learning. The instrumental rationale (I-rationale) is that the pupil wants to learn as it will pay out in examinations while the social rationale (S-rationale) is that the pupil wants to learn as knowledge has a value beyond its status as school knowledge. According to Mellin-Olsen these two rationales work together. The concept of a cognitive drive is not the same as the S-rationale. I interpret the cognitive drive as being a kind of subset of the S-rationale. This means that the cognitive drive is a search for knowledge for its own sake (not because of an examination) while the S-rationale furthermore includes, for instance, the application of mathematical knowledge.

Of emotional factors, Mason writes that three factors influence one’s mathematical thinking. These are: “your competence in the use of the processes of mathematical enquiry; your confidence in handling emotional and psychological states and turning them into your advantage; your understanding of the content of mathematics and, if necessary, the area to which it is being applied” (Mason, 1985, p. 146). Mason writes that practice is important but without reflection it may leave no permanent mark, but it also needs time. Mason also states that it is important to reflect on one’s successes as it builds up confidence, and that it is necessary but not sufficient to have an atmosphere where confidence can grow. To support mathematical thinking one needs an atmosphere that is questioning, challenging, and reflective (Mason, 1985, p. 153). Polya’s two first phases seem to be similar to what Mason (1985, p. 28) calls the entry phase.

2. Carry out the plan, practice

In relation to carrying out the plan, Polya (1971, p. 12) states that it is much easier than devising the plan as it only requires patience. A main feature is however that the student is convinced that each step is correct (Polya, 1971, pp. 12-13).

Here I will also turn to theories more aimed at explaining what the pupils can *do* to learn mathematics. According to Polya, it is a practical skill to be able to solve problems and

since we require all practical skills by imitation and practice, this also applies for solving mathematical problems: “Trying to solve problems, you have to observe and to imitate what other people do when solving problems and, finally, you learn to do problems by doing them” (Polya, 1971, pp. 4-5). He therefore recommends that teachers give their pupils plenty of opportunities for imitation and practice combined with questions and suggestions suitable for developing the students mental operations and through this guidance the “student will eventually discover the right use of these questions and suggestions, and doing so he will acquire something that is more important than the knowledge of any particular mathematical fact” (Polya, 1971, p. 5). Practice, observation, and imitation are therefore important aspects in learning mathematics.

Sfard (1991) also suggests practice and describes a three-stage process in concept development. At the first stage, the ‘interiorization’, the pupil gets acquainted with the processes and operations that will lead to concept development. At the second stage, the ‘condensation’, the pupil begins to refer to the process in terms of input-output relations. The understanding at the first and second stage, is called the ‘operationally’. The third stage, the ‘reification’, is an ontological shift “The new entity is soon detached from the process, which produced it ... Processes can be performed in which the newborn object is an input” (Sfard, 1991, pp. 19-20). This is called the ‘structural’ understanding. Sfard (1991, p. 18) also states that operational understanding is the only way to ‘get in touch’ with abstract constructs. A nearly similar process is described by, among others, Dubinsky, who focuses on: action, process, object, and schema (Asiala et al., 1996, pp. 9-12), where the fourth stage is not included in Sfard’s theory. Skemp (1993) also writes about the necessity of automatic manipulation, but he distinguishes between “routine manipulations and problem-solving activity; and unless the former can be done with minimal attention, it is not possible to concentrate successfully on the difficulties” (Skemp, 1993, p. 83). But the routine manipulations are not in themselves mathematics.

3. Looking back, reflecting

Polya’s last phase is the looking back part and he writes (1971, pp. 14-15) that if the student after finding the solution, writes down the answer and then shuts the book, he misses the consolidation of his knowledge and the development of his problem solving skills which he

could have had, had he looked back at the solution, reconsidered and re-examined the result and the path that led to it. Also Mason states that “the only way to learn is from experience but experience alone is not enough. The experience must leave its mark” (Mason, 1985, p. 115). In this sense reflecting is essential. He then turns to discuss how a student in mathematics can develop an internal monitor to act like a tutor. By monitor, Mason means to have an idea about if the calculations are still relevant, have an idea about the execution of a plan, recognise generalisation, evaluate ideas, notice when the pupil is stuck, suggest alternative perspectives, suggest to change the plan, critically examine arguments, prompt the student to review the solution, and look outward (Mason, 1985, p. 117). In creating this internal monitor, practice is important and also rubric writing (Mason, 1985, p. 119), which means that one writes notes to oneself where one particularly notices incidents of being stuck, aha, check, and reflect (Mason, 1985, pp. 17-18). This development of an internal seems to be related with Schoenfeld’s (1985, 1992) discussion about metacognition as regulation of cognition and Krutetskii’s (1976) discussion of an adapter (see Chapter 1). Vejleskov (1998, p. 110) argues that also according to Piaget reflection is necessary.

3.2.2 SECOND THEME: THE UNCONSCIOUSNESS

The roles of the unconsciousness and the consciousness are however not mutually exclusive. As stated above, conscious work is a condition for the unconscious work, and vice versa. Mason writes, for instance, that for practice to leave a permanent mark, one needs reflection and *time* (Mason, 1985, p. 153). Also Polya, who talked a lot about the conscious effort’s importance, also discusses the unconsciousness.

One of the most famous examples of the impact of the unconsciousness for mathematics is Descartes’ vision and his three dreams on November 10, 1619. What happened was that he crawled into a wall stove and when he was well warm he had a vision. Preceding this had been a long time of intense concentration and anxiety over huge problems:

He was possessed by a Genius, and the answers were revealed in a dazzling, unendurable light. Later, in a state of exhaustion, he went to bed and dreamed three dreams that had been predicted by this Genius. ... He tells us that his third dream pointed to no less than the unification and the illumination of the whole of science, even the whole of knowledge, by one and the same method: the method of reason.
(Davis & Hersh, 1988, pp. 3-4)

Also Polya explains that “a problem, after prolonged absence, may return into consciousness essentially clarified, much nearer to its solution than when it was dropped out of consciousness” (Polya, 1971, p. 198). The work of the unconsciousness is experienced as ‘the idea came to me’ or ‘a sudden flash of lightning’, and Hadamard (1945, p. 56) states that there are four stages in learning: preparation, incubation, illumination, and verification.

1. Preparatory conscious work

Conscious work is preparatory to the illuminations. Hadamard writes that the unexpected inspirations are the result of intensive and lengthy work of the unconsciousness (Hadamard, 1945, p. 44). To Hadamard, discovery is not only a product of chance, but depends on preliminary work of the conscious (Hadamard, 1945, p. 46). Polya also states that “only such problems come back improved whose solution we passionately desire ... conscious effort and tension seem to be necessary to set the subconscious work going” (Polya, 1971, p. 198).

2. Incubation and illumination

The illumination is generally preceded by an incubation stage where the solving of the problem is completely interrupted (Hadamard, 1945, p. 16). Often the solution appears without any relation to previous attempts of solving the problem and could therefore not have been elaborated by the previous conscious work. But the illuminations cannot be produced without unconscious mental processes (Hadamard, 1945, p. 21). Two hypotheses have been set forth in relation to the phase of incubation. One is the so-called ‘rest-hypothesis’ where it is argued that a fresh brain in a new state of mind makes illumination possible. Another hypothesis is the ‘forgetting-hypothesis’ where the essential cause of illumination is the absence of interference to block progress. The incubation phase gets rid of false leads and makes it possible to approach the problem with an open mind (Hadamard, 1945, p. 33). In relation to the forgetting-hypothesis, I would speak of discovery and not illumination, as even though the solution appears unexpectedly, it is the result of new work. Instead, “the illumination process is not of the same nature as the previous conscious work” (Hadamard, 1945, p. 37).

The first stage in solving a problem is therefore to work in a very concentrated manner on it. One could state that this phase is related to the discussion in the first theme. After working, one puts the work aside, and then, following Skemp (1993, p. 86), unconscious mental activities continue and then suddenly an insight related to the problem come into consciousness at a time when no deliberate work on the problem is being done. Conscious work is therefore preparatory to the illuminations, and it is only the problems which one “passionately desire” to be solved that are improved (Polya, 1971, p. 198). Tall also argues that “working sufficiently hard on the problems to stimulate mental activity, and then relaxing ... allow the processing to carry on subconsciously” (Tall, 1991, p. 15). What is experienced as sudden inspiration “despite the apparent absence of a connection with his former experience, is the result of previous protracted thinking, of previously acquired experience, skills, and knowledge; it entails the processes and use of information the person amassed earlier” (Krutetskii, 1976, p. 305). It therefore seems that what is required for learning is: (1) work hard, (2) desire solution, and (3) relax/time.

3. Verification

Hadamard’s fourth stage consists of (i) verifying the solutions, (ii) state them precisely through calculations, and (iii) continuation of the work, how to utilise it (Hadamard, 1945, pp. 57-60). I will not discuss this phase further, as it is not directly related to how to *learn* mathematics. This means that in this thesis I work with a particular part of the concept of learning. Others might argue that it is an important part of the learning to be able to communicate or use the results, and it is, but this was not the scope of this thesis.

3.3 SECOND PAIR OF THEMES: LANGUAGE - TACIT

Besides the role of various types of conscious and unconscious work, one could also discuss the role of language and words in learning as well as discuss concept formation. I would assume that language has a role in the more conscious part of learning mathematics, but not in the unconscious part, which is also what Skemp (1993, p. 86) seems to argue. The opinions of the role of words and language can be divided into two groups. To some theoreticians words

are a necessary thinking-tool; to others it almost kills or obstructs thinking. I have therefore created the opposite-pair ‘language-tacit’ where language is the heading of views which talk about the positive/necessary aspects of language for learning mathematics, while ‘tacit’ is the heading for both the views that claim that language mainly obstructs learning and views that emphasise that language is not the *main* tool for learning, subsidiary, that people cannot express what they already know. I will now describe these views in more detail.

3.3.1 THIRD THEME: THE LANGUAGE

I have chosen to label this theme ‘language’ and by this I mean spoken and written language/words, not other languages such as gestures.

1. Language and thought

Polya finds questions posed by the teacher to be of great importance. The nature of these questions is “natural, simple, obvious just plain common sense” (Polya, 1971, p. 3). According to Hadamard, Polya also says that “the decisive idea which brings the solution of a problem is rather often connected with a well-turned word or sentence. The word or the sentence enlightens the situation, gives things, as you say, a physiognomy” (Hadamard, 1945, p. 84). Also Russell’s talks in a positive way about language: “Language serves not only to express thoughts, but to make possible thoughts which could not exist without it. ... I hold that there can be thought, and even true and false belief, without language. But however that may be, it cannot be denied that all fairly elaborate thoughts require words” (Russell, 1948, p. 74). Hadamard quotes Muller for saying that “no thought is possible without words” (Hadamard, 1945, p. 66). Russell and Polya do thus have a slightly less positive view of language than Muller.

Muller’s view might be considered to be connected with Vygotsky’s description of language as the logical and analytical thinking-tool (Vygotsky, 1962, p. viii) and that thoughts are not just merely expressed in words but come into existence through the words (Vygotsky, 1962, p. 125). Vygotsky also says that “Language does not of necessity depend on sound” (Vygotsky, 1962, p. 38). And further: “Thought development is determined by language, i.e.,

by the linguistic tools of thought and by the sociocultural experience of the child. ... verbal thought is not an innate, natural form of behavior but is determined by a historical-cultural process” (Vygotsky, 1962, p. 51). Thus, to Vygotsky, language is indispensable for being able to learn and furthermore the verbal form of language is not something that comes natural to a person.

These “positive” arguments are basically that thought and learning cannot take place without the use of language.

2. Concept formation

I decided to discuss concept formation under the heading of ‘language’ as concepts inevitable become part of a language regardless of how they are being created. Mathematics is furthermore itself regarded as a language (Pimm, 1990, p. 2; Dahl, 1995; 1996a&b), which means that the formations of concepts are an essential part of learning mathematics. Also Dowling discusses this and he states that school mathematics is discursive: “In my terms playing with teddy bears is not a discourse to the extent that its principles are always context-dependent and so non-explicit. School mathematics, on the other hand, is more discursive, because its principles are comparatively explicit and context independent” (Dowling, 1998, p. 97). In relation to the learning of mathematical concepts, there are, according to Skemp, two basic principles:

(1) Concepts of a higher order than those which people already have cannot be communicated to them by a definition, but only by arranging for them to encounter a suitable collection of examples. ... (2) Since in mathematics these examples are almost invariably other concepts, it must be ensured that these are already formed in the mind of the learner.
(Skemp, 1993, p. 30)

The examples must be alike in the features that should be abstracted, and different in the ways which are irrelevant for the particular concept. All concepts except the primary ones are derived from other concepts and they take part in the formation of other concepts (Skemp, 1993, p. 35). Therefore it is important that more basic concepts are learnt before going further. This conceptual structure is called a schema, and a schema is therefore a tool for learning as it integrates existing knowledge (Skemp, 1993, p. 37). It therefore seems that to Skemp, language is not essential for the creation of the basic concepts, but the higher concepts build on the basic concepts, which after being created, or discovered, become part of

the language. Rote-memorising is meaningless and an integrated conceptual structure is easier to remember than unconnected rules (Skemp, 1993, pp. 29-30). However, “rote and meaningful learning are not completely dichotomous” (Ausubel et al., 1978, p. 24). Schematic learning has the disadvantage that it may take longer. New knowledge that fits an existing schema is much better remembered, and therefore a schema is very selective and can be a hindrance to learn if it does not fit the new knowledge. An individual may resist changing his schema if he feels threaten, but a change is always difficult, whereas the assimilation of new knowledge to some existing schema gives one a feeling of mastery (Skemp, 1993, pp. 41-42). Similarly, Tall writes that “students struggle to come to terms with ideas which challenge and contradict their current knowledge structure” (Tall, 1991, p. 3), and he also refers to Piaget’s notions of “assimilation to describe the process by which the individual takes in new data and accommodation the process by which the individual’s cognitive structure must be modified” (Tall, 1991, p. 9). About learning, Piaget explains:

To know is to assimilate reality into systems of transformations. To know is to transform reality in order to understand how a certain state is brought about. By virtue of this point of view, I find myself opposed to the view of knowledge as a copy, a passive copy of reality. In point of fact, this notion is based on a vicious circle: in order to make a copy we have to know the model that we are copying, but according to this theory of knowledge the only way to know the model is by copying it ... knowing an object does not mean copying it - it means acting upon it. It means constructing systems of transformations that can be carried out on or with this object. Knowing reality means constructing systems of transformations that correspond, more or less adequately, to reality. ... Knowledge, then, is a system of transformations that become progressively adequate.
(Piaget, 1970, p. 15)

Thus, there are concepts of different kinds learning, which seem to be characterised either as rote-learning, assimilation into existing schemas, or accommodation and change of a schema.

3.3.2 FOURTH THEME: THE TACIT

In relation to the “negative” views on language, Berkeley argues that “words are the great impediment to thought” (Hadamard, 1945, p. 68). Krutetskii writes that the thinker arrives at an answer with little awareness of the process by which he reached it (Krutetskii, 1976, p. 307). Furthermore, a pupil who “behaves the right way usually does not care to express his behavior in clear words and, possibly, he cannot express it” (Polya, 1971, p. 3). Galton explains that results can be perfectly clear to himself but “when I try to express them in

language I feel that I must begin by putting myself upon quite another intellectual plane. I have to translate my thoughts into a language that does not run very evenly with them” (Hadamard, 1945, p. 69). Hadamard states that “thoughts die the moment they are embodied by words” (Hadamard, 1945, p. 75). Hadamard also writes that a thought “can be accompanied by concrete representations other than words. Aristotle admitted that we cannot think without images” (Hadamard, 1945, p. 71). Later Hadamard writes that he “feel some uneasiness when Locke and ... Mill consider the use of words necessary whenever complex ideas are implied. I think ... that the more complicated and difficult a question is, the more we distrust words, the more we feel we must control that dangerous ally and its sometimes treacherous precision” (Hadamard, 1945, p. 96). But still he acknowledges that “signs are necessary support of thought” (Hadamard, 1945, p. 96). Piaget (1970, pp. 18-19) states that “This, in fact, is our hypothesis: that the roots of logical thought are not to be found in language alone, even though language coordinations are important, but are to be found more generally in the coordination of actions, which are the basis of reflective abstraction”. Here Piaget seem to disagree with Vygotsky who, see above, stated that the roots of logical thinking is language alone. Piaget (1970, p. 15) states that to him knowing an object does not mean to copy it, but to act upon it. To Piaget, an abstraction is “drawn not from the object that is acted upon, but from the action itself. It seems to me that this is the basis of logical and mathematical abstraction” (Piaget, 1970, p. 16).

One could here discuss if the lack of ability to explain what one is doing is because the knowledge is tacit and one therefore knows more than one can tell, or because the pupils have nothing to say. In relation to tacit knowledge, one can observe that a person has a certain kind of knowledge, but “on questioning, it appeared that he did not know he was doing this. Here the subject got to know a practical operation, but could not tell how he worked it” (Polanyi, 1967, p. 8). Wachterhausen (1991, p. 90) describes the so-called technological logic which he defines as the view that all knowledge is explicit linguistic knowledge and that reality can be described and explained completely in an explicit linguistic way. In combination with this is, according to him, tacit knowledge and situated learning.

About having something to say, Schoenfeld (1992, pp. 356-357) describes that after constantly asking his students to explain what they were doing and why, while solving problems, they became able to articulate their reasons, which also helped their problem-solving behaviour.

The “negative” arguments are thus centred on the general uselessness of words in thinking and learning, as well as the lack of ability to describe what one is doing.

As this thesis is about how pupils say they learn mathematics on the cognitive level I will delimit myself from non-cognitive ways of tacit knowledge about learning. This means that when I write about tacit knowledge I concentrate about four things: (1) Notice if/when the pupils are unable (or perhaps unwilling) to speak about their learning process. (2) Notice if they even say that something is “difficult” to talk about. (3) Notice if they say that the language is not an important thinking or learning “tool”. Also if they state that examples come before words. (4) Interpret from for instance their actions in the intersection (for the English pupils) that they perhaps know something that they are not conscious about that they know. It could also be that they know a certain “method” works but they are unable to explain why it works.

3.4 THIRD PAIR OF THEMES: INDIVIDUAL - SOCIAL

Across the above features for the learning of mathematics runs a duality of the individual and the social. I will now discuss the importance of the individual and the social interaction. One might argue that this is also about the consciousness, but I do not place it in the first theme, as the emphasis is not on a specific plan or strategy.

3.4.1 FIFTH THEME: THE INDIVIDUAL

This perspective is represented by for instance Glasersfeld and Piaget. According to Glasersfeld, Piaget is the author who has influenced his later thinking more than any other (Glasersfeld, 1995, p. 12). Glasersfeld’s epistemology is that “knowledge, no matter how it is defined, is in the heads of persons, and that the thinking subject has no alternative but to construct what he or she knows on the basis of his or her own experience. ... all kinds of experience are essentially subjective” (Glasersfeld, 1995, p. 1). For Glasersfeld constructivism “the subject cannot transcend the limits of individual experience. This condition, however, by no means eliminates the influence and the shaping effects of social

interaction” (Glaserfeld, 1995, p. 2). The two basic principles: “knowledge is not passively received but built up by the cognizing subject; the function of cognition is adaptive and serves the organization of the experiential world, not the discovery of ontological reality” (Glaserfeld, 1995, p. 18). Knowledge is thus personal.

Piaget discusses what mathematical-logical knowledge is abstracted from and then discusses two possibilities: one is that we get knowledge from the object itself through performing acts on it. This is not the basis of mathematical-logical abstraction. Individual actions (for instance throwing) give rise to abstractions from objects (Piaget, 1970, pp. 17-18). Instead Piaget finds that the basis of abstraction comes from the action itself, not the object (Piaget, 1970, p. 16). The individual who is learning is therefore active and the acknowledgement comes as the individual manipulates with the objects and reflects on this manipulation. Piaget talked in this connection about reflective abstraction, which *inter alia* means the transposition from one level of a hierarchy to another, and it means the mental process where a reorganisation of thoughts takes place. Reflective abstraction is based on coordinated actions, not individual. As quoted above, Piaget (1970, pp. 18-19) argues that (1) language is not the main thinking-tool, (2) both the individual actions and the individual performs coordinated ones lead to abstraction, but it is the latter that leads to reflective abstractions and then to logical-mathematical knowledge. Piaget therefore finds that logical-mathematical abilities do not arise from language or linguistic competency, but from the ability to coordinate actions and operate with objects. However, according to Vejleskov (1998, p. 109) Piaget did actually emphasise that adults should help children to structure their experience. Furthermore, the central aspect of activity is not the manipulation, the central feature aspect is interest, or engagement. It is therefore only activity if the child is personally engages and involved, which is a lot similar to the concept of activity that is tied to meaning and motivation in Soviet psychology.

In general, Glaserfeld and Piaget seem to argue that the basis of learning is the individual acts and that the ability to perform these acts is inborn. This is different from Vygotsky. But Piaget does not talk about any kind of activity but activity where the learner is personally motivated and where a knowledgeable person plays a role, which is more in line with Vygotsky.

3.4.2 SIXTH THEME: THE SOCIAL

The discussion of the role of the social will be in relation to internalisation, verbalisation, discussion, and the zone of proximal development (ZPD).

1. Internalisation and discussion

For Schoenfeld, social interaction plays a fundamental role in shaping pupils' internal cognitive structure (Schoenfeld, 1985, p. 141). Also Vygotsky writes about this and he states that this process has two levels, the social and the individual: "first between people (interpsychological), and then inside the child (intrapsychological). ... All higher functions originate as actual relations between human individual" (Vygotsky, 1978, pp. 56-57). In this theory, the process of internalisation is gradual. In the beginning a teacher controls and guides the pupil's activity, but later they begin to share the problem-solving functions, and here it is the pupil who takes the initiative while the teacher corrects and guides. At last, the pupil is in control and the teacher's role is mainly supportive (Confrey, 1995, p. 40). This could be linked to the discussion of practice and imitation in Theme 1 and Ernest's social constructivist view that pupils reconstruct the objective mathematical knowledge as subjective knowledge through social negotiation with teachers, a text, or other pupils. It is particularly the negative feedback from others help to develop a reconstruction, i.e.: fit, between the subjective and objective knowledge (Ernest, 1991, p. 81). Objective knowledge of mathematics is social and is not in recorded material or in an ideal world. Instead it is in the shared rules and meaning of the individuals and in their interaction, and objective knowledge is therefore constantly being recreated by the growth of subjective knowledge in the heads of individuals, but objective knowledge is not just the sum of all subjective knowledge (Ernest, 1991, pp. 82-83). To Ernest, knowledge is therefore partly in the heads of individuals and partly "between" individuals.

Following Schoenfeld, working as an individual, one might perform up to a certain level, while working under guidance, or in collaboration, one might perform at a higher level. Schoenfeld states that "more progress takes place when children with different cognitive strategies work together than when children with the same strategies do so, and that not only the less advanced but also the more advanced child makes progress when they interact with

each other” (Schoenfeld, 1985, p. 142). Others argue that “the gain of skill is always greatest among low-ability pupils and among pupils working with superior partners” (Ausubel et al., 1978, p. 468). The main difference here seems to be how much the high-achieving pupils benefit from interacting with low-achieving pupils.

According to Vygotsky, the potential for learning is furthermore limited to the “zone of proximal development (ZPD)” (Vygotsky, 1978, p. 86). ZPD is the area between the tasks a pupil can do without assistance, and those, which require help (Confrey, 1995, p. 40).

It therefore seems essential for learning that pupils are active and have the opportunity to be guided by a knowledgeable person.

2. Verbalisation

According to Skemp, verbal thinking, through either spoken or written word, is an example of a social activity. Some pupils have a strong visual imagination whereas others are more verbal. It is easier to communicate auditory images than visual ones, as one just turns thinking into speaking aloud, but to communicate visual thoughts, one must draw a painting. When the pupil speaks aloud, the “audible speech brings ideas into consciousness more clearly and fully than does sub-vocal speech” (Skemp, 1993, pp. 91-92). This relates to the experience that we sometimes are able to solve a problem after talking about it loudly even without the listener interfering. In a discussion this subjective effect is on both sides. Vision is therefore individual, while hearing is collective (Skemp, 1993, p. 104).

3.5 DISCUSSION

I will here briefly summarise some of the arguments from above and discuss the six themes in the CULTIS model for analysis.

The first theme is termed the consciousness. It does not mean that people are not conscious about anything but what is written under this headline. It is important that pupils (or at least teachers) are consciously aware of the possibilities and ways of learning (what I will term *methods*) offered by the various theories mentioned in the themes. In that sense being conscious is the general overall *method* in all the themes. Thus, the unconscious theme is

about ‘being conscious about the unconsciousness’, the language theme is about ‘being conscious about the language’ etc. Hence, the conscious theme is about ‘being conscious about the consciousness’ which denotes that for the conscious theme being conscious is also the *tool*. This means that for instance the ‘understanding’ before beginning to solve a problem, the ‘practice’ while one learns, as well as the ‘reflection’ towards the end, all are conscious efforts. They are things one has control over; for instance the order in which one does things. This is contrary to for instance the language theme, where the tool is ‘language’ and one probably knows methods of which to use language, but one does not have control over the ‘language’ as such. One is subject to the law of language and can only hope that the methods to handle that the language is a means to learn, are good enough. Similar things can be said about the other themes.

In summary, what the conscious theme mentions is mainly from Polya and Mason. I described three of conscious work phases: First, before starting to work with the mathematics: understanding, motivation, and planning; second, carry out the plan, practice; and third, look back and reflect. All the steps are cognitive, except the second. There is interplay between the conscious and the unconscious factors. They are not distinct themes but the conscious effort is a condition for the unconsciousness. The work of the unconsciousness is furthermore a necessary input for the further conscious work. The role of language is mainly seen in relation to the conscious work. These factors relate to how the individual work with mathematics. However, the role of interaction plays a role for particularly the language and the concept development.

I have termed a second aspect of learning for unconsciousness, and here is Hadamard particularly relevant. The tool here is that preparatory hard conscious work, where one really wants to find the solution, can be followed by an incubation phase. This phase consists of time and/or rest and this is where the unconsciousness works. This leads to the illuminations. Two hypotheses have been set forth in relation to the phase of incubation, the ‘rest-hypothesis’ and the ‘forgetting-hypothesis’.

The third theme is the language (spoken or written) and the fourth is tacit. The opinions of the role of words and language can be divided into two groups. To some, words are a necessary thinking-tool; to others it obstructs thinking or at least play a minor role. ‘Tacit’ is the heading for the two latter views; i.e. that language is not the *main* tool for learning. There might be a problem with the tacit theme for this study as the tacit theme may not be rightfully

investigated in a study like the present, which build on interviews. On the other hand, what is relevant to investigate is whether the views mentioned in the fourth theme form part of the pupils' verbalised metaknowledge, to which the CULTIS model for analysis is useful.

In relation to the last four themes, each pair of themes seem to be "opposite" to a larger extent than the first two themes who support each other. It will therefore be interesting to see if the pupils talk about speaking aloud, and the role between visualisation and verbalisation, examples, language, and learning the basics. Some theorists have also considered the role of interaction essential and it would be important to see how this fits the pupils' explanations, particularly in relation to how they talk about the role of the individual's activities. Anyway, the relationship between some of the theories will be discussed more in the following chapter.

The four last themes seem to be connected in that the language and social have some intersection and the same can be said about the tacit and the individual theme. The angles are however different. What seems to be the trend is that the language-social themes seem to be dominated by Vygotsky who (briefly) states that social interaction plays a fundamental role in shaping pupils' internal cognitive structure as well as that language is the main thinking-tool. Knowledge is partly in the heads of individuals and partly "between" individuals. Discussion and internalisation are key words, and verbalisation is an example of a social activity. The potential for learning is limited to the ZPD. Particularly about language, Polya, Vygotsky, and Skemp talk about that words posed by the teacher is of great importance and language as being the logical and analytical thinking-tool and that thoughts are not only expressed in words but come into existence through the words. School mathematics is, according to Dowling, highly discursive and mathematics itself to a great extent consists of concepts. In relation to the learning of mathematical concepts, there are, according to Skemp, two basic principles, and all concepts except the primary ones are derived from other concepts and they take part in the formation of other concepts. Therefore it is important, following Skemp, that more basic concepts are learnt before going further. This conceptual structure is called a schema, and a schema is therefore a tool for learning as it integrates existing knowledge. Rote-memorising is meaningless and an integrated conceptual structure is easier to remember than unconnected rules. There are therefore different kinds of learning, which seem to be characterised either as rote-learning, assimilation into existing schemas, or accommodation and change of a schema. However the latter concepts about assimilation etc. are concepts from Piaget.

Piaget, however, seems to be a main figure in the relationship between the individual and the tacit theme. The individual-theme is represented by for instance Glasersfeld and Piaget. The general view is that knowledge is in the heads of persons, and a person has to construct his knowledge basis on his experience. This therefore means that (1) language is not the main thinking-tool, (2) both individual actions and the individual performed coordinated ones lead to abstraction, but it is the latter that leads to reflective abstractions and then to logical-mathematical knowledge. Piaget therefore finds that logical-mathematical abilities do not arise from language or linguistic competency, but from the ability to coordinate actions and operate with objects. The fifth theme is therefore to some extent linked to the fourth. The theories in the fifth theme do also seem to argue that the ability to learn is inborn, in each individual, while activity theory in the sixth theme argue that this is not the case. Instead they argue that all higher functions originate as actual relations *between* human individuals. In relation to the tacit theme, it means that a pupil perhaps behaves the right but then he sometimes cannot express his behaviour in clear words. But concepts can be necessary support of thought. Also Piaget stated that the roots of logical thought are not in language alone but in the coordination of actions, which are the basis of reflective abstraction. The tacit theme therefore concentrates on arguments centred on the general uselessness of words in thinking and learning, as well as the lack of ability to describe what one is doing.

Another link between the themes is between the first and third theme: one could argue that to get a conscious planning (Theme 1) it is necessary with metaknowledge which again might presuppose that this knowledge has been made linguistic - unless metaknowledge can be tacit.

According to the Diversity Thesis, high-achieving pupils learn differently and they are different from other pupils. Furthermore one could expect the high-achieving pupils to be good at describing their own learning process. This is owing to that high-achieving pupils are able to make the generalisations, which are necessary for being able to communicate one's experience. These pupils have stable conceptual models and have internal dialogues. One could therefore assume that some of these pupils are able to talk about how they learn mathematics. Others might not be able to do so. I would expect to see evidence of pupils who perceive a new mathematical concept in a fast analytical-synthetic process, and are able to generalise from analysing one phenomenon. The pupils might enjoy the learning activity and have systematic and motivated trials. They might not have a particularly good memory,

especially for details, but they are able to remember the general character of their learning process.

As stated above, the CULTIS model for analysis does not favour any particular theory but aims at being neutral. However, the CULTIS model for analysis is in itself an analytical construction. For instance in Theme 1, I mention a number of factors important for a successful learning. Here, the motivation is one item of this list, which according to activity theory is fundamentally wrong; motivation is a central feature, not just a point on a list. Furthermore, about motivation, according to Mellin-Olsen (1989, p. 18) both Piaget's and Vygotsky's work are activity theories in the sense that they both emphasise the importance of the learner being active and that the teacher cannot do the learning for the pupil, he has to do it himself. However, a Vygotsky perspective does also encompass the importance of that the pupils' goal and motives for learning is incorporated into the planning of the education. Mellin-Olsen argues that "there are still some relationships missing from didactical theory based on Piaget's general epistemology" (Mellin-Olsen, 1987, p. 20). He then mentions several things that he is missing, such as the pupil's evaluation of the kind of learning situation he is confronted with, recognition of the fact that he is in a position to reject the kind of activity he is invited to participate in, and Mellin-Olsen also misses the concept of object-oriented communication or dialogue in which both the teacher and the pupil participate. Therefore the CULTIS model for analysis in Theme 1 is not "neutral" as it is "anti-activity theory" in the Vygotskian sense. The reader should not read anything into this besides that a choice was necessary. I could either choose to mention motivation as one item on a list *or* to present it as something central. To do both would seem odd. Regardless of the choice I would have made, I would have chosen one rather than the other; and thus someone will always lose; this time it was activity theory. In this sense, the CULTIS model for analysis is a non-neutral theoretical de-construction of different theories.

An advantage of having a model for analysis such as the CULTIS is both practical and qualitative. If the goal of a piece of research is to compare pupils' statement with a number of theories, then some systematism is necessary. Otherwise the overview is easily lost; where do you begin and where do you end such an analysis. Such a model also provides an easier tool for spotting differences and likeness between the theories. Besides the practical side, the model does also make it possible to focus on some particular issue for discussing how a pupil learns mathematics, without forgetting other issues.

3.5.1 THE CULTIS MODEL FOR ANALYSIS

There are therefore six themes in the CULTIS model for analysis to which the pupils' narratives can belong. To some extent they overlap and interact with each other. The six themes are:

Theme I: Consciousness (practice, reflection, planning)

Keywords: *practice; planning; reflection; monitoring; confidence; positive atmosphere; motivation.*

Theme II: Unconsciousness

Keywords: *preparatory work; incubation; illumination.*

Theme III: Language

Keywords: *language as basic thinking-tool; basics important; schematic understanding; rote-learning; assimilation, accommodation.*

Theme IV: Tacit

Keywords: *words obstruct thinking; cannot explain, but do.*

Theme V: Individual

Keywords: *construction; self-activity, visualise.*

Theme VI: Social

Keywords: *internalisation; guidance, interaction; ZPD; speaking aloud; verbalise.*

F is for Failure, a horrible curse.
Success is the only thing known that is worse.
People like goof-offs, losers, and quitters.
Towards champions and victors they feel little but bitter.
Pretend you succeeded and say that you spurned it.
But if you succeed, don't let on that you earned it.
There's something for which folks have more hate reserved
than for chance success. It's success deserved.
(O'Rourke, 1987, pp. 157-158)

4. A SYNTHESIS OF DIFFERENT PSYCHOLOGICAL THEORIES?

The creation of the six themes in Chapter 3 does *inter alia* reflect that there exists various theories, and that the theories can sometimes seem contradictory. This is mainly seen in Theme 4 to 6. The first two themes are different as they each use the other consciousness as an indispensable phase in the learning process. In contrast, one sees that in themes like language, and the role of the individual and the social, there is a disagreement. Two representatives of these disagreements are Piaget (1896-1980) and Vygotsky (1896-1934), the former representing the constructivism perspective who sees learning as construction, and the latter representing the activity theory perspective that sees learning as appropriation. An example of someone who thinks that these views are mutually exclusive is Lerman who states that the constructivist perspective has limitations as it “does not offer enough as an explanation of children’s learning of mathematics” (Lerman, 1996, p. 133). Lerman’s main problem with constructivism is its lack of “adequate explanation of intersubjectivity” (Lerman, 1996, p. 134). He states that all types of constructivists

draw their inspiration from Piaget, for whom the individual is the central element in meaning-making. ... I will draw on Vygotsky's psychological theories as contrast to Piaget's. ... Vygotsky's and Piaget's programs have fundamentally different orientations, the former placing the social life as primary and the latter placing the individual as primary ... the assumption of complementarity leads to incoherence.
(Lerman, 1996, p. 133)

Lerman has previously (Lerman, 1989) been in favour of radical constructivism but later, as seen above; he shifts to a more activity theory perspective. He does not even think that these theories could be complementary. I agree with Lerman (1996) in that if one sees the Piaget perspective and the Vygotsky perspective as two bodies of knowledge that are built up as a mathematical-logical formal system with different basis, then the theories are mutually exclusive. But then one might ask, what is the basis of these theories, and does it matter if they are different? In Section 4.1 I will therefore discuss if the basis of the theories of Piaget and Vygotsky are different and, if so, whether this means that they cannot somehow be “united”. Following this, Section 4.2 will discuss if various learning theories imply an ontological commitment as well as the possible synthesis of the theories, as a theoretical possibility. The subsequent sections discuss various ways of solving the problem of mutual exclusive theories and duality between theories. Here the concept of complementarity will be discussed.

4.1 ARE VYGOTSKY AND PIAGET REALLY THAT DIFFERENT?

Vygotsky and Piaget’s work is spread out on a great number of areas and books. This section will focus on one of the main areas, namely the question of language and its importance for thoughts and learning. There will also be a discussion of the relationship between the individual and the social. Below are some rather long quotations from both Vygotsky and Piaget when they discuss each other’s argumentation on this topic. The purpose is, besides telling what they think, to show the “tone” between them.

In the beginning of the book “Thought and Language” Vygotsky talks about Piaget. He begins by stating that

Psychology owes a great deal to Jean Piaget. It is not an exaggeration to say that he revolutionized the study of child language and thought. He developed the clinical method of exploring children’s ideas which has since been widely used. He was the first to investigate child perception and logic systematically; moreover, he brought to his subject a fresh approach of unusual amplitude and boldness. Instead of listing the deficiencies of child reasoning compared with that of adults, Piaget concentrated on the distinctive characteristics of child thought, on what the child has rather than what the child lacks.
(Vygotsky, 1962, p. 9)

However, Vygotsky also has some criticism of Piaget around the concept of egocentrism⁸ and egocentric speech:

Since Piaget's conception of child egocentrism is of primary significance in his theory, we must inquire what facts led him not only to accept it as a hypothesis but to put such great faith in it. We shall then test these facts by comparing them with the results of our own experiments. The factual basis of Piaget's belief is provided by his investigation of the child's use of language. His systematic observations led him to conclude that all conversations of children fall into two groups, the egocentric and the socialized. The difference between them lies mainly in their functions. In egocentric speech, the child talks only about himself, takes no interest in his interlocutor, does not try to communicate, expects no answers, and often does not even care whether anyone listens to him. It is similar to a monologue in a play. ... In socialized speech, he does attempt an exchange with others. ... Piaget's experiments showed that by far the greater part of the preschool child's talk is egocentric. ... In his description of egocentric speech and its developmental fate, Piaget emphasizes that it does not fulfil any realistically useful function in the child's behavior and that it simply atrophies as the child approaches school age.
(Vygotsky, 1962, pp. 14-15)

In contrast to this view, Vygotsky states that his experiments suggest that egocentric speech has a very specific role (Vygotsky, 1962, p. 16). He writes:

In order to determine what causes egocentric talk, what circumstances provoke it, we organized the children's activities in much the same way Piaget did, but we added a series of frustrations and difficulties. For instance, when a child was getting ready to draw, he could suddenly find that there was no paper, or no pencil of the color he needed. In other words, by obstructing his free activity we made him face problems. We found that in these difficult situations the coefficient of egocentric speech almost doubled, in comparison with Piaget's normal figure for the same age and also in comparison with our figure for children not facing these problems. The child would try to grasp and to remedy the situation in talking to himself: "Where's the pencil? I need a blue pencil. Never mind, I'll draw with the red one and wet it with water; it will become dark and look like blue. ... Our findings indicate that egocentric speech does not long remain a mere accompaniment to the child's activity. Besides being a means of expression and of release of tension, it soon becomes an instrument of thought in the proper sense - in seeking and planning the solution of a problem.
(Vygotsky, 1962, p. 16)

Thus, to Vygotsky egocentric speech, besides its communicative role, has an important role as a thinking-tool and as a tool to solve problems. According to Vygotsky, Piaget sees the development of thought as "gradual socialization of deeply intimate, personal, autistic mental states. Even social speech is represented as following, not preceding, egocentric speech" (Vygotsky, 1962, p. 18). But following Vygotsky, the main function of speech is instead communicative, to create social contact. "Egocentric speech emerges when the child transfers

⁸ *The notion of egocentrism in Piaget's work is "quite unrelated to the common meaning of the term, hypertrophy of the consciousness of self. Cognitive egocentrism, as I have tried to make clear, stems from a lack of differentiation between one's own point of view and the other possible ones, and not at all from an individualism that precedes relations with others" (Piaget, 1962, p. 4).*

social, collaborative forms of behavior to the sphere of inner-personal psychic functions” (Vygotsky, 1962, p. 19). Vygotsky then continues and states that:

Thus our schema of development - first social, then egocentric, then inner speech - contrast both with the traditional behaviorist schema - vocal speech, whisper, inner speech - and with Piaget's sequence - from nonverbal autistic thought through egocentric thought and speech to socialized speech and logical thinking. In our conception, the true direction of the development of thinking is not from the individual to the socialized, but from the social to the individual.
(Vygotsky, 1962, p. 19-20)

To Vygotsky, inner speech “is not the interior aspect of external speech - it is a function in itself. It still remains speech, i.e., thought connected with words. But while in external speech thought is embodied in words, in inner speech words die as they bring forth thought. Inner speech is to a large extent thinking in pure meanings” (Vygotsky, 1962, p. 149). Thus to Vygotsky a main and basic criticism of Piaget is that thinking develops from the social level to the individual, while it is opposite for Piaget. Furthermore egocentric speech has a main role in problem-solving.

Vygotsky's book “Thought and Language” was first published posthumously in 1934 in Russian but the book was suppressed in the Soviet Union from 1936-1956.⁹ It was not until 1957 that an English translation was begun on Luria's initiative (Vygotsky, 1962, p. xi). This translation was published in 1962 and following this Piaget wrote in 1962 a “Comment” on what Vygotsky once had written about him. Piaget begins as follows:

It is not without sadness that an author discovers, twenty-five years after its publication, the work of a colleague who has died in the meantime, when that work contains so many points of immediate interest to him which should have been discussed personally and in detail. Although my friend A. Luria kept me up to date concerning Vygotsky's sympathetic and yet critical position with respect to my work, I was never able to read his writings or to meet him in person, and in reading his book today, I regret this profoundly, for we could have come to an understanding on a number of points. Miss E. Hanfmann, who is one of Vygotsky's closest followers, has kindly asked me to comment on the reflections of this distinguished psychologist concerning my early work. I should like to thank her, but also confess embarrassment, for while Vygotsky's book appeared in 1934, those of mine he discusses date back to 1923 and 1924. On thinking over the question of how to carry out such a discussion in retrospect, I have, however, found a solution that is both simple and instructive (at least for me), namely to try and see whether or not Vygotsky's criticisms seem justified in the light of my later work. The answer is both yes and no: on certain points I find myself more in agreement with

⁹ The reason for the suppression was that “he would not brook either materialist reductionism or mentalism, nor the easy Cartesian dualism” (Vygotsky, 1962, p. vi). The fact, that Vygotsky's work was forbidden seem to be owing to him not being a “real” Marxist, however: “From the Marxist ideological perspective, he is celebrated as the man who recognized the historical determination of man's consciousness and intellect. But looking at Vygotsky's place in world psychology, his position transcends either the usual functionalism of the Dewey-James variety or the conventional historical materialism of Marxist ideology. Vygotsky is an original. It is a disservice to him ... to find his significance solely in developing Soviet conceptions of man” (Vygotsky, 1962, p. vi).

*Vygotsky that I would have been in 1934, while on other points I believe I now have better arguments for answering him.
(Piaget, 1962, p. 1)*

As a response to Vygotsky's critique of Piaget's previous view of egocentric speech, Piaget has a rather long preamble, but then continues:

*This long preamble has seemed necessary to bring out how much I respect Vygotsky's position on the issue of egocentric speech, even though I cannot agree with him on all points. First, Vygotsky did realize that a real problem was involved, and not merely a question of statistics. Second, he himself verified the facts in question, instead of suppressing them through the artifices of measuring; and his observations on the frequency of egocentric speech in children when their activity is blocked and on the decrease of such speech during the period when inner speech begins to form are of very great interest. In the third place, he proposed a new hypothesis: that egocentric speech is the point of departure for the development of inner speech, which is found at a later stage of development, and that this interiorised language can serve both autistic ends and logical thinking. I find myself in complete agreement with these hypotheses. On the other hand, what I think Vygotsky still failed to appreciate fully is egocentrism itself as the main obstacle to the co-ordination of viewpoints and to co-operation. ... In brief, when Vygotsky concludes that the early function of language must be that of global communication and that later speech becomes differentiated into egocentric and communicative proper, I believe I agree with him. But when he maintains that these two linguistic forms are equally socialized and differ only in function, I cannot go along with him because the word socialization becomes ambiguous in this context: if an individual A mistakenly believes that an individual B thinks the way A does, and if he does not manage to understand the difference between the two points of view, this is, to be sure, social behavior in the sense that there is contact between the two, but I call such behavior unadapted from the point of view of intellectual co-operation. ... As far as I know I have never spoken of speech 'not meant for others'; this would have been misleading, for I have always recognized that the child thinks he is talking to others and is making himself understood. My view is simply that in egocentric speech the child talks for himself.
(Piaget, 1962, pp. 7-8)*

What might be surprising in the quotation above is that Piaget declares that he agrees completely with Vygotsky in that for instance egocentric speech is the point of departure for the development of inner speech and that it is this inner speech that can serve logical thinking. Vygotsky emphasises that language is not just a means of expression; it is an instrument of thought. Whether Piaget's expression "serve logical thinking" is the same as Vygotsky's "instrument of thought" is, however, not certain since Piaget's expression seem to grant language slightly less significance for the development of thought than Vygotsky's. However, the difference is small. A place where they do disagree is, according to Piaget, that Vygotsky still failed to understand that egocentrism itself could be a main obstacle for learning. This means that language, as discussed in Theme 4, can also hamper learning. Regarding if egocentric speech is "for others" or not, there seem to have been some kind of misunderstanding between the two; both seem to think that egocentric speech is social. They

do not agree about the correct sequence of ‘egocentric speech, ‘inner speech’, and ‘socialised speech’. However, what is clear is the mutual admiration they have for each other.

A related, but different, topic is the discussion of the role of the individual and the role of the social in learning, which links to Theme 5 and 6 of this thesis. Piaget (1969) discusses a doubleness in the teaching and states that on the one side is the rising individual and on the other side the social, intellectual, and moral values that the educator tries to convey. He criticises “old” ways for mainly focussing on the *goal* of the upbringing, instead of its *methods* (Piaget, 1969, p. 131). His (new) method’s aim is to make children try to approach the grownup stage not through overtaking readymade reasons and rules for the right action, but by capturing it through own force, self-regulation, and personal experiences (Piaget, 1969, p. 132). Piaget does thus not say that learning is social, only that the *individual* himself and by himself takes over some of the surrounding world’s knowledge. Also social-constructivism can be seen as such an attempt.

According to the culture-historical school and activity theory, to which Vygotsky belongs, learning is a question of appropriation of the culturally created surroundings. In line with Vygotsky, Leontiev¹⁰ says: “The child is not adapted to the world of human objects and phenomena surrounding him, but takes it to himself, i.e. appropriates it. ... This is a process which has as its result reproduction in the individual of the historical formation of human qualities, abilities and characteristics of behavior” (Eriksen, 1993, p. 43). Knowledge and concepts are not seen as constructions that are created by the single individual, but it is historical and cultural founded mental artefacts and phenomena, which the individual takes up and makes his own. So far, there seems to be quite agreement between Piaget and Vygotsky, i.e. that what the individual must learn is a social product of past generations. However, Eriksen (1993, p. 45) writes that basically Leontiev argues that learning is an active process from the point of view of the child, and the child can with social support of the surroundings reproduce culturally accumulated knowledge. Eriksen (1993, p. 45) quotes Leontiev for saying:

This process takes place in the child’s activity in relation to objects and phenomena of the surrounding world in which are embodied the achievements of mankind. Such activity, however, cannot be developed by the child himself, it develops in practical and verbal intercourse with people surrounding him, in

¹⁰ The quotes of Leontiev come from: Leontiev, A. N. (1963) ‘Principles of Mental Development and the Problem of Intellectual Backwardness’ In B. Simon (eds.) *Educational Psychology in the U.S.S.R.* (London, Routledge & Kegan Paul). The quotes are from pp. 72-75. The original source could not be found.

combined activity with them; when the aim of such activity is specifically to transmit to the child certain knowledge, skills and habits then we say that the child learns, the adult teaches.

The main difference in Piaget and Vygotsky is therefore if the ability to learn, construct, is inborn or if other people are necessary for the learning process. The answer to the question of whether Piaget and Vygotsky are different is Yes. However, the difference is not about, as formulated by Cole and Wertsch (1996), a primacy of individual primacy of individual psychogenesis versus sociogenesis of mind, but, briefly stated, more that Piaget sees the individual as *the* source of learning, and that children learn by continuous interaction and experience with their environment, the egocentric speech is valuable for logical thinking but it can also obscure the meaning. Instead Vygotsky emphasises that one cannot learn without the verbal interaction and activity with others. The dualism is thus still there, but not as distinct as seems at first. According to Vejleskov (1998, p. 117), some has suggested to build a bridge between the two, by naming in co-constructivism. Furthermore Vejleskov quotes Bruner for, at a Piaget-Vygotsky congress in 1996, having said that Piaget owes us an explanation of how the self-regulation is taking place and Vygotsky owes us an explanation to why we do not all become a copy of the socio-cultural context in which we grow up. Another pair of authors who describe the difference between Vygotsky and Piaget are Cole and Wertsch who argue that

For Vygotsky, like Piaget, the relationship between the individual and the social is necessarily relational. However, by placing cultural mediation at the center of adult cognition and the process of cognitive development, social origins take on a special importance in Vygotsky's theories that is less symmetrical than Piaget's notion of social equilibration as 'resulting from the interplay of the operations that enter into all cooperation'. For Vygotsky and cultural-historical theorists more generally, the social world does have primacy over the individual in a very special sense. Society is the bearer of the cultural heritage without which the development of mind is impossible. (Cole & Wertsch, 1996)

The difference between Piaget and Vygotsky are thus not that one puts an emphasis on the individual side and the other on the social side, but instead that one, Piaget, seem to balance the two aspects more equally than the other.

Another point about the difference is put forward by Vejleskov (1998, p. 111) who argues that Piaget is being criticised for only working with the cognitive level and not, for instance, including social relations. Vejleskov instead argues that as Piaget's theory is not a pedagogical theory, but a psychological theory, it is therefore legitimate to choose to study one element. Following this, I would argue that perhaps the basic disagreement is to whether

a “sole” focus on psychology is legitimate at all or whether other factors are necessary, i.e. a discussion of if psychology or pedagogy should be the main contributor to a discussion of the education. According to Mellin-Olsen (1989, p. 13) Piaget tried, with his background in biology to investigate epistemology but his aim was never to build a basic pedagogical theory. Vygotsky, on the other hand, had a background in literature research and had as his purpose to put a psychological and pedagogical ground for creating the new Soviet Man.

However, the dualism still exists. The dualism has resemblance with the traditional actor-structure dualism also seen in sociology and philosophy. Whether or not this dualism can be overcome, will be discussed below. But first I will discuss if it is possible to somehow “synthesise” the two theories without violating that they each, on the level of ontology, are completely different. In other words, does a learning theory automatically imply that one has to “buy” the theory’s epistemology and ontology? If not, it might be less problematic to synthesise the theories.

4.2 DOES A LEARNING THEORY IMPLY AN ONTOLOGICAL COMMITMENT?

When discussing theories about learning, one basically operates with three levels - ontology (the nature of reality), epistemology (the nature of knowledge), and the “learning theory” level (how one learns). This section will discuss constructivism, with Piaget, and activity theory, with Vygotsky, in relation to these three levels.

In terms of the ontological level, the background of constructivism is “non-realism” which means that there is no reality that exist independently of human thinking while the philosophical background of activity theory is dialectical materialism, which is a particular type of metaphysical realism that stands for that everything that exist does not *only* have physical characteristics, there are several levels in reality such as the physical, the organic, the conscious, the socio-economical, etc. (Lübcke, 1993). On the epistemological level, constructivism states that knowledge is in the heads of persons and the thinking subject has no alternative but to construct what he knows on the basis of his own experience. Contrary to this is the activity theory that emphasises that knowledge is created in a negotiation/interaction among people and that people appropriate knowledge.

One problem in creating a “synthesis” between Piaget and Vygotsky is that *if* we for instance “buy” constructivism on the level of “learning theory” do we *then* have to follow it all the way back to an ontology of non-realism? *If* this is the case, *then* a synthesis becomes impossible as Vygotsky rests on a branch of realism. To answer this question I will discuss the connection between ontology and epistemology for Piaget and Vygotsky’s works:

For Piaget: Constructivism claims that we *always* and *only* learn through constructing. *If* this is the case, *then*, I will argue, it must follow that this is how we learn regardless of how we are being taught and regardless of the nature of reality. Piaget does not separate the level of epistemology from the level of learning theory in his work. He states that his genetic epistemology deals with both the formation of and the meaning of knowledge (Piaget, 1970, p. 12). Furthermore, he writes:

From the empiricist point of view, a ‘discovery’ is new for the person who makes it, but what is discovered was already in existence in external reality and there is therefore no construction of new realities. ... By contrast, for the genetic epistemologist, knowledge results from continuous construction, since in each act of understanding, some degree of invention is involved; in development, the passage from one stage to the next is always characterized by the formation of new structures which did not exist before, either in the external world or in the subject’s mind.
(Piaget, 1970, p. 77)

The above quote does not show a rejection of realism as what Piaget here discusses is epistemology, and what he says is that *knowledge* does not exist beforehand in the external world; but he does not say that the external world does not exist independently of man.

Hence, being a constructivist on the level of epistemology is not synonymous with having non-realism as one’s ontology.

For Vygotsky: (1) *If* Vygotsky is right in saying that learning activity cannot be developed by the child himself but only through social interaction with other people, *then* external reality must exist as we must assume that it is in reality that these other human beings resides. Vygotsky says himself:

If we compare the early development of speech and of intellect ... with the development of inner speech and of verbal thought, we must conclude that the later stage is not a simple continuation of the earlier. The nature of the development itself changes, from biological to sociohistorical. Verbal thought is not an innate, natural form of behavior but is determined by a historical-cultural process ... Once we acknowledge the historical character of verbal thought, we must consider it subject to all the premises of historical materialism.
(Vygotsky, 1962, p. 51)

One of the premises of historical materialism is realism. (2) *If* reality does exist independently of us, *then* it does not automatically imply that knowledge is of a certain kind or that the way one gains knowledge of the world is through interacting with this reality. A Platonist might argue, that knowledge is in an imaginary world.

The conclusion on this discussion must be that being a constructivist on the level of epistemology and/or learning theory *does not imply* an ontological commitment, whereas being a Vygotskian on the level of epistemology or learning theory *does imply* an ontological commitment to realism. One could therefore conclude that to create a synthesis of Piaget and Vygotsky on the level of learning theory one must:

- Include realism on the level of ontology as Vygotsky's theory is indispensable without it, but Piaget's epistemology and learning theory does still "work" in a realist world.

A conclusion is thus that a synthesis seems less difficult to create as they do not necessarily have a different ontological basis. Thus, the CULTIS model for analysis developed in Chapter 3 using various theories, might not be self-contradictory. However, as the theories stand today, the dualism still exists. From the "tone" between Piaget and Vygotsky, their critique of each other never seemed personal, but was carried by mutual respect and driven by a desire for finding the truth.

4.3 GIDDENS' ATTEMPT TO SOLVE A DUALITY IN SOCIAL SCIENCE

It is not just within psychology that one sees dualism between theories. Giddens describes several of what he calls dilemmas, and one of them seems related to the above mentioned between Piaget and Vygotsky:

One dilemma concerns human action and social structure. It is: How far are we creative human actors, actively controlling the conditions of our own lives? Or is most of what we do the result of general social forces outside our control? This issue has always divided, and continues to divide, sociologists. Symbolic interactionism stresses the active, creative components of human behaviour. The other three (eds.: Functionalism, Structuralism, and Marxism) emphasize the constraining nature of social influences on our actions.
(Giddens, 1993, p. 718)

Overcoming such a duality might be a first step in finding an overall grand theory of the field. In this connection also Skinner (2000) writes about several attempts on finding a grand theory in the human sciences towards the end of the 20th century. Skinner (2000, p. 3) quotes a book written by the sociologist Mills in 1959 where Mills stated his scepticism of the goal that the human sciences should seek a grand theory, and thus construct a systematic theory of ‘the nature of man and society’. According to Skinner, “this hostility towards the construction of abstract and normative theories of human nature and conduct was an attitude he [Mills] shared with most of the leading practitioners not merely of sociology but of all the human sciences in the English-speaking world at that time” (Skinner, 2000, p. 3). Particularly for psychology, Skinner writes that:

even more vociferous doubts about the normative presuppositions of positivism have been voiced of recent years by the psychologists. To perceive all human behaviour in lawlike, causal terms ... presupposes that the question to ask about abnormal behaviour must always be what malfunction is prompting it. But this it to overlook the possibility that the behaviour in question may be strategic, a way of trying to cope with the world. And this oversight ... has the effect of reducing the agents involved to objects of manipulation when they deserve to be treated as subjects of consciousness.
(Skinner, 2000, p. 9)

Skinner later argues that all the sceptical stands against creating a grand theory actually contribute to a return of grand theory. He argues as follows:

Although they [the sceptics] have given reasons for repudiating the activity of theorising, they have of course been engaged in theorising at the same time. There is no denying that Foucault has articulated a general view about the nature of knowledge, that Wittgenstein presents us with an abstract account of meaning and understanding, that Feyerabend has a preferred and almost Popperian method of judging scientific hypotheses, and even that Derrida presupposes the possibility of constructing interpretation when he tells us that our next task should be that of deconstructing them. ... We next need to note that, during the past two decades, there has also been an unashamed return to the deliberate construction of precisely those grand theories of human nature and conduct which Wright Mills and his generation had hoped to outlaw by from any central place in the human sciences. This can be seen most obviously in the case of moral and political philosophy. ... One has been a renewed willingness directly to address the most pressing evaluative issues of the day. As a result, such topics as the justice of war, the social causes of famine ... all these and many other kindred questions of obvious urgency have again become the staples of philosophical debate. But the other and even more startling development has been a return to Grand Theory in the most traditional and architectonic style, the style employed by the great normative systembuilders of earlier centuries. Moral and political philosophers have ceased to be in the least shy of telling us that their task is that of helping us to understand how best to live our lives.
(Skinner, 2000, pp. 12-14)

In relation to the dualism of individual-system between Piaget and Vygotsky, Giddens tried in the area of sociology to abolish this classical dualism and create a grand theory. Giddens puts

the actor in the centre and with the notion of structure-duality he tries to transgress the sociology's traditional structure/actor dualism. With this notion Giddens wants to emphasise that social system's structural characteristics at once is a medium for and a result of the individual actors actions:

Structure is not to be equated with constraints but is always both constraining and enabling. This, of course, does not prevent the structure of properties of social systems from stretching away, in time and space, beyond the control of the individual actor. Nor does it compromise the possibility that actors' own theories of the social systems which they help to constitute and reconstitute in their activities may reify those systems.
(Giddens, 1986, p. 25)

Giddens' theory has however been criticised for being so abstract that it could not be employed in empirical research in practice (Gregson, 1989).

4.4 BOHR'S ATTEMPT TO SOLVE A DUALITY IN NATURAL SCIENCE

We could also look at what physicists do faced with the problem of what light is. Some theories state that light is a wave (which means a field spread out in a large space), others that it a particle (which means that the substance is limited to a very little volume). Which one is it? The theories are mutually exclusive, but still physicists use both, they exists side by side. What physics do is to use the theory that "fits" the given problem they are solving. Furthermore: "Niels Bohr's Principle of Complementarity ... states that each description excludes the other, but both are necessary - they complement each other" (Marshall & Zohar, 1997, p. 101). Russell seems to discuss something similar when he describes Einstein's general theory of relativity. According to Russell, Einstein's theory does, *inter alia*, lead to the conclusion that

the universe is finite but unbounded, like the surface of a sphere, but in three dimensions. All this involves non-Euclidean geometry, and is apt to seem mysterious to those whose imagination is obstinately Euclidean. ... Professor Milne holds that there is no need to regard space as non-Euclidean, and that the geometry we adopt can be decided entirely by motives of convenience. The difference between different geometries, according to him, is a difference in language, not in what is described.
(Russell, 1948, p. 34)

Russell does here argue for that fundamental different approaches can be used to describe the same thing. This remark might be seen as being very surprising considering that non-Euclidean geometry per definition is any geometry which denies one of Euclid's five basic postulates (Euklid, 1959; 1st edition about 300 BC); in practice the Parallel Postulate. If this approach is possible for natural scientists, it perhaps ought to be possible for psychologists or researchers in education as well. It is a paradox, but Marshall and Zohar quote the physicist Feynman for saying: "A paradox is not a conflict within reality. It is a conflict between reality and your feeling of what reality should be like" (Marshall & Zohar, 1997, p. 387). The idea of complementarity is, however, also criticised by Marshall and Zohar who argued that Bohr's idea of complementarity rests on ideas of the old worldview. In my view, old ideas are not wrong just because they are old. Old-fashioned and outdated is not the same. What is important must be which view is true.

In relation to Bohr's idea of the range of application of the Principle of Complementarity, Marshall and Zohar writes that:

Bohr himself applied his Principle of Complementarity widely in fields outside physics. ... thought and action, subjectivity and objectivity, feeling and reasoning, male and female, the truths and values of one culture and those of another. Physics and philosophers of Bohr's generation liked this way of thinking because it rested within the dualist either/or paradigm of the old world view and required no revolution in thinking. ... To accept that light is both a wave and a particle, is one of the creative leaps quantum physics calls upon us to make. Applied in other fields, both/and thinking requires us to see that there may be two or more mutually contradictory ways of doing something, or of looking at something, all which are valid. Seeing the truth of all tells us something more profound about the situation.
(Marshall & Zohar, 1997, p. 102)

Bohr did therefore not only use the concept of complementarity within a context of quantum physics. Also Cole and Wertsch argues within the area of psychology that "There is little doubt in our view that there is still much to be learned from both Piaget and Vygotsky, and in many cases the strengths of one theorist complement the weakness of the other" (Cole & Wertsch, 1996). Also Piaget himself used a concept of complementarity: "I shall begin by making a distinction between two aspects of thinking that are different, although complementary" (Piaget, 1970, p. 14). The concept of complementarity is also used by researchers in mathematics education. For instance does Vithal (1999) discuss the connections between mathematics education and democratic society and hence the relation between democracy and authority, which according to Vithal, is best understood and explained with

reference to the idea of complementarity. And Sfard (1991, p. 4) writes that “operational and structural conceptions of the same mathematical notion are not mutual exclusive. Although ostensibly incompatible ... they are in fact *complementary*”. In that sense one could argue that the concept of complementarity has a more general application into the range of mathematics education.

4.5 SO WHAT DO WE DO NOW?

If one wants to use various, sometimes contradictory, theories, it seems that one has three options: (1) Find/invent the unifying grand theory, (2) use a concept of complementarity, or (3) follow the recommendation of Marshall and Zohar (1997, p. 102) and accept a both/and thinking and seeing the truth of all. I will now discuss these options.

4.5.1 A GRAND THEORY OF THE PSYCHOLOGY OF LEARNING MATHEMATICS?

Vygotsky thought that psychology ought not to be divided into different schools; he states:

As long as we lack a generally accepted system incorporating all the available psychological knowledge, any important factual discovery inevitably leads to the creation of a new theory to fit the newly observed facts.
(Vygotsky, 1962, p. 10)

He can therefore be interpreted as talking about a grand theory. To look for a unifying theory seems to be in line with Descartes’ dream. Descartes finds reason to be the method to unify all sciences. According to Davis and Hersh:

The vision of Descartes became the new spirit. Two generations later, the mathematician and philosopher Leibnitz talked about the ‘characteristica universalis’. This was the dream of a universal method whereby all human problems, whether of science, law, or politics, could be worked out rationally, systematically, by logical computation. In our generation, the visions of Descartes and Leibnitz are implemented on every hand.
(David & Hersh, 1988, pp. 7-8)

We can therefore ask if this reason is still the method, or do we need an additional vision? Milne (quoted above) stated that the different descriptions of reality are just different

languages. In line with this view, we have to find the unifying language, as this could be the appropriate tool. As an example, mathematics took a big step forward in development after Newton and Leibniz, separately, had developed differential and integral calculus towards the end of the 17th century. They had created/discovered a very useful tool that was further developed by the next generations of mathematicians (Andersen, 1978, p. 48). Hence we might need an appropriate language and/or tool to be able to find the grand theory.

We could seek inspiration from areas such as brain research or physics. The former has the later years made huge discoveries (see for instance Gade, 1997) and, as written above, some researchers in physics talk about the principle of complementarity, but others seem to be looking for a grand theory. In relation to the latter: “Physicists seek a theory that will unify all known forces of nature” (Nozick, 2001, p. 161). Hawking writes that “we might be near finding a complete theory that would describe the universe and everything in it” (Hawking, 1994, p. 29). Deutsch writes that: “quantum physical investigations of shadows and light have extraordinary consequences, and to explain these demand not only new physical laws but also a new level of description. It first and foremost reveals the existence of parallel universes” (Deutsch, 1998, pp. 32-33). I do not want to go any deeper into the discussion of parallel universes but only draw a conclusion from these quotes, namely that physics scientists expect to find a grand theory and that new research in physics suggests not only new laws, or theories; but more radical changes of ways of thinking and describing.

A grand theory of psychology of learning mathematics might therefore exist, but to my knowledge it has not been found/invented yet. To find it requires not only a consideration of the Section 4.2, input from other areas such as the latest brain research, but also that we find some new language, and a new ground on which to built the theory. The extent to which one believes that a unifying theory exists, also rests on the modern way of thinking. Modernism refers to a long and dominating cultural tradition which *inter alia* had as characteristics:

the ideal of a complete and scientific explanation of physical and social reality. Though this might not in practice be possible, it remains an intelligible ideal. ... there is thus a 'grand narrative' which we have subscribed to, namely, the 'enlightenment' view that reason, in the light of systematically researched evidence, will provide the solution to the various problems we are confronted with.
(Pring, 2000, p. 110)

Also the positivist tradition seems to be in favour of grand theories. Pring argues, when he discusses positivism, as follows:

First, there can be no clear logical distinction between research into physical phenomena and research into social institutions and structures. Society can be studied scientifically. There are social facts, just as there are physical facts. People, despite their individuality, fall into types or groups, and general statements can be made about these types. Such generalisations can be verified. Gradually a theoretical picture can be built up which relates types to social structures, such that to explain why certain people act in the way they do one refers to the social structures which could be said to cause that kind of behaviour. Such social explanations contradict those which seek to explain behaviour in terms of personal choice or individual psychology. Of course, one cannot deny that there is some personal choice, but, first, such choice will be exercised within parameters determined by the social facts, and, second, typical behaviours are what are being explained - there can always be exceptions. ... Second, the positivist spirit requires a clear distinction between the aims and values of education, on the one hand, and the means of reaching those ends, on the other. Matters of value are not open to empirical enquiry (and are thus outside the bounds of meaningful discussion) whereas the means of realizing those values are. Researchers are required to show how certain ends might be reached, not to say what those ends ought to be.
(Pring, 2000, pp. 93-94)

Furthermore, positivism can be seen as a guard against dominating suppressing ideas: “Those who now decry the positivist agenda need to remember the spirit and motives which drove it. There was a deep suspicion of those explanations, without evidence to support them and not open therefore to counter argument, which sustained the social order as it was, despite the obvious injustices and evils” (Pring, 2000, p. 90).

4.5.2 COMPLEMENTARITY IN THE PSYCHOLOGY OF LEARNING MATHEMATICS?

When discussing the possibility of using the concept of complementarity in this work, one needs to discuss two things. (1) There might be qualitative differences in the nature of physics and the nature of psychology of learning mathematics which means that even if a grand theory exists in physics, it does not mean it exists in the psychology of learning. (2) How does a concept of complementarity influence our logic?

In relation to question (1) of whether the physical problem of light and the problem of a psychological learning theory are different:

First, the former is a natural science phenomena, the other a psychological phenomena. For instance Berger and Luckmann argues that social phenomena are not as the physical, and that human reality is a socially constructed reality (Berger & Luckmann, 1984, pp. 210-211). Furthermore according to, among others, Skinner there has within the philosophy of science been a critique of the positivist account of what constitutes an explanation:

the widespread reaction against the assumption that the natural sciences offer an adequate or even a relevant model for the practice of the social disciplines. The clearest reflection of this growing doubt has been the revival of the suggestion that the explanation of human behaviour and the explanation of natural events are logically distinct undertakings, and thus that the positivist contention that all successful explanation must conform to the same deductive model must be fundamentally misconceived. From many different directions the cry has instead gone up for the development of a hermeneutic approach to the human sciences.
(Skinner, 2000, p. 6)

However, following Descartes and the modern dream, there is something that unites *all* sciences.

Second, in the theory of light, the two views of light are mutually exclusive as a certain thing cannot be a particle and a wave at the same time. However, according to Heisenberg, the dualism here is not problematic as we know from the mathematical formulation of the theory that there cannot arise contradictions. By a simple transformation one can rewrite the equation of motion for the co-ordinates and the momenta of the particles to make it look like a wave equation for an ordinary 3-dimensional matter wave. “Therefore, this possibility of playing with different complementary pictures has its analogy in the different transformations of the mathematical scheme; it does not lead to any difficulties in the Copenhagen interpretation of quantum theory” (Heisenberg, 2000, pp. 18-19). This means that since both theories of light build on the same basis and language, which is mathematics, it is unproblematic to say that they can complement each other. I would therefore call such an incidence for *even complementarity* to denote that both, compared to reality and the general mathematical knowledge, are equals. But the various psychological learning theories do not share such a common ground; at least not on the level of epistemology. Therefore, “even complementarity” is not possible here. Instead I will call for a term of *odd complementarity* to denote that neither theory is completed, but they might not be equally *dis*-completed. In other words, I call for choosing a small preference for either the hen (or the egg) and then subsequently state that the egg (or the hen) is indispensable compliments.

In relation to (2), how this influences our logic, one can argue that perhaps one does not need to have problems with having two different theories complement each other. Mathematics itself is not a foolproof consistent system. Gödel’s Theorem from 1931 set out to prove if it is possible to formulate a rich or interesting mathematical system that could contain the proofs of all its own truths: “Gödel proved that any consistent logical or mathematical ‘formal system’ rich enough to contain the natural numbers (1, 2, 3 ...) would also contain a statement that could be neither proved nor disproved from within the system itself” (Marshall

& Zohar, 1997, p. 176). One could argue, that if this is the case for mathematics, which obviously works, then why not for social sciences. We also know from the double-slit experiment that *one* photon can enter *two* different holes at the same time (unless we observe it) (Gribbin, 1984, pp. 163-171); perhaps unbelievable, but yet true. This does not eliminate the concept of truth/false; it merely teaches us more about the truth and makes us understand the truth better.

If we can accept that a photon can be two places at the same time, perhaps we can accept to use two different theories in a sort of (odd) complementarity until we might find/invent the grand theory. At least when these two different theories are not that different. The war on theories is then not on one *or* the other, but more on which is primary and which is secondary. Furthermore, if one is a modernist and still in favour of the principle of complementarity, one needs to include the concept of odd complementarity; otherwise one is inconsistent. Mellin-Olsen (1989, p. 18) furthermore argues that the relationship between Vygotsky and Piaget can be interpreted as being dialectical. It is not either-or. Instead it is about, while teaching, to have these two theories in one's mind (as well as other theories) and then balance wisely.

4.5.3 EVERYTHING IS TRUE?

Marshall and Zohar argue for accepting “both/and” thinking and accept the truth of all explanations. This view seems postmodern, which according to Pring is a questioning of the modern premises. Pring describes postmodernism as follows:

Rival disputes about what is to count as a rational view of the world cannot be settled by appeal to reason. There is no 'meta-narrative' of rationality to which we can appeal and which will bring a certain unity to this diversity. ... There is no grand narrative which legitimate one set of values rather than another or one way of organising knowledge rather than another. Therefore we need to come to terms with pluralism, not simply in recognizing the diverse modes of rationality and of perspective. Is not reason, too, a social construct?
(Pring, 2000, pp. 110-111)

One could state here, that saying that there is no grand theory is in itself a grand theory. And that following the postmodern way of arguing one might end up with accepting any explanation. In line with Marshall and Zohar is Eisner who stated that “there is no single legitimate way to make sense of the world. ... Insofar as our understanding of the world is our

own making, what we consider true is also the product of our own making” (Eisner, 1993, p. 54). A critique here is that “the social items that are claimed to generate social facts must themselves be understood to be generated by other social items, and so on *ad infinitum*” (Collin, 1997, p. 78). I could also argue that the view of Eisner is internally illogical as it with certainty rejects “objective universal truth”, only to replace it with a new universal truth, namely that the universal truth does not exist. In connection with this, Nozick argues (2001, p. 15) that he feels uncomfortable with this kind of quick refutation of relativism; i.e.: that if the relativist position, that all truth is relative, itself is nonrelative, then it is false; and if it is not a general position, but instead says that all other truth except itself are relative, then what makes it so special. Nozick (2001, p. 16) then defines the ‘relaxed relativism’ as “the relativist granting that some statement is nonrelative, namely, the statement of the relativist position itself (along with its consequences)”. He continues: “This makes it look as though relativism about truth is a coherent position. ... To say that relativism about truth is a coherent position is not to say that it is the *correct* position” (Nozick, 2001, pp. 16-17). Nozick also argues that the ‘weak absolutist’ can hold that some truth are relative (Nozick, 2001, pp. 20 & 65). Thus relativism does not undercut itself if we take into consideration its domain of application. Nozick then introduces the concept of ‘alterability’: “the relativity of a truth is not the same as its alterability. Even if it is a nonrelative truth that my pen is on my desk, that is a fact easily changed. Whereas if it is merely a relative truth that New York City is adjacent to the Atlantic Ocean or that capitalism outproduces socialism, these are not facts that are changed easily” (Nozick, 2001, p. 23). Following this line of reasoning, I would argue that even if relativism about truth is a true position, it does not change the fact that there are ways of working with mathematics, or setting in which to work, that are “unhelpful” (or more helpful) if the desired “output” of the activities is that the pupils should have learnt certain things. These facts are not easily changed unless one can genetically change the nature of man. Thus, even talking Nozick’s argumentation into consideration, the truth about how to learn mathematics might still exist.

I would also like to follow Phillips when he argues that truth exists independently of us but we can never reach it. Objectivity and truth are thus not synonyms, but through criticism we can approach truth and the, at any time, most rational theory is thus the most objective (Phillips, 1993, p. 61). This is in line with Popper’s view that we can never verify a theory but only falsify it: “*we do justify our preferences by an appeal to the idea of truth: truth plays the*

role of a regulative idea. *We test for truth*, by eliminating falsehood” (Popper, 1979, pp. 29-30). Kuhn might here pose the counter argument that with this type of falsification one is still within the same paradigm and progress is caused by paradigm change (Kuhn, 1970, pp. 52-66). In relation to this, Hollis argues that “the difference is a matter of degree of entrenchment, with normal science more willing to question its core theories than Kuhn recognised” (Hollis, 1994, p. 88). Thus we can never reach truth, but this does not mean that any version of reality is as good as any other. I would argue in line with Pring: “The acceptance of a reality independent of the researcher does not contradict the possibility of many interpretations of that reality” (Pring, 2000, p. 114).

I would also like to criticise Marshall and Zohar for not being ambitious enough for natural sciences. Giving up on finding a grand theory on psychological level is, in my view, and being inspired from a discussion of Hawking, the same as looking away from the fact that even though the human brain is subject to Heisenberg’s Uncertainty Principle,¹¹ and therefore has an element of quantum mechanical randomness, it is little energies that are transformed in the brain, so the quantum-mechanical uncertainty has only a minor effect. The real reason why we cannot (now) predict human actions has more to do with that it is too difficult. We already know the basic physics laws that govern the brain’s activity, and they are rather simple, but it is too difficult to solve the equations when there are more than a few particles involved. Even in the simpler Newtonian theory of gravitation one can only solve the equations exactly if there are no more than two particles present. For three or more particles one has to rely on approximations and the difficulties rises by the number of particles. The human brain contains approximately 10^{26} particles which is far too many for us to ever solve the equations and predict the brain’s behaviour (Hawking, 1994, pp. 120-121). Or as Hawking puts it elsewhere: “Although in principle we know the equations that govern the whole of biology, we have not been able to reduce the study of human biology to a branch of applied

¹¹ This principle is basically that “The Uncertainty Principle asserts that it must always be so; we must always content ourselves with partial truth and ambiguity when dealing with fundamental physical reality. A particle was always thought to have both position and momentum. A given particle should always be somewhere (have a location) and is always travelling at a certain speed. But we can never know both. If we measure, or focus on, the position, the momentum becomes unfixed; if we measure the momentum, we lose the position” (Marshall & Zohar, 1997, pp. 182-184). Davies writes in an introduction that “This unpredictability of quantum systems does not imply anarchy, however. Quantum mechanics still enables the relative probabilities of the alternatives to be specified precisely” (Heisenberg, 2000, p. x). Furthermore: “what the uncertainty principle tells us is that, according to the fundamental equations of quantum mechanics, there is no such thing as an electron that possesses both a precise momentum and a precise position. ... quantum theory cuts free from the determinacy of classical ideas. To Newton, it would be possible to predict the entire course of the future if we knew the position and momentum of every particle in the universe” (Gribbin, 1984, p. 157).

mathematics” (Hawking, 1994, p. 43). I would argue, that if the real problem was that of solving equations, it would just be a matter of time until we invent larger enough computers. The essence of the problem of a grand theory of psychology, I would argue, is instead whether it at all is possible to predict human behaviour. If one cannot (always) predict human behaviour in particularly a learning situation (neither the human actions nor the brain behaviour) then a complete and all-including grand theory is impossible. Hawking talks in this connection about free will, and writes “The ultimate objective test of free will would seem to be: can one predict the behaviour of the organism? If one can, then it clearly doesn’t have free will but is predetermined. On the other hand, if one cannot predict the behaviour, one could take that as an operational definition that the organism has free will” (Hawking, 1994, p. 120). Hence, I would argue, that the question of a grand theory in psychology might boil down to the (theological) question of whether we as human have a free will.

I would therefore argue that the option, given by Marshall and Zohar, of thinking of “all as truth” is partly inconsistent and partly not necessary (if we do not have a free will); at least in the longer run where I will expect science to know more. Even if we as humans do have a free will, it does not rule out that we can get *more* understanding of how we act and learn as the quantum mechanical uncertainty only has a *minor* effect; thus, it is not anarchy either.

4.5.4 PRAGMATISM FOR NOW: ODD COMPLEMENTARITY

As Hawking writes: “However, it is too difficult to think up a whole theory of everything all at one go. ... What we do instead is to look for partial theories that will describe situations in which certain interactions can be ignored or approximated in a simple manner” (Hawking, 1994, p. 46). Whether there is a grand theory of the psychology of learning mathematic, and whether we in that case can find it, does not solve the immediate problem of today, namely that we do not know it *yet*. We only have partial theories so far. So for this thesis I will settle with Bohr’s Principle of Complementarity, in the sense of Odd Complementarity.

In this connection one might look at the parable about the blind man and the elephant (Shah, 1984). The core of the parable is that a blind man is asked to give a description of something he does not know what is. He touches the elephant at different places and say different things every time, but he does not know, that what he is touching is the same thing.

Following this parable, I am interested in the elephant, and until I can see the whole of it, I will settle with having slightly different ideas of what an elephant is, depending on where I touch.

*'Logic!' said the Professor half to himself.
'Why don't they teach logic at these schools?
There are only three possibilities.
Either your sister is telling lies,
or she is mad, or she is telling the truth.
You know she doesn't tell lies and it is obvious
that she is not mad. For the moment then
and unless any further evidence turns up,
we must assume that she is telling the truth.'*
(Lewis, 1996, pp. 213-214)

5. DATA PROCESSING AND FIRST ANALYSIS

The analysis takes place mainly from the transcription of the interviews and, for the English interview, also the notes from the observation. I will first discuss how I created the transcriptions. A description of the process of analysis follows after this.

5.1 HOW THE RECORDING AND TRANSCRIBING TOOK PLACE

One should think that transcribing a tape would be quite simple. Listen to the tape and write down whatever is heard and then one has a complete picture of what was said. However, as is also described by Kvale, the transcription is a frozen form of a lived conversation (Kvale, 1996, p. 166) and transcribing is an interpretative process (Kvale, 1996, p. 160). The reason for this is that a central feature in the oral culture is that it is situational, empathic, and participatory. Body language and vocal intonations gives nuances to what is said, whereas written culture is analytic, abstract, and has an objective distance. It is therefore impossible through the transcription to have a complete picture of the conversation and what took place there. Below I will therefore discuss how I chose to record and transcribe as well as discuss the validity and reliability of the transcriptions.

5.1.1 RECORDING

I used audiotape to have a record of not just the words, but also pauses and tone of voice. As the pupils during the interviews talked about their teachers or other pupils in the class in a negatively way, I assume that they felt comfortable, and that the tape recorder or microphone did not seriously influence the way they talked and expressed their views. Below are some examples of this.

*A: Er, some people like to learn by doing [2 sec silence] you know sort of finding a general, they'll do load and loads of examples like E [laughs] and and you know sort of just get been told how to do something, and then just practicing and practicing and practicing over and over again [C laughs silently]. Really bor
(IE1, 659-662)*

*A: Mr. X has a, he is, I think he IS a good teacher and I like this sort of informal discussion style, when we have in our lessons, he does have a bit of a tendency to sort of prove prove things [A & C laugh]. It can be, it can be quite useful to see, like I said, to see where the methods of comes from (I: mmm), but sometimes it does little get carried away [smiles] (I: Mmm [Laughs a bit]). You say, a tendency to sort of prove things which might not become relevant till say, like the THIRD [C giggles] year of our maths-degree or something like that (I: mmm) which is what he went into the other day [C giggles] he tends to get a bit carried [laughs] get a bit carried away, which CAN be interesting sometimes (C: Uh [giggles])
(IE1, 689-699)*

*E: But by that time you want to do maths and you are interested in it and it's your choice to do it (D: Yea). But I think people, I think a lot of people here are quite intelligent and don't do well in maths when they were younger, and don't go on, its because of their, er, I'll blame it on the teacher though, I don't know, they are all lacy people and all sorts (D: mmm) of different reasons for that
(IE2, 376-380)*

I did not use videotaping, neither in the intersection phase in the English interviews. My main interest was what the pupils said, and during the observation phase I audiotaped the pupils' discussions and other 'noises' and I took notes of what I saw them do. I assumed that videotaping would have been even more disturbing than audiotaping. I might therefore have more and (formally) "better" data but I risked that it was of lower quality. Even if the videotaping would not have disturbed that much, it would not have added much extra essential information as the research is about how the pupils *say* they learn a mathematical concept that is new to them.

To avoid noise on the tape, I did not bring the pupils glasses for their drinks but they all either drank from plastic bottles or got plastic cups. In the English interviews I also tried not to say "mmm" or interrupt, when they spoke as I had experienced that the many "mmm's" I

said in the Danish interview had been quite disturbing while transcribing. It was difficult, but instead, I nodded every time I understood a statement, as I wanted to be encouraging and underline that I wanted to convey a level of understanding.

5.1.2 TRANSCRIBING

In this subsection I will describe and discuss the style I chose for the transcriptions, why I transcribed the interviews myself, and how I experienced this as well as some ethical considerations.

Verbatim style

For different kinds of analysis, different types of transcription are necessary and useful (Kvale, 1996, p. 166). I chose to transcribe the entire interview very detailed as I did not want to discard any information beforehand. I therefore decided on a verbatim style instead of a more formal written one. The verbatim style is a word-by-word reproduction including the frequent repetition in daily language as well as pauses, emphases in intonation, and emotional expressions. Instead the formal style is a condensed summary of the relevant parts. Another reason for choosing the verbatim style was that I wanted to be able to judge to what extent I might have been leading and therefore I needed to see how the dialogue started, developed, and ended. Furthermore I wanted a sort of psychological analysis where “the many ‘hm’s of an ordinary conversation, disturbing when reading a transcript, can be relevant for later analysis” (Kvale, 1996, p. 171). For that reason I also noted the length of periods of silence.

To get a high validity of the transcriptions I used a lot of time transcribing, and whenever in doubt, I chose to rewind the tape an extra time. For the same reason, I also wrote “inaudible” if I had doubts. The tapes were of a good quality, but even so, it was in the English interviews sometimes difficult for me to understand abbreviation, slang, mumbling etc. I also experienced that words in statements, I remembered having understood during the interviews, had become incomprehensible. This was perhaps due to loss of information from body language, eye contacts etc. In almost all cases, after listening to the piece, perhaps five

times, leaving it for a while and then coming back, I had an ‘aha’ experience - ‘so that’s the word!’ These problems may be connected with that as non-native speaker one has a smaller stock of words which one’s brain can go through to see to which word the “strange sound on the tape” fits best. This was not a problem in the transcriptions of the Danish interview.

No transcription will include everything and a transcription is an artificial written construction of an oral conversation (Kvale, 1996, p. 168). Kvale argues, that what a correct transcription is, cannot be answered, and is not relevant. Instead Kvale wants to know what would be a useful transcription for a given purpose (Kvale, 1996, p. 166). I agree with Kvale to a certain extent. An example: in IE1, 305, I have written in the transcription that a pupil said ‘it’ four times. But was it really exactly four times, and not three or five? In such cases I decided that the exact number was not relevant for answering my research question. Therefore, in that sense, writing ‘it’ four or five times are equally valid, for this piece of research. I therefore find that once one has determined the necessary style of one’s transcriptions, then within these frames one can certainly talk about what the correct transcription is, or at least (as argued in Chapter 4) that any version is not as good as any one else.

I transcribed all the words and narratives in one long sequence. I used full stop when the discussion took another direction, and commas to ease the reading and to indicate pauses or smaller changes in direction. When I write “mmm” this is meant to cover two types of “sounds”: (1) a nod in approval or (2) indicating, so that that is what it is. The context will make it clear which type it is. I used following notation: [4 sec silence] means an observation or sound that characterises the interview, for instance that there were four seconds of silence. (C: but) means that a pupil said “but” something in the middle of something another pupil said. Words written in ‘CAPITAL LETTERS’ means that the words were either being emphasised or said in loud voice; ‘D/E’ means that either Pupil D or E said something but it was impossible to hear who; ‘F & B’ means that pupil F and B said the same at the same time. ‘Z?’ indicates that it is most likely said by Pupil Z. Each time a new pupil said something, I changed line, except if it was just a short interruption in what another pupil was saying. When I said something I use the letter *I* to denote ‘interviewer’. I found this level of detail adequate for my analysis.

Transcribing myself

I transcribed all the interviews myself to get to know the data better. I could, as Kvale (1996, p. 169) suggests, have had a typist do it, and then perhaps edit this, but I did not have the financial resources to do so. Having others to transcribe would furthermore require a quite detailed description about how exactly they should transcribe. As I did it myself, I developed the “rules” and a “style” that I found suitable to give readers and analysts an adequate picture of the interview. The style was developed as I went along, I went back and edited, whereby I ended up with what I mentioned above.

A method to validate transcriptions is through respondent validation (Kvale, 1996, p. 171). I could during or after the transcribing let the pupils listen to the tape and tell what they said. This is complicated to arrange and I would get a lot of extra information and explanations about what they said during the interviews. And what do I then do to this new information: tape it and transcribe it? *Ad infinitum*. One could check the reliability of the transcriptions by letting someone else type the transcriptions and then compare, perhaps using statistics to calculate the correlation coefficient. Another possibility is that I could have transcribed twice. A very detailed description on how to transcribe, would, in theory, make it possible to get a high reliability. A lack of high reliability might cause problems for the interpretation of the narratives. Kvale (1996, pp. 163-164) gives an example of this where two different people transcribe the same passage from an interview. One writes in a more verbatim style, the other records only what is clear and distinct. Some discrepancy between the two versions is seen and causes problems for the interpretation. I did not have the resources to let someone else transcribe and I did also not have the time to do it myself twice. Instead I tried to be precise.

While transcribing, I wrote down things I had not noticed during the interviews, which is a further argument for transcribing oneself. Some of my experience was the following:

ID

To me, Pupil Å had seemed very little articulate in the interview and it had seemed that she hardly said anything relevant for this study. However, re-hearing and transcribing the interview I realised that she may not have said much quantitatively, but that what she said in a “second light” seemed quite relevant. This first impression may have something to do with

that particularly Pupil Z was very articulate and therefore she “took the picture”. Also, Kvale (1996, p. 161) states that the “interviewer’s immediate memory will, however, include the visual information of the situation, which to a large extent is lost in the audiotape recording”. In this case it may have been an advantage only to have the interview on tape, as I assume this helps the analysis to remain focused on what actually *said*. *Not* having a video recording of the event may therefore help remove information that, firstly, is not that relevant for the research question (how the pupils *articulate*), secondly, the extra information may disturb the picture.

IE1

Pupil A seemed a bit domineering, as he often interrupted C. This was the first English interview and I was just happy that someone wanted to talk. As I did not notice this during the interview, I did not do anything about it, which therefore influenced the development of the interview. However, Pupil C did not raise his voice against it. The teachers had furthermore put them together, and they had just come from playing squash. So they may know each other so well and what to me looked like one dominating the other might just be a sign that they know each other very well. This anyway shows the necessity to be firmer in the beginning of an interview in setting some standards for behaviour, but this should be balanced with having them to respond to each other.

IE2

The pupils talked a lot about irrelevant things - at least this was what I thought during the interview. But when transcribing, some of these things gave me the idea that how one is taught mathematics to begin with influences how one later works with it and is able to learn it. Furthermore about following remark by I:

E: I don't know, it's going to be really logic when everything's gonna (D: [inaudible]) be in the right order [D laughs] looking for [inaudible]. Er I do work slowly because of it, Er cause I am so er I always have to have everything in logical order and everything perfect I can't just sort of er [1 sec silence] work at a textbook that jumps from here to there. (I: Mmm) I like things to follow on. I can jump back when I am using text but, when it comes to revision I like everything sort of, in a nice package and (? : [inaudible]) if it is not in a package I try I like to make it to a package, I go and research it, makes sure it is all nice [laughs].

[1 sec silence]

I: OK, er, I mean, I just say, I am just interested in what it is you have to say, so, there is not any right or wrong answer (D: Yes) it is just, I am just curious to know how you (D: The way we work) yea, the way you work (IE2, 138-150)

This sounds odd on the tape, but felt right at the moment. This was probably connected with observation of eyes, hands etc. which cannot be “heard” on the tape.

IE3

It was sometimes difficult to distinguish two voices (two boys), which may have caused faults in the transcription. This was also a difficulty in the first interview (two boys), but only when they discussed the knots in the intersection while mumbling and laughing. I was also different in the third interview, more directive, as they were not so talkative, which might have caused them to become even less talkative. I also got a bit annoyed that they did not say more, which actually might have affected them further. It did not help that Pupil B was late and that his mobile phone at some point rang (IE3, 216). Actually Pupil G was supposed to be there, but he had forgotten, instead B came, which I did not know.

5.2 CODING OF THE TRANSCRIPTIONS

The method of analysis is to use the CULTIS model for analysis developed in Chapter 3 to identify what the pupils say or do, and identify what they say or do which is not supported by these theories as represented in the CULTIS model for analysis.

In the coding I (1) identified statements where the pupils said things that were relevant for the research question. This meant that when I read the transcriptions, I underlined phrases where the pupils said something about how they learn mathematics. (2) For each of these underlined statements I determined which of the six themes in the CULTIS model for analysis the statement belonged to or if it was not covered by the CULTIS model for analysis.

I coded several times, with a time span between the coding, as I did not want to rush into coding but instead keep my mind open. In developing the final matrix I looked at all the previous codings. Furthermore, rushing into coding is a rush into judgement and might therefore affect the validity (Robson, 1993, pp. 204-205).

Kvale (1996, pp. 208-209) discusses two ways of controlling the interview analysis and thereby increase the reliability: (1) multiple interpreters and (2) explication of procedures. In relation to the first, one could use different people to code and interpret, but this was in practice not possible to arrange. Instead I coded several times, with a time span in between and then compared the coding. Each coding was from a “fresh” print of the transcriptions and there was also a time span of about a year between the codings, which following Hadamard’s (1945) rest and forgetting hypothesis might make me able to look at the transcriptions with new eyes. I also coded at various locations which, following Mellin-Olsen, might have as result that “reading the same text at a beach, in a train, and in a cafe helps one to look at the text with fresh eyes, and to discover what might be in it” (Mellin-Olsen, 1993, p. 151). I experienced the subsequent codings as easier. I had got more used to code, while I was a novice in the first try. I felt I had become better to distinguish between clear statements and less clear, and I was less in doubt about which theme various statements belonged to. One could assume that this gave the coding a higher reliability. Another advantage was that the time distance between the codings meant that I had forgotten most of the pupils’ faces and my own emotions around the interview and I felt that I could look at the interviews more “objectively”. In relation to the second option, about being explicit about procedures, one could show examples of the “material used for the interpretations and explicitly outline the different steps of the analysis process” (Kvale, 1996, p. 209). The advantage is that the reader can see what the researcher did, and decide whether or not he agrees with it. Below I have attempted this through, firstly, giving examples of codings (Section 5.2.3), secondly, through displaying the pupils’ narratives in checklist matrices (see Appendix A).

5.2.1 PROBLEMS WITH DETERMINATION OF THE RIGHT THEME

While I was coding, I experienced some problems with overlaps between the themes. One example was in IE2, 199-200:

E: We just sit and talk about it and try different ways, trial and error sometimes. (D: Yea) We get

The problem was whether this belongs to the Theme 1, as it had to do with how to plan and evaluate or if it belonged to the Theme 6 as the pupils discussed this? After consideration I

chose that it mostly belonged to the Theme 1, as E was more describing the way they tried and planned than he was describing how exactly they discussed and why that was helpful.

Another example is from IE1, 556-562. When I asked the pupils how they work out the personal tricks, the answer was:

C: It just happens [1 sec silence] ss, you just [1 sec silence] you happen to do it and it works. It is general for most people, I don't know (I: No) er some people like writing it out, everything, the whole page [A coughs] read from that. Some people make notes from whatever, some people just memorizing notes they get in class, some people er look at it and memorize [both laugh]. Er different people different things (I: mmm).

Did this belong to Theme 2, as Pupil C is not conscious about the way he found the method? It was more “something” that happens. Did it perhaps instead belong to Theme 4, as the pupil can clearly not explain what he is doing, and it could therefore be seen as tacit knowledge? Or maybe Theme 5, as the statement clearly says that the methods are individual. I decided that the “main” theme is not Theme 5 as even though the pupil states that the methods are individual the question posed by I was not about the nature of the methods (i.e. individual or general) but the question is how the methods are *created* and the pupil’s remark about the nature of the method can therefore be regarded as a side-remark. It is still important and worth taking into account that something was said about Theme 5, and this will also be remembered in the further analysis, but in the determination of theme, one must choose the “closest”. It is therefore also not Theme 4 as the “Tacit-theme” has more to do with that one *knows* that ‘something’ will work, but one cannot articulate why, whereas Theme 2, the Unconsciousness, has to do with that one does *not* know where the “answer” came from, it just came; and later, looking back and analysing, one be able to explain why it works. In that light I have therefore placed this remark in Theme 2.

Another example from IE1, 791-801, is when Pupil A, after being asked what he would do (to learn the mathematics given to them in the intersection) if he had to present it to their classmates says the following:

A: I would draw them out myself. I would try and understand it, I just take each paragraph or each you know set of ideas like I said, [inaudible] ups I seem to have [can't find the pen, 'I' helps and he finds it]. Er and then sort of take each set of paragraph and draw out for myself you know some things and how they [1 sec silence] sort of how they link together (I: mmm) er and then you know sort of like try to understand write it out for myself what different things are and just draw out some sort of random, sort of knot and try to work out how they fit into the arguments I've got on page [sniffles] because [3 sec silence] I definitely find that helps me to understand the concepts an idea [1 sec silence], sort of method to doing something if I can see how it's actually used (I: mmm) rather

that just bah sort of method or whatever it, I need to be able to see how it is applied [1 sec silence] and be taken through that applied or going through myself as applied [1 sec silence] before I can get proper understanding.

The pupil clearly talks a lot about doing something *himself*, which could speak in favour of putting this statement in Theme 5. Furthermore he does not say anything about asking someone else, a teacher, or me, to explain it to him. But the core of what he says is what he says in the end. *“I definitely find that helps me to understand the concepts an idea [1 sec silence], sort of method to doing something if I can see how it’s actually used (I: mmm) rather that just bah sort of method or whatever it, I need to be able to see how it is applied [1 sec silence] and be taken through that applied or going through myself as applied [1 sec silence] before I can get proper understanding”*. One can here see that what makes him understand is that he sees how it is applied, getting “through” it. Whether he finds it out by himself or someone has told him means in that respect less. The way to understanding is seeing how it is applied and be taking through it, which is much more Theme 1, which has to do with practice and how things are used. In general, if the pupil talks about doing examples or exercises, the utterance is place in Theme 1, and if the pupil talks about pictures or diagrammes, the utterance is placed in Theme 5.

I experienced that most narratives, about four out of five, were quite obvious to place within a theme. The rest needs a bit more thinking. It also happened a few places that I had to place narratives in two themes at the same time. One example is in IE2, D2&5:

-probably, except that writing things down is much more of a conscious effort, trying to understand. When you understand subconsciously it’s like an immediate shedding of light on the problem, when I write something down, that’s much more consciously working it through in your mind. I understand this bit and then linking all together on paper (1232-1235).

Pupil D does here talk about the difference between subconscious and conscious work, and as she links conscious work very much to individual self-activity, I chose to place this utterance in Theme 2 (unconsciousness) as well as Theme 5 (individual).

5.2.2 THE USEFULNESS OF THE THEMES IN THE ANALYSIS

In the light of the above mentioned examples of some of the problems in determining which theme some of the remarks belong to, one could perhaps argue that the themes do more harm than good as it forces the researchers to oversimplify matters that are naturally complex. On the other hand, putting the narratives “in boxes” were far from always as difficult as the examples shown above. Often the case was much more clear. Even so, placing an utterance in one “box” does always mean “cutting” some information off. But when I later describe the pupils, I will not be too “squared”. On the other hand the placing of the narratives in themes forces the researchers to think more of what the pupils actually talks about. Furthermore it is also a way to test and refine the content of the themes. Also it might be a way to (dis)validate the research method of asking pupils how they have learnt mathematics if each sentence they say fit either all or none of the themes, alias the theories.

To ease references to a certain theme in a particular matrix I will refine the previously used notation: “IE2-D2, 690-692” means that something that was said about Theme 2, the unconsciousness, by Pupil D in the second English interview transcribed in line 690-692. For the Danish interview it would for instance be “ID-Ø5, 1256”, meaning that Pupil Ø said something about Theme 5, the individual, in line 1256. Inside each box will be a reference to which line a given statement can be seen in the interview, for instance (432-437). In Appendix F, G, H, and I, the reader can find all four interviews in their full length. Each utterance in a box begins with a – (dash) and other comments are in (brackets).

5.2.3 EXAMPLES OF CODING AND HOW THE MATRICES WERE CREATED

An important part of the analysis is the use of a checklist matrix. Each pupil’s coded narratives are here displayed to give an overview of the distribution and range of opinions. I have rephrased slightly as the transcripts are in a verbatim style. According to Miles and Huberman (1984, p. 96) “Two or three brief quotes per cell are enough to communicate”. I will however show more than just two or three statements if this does not cover the discussion about the topic. I have also added the notes from my observations in the English interviews in one of the rows.

I will now show some example of how I transformed the pupils' narratives into the matrices. As mentioned above the coding had two steps. (1) The first was an identification of statements where the pupils said things that were relevant for the research question. I therefore underlined key statements. (2) Afterwards I determined which of the six themes in the CULTIS model for analysis the statement belonged to, if any. In the boxes below are given examples of the coding from different parts of the interviews.

In the first column is either a number from 1-7 or the letter Q. The numbers 1-6 each stand for a theme in the CULTIS model for analysis (1. Consciousness, 2. Unconsciousness, 3. Language, 4. Tacit, 5. Individual, 6. Social), the number 7 stands for a statement that did not fit any of the themes. 'Q' is an identification of a key question posed by the interviewer. The example from the Danish interview is not translated, but I will illustrate how some coded bits were translated into English.

The first example is taken from IE1, 491-518.

6	C: Yea (I: mmm), otherwise if you just read it in your head it like a just read it a feel [snaps his fingers] I'm just read it and, I mean, don't understand. If you read it aloud, and you come across like a phrase in which, <u>sound really awkward, like, you know, the one here, where it says: "in the plane whose interior intersects the diagram in one of the configurations involved". Like that (I: mmm). You just read it over, you read it, but you don't think about it, but if you read it out, then er, you think about you think about the disc, and then you visualise the disc, and then you visualise the plane.</u> (I: mmm) (A: Yea yea) One by one (I: mmm), like in er, my, I do biology, it helps cause you've got to memorise a lot of sequences of event like, say neurotransmissions or the seven steps, if you just read it, you just read it, er if you like, think about it, what is it, (I: mmm) you know, you read it aloud and you think about what's [A: Laughs]
Q	I: Are there other techniques like er reading aloud, are there other techniques or tricks [interrupted]
5	C: Er, yea (I: you use?) if, <u>if you can put on paper</u> (I: mmm) <u>then draw out er,</u> [laughs] like (A: Yea) in chemistry like drawing out certain equations.
3	A: <u>If I have to memorize,</u> [1 sec silence] or not necessarily, er, if I, if I say memorizing from notes, <u>it doesn't happen so much in maths, cause it is more understanding of the method rather than you know sort of facts you have to learn</u> (I: mmm). Not the same thing in physics which is really a combination of both (I: mmm). If, <u>if I am revising from my notes, I find that helps if I actually write the notes out again, just, just copying them,</u> out out from you know [C coughs] the previously. Because, <u>if you just sort sit down to revise, you read through, even if you read it aloud, you pick up some things, remember some things. But if you write it out, you sort of read again and then write again</u> (I: mmm) and it sort of reinforces it. And I definitely found er, if I, <u>if it is something where I have to memorize</u> [1 sec silence], you know sort of examples, sort of methods, equations, <u>how things work, I definitely find it easier if I write, er, as an aid to memory</u> [2 sec silence] (I: mmm) er, [1-2 sec silence] Yea.

In the matrix for instance the utterance by Pupil A belonging to Theme 3 therefore looks as follows in IE1-A3:

(In connection with IE1-C5)

-if I have to memorize from notes, it doesn't happen so much in maths cause it is more understanding the method rather than facts you have to learn. If I am revising from my notes, I find that it helps if I

actually write the notes out again, just copying them. Because if you sit down to revise, even if you read it aloud, you remember some things. But if you write it out, you sort of read it again and then write it again and it sort of reinforces it. And I definitely found that if there is something I need to memorize, sort of examples or methods, equations, how things works, I definitely find it easier if I write as an aid to memory (508-518).

The next example is taken from IE2, 918-968.

Q	I: So, what could they have done, let's say, if they wanted to describe knot theory.
6	D: I think this is, <u>this is the kind of thing where it is very difficult to talk about in a book and to represent 3-dimensional object within a 2-dimensional way, and it's where it would help to have a teacher explaining something and say pointing all this is the vertex and this is an edge</u> (E: [inaudible]) [inaudible]
	I: Pointing at, I mean
6	D: Yea, <u>drawing it</u> or tiny little knots <u>and say this is</u>
5	E: <u>Depends who they are targeting it at</u> [1 sec silence], don't be so [1 sec silence] so stuck up [inaudible]
4	[laughs] and so you get to a, I don't know, <u>don't use such big words, they are aiming to people who don't understand it</u> (I: mmm, well it's part of er) and use basic, yea (I: so I mean) no, I know, but I it would seem a bit sort of [1 sec silence] if it if it was in a GCSE and A-level course (D: mmm) all this would have, language, <u>it wouldn't be right, it's the the way they approach it, the language, it's just too, people would struggling with the language when they are suppose to be learning the maths.</u>
4	
Q	I: So is there a diff, I mean, er, so maths has nothing to do with the language? Or, can you learn maths without language.
4	D: <u>Yea.</u>
3	E: <u>No, but you can use different language, simple language to convey a point.</u>
4	D: <u>Cause the maths in it is quite easy,</u> I think, well, it's not (E: I'm sure it is [giggles]) [laughs] What do you mean it is nothing really difficult what it is saying is this is what a knot is, this is (E: Yea) what a link is, and, OK, that really really simplistic, it takes a long time [laughs [inaudible]] <u>it took me a long time to work out what they were trying</u> (E: <u>Yea what they were explaining</u>) whereas the fact <u>as soon as I, kind of translated it, I thought oh well, that's what a knot is, find that's easy.</u>
Q	I: What did you translated it [inaudible]
4	D: <u>Into simple language</u> [laughs] er, it er [inaudible]
Q	I: You translate it before you understand it, er, so (D: [inaudible]) if you have understand, then, it, you don't need to translate it.
6	E: I think <u>it here would be easier if the author translated</u> (D: Yea [laughs]) <u>rather than er leaving the reader to er [1 sec silence] to do it,</u> I mean.
4	D: <u>You have you have do the two together, you have to translate while you're trying to understand</u>
5	E: Depend, <u>it depends on your audience</u> though, er, I I mean, if you've got an audience who are used to, this approach and don't understand then it's fine [1 sec silence] and I don't think many people, and I think people, <u>everyone will struggle with this, in our school in our</u> (D: In our, yea) set. Er, I mean <u>some people would would take approach and probably solve it and er you know understand it virtually</u>

As an example, Pupil D's narratives in Theme 4 (second half) looks the following in IE2-D4:

*(When asked if mathematics has nothing to do with the language, or if one can learn mathematics without language, 937-938)
-yea (940).
(In connection with IE2-E3, 942, about if it is possible to learn mathematics without the use of language)*

-cause the mathematics in it is quite easy, what it is saying is what a knot is, what a link is. It took me a long time to work out what they were trying, whereas the fact as soon as I kind of translated it, I thought oh well, that's what a knot is, find that's easy (944-949).
 (When asked 'you translate it before you understand it, so, if you have understood, then you don't need to translate it, 955-956)
 -you have to do the two together, you have to translate while you're trying to understand (961-962).

The next example is taken from IE3, 98-139.

	B: It's the same thing with just different numbers basically
6	F: Yea, that's so, er, <u>if we are given more teaching time in class er and fewer repetition of the same question different guys [inaudible] then er I think we can go a lot faster. That's my view.</u>
Q	I: So you don't need all this exercising?
1	F: <u>You need an example of each one and you need to get it right and not just er our books have answers in the back as well, you do need to make sure that they are not just looking in the back.</u>
1	B: <u>I think it is not just one of every one, I mean you need a few just to get the sort of process into your head when you don't [inaudible] but sometimes we get a set far too many just repeating the same thing, and when you've done it for half an hour and you've learnt it and you have to carry on doing it hours of work on top of it, it's just the same thing again (I: mmm)</u>
1	F: It's sometimes crunching, er it's a bit boring and also I don't in er biology and Latin and Greek we er get actual exam questions every week - you don't get set back in maths, really, I mean some of the book questions are exam questions but most of them aren't.
1	B: <u>I actually find it more useful when we ARE doing sets of exam questions</u>
1	F: <u>Exactly.</u>
Q	I: Why?
1	F: Er, cause they they sort of, [inaudible] <u>the exam question, the whole topic area is condensed into one question, it has a bit of each sort of part. And er you get used to combining the whole lot together rather than just using one specific bit and repeating it over like you're doing in exercise.</u>
Q	I: Mmm, but do you need exam er questions to do this (B: Er) or is it more the type of questions?
1	B: <u>It's getting use to the type of questions.</u>
	I: Mmm
1	F: <u>Yea, it's the whole format cause one exam question can, if there is 6 chapters in a book you're studying er and each one is about a different subject er an exam question can test four of them all at once in one question and that's a lot more useful than doing each chapter and then forget the chapter and then you go to the next one, then you come up with an exam paper and you have to link them all together, that's the problem I think.</u>

The utterance by Pupil F in Theme 1 therefore looks as follows in IE3-F1, 105-139:

(When asked if they do not need all this exercising, prompted by the discussion in IE3-F6, 100-101)
 -you need an example of each one and you need to get it right and our books have answers in the back as well, you do need to make sure that they are not just looking in the back (105-106).
 (In connection with the discussion in IE3-B1, 108-111, just after that F (113-115) has said that some of the questions in the book are exam questions but most of them aren't)
 -I actually find it more useful when we ARE doing sets of exam questions (117).
 (When asked why it is more useful, 121)
 -in the exam question, the whole topic area is condensed into one question, it has a bit of each sort of part. And you get used to combining the whole lot together rather than just using one specific bit and repeating it over like you're doing in exercise (123-126).
 (In connection with IE3-B1, 128-131)

-yea, it's the whole format. An exam question can test four of them all at once in one question and that's a lot more useful than doing each chapter and then forget the chapter and then you go to the next one, then you come up with an exam paper and you have to link them all together, that's the problem I think (135-139).

The Danish interview represents a problem as it is in Danish and therefore only understood by about 5 million Danes as well as about 15 million people in the Nordic countries. For the Danish interview I coded just like in the English interviews but when the codes were transformed into the matrix, I translated it into English. The following example shows this translation process and I apologise for the non-Nordic reader who cannot follow this but are left to trust the author's abilities to translate. The example is taken from ID, 409-503.

Q	I: Hvad, hvad, hvordan, <u>du siger, du prøver dig frem. Øh, gør du det på en bestemt måde, eller er det mere sådan tilfældigt?</u>
5	Z: <u>Det tror jeg faktisk er meget individuelt fordi</u> (En anden: Ja) <u>det er tit sådan hvis, hvis vi for eksempel skal sidde, hvis mig og Æ, vi har matematik sammen ikke, skal sidde og snakke lidt om en opgave. Så er der tit forskel på, så følger vi li'som den samme erkendelse gennem at vi sidder og snakker, men så er der li'som forskel på, hvornår erkendelsen kommer fra den enkelte (Æ: Ja) til den enkelte. Fordi jeg tror meget det der med at finde den måde, som man helt selv personligt kan forstå et eller andet på. (En anden: mmm) Og jeg tror ikke vi for eksempel sidde og sætte sådan en systematik op sådan for alle. Først, så gør man sådan, og så gør man sådan. Jeg tror mere det er sådan noget med at, at jeg har også nogen andre hylder og ligge tingene end på end Æ har igås. (En anden: mmm) Det kan være, jeg har talt med M eller N [to andre elever] om noget andet før ikke, der gør at [inaudible] – jo det kan jeg egentlig godt benytte her, hæii, sætte ind på den her hylde som Æ måske ikke har, hun har en anden hylde, hvor hun kan lægge det ind på (En anden: mmm). Og hvis man ikke har li'som det samme udgangspunkt så, så gør vi [inaudible] så nogen forskellige ting, jeg har [inaudible] det er sådan en eller anden gnist (Æ: Ja) der li'som tænder, igås. Så forstår man det bare. På en eller anden måde, man skal li'som finde SIN indgangsvinkel til det igås. Og så lige pludselig så går det så op [inaudible]</u>
5	Æ: <u>Jeg har det bare meget sådan at, at det er vigtigt for mig at, at jeg, jeg kan se, hvad det er, altså. Jeg har mange gange jeg li'som laver en tegning, fordi jeg, jeg VIL kunne se, altså. Det der med at man bare har det, noget teori man sidder og læser om, altså. Det er vigtigt for mig for at kunne forstå noget, at jeg har det på papir, jeg har en tegning af det. Altså det, det er i hvert fald det første hvis jeg der er noget jeg ikke forstår, at jeg har det på tryk eller tegning. [afbrydes af Ø]</u>
5	Ø: <u>Hvor der så er andre der gerne vil se et eksempel. (En anden: mmm) Og sådan, nogen opgaver og så er det der igennem at (I?: mmm) det li'som kommer, ikke. Og hvor nogen de sådan mere kører sådan i retning af beviserne og alt det der, det er DEM man vil forstå for at forstå sammenhængen, ikke altså. [inaudible]</u>
Q	I: <u>Det vil sige, der er også nogen der, der faktisk forstår matematik gennem beviserne?</u>
5	Ø: <u>Ja DET tror jeg da i hvert fald [inaudible]. Ja det kan man godt have nogen gange, at man gør ikke. Ved nogle mere simple beviser, men måske igen fordi de ligner noget man har haft før. (En anden: mmm) Eller at man kan finde nogen lighedspunkter i den der erkendelses- eller forståelsesproces. <u>Men det er nok meget forskelligt, hvordan man forstår, også (En prøver at afbryde (Z?), men det mislykkes) om man er kreativ eller som det Æ siger, fordi det er jo egentlig det, det handler om, ikke. At skulle se det visuelt. (En anden: mmm) Og en anden måde det er også, at man gerne vil læse tingene i stedet for. Det sådan kan jeg godt have det nogen gange, at jeg forstår simpelt ikke når der er [griner kort - hæ] en der står og siger tingene til mig (En anden ?I: mmm) altså jeg skal se det på skrift for at forstå det.</u></u>
Q	I: <u>Du skal li'som selv altså (En anden: Ja) (Ø: Ja) fortælle dig det, fortælle det for (Ø: Ja) dig selv (Ø: Ja)</u>
5	Å: <u>[inaudible] Når sidder og læse i bogen, så er der nogen man li'som skal tænke mere over det (En anden prøver at afbryde med: Jeg tror heller ikke) skrive lidt og læse lidt videre når man så har forstået</u>

	det første altså. <u>I stedet for når man sidder i klassen og det hele går så stærkt.</u> [inaudible] Når der er noget man ikke forstår så kører læreren jo bare videre til det næste, ikke.
	?: Ja
5	Z: <u>Jeg tror ikke helt, jeg har den der, den der visualiseringsting så meget. Men jeg tror meget, jeg har sådan noget med, at li'som Ø siger med, at det skal helst være på skrift</u> [inaudible] <u>det skal være noget jeg kan se, ikke.</u> (En anden: mmm) Det er altså meget fordi, jeg tror måske også meget, <u>vi er opdraget til at vores erkendelse, den er visuel på en eller anden måde, ikke, altså.</u> (En anden: mmm) <u>Mange af de ting vi skal lære, de foregår altså i, via, ved hjælp af vores øjne.</u> (En anden: Ja) Og der kan jeg, derfor synes jeg også det er vigtigt, <u>det der med nogen gange, når man, nu sætter jeg mig altså ned, så selv, så kigger jeg bare på det her igås.</u> Så tror jeg meget, det er der, hvor jeg har den der, så kommer punkt 1 punkt 2 punkt 3. Så tror jeg, også en anden ting for mig som også er god, det er også det der med og, og <u>kunne snakke med andre folk, og det har vi sådan set, når vi regner vores opgaver.</u> [de regner meget i grupper når de regner opgaver] (En anden: [inaudible]) Og der er det, <u>der er det utrolig fordelagtigt, også at snakke med nogen som heller ikke kan finde ud af det, om det.</u> (En anden: mmm) <u>Fordi at man på den måde også får nogle andre indgangsvinkler til det, som måske kan være den indgangsvinkel, man selv manglede.</u>
7	
5	
6	
Q	I: <u>Hvad så med at snakke med andre, som kan finde ud af det, er det, det er ikke</u> [afbrydes af Z]
6	Z: <u>Det er ikke, det er ikke altid lige godt vel, fordi så får man tit den her forklaring, og de er rigtig grebet af, at de har forstået det, og så kommer de aw-aw-aw det er da nemt</u> [inaudible] <u>ha-ha-ha</u> [inaudible] [afbrydes af Å]
6	Å: <u>Og bagefter så sidder man bare, sådan lidt, ok</u> [inaudible] <u>det var selvfølgelig fint nok</u> [inaudible] <u>Det er bedre, hvis man kan få diskuteret det, sådan på lige fod med en eller anden</u> (En anden: mmm) <u>som har det li'som én selv.</u> Så sige, synes du man kan gøre det sådan, nej måske kan man gøre det sådan. Så når man også frem til [inaudible]
6	Æ: <u>Men så også det, hvis man egentlig, æh, mange gange hvis man prøver at forklare en person det, det handler om, så pludselig så når man selv er i gang med at forklare det, så forstår man det.</u> (En anden: 1-2 bryder vist bifaldende ind [inaudible]) så, så er det, <u>det er egentlig ikke sikkert man HELT har forstået hvad det er, det drejer sig om, når man BEGYNDER at forklare den anden person, men, men så som man egentlig, man står egentlig også og forklare det for sig selv</u> (En anden: mmm) så forstår man det.
Q	I: [inaudible] <u>Hvad, hvordan kan det være? Øh, hvorfor, jeg kan godt, jeg kan godt genkende det men hvordan tror du det kan være?</u>
	[Flere prøver at "komme til", men man "bliver enige" om, at Å svarer]
2	Å: <u>Li'som man sidder med en opgave, ikke forklare dig selv</u> [inaudible] <u>lavet før</u> [inaudible] <u>mangler en eller anden ting, og så når man ser det igen, ikke</u> [inaudible] <u>det kan være</u> [inaudible] <u>måske været gået lidt væk fra det og så noget</u> [inaudible] <u>et eller andet man pludselig lægger mærke til, man slet ikke tænkte over før.</u>

As an example the utterance by Pupil Z in Theme 5 looks as follows in ID-Z5, 409-426:

*(When asked how they feel their way through the maths, if they had a specific technique, 409-410)
-I actually think this is very individually (someone else says yes). When we sit and discuss there is often a difference in when the understanding comes to the individual. I think it is very much about finding the way that you personally can understand something, I do not think we can draw up a systematic which works for all. Somehow you have to find YOUR angle, and then suddenly you understand (412-426).*

In Appendix A is four matrices for each of the interviews.

S is for Scholastic Aptitude Test.
Be sure to do better than all of the rest.
That way you'll get into Harvard or Yale,
And land a job in the government if you pass or you fail.
And government is a lucrative field
With loads of influence and power to wield.
Plus a government job insures that eventually,
When you're caught, you'll serve time in the best penitentiary.
(O'Rourke, 1987, p. 161)

6. FIRST PAIR OF THEMES: CONSCIOUSNESS AND UNCONSCIOUSNESS

This chapter discusses those of the pupils' utterances which fell within the two first themes. I will discuss these themes separately as well as jointly. When I describe the pupils' self-understanding I will leave a reference to the appropriate matrix, for instance ID-Ø2, which, to recapitulate, stands for the Danish interview where Pupil Ø said something which belongs to the second theme. IE3-F8 refers to something Pupil F said in the third English interview during the intersection and thus not directly in the interview. The number '7' denotes something said outside of themes. These matrices can be found in Appendix A, and here there are references to which lines this was said in the transcribed interview, see appendices. As described in Chapter 2, I will begin the analysis by phrasing a pupil's self-understanding and here, to some extent, preserve the pupil's own words. Then I will use the theories from Chapter 3 in a theoretical understanding. I give my perception of what the pupils say in each theme. At various places some of the discussion have already have taken place during the presentation of the self-understanding. This will depend on the nature of the utterances. There is a discussion of each of the themes as well as a discussion of the pair of themes seen together. This is how the next three chapters are organised. There are minor variation from the general structure owing to the different nature of the theme as well as the amount of information the pupils provided. In Chapter 6, I will discuss the first pair of themes: the consciousness and the unconsciousness, in Chapter 7, the second pair of themes, the language and the tacit; and Chapter 8 will discuss the third pair of themes, the individual and the social.

The length of Chapter 6, 7, and 8 depend on how much was said in the interviews it does not signal any priority on my behalf.

6.1 CONSCIOUSNESS

To get an overview of the pupils' narratives within this theme, I split them up in three areas depending on firstly, if it is about the phase before starting to work with the mathematics, for instance planning or motivation; secondly, about carrying out the plan; or thirdly, looking back, monitoring, or reflecting afterwards.

6.1.1 BEFORE STARTING TO WORK WITH THE MATHEMATICS

This section will discuss the issues of planning and motivation.

Planning

The pupils who seem to talk about a planning phase are Pupil A, D, and E. Indirectly also Pupil Æ talks about it.

Pupil Æ said in her self-understanding (ID-Æ1) that sometimes when one has a problem and one does not know in which direction to go, then one tries different directions and perhaps she is lucky.

The theoretical understanding could therefore be interpreted to be that she does not seem to have a particular plan as she says that sometimes when one has a problem one does not know in which direction to go. However, she says *sometime*, which may suggest that at *other times*, she can and thus has a plan.

Pupil A's self-understanding (IE1-A1) is that he does not go through the problems systematically but instead just looks at it and makes connections between this and something

that he might have seen before. He tends to think around it and see ways of doing it. He relaxes and tries to think about the answer rather than beginning to work it out right away.

About his theoretical understanding; when he says that he does not go through the problems systematically but looks at it first, it seems to be linked to Mason's (1985, p. 28) and Polya's (1971, p. 9) theories stating that an early phase of problem-solving is to have a plan that is based on past experience and formerly acquired knowledge.

In Pupil D's self-understanding (IE2-D1), she refers to the mathematics in the intersection, the knot theory, and to learn it, she would go through it all and perhaps take another book to learn the basic bits and then, once she completely understands it, she would go on to the harder bits. She would not go into each bit straight away. Later she corrects herself and says that she actually thinks that when she went through it in the intersection, she *did* try to make sure she understood each bit first. One could then discuss which of the statements are more correct, in terms of what actually works for Pupil D, and what she really does. Perhaps this is an example of that even when we humans know that something is the right thing to do, we do not always do it. I would therefore regard her latter statement as one showing just that, and I will therefore assume that the way to learn for her, is to first go through it all, perhaps supported by other material, and then when she completely understands that, she would go on to the more difficult bits. Pupil D also states that she finds it quite interesting that when I gave them the knot theory, she did not take any notes, she does not know why, but sometimes she finds it helps to write notes.

In relation to theoretical understanding one could argue that to Pupil D the planning part has both to do with using past experienced *methods* as well as past experienced *mathematics*. This seems to fit with the experience of Pupil A. The way of learning for Pupil D is not to go into each mathematical detail at once, here she is much alike with Pupil A. What makes them different is that Pupil A seems to draw his planning phase on past experienced *mathematics* whereas Pupil D more builds on past experienced *methods* of learning.

Pupil E states in his self-understanding (IE2-E1) that when they sit in class and work together, they talk about the mathematics and try different ways, sometimes trial and error. The learning does therefore not always seem to be a result of careful planning but also "luck" and

coincidence. When the pupil talks about getting to know the knot theory, he explains that after he has read the mathematics a couple of times, to have a general overview, he would go through each little bit in term to try to understand it bit by bit. He would not do that the first time, as there may be something later on that will help him. He finds it is easier if one has a whole picture, even if one does not understand it all, before one starts trying to attack each little bit at a time. These latter lines have resemblance with what Pupil A and D states above. He also explains that if one has a problem to solve, it is helpful to get a piece of paper and scribble one's ideas down. It therefore seems that when he works for himself, he has a clear strategy for how to learn mathematics, whereas when he works with others, it is more a coincidence what they do.

The theoretical understanding could thus be argued to be as follows; the planning phase when working with others has sometimes character of being blind, which usually is a characteristic of lower-achieving pupils. Some examples, not necessarily diagrams, are good. Otherwise Pupil E prefers the "opposite" approach, namely to have a whole picture first, even if he does not understand it completely. One is better off having a vague whole picture before one starts trying to attack each little bit at a time.

The pupils thus fall into the following two groups:

I: Planning builds on past experience {A, D, E}

II: Supports very indirectly {Æ}

III: Says nothing {Z, Ø, Å, C, F, B}

There is a slight difference between Pupil A and D in whether the main inspiration to planning comes from past experienced methods (D) or past experienced mathematics (A). Pupil E tells that in practice, planning is not what he does when he works with others, only when he works alone. I have chosen to put him in the same group as Pupil A and D as it seems that Pupil E's learning phase is more "planned" when he works alone. The English pupils refer to the knot theory as examples of a topics where planning works. This does, however, not imply that this is the only topic within mathematics where such an approach is useful, but it shows that the knot theory was a useful tool to prompt a discussion about planning, at least among some of the pupils.

Motivation

The pupils who seem to talk about this issue are: Pupil Z, Æ, Ø, Å, A, C, D, and E, which is all pupils except the ones in IE3.

Pupil Z's self-understanding (ID-Z1) is that it is very important to be motivated. Motivation seems to be something that to some extent comes from outside the pupil and from the mathematical context as she states that it is necessary to be forced, or thrown into it. One needs to be challenged to learn. She goes as far as to say that one would not learn anything without being challenged, and one is only challenged the moment it is exciting. What she exactly means by 'being exciting' is however not so clear. But later she describes mathematics as a game and says that one needs to think that it is fun in itself. Personally she does not care what she can use mathematics for. She thinks it is fun; it is a game, like playing with Barbie dolls. This could be an example of the necessary 'excitement' she mentions above.

Another motivation is that if one can work it out, she finds that it is "cool"; thus it has a kind of show-off-effect. Another part of the motivation is the fascination of something that is difficult. For instance a proof where the teacher says that 'this is very difficult', and then she becomes excited and wants to learn it. But it is not an advantage if the teacher says that it is difficult in the beginning, for instance the first months in the first year of high school. It is also not good if the teacher introduces something as trivial and then one cannot figure it out. It is better if the teacher motivates the pupils by saying that this is difficult but one can learn it and it is really interesting to learn. Then one becomes more motivated.

This outside motivation is "paired" with an inner wish to learn as the pupil states that there is a motivation in the confusion if one really wants to learn it. The confusion is according to Pupil Z necessary as one does not learn by just sitting and learning the whole mathematics language/notation first, for instance that f is a function, A is a vertex. It is furthermore good to have the attitude, that mathematics is something where one has to work to understand it. Basically the pupil here talks about the importance of pressure, the excitement of mathematics as well as a kind of social drive (i.e.: to show off). The term "social drive" arises from the pupils' explanation and is not as such supported by the theories mentioned in Chapter 3. This social drive is a non-cognitive factor of which Carlson (1999)

describes some others, but not this one (see Chapter 1). One might call this a “grounded theory” element of this study as this concept arose from the pupil’s statement.

In relation to theoretical understanding of the role of motivation she seems to mention three important issues (1) External forces: either by forced/being thrown into it or being challenged. (2) Cognitive drive: either a motivation in the confusion if one really wants to learn it, or mathematics is like a game and one needs to think that it is fun in itself. She does not care what she can use mathematics for. (3) Social drive: a show-off-effect.

Pupil Æ ’s self-understanding (ID- $\text{Æ}1$) is that learning mathematics is a game, but at the same time it is also a challenge. Calculating at the grocer’s shop is not a challenge. Learning mathematics is fun and it is the game that is the fun part of it. The fact that there is something one cannot understand is in itself a challenge. There is some mathematics that is not so easy to understand. She accepts that there are things that she does not understand at once. There are also things that she does not understand completely, but after all she understands some of it, and she knows what it is that she just accepts. If she feels that she cannot get through, she thinks that she must and then she does it. This is a difference between Pupil Z and Pupil Æ . Pupil Æ seems to take the challenges more negatively, it is something she has to do and then she puts herself together and does it whereas Pupil Z perceive the challenge more positively.

In relation to the theoretical understanding, the motivation seems partly to be: (1) the cognitive drive, which means that mathematics is a game and a challenge and partly (2) the external pressures that she accepts the situation and even if she does not think she can do it she goes on, as she *has* to.

Pupil Ø finds, in her self-understanding (ID- $\text{Ø}1$), that there is a barrier when one does not think that one can do the mathematics. When she experiences this, she tells herself that of course she can do it. She does, however, not explain “the opposite way” namely if self-confidence causes motivation. But still she seems to argue that self-confidence is an important pre-condition for motivation to arise.

In this sense, she, in the theoretical understanding seems to be in line with what Mason (1985) writes about the necessity of having an atmosphere where confidence can grow.

Pupil Å finds, in her self-understanding (ID-Å1), that one needs to be interested in the mathematics because if one just continues with ‘what can I use it for’ or ‘I cannot understand this’, then one is not going to learn it. A wrong way to motivate is if the teacher says that this is easy and everybody can understand it, as then one feels stupid if one cannot understand. Here she is in line with Pupil Z. Another motivation is that mathematics is old and one always hears about the old scientists, which Pupil Å finds is so great. The motivation therefore needs to come partly from oneself, one needs to take an interest, and partly from the teacher’s way of presenting the topic.

The theoretical understanding can thus be interpreted to be that the cognitive drive is a better motivation than knowing how it can be used. But the teacher’s way of presenting is also important.

In Pupil A’s self-understanding (IE1-A1), confidence is important and included here is the will of finding, and knowing if every step is correct. The pupil states that it is better if the teachers just hint them in which direction to go instead of writing out the whole solution. It motivates the pupil to give it an extra try as well as it gives the pupil confidence in that he can do it. He furthermore finds that it is important to accept that something is true, otherwise he cannot move on. After thinking a little longer Pupil A says that a proof or something else written out can be very helpful to make one accept. The reason that it is helpful is that one then knows where the method comes from. Even though he does not understand the whole thing, it is interesting. Pupil A mentions solving a differential equation as an example when he explains that if one knows the next step in the method of doing it, or the next step in a proof. If one can see where it is going to be and one can explain it, then one has understood. Pupil A says that differentiation was totally new and different and that one sometimes has to accept something, and he again refers to differentiation. He explains, that when one is told that something like $3x^2$ differentiate $6x$, then one has to accept that this is how one does it, before one can move on.

About the theoretical understanding, following Hadamard (1945, p. 31), one could argue that seeing it as true drives the motivation for Pupil A. In the words of Polya (1971, p. 6), what makes Pupil A “desire” the solution of a problem is that he understands the problem before he starts to work on it. It is important for him to accept that it is actually true, and that he can see where it comes from. This is slightly different from what Polya means as Pupil A

talks about *knowing* that either the method of problem-solving or the whole piece of mathematics is *true*, whereas Polya talks more about *understanding* a specific problem. According to Polya (1971, pp. 12-13) a main feature in problem-solving is that the pupil is convinced that each step is correct, which seems to be supported by what Pupil A says about wanting to know that things are true.

In terms of the question of motivation, it therefore seems that Pupil A needs to be motivated/personally encouraged and have self-confidence. Furthermore, Pupil A also states that even though he does not always understand it, it is still interesting, which supports the role of the cognitive drive.

Pupil C says in his self-understanding (IE1-C1) that the situation in the intersection with the knot theory is unusual as it is individual, and they are not being taught. It is self-learning, and he cannot teach himself something new unless he really wants to, but he finds it difficult to motivate himself to something like this as it does not trick anything he knows from before.

The theoretical understanding could thus be interpreted to be as follows: Pupil C needs external motivation, probably other than the cognitive drive, as he says that he cannot motivate himself. He needs to know that it is right to be able to accept it.

Pupil D tells in her self-understanding (IE2-D1) that she thinks that at A-level the teachers are confident in that they can teach well, but up until then, in order to inspire an interest in mathematics, one needs someone who is nice. But it is very difficult to be interested in mathematics. When she was younger she liked logic puzzles. She finds that it is good to be pushed hard because one is pushed to the edge of one's ability where one just cannot get away with doing one's minimum. The teachers push them to do more and that is good because it prepares them for going to university and study the mathematics more. It is like tightening up all the loose ends, which is something one does not have to do to pass the exam, but it is more satisfying. It therefore seems that the role and importance of motivation has to do with the maturity of the pupil. She seems to suggest that when one is "young" the teacher has to be "nice" but when one is "older" (her age and above) one needs a more "firm push".

The theoretical understanding could be interpreted to be that it is more important to be motivated when one is younger where one needs encouragement and self-confidence. At their

age an external firm push is necessary as it is difficult to take an interest in mathematics. The cognitive drive is thus not functioning that well here, though it is still a force.

For Pupil E (IE2-E1) it seems that the motivation to some extent comes from outside as he tells in his self-understanding that the teachers have to be enthusiastic. He furthermore tells that he hated mathematics until his last year of GCSE. What changed was that he got a really “cool” mathematics teacher. Pupil E describes this teacher as intelligent, a good guy, knowing his stuff, and that he made mathematics fun. Pupil E finds that he has learnt more with him in one year than he has done ever before because the teacher paid attention. Pupil E further explains that there might be differences in how the teacher should behave depending on the pupils’ age. When one is at A-level, one is interested in the mathematics and it is one’s choice to do it. It was difficult to be interested in mathematics before, but he was interested in applied mathematics by which he means mathematics that had a purpose, or a problem that seemed impossible, like a brainteaser. He does not like basic addition and subtraction. He thought trigonometry was useless until he found out that one could solve other things with it. When it is just three graphs and that one is forced to memorise them, one cannot cope with that. He finds that it is good to be pushed harder and to learn things that are not in the textbook as with those extras things one can explore mathematics and that is a lot more interesting to Pupil E. When he says that he writes his own notes he means that he has got all the little bits together and then has got a nice picture.

In relation to the theoretical understanding, one could argue that the teachers have to be enthusiastic to encourage the pupils, but not so much at A-level where one is supposed to be interested mathematics. It is good to be pushed harder as all those things that are not in the textbook but those little extras makes it more interesting. He therefore speaks more of the cognitive drive and external force.

The pupils thus fall into following groups:

I: Cognitive drive {Z, Æ, Å, A, D, E}

II: External forces (some “nice” some “firm”) {Z, Æ, Å, A, C, D, E}

III: Social drive (invented notion) {Z}

IV: Self-confidence {Ø, A}

V: Says nothing {F, B}

Here, the pupils also mentions specific mathematical fields such as trigonometry, differentiation, calculation, functions, logic puzzles, applied mathematics, brain-teasers, and the knot theory. These fields could be perceived to cover both more algebraic branches of mathematics as well as more geometrical and thus one might pose the cautious hypothesis that these four types of motivations cover all fields of mathematics. However, not every pupil mentions all kinds of mathematics, and the sample is small, but it could be a hypothesis to develop in future studies.

According to Mellin-Olsen's concepts of the instrumental rationale (I-rationale) and the social rationale (S-rationale) it seems that the eight pupils who talk about motivation describe something which has mainly similarities to the S-rationale, namely that the pupils want to learn as knowledge has a value beyond its status as school knowledge. The pupils describe a kind of "love for knowledge", which could be interpreted as the cognitive drive, and the teachers can use different strategies. Only Pupil Z seems to talk about something related to the status of mathematics, the social drive, where being good at mathematics has a kind of show-off effect. This effect has similarities with the I-rationale. It is, however, clearly combined with a number of other drives pointing at an interest in mathematics and, my perception of what she says is, that the social drive is not as important to her as the other drives. Pupil Å does furthermore explain that it is not helpful for the learning if a pupil keeps asking about what he can use it for. What she means exactly by "what can I use it for" is not clear. Does it refer to applying mathematics or if it is useful in terms of exams is not clear. Either way, Pupil Å seems to argue for that the cognitive drive is a better motivation for learning. The cognitive drive is not exactly what Mellin-Olsen terms the S-rationale is a better motivation for learning. It might be surprising that the eight (of ten) pupils, more or less, only mentioned something that has similarities with the S-rationale. Whether the S-rationale *causes* that the pupils become high-achievers, or that being high-achievers develops a love for knowledge, is not possible to answer adequately from this study as this study does not investigate such a causal link.

6.1.2 CARRY OUT THE PLAN, PRACTICE

In this section the only heading is ‘practice’. All pupils talked about the necessity of practice, but there were some differences in what they said.

Practice

The self-understanding of Pupil Z (ID-Z1) is that exercises have the effect that one understands a lot of single cases and one has also seen how things “move around”, for instance when working with planes, lines, that $r_1 \times r_2$ is the normal vector, and differential equations. Thus it sometimes becomes possible to understand them more overall. After having worked with them and looked at them, and perhaps drawn some pictures and imagined some things, one begins to be able to understand the bigger picture, for instance with spatial geometry and the distance formula. It is very difficult to be able to see the mathematics as one big whole from the start. It is possible to solve the problems and exercises without having completely understood the overall meaning. The exercises and problems mean that one has worked with the mathematics within oneself and have therefore understood more. Even though doing exercises mean that one works with mathematics “within oneself” this does not mean that what is said here belong to the individual theme, as the *method* of learning explained here, is the exercising.

The theoretical understanding of Pupil Z could be interpreted to be that what Pupil Z explains about the importance of practice it is as taken out of the theories of for instance Polya (1971) and Sfard (1991). For instance when Pupil Z says that she is able to do the exercises without having completely understood the overall meaning, but that she after having worked with the exercises, perhaps drawn some pictures and imagined some things, she begins to be able to understand the bigger picture. These keywords of Pupil Z (exercise, looking at drawings, imagine) are quite similar to the keywords of Polya (1971, pp. 4-5) that were practice, observation, and imitation. There are however a little difference in relation to Sfard’s theory. At the interiorization-stage, the pupil gets acquainted with the processes and operations that will lead to concept development, which is what Pupil Z talks about as well. But Pupil Z does not mention the condensation-stage, were the pupil should begin to refer to

the process in terms of input-output relations. Instead Pupil Z jumps directly to the third stage, the reification. However one could perhaps interpret that the second stage is included when the pupil says that the exercises and problems mean that one has worked with the mathematics within oneself and has therefore understood *more*. Doing exercises means that one understands a lot of single cases and one has also seen how things “move around” and it therefore sometimes becomes possible to understand it more overall. This ‘more’ as well as the understanding of all the single-cases could be understood to be the kind of “middle-stage” between the first and third stage.

The self-understanding of Pupil Æ (ID-Æ1) is that she thinks that practice is necessary for learning. When one revises things that a year ago was chaos, it begins to look easier. The reason is that one has been doing problems, calculating. It is like learning how to cycle. In the beginning one tumbles over, then some things begin to become automatic, and in the end one does not think about all these things, they are just in the head and one uses them automatically.

The theoretical understanding could thus be argued to be that exercises and calculating help to create order in the chaos as it puts things in one’s head and make it routine so one does not need to think so much about. This is much in line with Skemp (1993) when he writes about the necessity of automatic manipulation as unless these cannot be done with minimal attention, it is not possible to concentrate successfully on the difficulties.

The self-understanding of Pupil Ø (ID-Ø1) is seen in that she tells that it is necessary to do exercises.

The theoretical understanding could be interpreted to be that exercises are necessary.

Pupil Å’s self-understanding (ID-Å1) is that she finds it good to do exercises to learn, but that one can sometimes also use the mathematics without understanding it.

The theoretical understanding could be argued to be that exercises are important and that one does not always need to understand the mathematics to be able to use it.

Pupil A finds, in his self-understanding (IE1-A1), that it is boring just to do practice-questions/examples over and over again. According to him, one only learns how to apply the

method by doing examples. When Pupil C interrupting says, “you don’t understand it”, Pupil A continues to say that it is helpful in terms of exams. It helps one to be able to work around slightly different forms of the problems, being able to apply the method quickly without making any mistakes, and it is therefore definitely useful in terms of exams. Then follows a discussion where it becomes clear that to Pupil A, understanding where the methods come from is helpful. Pupil A says that it definitely helps him to understand the concept or idea to see how it is applied and be taken through that applied or going through himself before, to get a proper understanding. He thinks it is better when he gets examples and understands each bit before he moves on. Pupil A and his classmate C here talk about exam and they distinguish between what is necessary for passing an exam and what here might be different from real learning.

An interpretation of the theoretical understanding is that practice without reflection is not mathematics. The testimony of Pupil A therefore supports the ideas of Polya, that practice, observation, and imitation are important aspects in learning mathematics (Polya, 1971, pp. 4-5) but not enough in themselves. The discussions also reveal that the first stage described by Sfard (1991) seems to fit. Sfard’s level of ‘interiorization’ explains that it is necessary that a pupil gets acquainted with the processes and operations that will lead to concept development. Pupil A tells that this gives him a proper understanding but whether this means a ‘reification’ is not clear. His explanations might therefore be more in line with Skemp’s (1993, p. 83), who writes that automatic manipulation are necessary and that if it cannot be done with minimal attention, it is not possible to concentrate successfully on the difficulties. But the routine manipulations are not in themselves mathematics, which Pupil A seems to know.

Pupil C finds in his self-understanding (IE1-C1) that hints are better than being told every step as one then has a second try. He explains that if one just got the answer, one still does not understand how to do it. What one needs is a written method of how to do it or a hint to see the method. When attacking any specific problem, he thinks of the general solution, the general formula or equation which he can put it into. He finds similar examples, and tries to fit it into that, in a book or in his notes. He finds practice boring and want to achieve with minimum effort but at the same time he is ambitious and is not satisfied with minimum effort and bad results. He goes for quick reasoning and quick approach instead of going through the

whole thing all the way through. If a question is asking for a detail, he just goes straight into it, instead of going all the way around and prove that the detail is valid. When asked to describe his learning process the first thing that comes to his mind is that it is hard to accept something new unless one knows it is right. The personal tricks have not been told to him, but it was something that seemed natural to do. Pupil C also expresses that if one knows the basic integration, for example, there is no point in going through it for revision. What one needs is to have loads of practice.

The theoretical understanding could therefore be argued to be that he needs to be shown the method, mainly through hints. As a contrast to Pupil A, he goes directly into the detail and he looks for a general algorithm to help him solve the problems. When he talks about learning the mathematics, he explains that he needs to know if something is right, otherwise it is difficult to accept/learn it.

Pupil D states in her self-understanding (IE2-D1) that one has to learn to calculate to be able to apply the mathematics, but it is boring. Pupil D explains that if one has problems with a question one just goes back and look at when one did an example in class, one reads the notes and make sure that one understood it there. Usually she just reads the book and make sure that she understands it. She works through the exercises in the book and if she gets it right, she moves on to the next exercise. She very rarely writes revision notes from the book. She sometimes calculates backwards from the answers in the book to see if she has understood. She explains that most of the times, there will just be one mistake, and she has actually understood it perfectly. Learning though doing exercise is a boring method.

The theoretical understanding could be interpreted to be that practice through exercises is important but it is not clear if this just leads to operationally understanding or also structurally She does not describe a reification in relation to doing the exercises, she is more vague and seems to argue that doing exercises are necessary to learn the mathematics.

Pupil E finds in his self-understanding (IE2-E1) that having some examples, not necessarily diagrams are good.

The theoretical understanding seems to be that practice alone does not lead to learning. Just learning automatic manipulations is not enough.

Pupil F explains in the self-understanding (IE3-F1) the basic learning process as being taught through the principles. One does not need a lot of exercising, but one needs an example of each one and one needs to get it right. He does not think that one can learn without examples as one realises going through examples. He finds that it is more useful to do sets of exam questions as in the exam question, the whole topic area is condensed into one question, and it has a bit of each sort of part. And one gets used to combining the whole lot together rather than just using one specific bit and repeating it over like one is doing in exercise. It is more owing to the whole format rather than it is the fact that the questions are made for the exam. This type of question can test four different chapters in one question and that is a lot more useful than doing each chapter and then forget the chapter and then one goes to the next one. He can see it in his head, which means that he knows how to do it without actually doing the question. One can look at it and determine what technique to use. They learn when the teacher tries to teach them a wider view of mathematics and therefore includes things that are not in the syllabus. The reason is that the process that they have been using is taken one step further so one can see the application, the object process. Pupil F says during the interview that it is good with examples. Explain by given examples.

The theoretical understanding could thus be argued to be that many exercises are not necessary, one is enough. Pupil F is therefore able to generalise from just one example, which characterises high-achieving pupils. Exercises do not make him learn to combine different part of the mathematics. When the teacher tries to teach them a wider view of mathematics the learning process is taken one step further so one can see the application, the object process. He does actually himself use the terminology “object process”. He therefore seems to be able to shift from the ‘interiorization’ to the ‘reification’ very quickly. This is seen in that he says that he only needs one exercise and then he wants to start combining the topic with other.

In relation to practice, Pupil B states in his self-understanding (IE3-B1) that it is not necessary with just one exercise of each type, one needs a few just to get the process into one’s head, but sometimes he finds they get too many exercises, as they are only repeating the same thing. He agrees with Pupil F in that it is more useful to do exam questions where one gets used to the type of questions. Pupil B finds that the learning process in mathematics is different than other subjects as it is learning how to use a technique rather than lists of meanings. The learning process is just practice; have it explained and then practice it to get it over into one’s

head so one can do it easily. He knows that he has understood some mathematics from looking at a question. If he can look at it and see in his head how to tackle it without being confused. Pupil B says that one needs to practice, as one cannot go straight into an exam knowing all the techniques but not having worked through each one several times mechanically. He does not know exactly how they could be explained without using examples to see the technique. But one does not need numbers to, for instance, learning calculus and differentiation, they are still examples when one uses a, b, and c, constants.

The theoretical understanding could be interpreted to be that through practice and exercises one gets operational understanding. It is not enough with one exercise of each type, one needs a few to get the process “into one’s head”. He does not seem here to talk about a ‘reification’ phase. This is for instance seen in that he does not seem to find any “meaning” in learning mathematics, it is only a lot of techniques.

The pupils thus fall into the following groups:

I: Practice several examples {Z, Æ, Ø, Å, D, B}

II: Practice necessary but not sufficient {A, E}

III: Hints better than being shown the whole method {C}

IV: Practice one example {F}

In the discussion of practice the pupils often referred to problem-solving more than actually learning the mathematical concept embedded in the problem. However, as for instance Pupil Z describes, problem-solving/exercising is a way to get in touch with the concepts.

6.1.3 LOOKING BACK, REFLECTING

In this section the only heading is ‘monitoring’. It is furthermore only Pupil Z, Æ, and Ø who says something here. They are all from the Danish interview.

Monitoring

Pupil Z's self-understanding (ID-Z1) is that people are different but that most people have a very clear feeling of when they have understood things. She has not yet met someone who was capable of convincing himself that he had understood a piece of mathematics, which he had not. What she here speaks of is that she thinks that "most" people know when they have learnt something. She does not directly say that people during the learning process know in which direction to go, but somehow they at least seem to know when they have reached the "end-station".

The theoretical understanding could thus be argued to be that Pupil Z does not mention the importance of understanding the problem before starting to work on it, which is a crucial point for Polya (1971, p. 6). On the contrary she directly says that she can solve problems without having a complete understanding of the whole theory. On the other hand it may be possible for the pupil to *understand* the "single-case". Pupil Z furthermore seems to testify that an internal monitor, as mentioned by Mason (1985), exists, but only in the sense that most people have a very clear feeling of when they have understood things, not necessarily how to *reach* this understanding.

Pupil Æ said in her self-understanding (ID-Æ1) that sometimes when one has a problem one does not know in which direction to go, then one tries different directions and perhaps one is lucky.

The theoretical understanding could be interpreted to be that she does not seem to have an internal monitor. But it does not rule out that at "other" times she has one. She does not say that.

Pupil Ø states in her self-understanding (ID-Ø1) that one does not know in which direction to go when one does not understand anything.

Regarding the theoretical understanding, this does not directly speak in favour of an internal monitor but one could argue that she says that to be able to get an internal monitor one must understand the mathematics. She also says that even for high-achieving pupils, it is sometimes difficult to know in which way to go in subjects which they do not really know.

The pupils thus fall into following groups:

I: A kind of internal monitor {Z}

II: Perhaps sometimes an internal monitor {Æ, Ø}

III: Says nothing {Å, A, C, D, E, F, B}

6.1.4 DISCUSSION

I will now discuss the pupils in relation to each other and in relation to a theme.

Planning

Some pupils do not talk about this issue, others more openly denies it. But there are also some pupils who speak in favour of having a plan.

Among the pupils who does only very indirectly talk about plan, is Pupil Æ who says that sometimes one has a problem and one does not know in which direction to go. However, again this ‘sometimes’ does not rule out the possibility that at ‘other times’ she may have a plan and it does also not say anything about what is most common and it furthermore does not say anything about that in cases where one has a plan, the learning is easier. Instead it more testifies to the existence of the internal monitor, and that at least when one does not know where to go, alias do *not* have a plan, the learning becomes *difficult*. But two ‘negations’ do not “neutralise” one another. Here one could perhaps also put Pupil E, as it seems that problem-solving with peers sometimes has the character of being blind, which usually is a characteristic of lower-achieving pupils. However, when he works for himself, he has a clear strategy for how to learn mathematics.

Among the pupils who perhaps more openly denied having a plan, and also denies the benefits of a plan, is Pupil Z who directly says, under the headline “monitoring”, that she can solve problems without having a complete understanding of the whole theory. Here she does not directly talk about a plan but about understanding a theory, but the central claim she makes is that she can solve something without having to consciously having an overview.

This may be connected with her having an internal monitor that at least tells her when she has reached the “end-station”.

Some pupils does not directly call what they do a *plan* but nevertheless they seem to have it. Pupil A is a pupil who says that he does not go through the problems systematically but looks at it first, it seems to be linked to Mason’s (1985, p. 28) and Polya’s (1971, p. 9) ideas that an early phase of problem-solving is to have a plan that is based on past experience and formerly acquired knowledge. Pupil D also seems to benefit from having a plan as she says that she would go through it all and perhaps take another book to learn the basic bits and then once she completely understands, that she would go on to the harder bit. The planning part has to do with using past experience *methods* as well as past experienced *mathematics*.

In summary, the, rather few, pupils who speak about a plan, Pupil A, D, and E, talk about the benefits of experience from previously learnt mathematics and methods. At least when they work on their own. Pupil Z and Æ’s utterances are not clear, and the remaining five pupils do not seem to speak of a plan, neither positively nor negatively.

Motivation

There seem to be various views to the nature of motivations.

Both Pupil Z and Æ mention the cognitive drive as important. Mathematics is here often mentioned as a game, that is fun in itself. They do not “worry” about what mathematics can be used for. This fits with what Pupil Å talks about when she says that it is necessary to take an interest in the mathematics to be able to learn it. Pupil A may also talk about this when he says that he has to be able to accept that something as true, otherwise he cannot move on. Pupil D seems to argue that the cognitive drive means more at their level.

A more “outer” motivation is to be forced into it. Various pupils perceive this differently. Pupil Z seems to take as a positive challenge. For Pupil Æ, the external challenge seems to result in a more passive acceptance of the situation and that she even if she does not think she can do it she goes on, as she *has* to. Also Pupil A seems to talk about this as he says that that it is better if the teachers hint them as it motivates the pupil to give it an extra try as well as it gives the pupil confidence in that he can do it. Pupil D also speaks of the important to be pushed hard, she however says that this is more important when one is younger where

one also needs encouragement and self-confidence. It also seems that to Pupil E, the motivation to some extent comes from outside as Pupil E tells that the teachers have to be enthusiastic and that it is good to be pushed harder as all those things that are not in the textbook but those little extras makes it more interesting. He therefore also speaks of the cognitive drive.

A more “inner” motivation, linked to this, is the motivation from being confused by something. Pupil Z here finds that there is a motivation in the confusion if one really wants to learn it. The confusion is necessary, as one does not learn by just sitting and learning the whole mathematics language/notation first. Also Pupil C seems to fit in here, as he needs motivation, probably other than the cognitive drive, as he cannot motivate himself. He needs to know that it is right to be able to accept it.

Pupil Ø is a bit different here as she does not directly speak of the nature of motivation but more about what can make motivation disappear. Here, lack of motivation can be caused by lack of self-confidence, and that if one tells oneself that one can do it, it is the first step.

In relation to the Vygotsky activity theory perspective about motivation, particularly Group II reflects that the pupils’ goals are not always incorporated in the planning of the education, at least the pupils do not intercept that this might have been the purpose of the teachers. Nevertheless they learn and, for instance, Pupil Z cannot learn if she is not being challenged. Opposite is perhaps Pupil C who cannot motivate himself and finds it difficult to learn something new if he cannot link it to something he already knows.

It also seems that none of the pupils mentions the I-rationale as something important for the learning. Only Pupil Z who talks about a show-off-effect, which I named the “social drive”, could fall into this category.

Almost all pupils mention something about the role of motivation, and it is therefore, to a greater extent than for “planning” possible to get a broad picture about this.

Exercise, practice, and reflection

All the pupils talk about doing exercises as important for the learning process. There are however variations. Pupil F thinks that one is enough; Pupil A and Pupil E think that exercises are important but not alone. One could argue that they may only have referred how

they learn now, in their school system, but this was also what they were suppose to talk about. One could also argue that their past school experience have made them learn to learn through exercises, and even though they have really learnt through exercises, this may not be the most efficient way. On the other hand they actually *do* learn this way, and the interviews illustrates that the pupils are not afraid of saying when they do not learn, neither are they afraid of criticising their teachers.

Monitoring

One might have expected them to do, as it, according to instance Krutetskii (1976) - see Chapter 1, is not of the signs of high-achievers. But only three of the pupils said something in this area. And what they said gives very little evidence of the existence of an internal monitor. Among the pupils who might talk about an internal monitor is Pupil Z who says that most people have a very clear feeling of when they have understood things. Here, she does not directly say that the internal monitor exists but at least it seems that ‘most people’ knows when they are at the “end-station”.

Among the pupils who seems to directly say that she does not have an internal monitor, at least not in all cases is Pupil Æ who says that *sometimes* when one has a problem one does not know in which direction to go. The ‘sometimes’ opens up to an interpretation that at ‘other times’ the internal monitor may be there, but nevertheless the evidence is much weaker than seen in for instance Pupil Z. Also Pupil Ø seems to fall in this group when she says that one does not know in which direction to go when one does not understand anything. One could argue that she says that if one needs to know in which direction to go - one must understand it.

Theme 1 as a whole

For Theme 1 as a whole one can thus say, based on what the pupils’ positively state, that the importance of having a planning phase prior to the learning is ambiguous, however, there seem to be a slight tendency among the pupils to support the idea of a planning phase,

particularly when working alone. Motivation is important, but it takes many different shapes. Some kinds of motivation “works” for some pupils, but might be destructive for the learning of others. Exercises and practice are however regarded by all as being very important, although there is not agreement about how it should be practiced. Not many pupils talk about the existence of an internal monitor, and among those who do, not all talk positively about it. Going back to Chapter 1 and what Krutetskii said about what characterises high-achieving pupils, it might be surprising that some of the pupils here say that the internal monitor does not exist and that many examples are necessary. The pupils do from time to time give examples from their daily life on what kind of mathematics they work with. Differential equations, trigonometry, calculations, and the knot theory have all been mentioned. The way they mention these branches of mathematics is too unsystematic to be able to generalise, but it nevertheless shows that the pupils can relate the rather abstract discussion in the interview with their daily work on mathematics and that it makes sense to them to draw on their experience. One could then ask which of the various areas mentioned in the first theme is most important. Judging from the amount of things said, it is clearly practice and motivation. These two seem to fit each other, to some extent, as the pupils explain that “outer motivation” is helpful and they tell that practice is important to learn, which shows that they put high priority on practice themselves. Different kinds of motivation seem to be what “drives” them through, sometimes boring, exercises. Thus motivation and practice seem to be complementary forces.

6.1.5 THE KEYWORDS FOR THEME 1

Z: exercise, cognitive drive, affective drive, internal monitor, external pressure.

Æ: exercise, automatic manipulation, cognitive drive, force, no internal monitor, no plan.

Ø: exercise, self-confidence, no monitoring.

Å: exercise, cognitive drive.

A: exercise but not alone, reflection, automatic manipulation but not alone, cognitive drive
but not alone, needs encouragement/self-confidence, plan based on past experience,
needs to understand that it is true.

C: needs to know it is right, exercise.

D: exercise, cognitive drive, force, planning using past experienced methods.

E: exercise but not alone, planning chaotic, first big picture, cognitive drive, force, no automatic manipulation.

F: exercise but one is enough, learn more than syllabus makes one see the application/object process.

B: exercise.

6.2 UNCONSCIOUSNESS

Eight of the pupils talk about the role of the unconsciousness. Two of them talk about it in more general terms. One is Pupil D, who directly says that a lot of the learning process is hiding, it is subconscious (IE2-D2). Also Pupil C says that his personal tricks not something he consciously works out, but it is something he happens to do and it works. He furthermore finds that different things work for different people (IE1-C2).

I have here grouped the statements according to which of Hadamard's phases of the unconscious work they mention.

Preparatory work, incubation and illumination

The pupils Z, Ø, Å, A, and E seem to talk about all three phases, namely the preparatory work, incubation, and the illumination.

Pupil Ø's self-understanding (ID-Ø2) is seen when she explains that when one re-reads something one can get an aha experience. One can also have degrees of understanding but if one has not understood all the things that are necessary to understand, one can still have an aha experience.

The level of theoretical understanding could therefore be argued to be that she talks about the incubation phase between the first time one reads something and the second, and after that illumination follows. The need for preparatory work is seen in that the pupils talk about what can happen when one *rereads* something.

Pupil Å's self-understanding (ID-Å2) is seen when she says that if one sit with a problem that one cannot solve, then if one is a bit away from it, one perhaps suddenly notices something which one did not see before.

The theoretical understanding of her work does therefore seem similar to that of Pupil Ø, as Pupil Å also mentions the incubation and illumination phase after a phase of problem-solving (preparatory work) where she had experienced problems. Her explanation does, however, more clearly show the three steps than Pupil Ø as she directly talks about being away from the problem.

The self-understanding of Pupil Z (ID-Z2) is that it is an advantage that some time passes if one has difficulties understanding something. Review has the effect that subjects one has only understood half during the year, but where one has still been able to solve exercises, one can now understand the mathematics and see the greater whole. The aha-experience is necessary for learning. The aha experience can sometimes come as a result of re-reading a topic after a break, and then one suddenly understands it. The reason is that one then has got the basic things right and one is able to see the point behind. It goes from not being there at all, to suddenly being there. She says that she can sit one day and feel that she has suddenly understood for instance spatial geometry, and then she reads it through and suddenly, bang, it is there, it is a kind of revelation, and she has understood it. She thinks that if she has only understood a little of it, then she has not understood it at all. There is no semidarkness in between. This view may be connected with what she says above, in Theme 1, namely that it is not possible to convince oneself that one has understood something, which one has not understood, as understanding to her has to do with a strong revelation.

The theoretical understanding seems to be that after preparatory work and an incubation phase illumination comes. The illumination is a necessary stage in getting to understand something, without it there is no understanding.

Pupil A's self-understanding (IE1-A2) is that sometimes when he cannot solve a problem, even after spending ages thinking about it, he begins to be frustrated and then cannot think clearly. But if he goes away and does something completely different, he experiences that he opens himself up to the problem and then thinks about it without trying hard to solve it. This

explanation is even more detailed than Pupil A's, which makes it possible with a wider theoretical understanding.

In the theoretical understanding, illumination seems to follow after relaxation (incubation) and some preparatory work. It furthermore seems that what Pupil A experiences is more in line with the 'forgetting-hypothesis' than the 'rest-hypothesis' as the incubation phase gets rid of false leads and makes it possible to approach the problem with an open mind (Hadamard, 1945, p. 33).

The self-understanding of Pupil E (IE2-E2) is seen when he explains that if one experiences problems with especially long calculations or algebra, one can sometimes look at it for hours and not see the little mistake one has made, and then someone else looks at it and sees it straight away. One then sees it with a fresh mind. One can also look at it with a fresh mind oneself if one goes away and come back. The pupil also explains that sometimes when one feels frustrated and do not understand, suddenly one gets it. This comes from having done something oneself, worked it out.

The theoretical understanding could therefore be argued to be that illumination can follow after some preparatory work if this work is interrupted in an incubation phase. It furthermore seems that Pupil E, contrary to Pupil A, is more in favour of the 'rest-hypothesis' than the 'forgetting-hypothesis' when he talks about the fresh mind.

It therefore seems that all five of them in more or less detailed manner talks about Hadamard's three phases. Furthermore the description of Pupil A and E are detailed enough to determine which kind of incubation they mean, the rest-hypothesis, or the forgetting-hypothesis. Pupil A seems to talk about something that sounds like the forgetting-hypothesis, Pupil E the rest-hypothesis. One could therefore argue that even though these five pupils agree in this theme, they are not completely alike either. Furthermore it is also clear that the five pupils consists of three Danish girls and two English boys, the latter two are from two different interviews.

Preparatory work and incubation

The self-understanding of Pupil C (IE1-C2) is that he thinks it is good to read things again as a word and a phrase can have more than one meaning. If, for instance, one has the completely wrong meaning and one then extends one's ideas based on this, then one can see a clear difference when one reads back.

The theoretical understanding seems to be that he talks about preparatory work as well as an incubation phase. The incubation phase is seen in that he talks about reading something again and/read back. He does not seem to talk about illuminations. One could also argue that the incubation phase he talks about is not the "usual" incubation phase, as "reading it again" does not necessarily mean that there is a (lot of) time span in between. Pupil C seems only to be talking about the first two steps of the process described by Hadamard, namely the preparatory work and the incubation phase. One could however argue that even though he does not explicitly mention the illuminations, I would argue that he must implicitly mean that the preparatory work and the incubation lead to learning, otherwise he would not have mentioned it. But the reason why I have chosen not to mention him above is that the learning, which I estimate he implicitly thinks of, is not mentioned as an *illumination*.

Preparatory work and illumination

One could argue that it is problematic to determine if a pupil talks about the preparatory phase if this pupil does not mention the two other phases. What is distinctive for the preparatory phase is that a pupil experience to be working hard on something and then still does not understand. This is exactly how Pupil D expresses her self-understanding (IE2-D2) and she further says that sometimes when one has been concentrated hard on something, she does not spot it. Pupil D furthermore stresses that sometimes when one is frustrated and one does not understand it, suddenly one gets it, and then there is light. It is as two wires that gather and a spark come. Writing things down is much more of a conscious effort, it is trying to understand. When one understands subconsciously it is like an immediate shedding of light on the problem, when she writes something down, that is much more consciously working it through in one's mind.

The theoretical understanding could therefore be argued to be that illumination can follow after some preparatory work, but she does not directly talk about an incubation phase, instead that something “suddenly” worked.

Incubation and illumination

The self-understanding of Pupil Æ (ID-Æ2) is that when she meets some new mathematics that she does not understand, she first and foremost tries to keep a distance to it and then perhaps later returns to it. She says that she can sometimes look at it with different eyes and understand it better. Pupil Æ also describes that to her, sometimes there is suddenly something that falls in place. One has a lot of bricks in one’s head that suddenly start to fit. She explains that it feels like there continuously is being built on this, like rings in the water, who spread more and more and suddenly more things are being put together, chained together.

The theoretical understanding is therefore interpreted to be that when she meets problems understanding she leaves it (incubates) and later returns and then she sees it with fresh eyes and illuminations can happen. She does however not talk about a preparatory phase. One could, however, argue that this is what she says she does when she does not understand the mathematics. This could point to that there has been a preparatory phase; how else could she know that she does not understand it. On the other hand, Hadamard’s idea behind talking about a preparatory phase is that this phase consists of very hard work and Pupil Æ does not say anything that would imply that she had been working *hard*. Writing that this is what she means, might be pressing her words too much.

6.2.1 DISCUSSION

Here I will discuss partly what the pupils said which related to Hadamard’s (1945) phases and partly discuss what the pupils say more directly about the role of the unconsciousness.

Preparatory, incubation, illumination

All pupils, except Pupil F and B, mention something within the second theme. Pupil Z, Ø, Å, A, and E talked about what may be interpreted as all Hadamard's (1945) three phases: the preparatory work, the incubation, and the illumination. The three remaining pupils mention only two of the phases, but it is not the same phases they mention. They represent each their own combination. Pupil Æ talks about the illumination phase and the incubation prior to this. She does not talk about a preparatory phase. Pupil C talks about preparatory work as well as an incubation phase but not about an unconscious illumination. Instead he talks about that one happens to do something and then it works. Pupil D says that sometimes when one has been concentrated hard on something, one does not spot it, but suddenly one gets it, and then there is light. Illumination can therefore follow after some preparatory work, but she does not talk about an incubation phase. One can therefore form the following groups:

I: Phase 1, 2, and 3 {Z, Ø, Å, A, E}

II: Phase 1 and 2 {C}

III: Phase 1 and 3 {D}

IV: Phase 2 and 3 {Æ}

V: Says nothing {F, B}.

One could argue that Pupil C could be put in under the heading of Group I even though he does not use a word similar to illumination he still explains that “you see a clear difference”. One might argue that it is a matter of degree to which this expression might denote another way of talking about illumination. It could be owing to his personal temper. However, if one wants to take the pupil's explanation for what he actually says, he does not talk about an illumination, it more sounds as if he *himself* was able to work something out after reading something a second time. The conclusion of this discussion is therefore that Pupil C does not talk about illumination precisely, but he does argue for the benefit of preparatory work and some kind of incubation phase for better understanding of something. I will therefore let Pupil C remain in a separate group. However, this is an example of the difference between using theory to enlighten empirical data or to use data to test theory. One could reframe it to “which

stands above the other” in case of some “conflict”. In the case of Pupil Æ, C, and D I have chosen to not let theory stand “above” the self-understanding of the pupils.

Pupil Ø says that one can have degrees of understanding, which is different from what Pupil Z said. To Pupil Z, it is either-or: complete understanding, or none. The aha experience is linked to the complete understanding. Instead Pupil Ø talks about “steps”/degrees of knowledge.

In general, it seems that the pupils are very articulate about this phase. They use their own words but to a large extent these words are easily translatable into Hadamard’s concepts.

Unconsciousness

In more general terms about the role of the unconsciousness, Pupil C explains that the personal tricks are not something he consciously works out, but it is something he happens to do and it works. Pupil D also says that a lot of the learning process is hiding, it is subconscious.

The pupils thus fall into the following two groups:

I: The unconsciousness plays a role {C, D}

II: Say nothing: {Z, Æ, Ø, Å, A, E, F, B}.

6.2.2 THE KEYWORDS FOR THEME 2

Z: preparatory work, incubation, illumination.

Æ: incubation, illumination.

Ø: preparatory work, incubation, illumination.

Å: preparatory work, incubation, illumination.

A: preparatory work, incubation, illumination.

C: preparatory work, incubation.

D: preparatory work, illumination.

E: preparatory work, incubation, illumination.

F: nothing.

B: nothing.

6.3 THE FIRST PAIR OF THEMES SEEN TOGETHER

More than half the pupils talk, in my view, in favour of an unconscious phase quite similar to the one Hadamard (1945) describes. All pupils also talk in favour of for instance practice, which is a conscious effort. Following the discussions in Chapter 3, the consciousness and the unconsciousness “needs one another”. In the semi-structured interviews of this study, the author’s purpose was to see what came up, and not to be leading, and it seems that the self-understanding of the pupils is that the consciousness and the unconsciousness both are important factors in learning. The pupils do not directly link them together but they nevertheless positively talk about both issues. This is an example of the “islands” mentioned in Chapter 2; i.e.: that even though the pupils know some essential features of their learning process and are able to articulate these, they may still not provide a *complete* picture. Existing theory (which is other researchers’ results of *inter alia* interviewing) can “fill the holes” if what the pupils positively say is “enough alike” with what the theories say. When statements are “enough alike” is not always easy to determine as the discussion in Section 6.2 shows. Also the fact that the pupils use their own words makes it possible to read too much into what they say. However, a more controlled interview would give a result that is more self-fulfilling. In Chapter 4, I discuss the possibility of a synthesis among various different theories. As Theme 1 and 2 themselves seem to draw on each other, the discussion, one might perceive Theme 1 and 2 as being more “advanced” as they have already “figured out” that “odd complementarity” among various different themes is necessary; here with conscious preparatory work as the basis.

Illiteracy in the developing world has fallen from about 75 percent for the people born in the early part of the 1900s to below 20 percent among the young today. (Lomborg, 2001, p. 81)

7. SECOND PAIR OF THEMES: THE LANGUAGE AND THE TACIT

This chapter will discuss what the pupils said which fell within the third and fourth themes. I will generally follow the structure of the previous chapter.

7.1 THE LANGUAGE

In this section I will discuss issues of language as a thinking-tool, which is in the Vygotskian tradition, and issues of the importance of the basic in concept formation as well as schematic learning which is more inspired by Piaget.

7.1.1 LANGUAGE AS A THINKING-TOOL

Pupil Æ, Å, and C are the only pupils who do not seem to talk about this area.

Pupil Z experience in her self-understanding (ID-Z3) that it is difficult to find the meaning without the language and it is important to have some terms and have them worked through. For instance in trigonometry it is very important to line up the concepts one works with to be clear about what the problem is about.

The theoretical understanding is therefore interpreted to be that language can function as a thinking-tool. It is difficult to find the meaning outside the language. The pupil here explains that it is *difficult* to learn outside the language, but she does, however, not say that it is *impossible*. As an example she mentions trigonometry.

Pupil Ø says in her self-understanding (ID-Ø3) that there can be many confusing things when one learns and one gets stuck in details that actually do not have any significance for understanding the issue. But she finds that it is difficult to see how to avoid this, as they have to be introduced to the new notation. It is difficult to find the meaning without the language. Language is therefore essential for learning.

The theoretical understanding is argued to be that language is a basic thinking-tool. In this respect Pupil Ø is very similar to Pupil Z. Furthermore both seem to state that it is *difficult* to learn

Pupil A says in his self-understanding (IE1-A3) that writing is an aid to memory. When Pupil A is learning something new, he tries to put it into a language, into terms, that he is “happy with” and understands. Then he tries to find out if he can apply that back to the more complicated explanation he was first given. If he can do that, he feels that he has understood.

One could, in the theoretical understanding, argue that to Pupil A the (written) language is a sort of thinking-tool. When he says that he tries to put it into a language he is happy with, this might be in line with the more positive view of language for instance mentioned in Section 3.3.1. This does not mean that he supports the Vygotskian view about language as a logical and analytical thinking-tool (Vygotsky, 1962, p. viii) and that thoughts are not just merely expressed in words but come into existence through the words (Vygotsky, 1962, p. 125). Pupil A instead uses the language as a kind of translation tool. Furthermore, in the discussion about if words come first or not, Pupil A says that he is not sure, but it probably depends on the problem. If it is a visual problem where one has to think it through maybe in 3D, he finds that it is probably better to have the picture first and maybe graphs as well. But with a linear algebra problem, it might be better to have the words first and then the pictures to help one understand, cause it is the words one is trying to understand. This means that although words and language has a great value they are by no means *the* way to get to understand all branches of mathematics.

Pupil D says in her self-understanding (IE2-D3) that she thought that reading one of the other books in the intersection helped her to understand the knot theory as it explained things in a slightly different way and it is a different approach and they sort of strengthen each other and

help one to understand. It therefore seems that different language helps one to understand if one explanation is difficult.

In the theoretical understanding language seems to be a thinking-tool and not just a means of translation. Language is a necessary tool for learning and she seems to argue that without the different explanations, she would not have understood the knot theory. Thus language is more important as a thinking-tool for Pupil D than for Pupil Z, Ø, and A.

Pupil E says in his self-understanding (IE2-E3) that it is not possible to learn mathematics without the use of language, but he finds that one can use different language, simple language to convey a point.

The theoretical understanding could be interpreted to be that language is indispensable for learning mathematics, but it has to be the right kind of language. This has similarities with what Pupil D says as Pupil E also states that it is an advantage to have things explained in another way. He is, however, more specific, as he does not just talk about *other* languages as being important, but *simple* languages.

Pupil F expresses during the intersection (IE3-F8) that the terminology is essential.

The theoretical understanding is argued to be, following the observations in the intersection, it is seen that the terminology is basics; he therefore expresses the importance of language as a thinking-tool for learning the knot theory.

According to his self-understanding, it is good for Pupil B (IE3-B3) if the mathematics is made simple, for instance in a simpler diagram, as one then understands it more quickly. The examples are explained using the mathematics terminology, but then he says that if they do not understand this terminology they have no hope of understanding the examples. So the terminology is the most important. Pupil B said during the intersection that to get a further understanding, one has to look what the terms mean. It is better with more simple ideas particularly if they are explained slowly and in an understandable language. In relation to the mathematics in the intersection, he would look for some more books with a more understandable summary, which maybe went through it a bit slower and let one into it through more understandable terms. It is important to define the terminology and know what the notation means and find something that will use basic mathematics notation about knots.

In the theoretical understanding it seems that the words are the major thinking-tool as without them one cannot understand the examples. To Pupil B, language is the learning-tool; particularly simple language. He is much in line with Pupil E when he talks about the need of simple language.

The pupils therefore fall into the following groups:

I: Difficult to learn outside language {Z, Ø, A}

II: Language is the thinking-tool {D, E, F, B}

III: Say nothing {Æ, Å, C}

The main difference between the two groups is thus whether learning is only “difficult” outside language or if it is “impossible” without language. There is also a discussion about what types of language might be most useful to facilitate learning. It also seems that language is not equally important for all branches of mathematics, but mainly useful for more algebraic expressions.

7.1.2 BASICS AND SCHEMATIC LEARNING IN CONCEPT FORMATION

Pupil F is the only one who does not seem to utter anything here.

According to Pupil Z (ID-Z3), the *best* learning takes place when one is allowed to have a slow learning where one builds up the things in the basic steps. It is easier, and one feels that one has a platform if one is allowed to begin from the bottom instead of having notions, which have not yet been explained. Learning sometimes means that one moves up step by step. This means that if, during a class presentation, there at some point is something which one does not understand, then one does not understand the rest of it. Then one has to stop and start from the beginning and take the whole way up again. Understanding means knowing where something came from. In line with the above, the best style of a learning process is a step-by-step process where one understands the steps “behind”.

The theoretical understanding is thus argued to be that knowing the basics and having a systematic process in a kind of schematic learning are the best and easiest way of learning. This suggests that there seem to be various ways of learning. Knowing the notation is also important; otherwise it is difficult to see the sense of the mathematics. It takes time to learn the notation.

Pupil Æ (ID-Æ3) states that there are several degrees of understanding.

The theoretical understanding could thus be interpreted to be that she probably talks about schematic learning when she states that there are several degrees of understanding but it is not certain as she actually does not talk about the *process* of learning but rather that one seem to be able to have learnt/understood something in different “degree”.

Pupil Ø states (ID-Ø3) that it is almost hopeless to learn when one does not have the foundation to understand where it is one is going. She furthermore says that she finds that it is necessary with the foundation, but her experience in class is that the teacher begins halfway. The aha experience to her is to have got the bricks underneath.

The theoretical understanding is thus argued to be that it is difficult to find the meaning outside the language. Without knowing the basics it is very difficult to learn what is above, schematic learning. Pupil Ø is thus very much in line with Pupil Z.

Pupil Å (ID-Å3) talks about rote-learning and she says that it bothers her a lot if there is something she does not understand but which she has to use to calculate something. This is different from not knowing how an engine in a car works, as she does not have a wish to understand how this works. It is important to know the basics.

In the theoretical understanding, Pupil Å seems to talk against rote-learning when she says that it is very annoying to have to use something without having understood it. On the other hand she does not say that she does *not learn* this way, only that it is annoying. In Theme 1 she seemed to argue that doing exercises are important. An interpretation of these utterances is that Pupil Å finds that one learns through exercises and rote-learning, but it is an annoying/hard way. It is furthermore important to know the basics. Her explanation is, however, not detailed enough to determine if she also talks about schematic learning.

Pupil A (IE1-A3) finds that mathematics has more to do with understanding a method than memorizing. About learning techniques he says that when he is revising/memorising from his notes he finds that it helps if he writes the notes out again; copies them. The reason for that is that even if one reads aloud, one only remembers some things, but if one reads it again and then writes it again, it reinforces itself. This method works if he needs to memorize for instance examples, methods, equations, and how things work. Pupil A finds that it is quite easy to learn for instance a general formula for a lot of different things like, for instance trigonometry. But if he then comes across a slightly different problem and if he has only memorized it, he does not understand how it got there, and therefore finds that he has no chance to work back. However, if he understands it he can see where it comes from and where he needs to change it to fit.

In the theoretical understanding, what the pupil talks about seems to be much in line with Skemp's ideas that rote-memorising is meaningless and an integrated conceptual structure is easier to remember than unconnected rules (Skemp, 1993, pp. 29-30). Routine manipulations do not lead to structural understanding without convincing knowledge (proof) of where the processes come from. Pupil A tries to fit the new knowledge into an existing schema, but new knowledge that is different from previous knowledge requires a change of the cognitive structure.

Pupil C (IE1-C3) finds that it is also important to have a good basis in mathematics; otherwise it is quite hard to learn mathematics. When he was asked to describe his learning process of mathematics, what actually goes on in his head, he said that he thinks of previous experience or perhaps goes back to what he knows and then fit the new thing to what he already knows. He states that if he can do it, it is easy, as it is just addition. If it is completely new it is more difficult, unless one is taught it so it is easy to understand. He cannot learn from a book. Pupil C gives an example with integration. If one can link it to something one already knows like $n+1$, then he can fit that into integration. Then it is easy, but if one just has an example, such as $2x$ integrates to x square, then it is quite hard to understand how this happened. When they were asked if they knew other tricks, Pupil C, by himself came up with the distinction: "remembering or learning". He then explained, that to memorise something, one needs to find some sort of sequence in order to remember it. But if one is "just" learning, then he finds this has to do with 'understanding' and one does not need to remember the detail. One just needs

to know the overall concept, the principles. This suggests that it seems to be easier to learn something than just to remember it.

In the theoretical understanding Pupil C seems to talk both about the difference between assimilation and accommodation when he states that he fits new things to what he already knows and if it is easy, as it is just addition. If it is completely new it is more difficult. He does also seem to talk about schematic learning, and that knowing the basics are important. Thus, one can argue that Pupil C tells that if some new knowledge fits his existing schema is then easily assimilated, but some new knowledge is different and then requires a change of the cognitive structure. His memory is selective and only remembers general structures, and it is easier to remember cognitive structures than rote memorising. It is important that basic concepts are learnt. Pupil C is thus much in line with Pupil Z and Ø.

According to Pupil D's self-understanding (IE2-D3), she turned to the book because she was interested in what kind of questions one might ask from the knot theory. She would look at it and then pick up the most important bit that one needs from it. That is the way she states that she works if works for an exam. She would do the same even if she was not suppose to learn this for an exam, because one has to understand the basics and then one would still want to know how one would apply this knowledge. If one knows *how* to do it, one has just learnt the method and one just changes the numbers. But one basically recognises it because one has done the problem lots of times before. Understanding *why* one does it, is different. There is also a big difference between learning *how* to do something and learning *why*. Some questions may be easier if one goes to a level that is more advanced than one actually needs and in the exam it is easier cause one can go beyond it.

In the theoretical understanding, one can argue that it is important to learn the basic concepts. This is actually Pupil D's own words. Her explanations of learning 'how' to do things sounds a lot like rote-learning, whereas learning *why*, seems to be schematic learning as she talks about 'going beyond' a stage. She is thus in line with Pupil Z, Ø, and C.

In his self-understanding, Pupil E (IE2-E3) would personally rather understand the mathematics and not care about what one is suppose to do with it. Pupil E finds that one cannot apply the mathematics unless one understands the theory behind it and the basics. The pupil explains that people in his class get frustrated if they do not know why one does

something, whereas he thinks that a lot of mathematics pupils would be happy to know just how to solve the problem and get the right answer. He actually finds that it is easier in an exam if one knows *why* one does something rather than just know *how* to do it, because if one is stuck on a question and one only knows how to do it, then one is completely stuck, whereas if one knows the principles, right from the basics one can go straight from the beginning and all the way through to the end. It may take longer, but at least one can work through it logically and one knows why one is doing it, and eventually one will get the right answer or at least one can make some progress and get some marks.

As a metaphor for explaining these things he says that the difference between learning how to do something and learning why is the building of a house. One can be taught how to build a house that will not fall down, but the teachers will not tell why one needs pillars here and there to keep it standing up. So if one is stuck by oneself trying to build a house, and one does not know why one was doing particular things, one would not be able to overcome problems, one just knows how to build a basic house. He also compares it with learning complex numbers and solving quadratic equations, which was quite simple the first time they did it, as they did not need to learn it, as there were other ways of getting around it. Learning is better when one breaks it down in a sentence at a time and understands it and then move on.

He does not have any problems understanding the knot theory, but he thinks that if he was given a question, especially going straight into notation rather than theory, then he would probably struggle first, probably need some help. This suggests that the learning process is in steps. Even though he talks about needing some help, which would be Theme 6, he only says that this would *probably* help. This seems to some extent to contradict what Pupil E says in Theme 1 as he then talks about the need to begin with having a general overview. On the other hand one could argue that what he says here in Theme 3 is an explanation of how he would get the first general overview, as he clearly says that once he has read it, he has got a *basic* idea of it. The pupil says that it helps when the teachers explains using the blackboard as it is logical, going through things from the first principles, it is something one rarely does when one is solving a problem. As an example of this, Pupil E mentions differential equations, particularly auxiliary equations and particular integrals, where they have just got the method for doing it. So one just writes it out and takes the integral, one does not go through the two pages of work to get it, cause one has got the rules and one has got the things in one's head.

Some of the discussions in the theoretical understanding has already taken place above. Besides these, Pupil E seems to talk about accommodation of new knowledge when he says that he logically reorganises it. One could also argue that he needs to have a schematic understanding, rote-learning is not enough. One needs to know the basics. He is much in line with Pupil Z, Ø, C, and D.

Pupil B (IE3-B3) finds that it is good when the modules are split in different areas of mathematics. It keeps them separate so one does not get too confused. It keeps one general topic area distinct to the others and one gets examined in each of them straight away, and one can put it behind oneself. Pupil B here talks about different parts of mathematics as disjoint elements but this does not rule out that *within* each area, there may still be systematic understanding or that the various elements of mathematics could form a hierarchy.

The theoretical understanding is thus interpreted to be that he also talks about schematic understanding when he finds it good when the modules are split in different areas of mathematics. He does, however, not talk about knowing the basics, unless one can deduce it from what he says about keeping things separate. Owing to this uncertainty in the interpretation of the self-understanding, Pupil B is put in the same group as Pupil Z, Ø, C, D, and E, but mentioned as “B”.

The pupils do therefore fall into following groups:

- I: Best learning is schematic and knowing the basics is important {Z, Ø, C, D, E, “B”}
- II: One can learn something in different degree {Æ}
- III: Important to know the basics {Å}
- IV: Rote-learning is an annoying way to learn {Ä, A, C}
- V: Schematic learning, easier to remember {A}
- VI: Say nothing: {F}.

7.1.3 DISCUSSION

Below I will discuss language as a thinking-tool and basic and schematic learning.

Language as thinking-tool

Pupil Æ says nothing directly about this.

Pupil Z and Pupil Ø find that knowing the notation is essential to be able to learn the mathematics. Language can function as a thinking-tool and it is difficult to find the meaning outside the language. They do not, however, say that it is impossible to learn outside of language. Pupil A seems to be quite similar in his *moderate positive* view of language as a thinking-tool. The pupil uses language as a translation tool, and he also emphasise that it depends on the branch of mathematics whether the language “comes first”, i.e.: is indispensable.

Pupil D, E, F, and B have a *strong positive* view of the role of language. Language is a thinking-tool and not just a means of translation, but language is a necessary tool for learning. Different languages are necessary for learning. Some pupils argue that it is simple language that is the best.

Basics and schematic learning

Pupil F says nothing regarding this area within Theme 3.

Pupil Z, Ø, and Å agree that learning basics are very important for the learning process. Pupil Ø actually says that it is almost hopeless to learn if one does not have the foundation. Pupil Z, Æ, and Ø also seem to agree in relation to learning as schematic learning, but here Pupil Z and Pupil Ø are quite explicit about this while one can only interpret that this is what Pupil Æ talks about when she states that there are several degrees of understanding and that it seem to be possible to have learnt something in different “degree”.

Thus, most of the pupils can actually, more or less elaborately, be interpreted to support the idea of schematic learning and that the basics are important. A few vary a bit around these

issues. The pupils do also mention several branches of mathematics where an approach like this would work; for instance differentiation, integration, trigonometry, complex numbers, and the knot theory. Thus, representative from most branches of mathematics, at least the ones the pupils would have met by the time of the interviews.

Not rote-learning

Pupil Å tells that rote-learning is a boring way to learn, but nevertheless a way to learn. But this does not rule out that there might be other ways to learn. Pupil A talks about that rote-memorising is meaningless and an integrated conceptual structure is easier to remember than unconnected rules. Routine manipulations do not lead to structurally understanding without convincing knowledge (proof) of where the processes come from. Pupil C said that it is easier to remember cognitive structures than rote memorising.

One could then discuss the relationship between rote-learning and exercising. They have similarities in the way that others determine the activity. On the other hand they are also a bit different as ‘rote-learning’ could be considered to be a more general and broad term, while ‘exercising’ is means solving standard problems using a specific algorithm. It could be interested, though, to compare what Pupil Å, A, and C says about exercising with what they say about rote-learning. To recapitulate from Section 6.1: Pupil Å states that it is good to do exercises to learn, but that one can sometimes also use the mathematics without understanding it. What she meant by “good” in Section 6.1 might therefore not mean “interesting” but rather “efficient”. But the statements do not seem to contradict one another as both emphasise that rote-learning/exercising is *a* way to learn, but perhaps not the best. Pupil A said in Section 6.1 that it is boring just to do practice-questions/examples over and over again. According to him, one only learns how to apply the method by doing examples. Automatic manipulations are necessary and that if it cannot be done with minimal attention, it is not possible to concentrate successfully on the difficulties. But the routine manipulations are not in themselves mathematics. This is much similar to what he says in Theme 3 about routine manipulations. Pupil C states in Section 6.1 that he finds practice boring but what one needs is to have loads of practice. To him, his learning process is that it is hard to accept something new unless he knows it is right. This seems a bit different from what he seems to express within Theme 3,

namely that it is easier to remember cognitive structures than rote memorising. One possibility is that he is not sure of how he learns. Another option is that he thinks that it is necessary to begin the learning with lots of practice, but the mathematics does not become easy to remember until one has build up a cognitive structure.

7.1.4 THE KEYWORDS FOR THEME 3

Z: understand basics, systematic slow process, schematic learning from basics is best, language as thinking-tool, difficult to find meaning outside language, ping-pong between language and seeing it.

Æ: schematic understanding.

Ø: language as thinking-tool, difficult to find meaning outside language, basics important, schematic learning.

Å: no rote-learning, basics.

A: schematic learning, no rote-learning, language as thinking-tool but not alone.

C: assimilation, schematic learning, basics.

D: basics, rote-learning, schematic learning (better than rote-learning), language is a thinking-tool.

E: accommodate, schematic learning, no rote-learning, language as thinking-tool but it depends on the kind of language.

F: language as thinking-tool.

B: language the major thinking-tool but it depends on the language, schematic understanding.

7.2 THE TACIT

7.2.1 LANGUAGE HAMPERS LEARNING

Pupil Z, Ø, D, E, F, and B says something around this issue.

The self-understanding of Pupil Z (ID-Z4) is as follows: In Section 7.1 Pupil Z expressed that it is difficult (but she did not say impossible) to learn without the language, and as an example she mentions spatial geometry and functions. She explains this further by saying that if one knows the notation, it is simply logical. According to her, one can perhaps learn mathematics even if one has not understood the notation, but it is difficult to learn it from someone where one begins on different levels. But if one knows the notation it becomes easier as one learns it from someone who knows the mathematics. The notation is what seems relevant when one does not understand what it is that is really relevant as one becomes stocked with what the things are called, the mathematics language, instead of what it means. Learning is ping-pong between seeing it and the language. She seems to argue that language/notations are necessary tools in learning, but that the language/notations in themselves are not mathematics and that an unsuitable use of language can block learning.

The theoretical understanding, besides what is being discussed above, can be interpreted to be that notions can hamper learning if one does not know them, especially if one does not know them in the beginning of the learning process. It may be possible to learn mathematics even if one has not understood the notation. It is ping-pong between seeing it and the language so the language has some use but not all. This is in line with Piaget (1970) when he states that the roots of abstract mathematical thinking are not found in language alone. This seems to some extent to contradict what Pupil Z says in Theme 3 where she states that one cannot find meaning without language. But it could also reflect that the (right) languages are *necessary* but not *sufficient*.

The self-understanding of Pupil Ø (ID-Ø4) is that language can also confuse the meaning.

The theoretical understanding can be interpreted as that notions can hamper learning. As Pupil Ø does not say much in this context it is difficult to determine how much she agrees with Pupil Z above. But in Theme 3 Pupil Ø expressed that it was difficult, hence perhaps not impossible, to learn outside of language, which could go hand-in-hand with her explanation here. Thus she consistently has a rather negative view of the effect of language for learning. Pupil Ø is therefore different from Pupil Z.

The self-understanding of Pupil D (IE2-D4) is as follows: About the knot theory in the intersection she says that it was very complicated language. This could scare them off. She

says that the language is always an obstruction for learning if one does not understand it. It is frustrating cause one thinks that the mathematics is probably quite simple. When they discussed the knot theory using their own words it was fine. She further tells that one can learn the knot theory without language, cause the mathematics in it is quite easy, what it is saying is what a knot is, what a link is. It took her a long time to work out what they were trying to tell, but as soon as she translated it, it was easy. One has to do the two together; one has to translate while one is trying to understand. To her, what is important in mathematics is not to know the most complicated way to explain something, it is to know something.

The theoretical understanding can therefore be interpreted to be that words can obstruct thinking, but is a necessary translation-tool. The language is always an obstruction for learning if one does not understand it. One can learn mathematics without language, at least when it is quite easy. One has to do the two together; one has to translate while one is trying to understand. What Pupil D says is thus quite similar to what Pupil Z said above, but it is different from Section 7.1 where Pupil Z and D were placed in two different groups, as language was more important as a thinking-tool for Pupil D than for Pupil Z. But for Pupil D, language is the thinking-tool but at the same time language can also confuse the meaning.

The self-understanding of Pupil E (IE2-E4) is as follows: Pupil E explains that sometimes one can have problems learning mathematics because of the way the question is worded. Usually it is not because they cannot do the mathematics, but it has to do with the notation and the way that one approaches the question. If one from the textbook learns one way and one practices it 50 or 60 times, and one then approaches it from a different angle, or if one has not learnt the actual theory really well, one is sometimes a bit stuck. The pupil tells that the knot theory in the intersection is not written in the style they are used to study from. Whether it would scare them or depends on what it is for. If it was seriously important, he would “run for help”. He finds that the English is worse than the mathematics here and the language is here an obstruction for learning. He furthermore finds that the text should not use too big words as they are aiming at people who do not understand. People would struggle with the language when they are supposed to be learning mathematics. Pupil E neither liked the way the notation on the sheet with knot theory was given nor the fact that it was given. But he had bigger problems with what came just before they were given. He says that one sees the notation and one knows they refer to this set but one does not know what they are talking about.

The theoretical understanding is interpreted to be that language can be an obstruction for learning. One needs to know what the language refers to. Language should be easy so one does not struggle with the language when one is supposed to learn the mathematics. In this respect Pupil E is quite similar to Pupil Ø. However, in Section 7.1, Pupil E is one of the pupils expressing that language is the thinking-tool, which seems odd as he in this Section states that language/notation confuses the meaning. There are two possible explanations to this paradox, apart from the more general concern in the whole thesis, that the pupils may not have very stable understanding of their learning process (see Chapter 2): (1) Pupil E does only speak of certain branches of mathematics. This seems unlikely as he does not only specifically mention the knot theory in the intersection but he does also more broadly talk about ‘mathematics’. (2) This is an example of Piaget’s (1962) view (see Chapter 4) that egocentrism could both be an obstacle to learning as well as the point of departure for the development of thinking. I chose the second option. However, since Pupil E does not himself describe this dual nature of the role of language (as for instance Pupil Z and D), he is in this section being placed in the same group as Pupil Ø who consistently has a more “negative” view of the usefulness of language for learning.

The self-understanding of Pupil F (IE3-F8) is seen in the intersection where he said that it is useful to work out if there is some simple mathematics that would be quite next to this knot theory without all this difficult language.

An interpretation of the theoretical understanding is thus that when he expresses a wish to find some mathematics without ‘all this’ (confusing language), it means that language can be an obstacle for learning. Pupil F is not very elaborate on this issue but he seems to express something which is similar to Pupil Ø and E.

Pupil B explains in his self-understanding (IE3-B4) that at A-level all the terminology is explained, but in the intersection (in the interview) he says that one cannot cleanse all these words, and understand what they are referring to. It is important with an understandable language, for instance the oriented link is an example they understand in the English language, but he does not know how one sees an orientation mathematically. Here it seems that words can also obstruct thinking, but this is not a contradiction to what Pupil B says in Theme 3 as he in Theme 4 talks about that he cannot *cleanse* all these words. He did not say

remove the words, or something similar. He, so to speak, wants to clean the “dirt” of the words so the “clean” words appears and then these clean words can assist his learning. During the intersection, Pupil B says that if there are issues in the knot theory, which a computer can do, then this is where one has to start by explaining to the class. And after that one can talk about the ideas that are involved. It seems that what Pupil B here talks about is to begin *showing* the other pupils something, and then afterwards *talk* about the ideas behind. This does not belong to Theme 6 even though Pupil B talks about presenting something - thus a dialogue. Rather it reflects that the pupils were asked by me what they would do if they had to *present* it to their class - what “order” did they expect would be the right one for making their classmates learn.

The interpretation of the theoretical understanding is therefore, besides what is being discussed above, that language can be an obstacle for learning. Pupil B expresses that the learning does not happen solely through the language but he also says that some language can obstruct learning. He therefore seems to fall in the same group as Pupil Z and D.

The pupils fall in the following groups:

I: Language can hamper learning, but language can also help learning {Z, D, B}

II: Language can hamper learning {Ø, E, F}

III: Say nothing {Æ, Å, A, C}.

7.2.2 CANNOT EXPLAIN VERBALLY

Only Pupil Æ seems to say something directly about this area of Theme 4, which does not rule out that the nine remaining pupils feel the same, but did not choose to express it. It could also reflect that the pupils do actually feel that they *can* say something about their learning process.

The self-understanding of Pupil Æ (ID-Æ4) is that she says that it is difficult to say what it takes to make one understand. Her teacher has been making illustrations in spatial geometry to make it possible for them to see what it is that is asked. It was that which made her

understand. It is such small things and she finds it is difficult to say what it actually is that makes one understand it.

The theoretical understanding could be interpreted to be that some of the knowledge of learning process is tacit as she says that it is difficult to say what it takes to make one understand mathematics, in this case spatial geometry.

The pupils fall into following groups:

I: Difficult to say how you learn {Æ}

II: Say nothing {Z, Ø, Å, A, C, D, E, F, B}.

7.2.3. DISCUSSION

Language can hamper learning

The pupils do here fall in two groups. First the ones (Z, D, B) who describe a dual nature of the role of language for learning. This dual nature could be interpreted to have to be in line with Piaget's (1962) description of that language can both hamper learning and help learning. Here, for instance Pupil Z, also talks about a ping-pong relationship between a visual side of learning and a language side. Language seem to exist side by side with visual picture which means that the language has some role, but not alone. The second group has either a more consistently "negative" view of the role of language for learning (Pupil Ø and F) or they do not directly describe the dual nature of the role of language (Pupil E).

7.2.4 THE KEYWORDS FOR THEME 4

Z: language alone is not enough, some languages can hamper learning.

Æ: cannot always explain what makes one learn.

Ø: language can hamper learning.

D: language can sometimes obstruct thinking, sometimes learn mathematics without language.

E: language can obstruct learning.

F: language can obstruct learning.

B: language alone is not enough some languages can hamper learning.

7.3 THE SECOND PAIR OF THEMES SEEN TOGETHER

There seem to be various views of language in relation to thinking. Some pupils say that language is the main thinking-tool, others that it hampers thinking, other that language seems to have a dual nature as it both facilitates learning and hampers learning, this does also depend on the kind of language. Following Section 3.3.1 and 3.3.2 and Chapter 4, there seem to be basically two views of the language. (1) A position, which is rather negative, represented by for instance Hadamard who states that thoughts die when they become embodied by words; however, the words are also a necessary support of thoughts. Here, also Piaget states that the roots of logical thought are not to be found in language alone, even though language coordinations are important, but are to be found more generally in the coordination of actions, which are the basis of reflective abstraction. Thus Piaget seems to argue for a dual nature of language as he also states that even though egocentric speech is the point of departure for the development of inner speech and this interiorised language can serve logical thinking, egocentrism is also the main obstacle to learning. (2) A position which is positive of the role of language: Polya talked about that language enlightens the situation and Skemp argues that language is not essential for the creation of the basic concepts, but the higher concepts build on the basic concepts, which after being created, or discovered, become part of the language. Here particularly Vygotsky is very positive of the role of languages and argues that thoughts are not just merely expressed in words but come into existence through the words and that thought development is determined by language.

Position 1 seems to complement the positive and negative roles of language, whereas particularly Position 2, especially Vygotsky, has a more consistently positive view of language. Some of the pupils discussed in this chapter did also seem to reflect a dual nature of language others seemed more consistently against language. When discussing Theme 3 and

the role of language for learning, two groups arose: A moderate positive group, which found language to be important for learning but who did not seem to rule out that learning could happen outside language, I: {Z, Ø, A}. A stronger positive group seemed to argue that language is a necessary thinking-tool, II: {D, E, F, B}. When discussing Theme 4, again two groups arose. One group argued that language can both help learning and hampers it, III: {Z, D, B}. A second group seemed to argue more for that language can hamper learning, IV: {Ø, E, F}. The members of these groups are criss-crossing. In general the pupils seem to have expressed that language have a dual nature, namely both as a thinking-tool and as an obstrucater of meaning. The three latter groups mentioned just above (II, III, IV) consisting of {Z, Ø, D, E, F, B} seem to argue that language is both a necessary thinking-tool but that it can also hamper learning, depending on the type of language. Pupil A seems to have a slightly less “positive” view of the role of language for learning. Above it is also argued that Pupil A seems to use the language as a kind of translation tool. The role of language does also seem to some extent to depend on the branch of mathematics in question. Referring to Piaget’s critique of Vygotsky in Chapter 4, Piaget declared himself to be very much in line with Vygotsky about the positive role of language, but Piaget also argues that Vygotsky fails to acknowledge the obstacles language can give rise to. Thus, based on what the pupils here have explained, Piaget seems to be right in his critique. This does also support the idea of odd complementarity mentioned in Chapter 4, but where the “basis” of the complementarity varies depending on the branch of mathematics in question. It seems that language is not equally important for all branches of mathematics, but mainly useful for more algebraic expressions. Here Pupil A directly says that he is not sure if “words come first or not” but that it might depends on the problem. For visual problems in 3D he finds that it is probably better to have graphs or pictures first, but for algebraic problems it is better to have the words first and then use the pictures to help one understand.

Following Schoenfeld in Chapter 2, high-achievers have metaknowledge, which partly the long interviews might suggest is correct, and also only one pupil (Æ) seemed to say that it was difficult to say what makes them learn. This does however not rule out that this metaknowledge has roots in tacit knowledge of experience, but, following Krutetskii in Chapter 1, high-achievers only need one example to be able to generalise and thus the pupils here might have been so fast in creating their metaknowledge that they have no recollection of a time before the metaknowledge or internal monitor. What is clear, however, is that the

pupils are able to talk about issues of language for learning in a way that makes sense to not only themselves but also to the other pupil(s) in the interview. They have a discussion about these issues and most of the things they say can be interpreted to be reflected in various psychological learning theories.

*Y is for Your future, supposedly paled
By nuclear-holocaust world-end nightmare.
Don't get disconcerted by apocalyptic jive.
It's been just about to happen since 1945.
And no matter the MIRVs, ICBMs, and SAMs,
It's not going to happen before final exam.
(O'Rourke, 1987, p. 162)*

8. THIRD PAIR OF THEMES: THE INDIVIDUAL AND THE SOCIAL

This chapter will discuss what the pupils said which fell within the fifth and sixth themes. I will generally follow the structure of the two previous chapters.

8.1 THE INDIVIDUAL

In this section I will discuss construction and self-activity as well as visualisation as part of the individual theme.

8.1.1 CONSTRUCTION AND SELF-ACTIVITY

Pupil F is the only pupil who does not talk about something within this area of Theme 5.

Pupil Z explains in her self-understanding (ID-Z5) that a teacher's talk does not in itself make her learn mathematics; she finds that she has to take the initiative to learn herself. The role of the teacher is more of a consultant she can use when she has localised what it is that she cannot understand. Basically the teacher cannot help her to understand mathematics. She says that one has to sit independently by oneself and work with the mathematics. One has to take the responsibility of the learning. Another issue raised by Pupil Z is that it is difficult to overcome that threshold of fear of not understanding when there is someone who just stands

and talks. Then one keeps a distance to it, but the moment where the cognition has to be internalised, then one cannot keep it at a distance but one has to relate to it. It irritates her if she does not understand the meaning of mathematics, as she then does not understand the connections within it. With 'meaning' she does not mean what she can use it for, but some inner meaning of the mathematics. She finds that most mathematics is beautiful, everything fits. Pupil Z furthermore explains that learning takes place in a very individually manner. She bases this on that when they sit and discuss there is often a difference in when the understanding comes to each of the individuals. She thinks it is very much about finding the way in which one personally can understand the mathematics. One cannot draw up a systematic which works for all. Somehow one has to find one's angle, and then suddenly one understands. The dividing line is where one is just receiving since when one is not just receiving, then one has understood the things. She finds that when one writes it out oneself, one is better at locating the places where one does not understand, and when one takes the step oneself it is possible to see what one does not understand and then one can do something about it.

One type of self-activity is to write it down, point by point in a kind of systematic process where one can work it through for oneself. One has to sit within oneself and then say that now I have understood that this is how things are connected. One can split it up in some phases and then try to make the cognition less and in smaller bits. It is more fun if one is allowed to explore the mathematics.

The theoretical understanding could therefore be interpreted to be that construction, self-activity, and self-initiative are important. The teacher is a consultant and basically the teacher cannot help her understand. Learning is individual and her own responsibility. It is a construction, and one does not just receive, but one has to relate to it. There does also seem to be a connection with what Pupil Z states in Theme 1 about motivation. Knowledge is inside the heads. People learn differently, and one has to find one's own way. In a discussion the understanding comes at different times to different participants, there is not *one* way of learning mathematics.

Pupil Æ talks says in her self-understanding (ID-Æ5) that sometimes one does not understand one's own notes, but if one by oneself, with the book as starting point, begin from the very beginning and write down point by point, then one can use the notes. She also tells that

usually when the teacher tells them to sit in groups and practice proofs, then most people sit by themselves, individually. Pupil Æ says that the reason is that what is obvious for one is not obvious for someone else, and it is important that one sits by oneself and work it through so one knows all the little steps.

In the theoretical understanding it seems that the pupil talks about construction and self-activity. Working individually is better than in groups as one can sit and work with the small steps oneself and then understand it better. She seems to reconstructs through writing notes from the book. Self-activity is important so one knows everything oneself. Her explanation is thus quite similar to that of Pupil Z.

Pupil Ø tells in her self-understanding (ID-Ø5) that other pupils would like to see an example and some problems and then through them understand. She further explains that others are more oriented towards the proofs and find that it is them one needs to understand to be able to understand the whole thing. She thinks it is very different how one learns. Sometimes she does not understand it when there is someone who explains it to her; instead she needs to see it in writing to be able to understand. She has to tell it to herself.

In the theoretical understanding one can argue that Pupil Ø talks about self-activity. There are individual ways of learning and learning is individual. Pupil Ø is to some extent similar to Pupil Z and Æ, but different in that she does not seem to describe something about construction - building up. It does also seem that the methods of learning are individual, learning is a complex phenomenon, and that learning primarily is an individual act although she opens a door for a more social side of learning as she states that *sometimes* she does not understand it when someone explains it to her. This could indicate that at other times, she does actually learn from others.

In the self-understanding of Pupil Å (ID-Å5) it is expressed that when she sits and reads in the book she has to think more about it - write a bit and read a bit - whereas when she sits in class, everything goes so fast.

In the theoretical understanding one can therefore argue that this seems to suggest that time and self-activity are important factors for learning, particularly that things should not go too fast. But she is not very elaborate on it and therefore she is mentioned as “Å”.

Pupil A finds in his self-understanding (IE1-A5) that it is better to work things out for oneself from looking at examples. It is better to be able to see for oneself when one has done something wrong instead of being told 'oh that's wrong, try again'. It is better to be told something like 'well that's close but here you've got minus instead of plus'. About how one knows that one has understood the mathematics, not the problem-solving, Pupil A says that one may have understood when one can skip ahead and see what the next step is going to be, for instance in a proof. If one can see it and work it out for oneself before it is up on the board, then he knows he has understood it for himself. In relation to learning techniques, he says that he learns better (i.e. there are therefore more ways to learn, of perhaps different efficiency) if he looks at a lot of examples for himself (here he mentions integration as an example) and deduces for himself how it is done and how it works. This is better than being given a general formula at the beginning and then fit everything into that. If he can work it out for himself from seeing examples of bits of equations and things like that, he finds that he can more easily deal with slightly different problems cause he thinks he has then a better understanding and now better can see of how something works. He finds that he can spot from looking at several different examples how it moves as he operates the method through. As he sees more examples he makes a connection between different parts. Pupil A says that if the class spend too much time doing examples he needs help outside the lessons. Instead he thinks it is better to go over the mathematics completely, discuss it and turn it around in his head and see how it works and where the methods come from before one starts to do too many practice questions.

In the theoretical understanding, one can argue that through encountering a collection of examples he deduces the method himself. He explains that real understanding for him is when he has understood something for himself. Self-activity does thus appear to be important. What he says fits well with Glasersfeld's (1995, p. 1) definition, that knowledge is in the heads of persons, and that the thinking subject has no alternative but to construct what he knows on the basis of his own experience. Piaget says that the *meaning* or value of knowledge lies in its function, which does not seem to be supported by what Pupil A is saying here (and in other themes). It seems that mathematics, to Pupil A, has a meaning in itself if it is true. Furthermore Glasersfeld states that it is through encountering a collection of examples he deduces the method himself. Self-activity, guidance, interaction, and construction are

important. This seems to fit with what Pupil A and Æ describe. Pupil A also mentions integration as a branch within mathematics where this approach seems to work.

In the self-understanding Pupil C (IE1-C5) states that one needs to make up one's own techniques that work to solve the problem and to see how the actual detail in the structure is. He says that it all boils down to the teaching method and the teacher. It is a two-way thing; it is about a person learning as well as one being taught properly. It is good if one is taught in a way one can fit in. Pupil C here suggests that learning is individual but that the social is important, especially he states that if the social side does not work, this would give problems for the learning.

In the theoretical understanding an interpretation could be that construction and self-activity are important. There is furthermore a duality between the individual constructing and him being taught properly. It is good to be taught a way one can fit in. In that sense he distinguishes himself from the pupils mentioned above in this section as he himself mentions this dual nature directly. The other pupils seem to only mention it indirectly, which I will return to in Section 8.3.

Pupil D says in her self-understanding (IE2-D5) that the whole point behind learning why one does something rather than how, is that when one uses one's own way without necessarily following the textbook example, one can get the right answer by doing it a different way than the textbook. It does not matter if one understands what one is doing or what one wants to achieve by doing it. She furthermore tells that sometimes if there is something she does not understand then she can use the approach of writing down bits. Then she writes it down and works it through on the paper and then she suddenly she understands it. She finds that writing things down is more a conscious effort; one tries to understand. Pupil D explains that when one understands subconsciously it is like an immediate shedding of light on the problem. But when she writes something down, this is much more consciously working it through in one's mind. She understands this bit and then links all together on paper.

In the theoretical understanding, construction and self-activity seem to be central features. For instance does she talk about the importance of finding one's own ways of learning and she also tells that she sits by herself and works through things to get an understanding. She is thus quite similar to Pupil Z, Æ, and A.

Pupil E says in his self-understanding (IE2-E5) that he works a lot from his notes. After he has revised the notes and rewritten some more notes he then goes back to the book, uses it as a guideline, and he makes sure that everything in the book is in the notes. He always writes his own notes for exams to make sure he understands it. If there is something in the book he does not understand he uses other sources. He basically makes it into a booklet. He is not sure that he has understood everything before he has done it himself. He finds that after he has rewritten the textbook himself, he knows it. And it is organised in his head because he learns what is on his notes in the order he makes it. He rearranges the book into the way his mind works, and he thinks his mind works in a way so that the things have to be logic. Everything is going to be in the right order. Pupil E acknowledges that he does work slowly because of it. He cannot work with a textbook that jumps from here to there. If it is not in a package he tries to make it to a package, makes sure it “nice”, in a logical order, and perfect. Pupil E finds that the use of language depends on the audience. If one has got an audience who are used to this approach then it is fine. People do it differently; it is all very individual, even if one works on something together and one is both aiming to solve the problem. He thinks that one will do it completely differently from someone else and quite often he finds that he does not like other people’s styles, but one always gets one’s own. People understand different things in different degrees. But if he cannot solve a problem and he borrows someone else’s work, it often does not help because the way they have written it out. One does not follow the same sort of logical thinking. For a start people have approached the problems differently. Some people will do the first two steps in their head and quite easily be able to go to the third step. Other people start right at the beginning, and when one writes it out. People do things in different orders and people have their own notation to things.

In the theoretical understanding, Pupil E can be interpreted to talk about the importance of self-activity and construction. Self-activity, construction, and individual ways seems very essential judging on the detailed way that Pupil E describes how and why he rewrites the book. He reorganises it into his way of thinking. The learning is individual even when one physically sits and works with others. Pupil E is thus quite similar to Pupil Z, Æ, A, and D.

During the intersection, Pupil B (IE3-B8) told that he needs to be able to see the reason behind the decisions, he is not satisfied if it seems arbitrary.

In the theoretical understanding, when he says that he needs to be able to see the reason behind the decisions, and he is not satisfied if it seems arbitrary it could be an indication of the need of a visualisation of the process. However it may also mean that he needs to “see”, i.e.: understand, the “logic” behind it himself. This comment could also be interpreted as meaning that he has to see the reasons *himself*; he is not satisfied with for instance being told it. He thus talks about self-activity but it does not seem that he talks about construction.

The pupils therefore group in:

I: Construct and self-activity {Z, Æ, A, D, E}

II: Self-activity {Ø, “Å”, B}

III: Construction and self-activity; learning is both individual and social {C}

IV: Say nothing {F}.

8.1.2 VISUALISATION

Only Pupil B did not say anything about visualisation.

The self-understanding of Pupil Z (ID-Z5) is that if there is something which she has not understood, then she cannot see it. She does not think that she is that visual, she prefers to have everything in writing; it has to be something that she can see.

The theoretical understanding seems to be that that her learning not that visual in terms of pictures and diagrams, but prefers to have things in writing, and then *see* that.

The self-understanding of Pupil Æ (ID-Æ5) is that for instance with spatial geometry; she did not understand anything the first week. But then she began to make the drawings. When one makes a drawing then very often one understands what it is one is working with. Reading books are not enough; one also needs to see it in one’s head. One can for instance see a proof in spatial geometry, but this is just on a flat piece of paper, one also needs to be able to see in 3-dimensional, within the head, to be able to understand what it is all about. To her it is very

important that she can see it. She is not saying that she absolutely has to have a picture, only if it helps her to understand, but she is able to understand it completely without.

The theoretical understanding can therefore be interpreted to be that she has a visual way of learning but she can also understand it without.

The self-understanding of Pupil Ø (ID-Ø5) is that a picture is important. As an example she mentions when working with a tangent and trigonometrically equations.

The theoretical understanding can therefore be argued to be that she has a visual learning. She seems to argue in general and says that pictures are essential. She mentions trigonometry as an example. Pupil Ø does therefore seem to need visual pictures more than Pupil Æ.

The self-understanding of Pupil Å (ID-Å5) is that when one calculates something, then if one can see it for oneself and it is a lot easier to find the solution.

The theoretical understanding does therefore seem to be that visualisation makes it easier to solve a problem. Pupil Å seems to be more like Pupil Æ than Pupil Ø as she only talks about that the learning becomes easier.

The self-understanding of Pupil A (IE1-A5) is that he finds it much easier to understand if he visualises the mathematics.

The theoretical understanding can therefore be interpreted to be that visualisation is important in terms of making it easier to understand. Pupil A seems to have the same relationship to visualisation as Pupil Æ and Å as he also says that it makes learning easier.

The self-understanding of Pupil C (IE1-C5) is that besides reading aloud, Pupil C also uses drawings. When he was asked if he did the same things as Pupil A, he answers that he tends to learn more when things are visual. He tries to make it more visual, but he would not just draw it out, he would also think about it. When asked what he would do if he had to present the knot theory in the intersection to his classmates, he said that that he would draw it out, this is how it would work best. He explains that one cannot do many worked examples, it is just getting to know the theory so if it is like definitions. He would write it out in his own words. Draw a diagram if that helps and try to understand for himself and read that thing again to see

if it is the same. He would then present it and get at the key bits and then just talk about that. Once one has got that frame then one can put all the details in afterwards.

The theoretical understanding does therefore seem to be that Pupil C also uses drawings and visualisation; he tends to learn more than Pupil A when things are visual. He has furthermore a stronger visual imagination. Pupil C does therefore seem to be more like Pupil Ø.

The self-understanding of Pupil D (IE2-D5) is that when asked to describe how they get the mathematics into their head, she said that she looked at the diagrams. In relation to the use of graphics in the sheet in the intersection, she asked Pupil E, if he did not think the graphics were in the wrong place. The graphics should be telling the pupil that this is a knot in the beginning not after the second paragraph. In the first paragraph it is telling what a knot is, but it does not actually show one a knot until a paragraph down.

The theoretical understanding does therefore seem to be that visual pictures can be helpful. At least can a lack of graphs make the learning more difficult. Pupil D does therefore seem to be in the same group as Pupil Æ, Å, and A.

The self-understanding of Pupil E (IE2-E5) is that the graphics were a big help for understanding the knot theory. He tells that he was relieved when he got to the first graphic.

The theoretical understanding is therefore interpreted to be that drawing can help understanding at least for the knot theory. He says that the graphics were a big help, which could suggest that visualisation is as important to him as to Pupil Ø and C. However, Pupil Ø and C seem to talk more generally about the essentiality of visualisation while the only time Pupil E talks about it is when he states that the graphics were a big help *here*. For this reason I have chosen to place him with Pupil Æ, Å, A, and D, and then mention him with quotation marks to indicate that his group-membership might not be unambiguous.

Pupil F said during the intersection (IE3-F8) that he would use clear diagrams if he was making a presentation of the knot theory to the class. The first thing he could do would be to draw some knots and then show then respected oriented diagrams.

The theoretical understanding does therefore seem to be that since he talks about examples and diagrams it suggests that visualisations are helpful for learning. He is therefore being placed with Pupil Æ, Å, A, D, and E.

The pupils therefore group in:

I: Do not need to see pictures, but to see things in writing {Z}

II: Pictures are sometimes helpful, they make it easier {Æ, Å, A, D, “E”, F}

III: Visualisation is very essential {Ø, C}

IV: Say nothing {B}.

8.1.3 DISCUSSION

Below I will discuss self-activity and construction as well as visualisation.

Self-activity and construction

All pupils talk in favour of self-activity. Some of them (Pupil Z, Æ, A, D, and E) mentions this combined something that might be interpreted as construction, among these, Pupil C explicitly states that learning is both individual and social.

Self-activity is important for Pupil Æ. She seems to say that individual learning is the best way as she states that working individually is *better* than in groups as one can sit and work with the small steps oneself and then understand it better. Pupil Z seems to have a stronger view of the individual learning as she seems to say that learning individually is not only the *best way*, which Pupil Æ says, but it is also the *only way* one can learn. This is illustrated when she says that knowledge is inside the heads. People learn differently, and one has to find one’s own way. It is also important to Pupil Z but she furthermore mentions ‘self-initiative’ as being important. To her, the teacher is a consultant and basically the teacher cannot help her understand. Learning is individual, her own responsibility. Based on that she seems slightly more “independent” than Pupil Æ, as what Pupil Æ talks about could also

include self-activity on things the teacher told her to do. But then again, absence of evidence is not evidence of absence. I cannot know this, as Pupil Æ is not being that explicit. Pupil Ø seems to be in line with Pupil Æ as self-activity is also important for Pupil Ø. She further tells that she does not understand it when someone explains it to her; instead she needs to see it in writing to be able to understand. She has to tell it to herself. Pupil Ø does also seem to be mostly in line with Pupil Z on another area, namely that Pupil Ø says that there are individual ways of learning and learning is individual. Pupil Å suggests that time and self-activity are important factors for learning, particularly that things should not go too fast. The fact that she says that things should not go too fast could indicate that the importance of time might speak in favour for a view close to Pupil Z's "self-initiative" view. To Pupil A, self-activity is important and knowledge is in the heads of persons. To Pupil C, construction and self-activity are important. It is a two-way thing, the individual constructs and he has to be taught properly. It is good to be taught a way one can fit in. In that sense he distinguishes himself from the pupils mentioned above in this section as he himself mentions this dual nature directly.

Pupil Æ states that the individual learning is sometimes a reconstruction through writing. Also to Pupil Z, learning is a construction and one does not just receive passively. Pupil A seems to talk about construction when he explains that he deduces the methods himself encountering a collection of examples. Pupil D explains that she reconstructs the mathematics through writing down bits and then she understands the different bits and then link all together on paper. Pupil E also uses a method of writing, and he revises his notes. He is not sure that he has understood everything before he has done it himself.

Visual

There seem to be different views of the visual part. Pupil Z does not need to see pictures, but to see things in writing; she is not that visual, in terms of pictures and diagrams, but prefers to have things in writing, and then *see* that. The main group consist of Pupil Æ, Å, A, D, E, and F and is relatively positive of the role of pictures, as they state that pictures are sometimes helpful, they can make it easier to learn. Pupil Ø and C find visualisation to be very essential.

8.1.4 THE KEYWORDS FOR THEME 5

Z: self-activity, learning is individually, construction, one does not learn only through receiving, self-activity, knowledge is inside the head, visual in relation to writing/reading, own responsibility for learning.

Æ: visual but learning can also happen without, work individually better than in groups, reconstruct, self-activity, visual but for more than reading books.

Ø: visual, more than auditory, individual ways of learning, self-activity.

Å: self-activity, visualisation.

A: self-activity, knowledge is inside the head, the meaning/value of knowledge lies not in its function, visualisation.

C: drawing, visual, construction but not alone, self-activity, individuality.

D: visual, self-activity, construction.

E: self-activity, construction, visual.

F: visual.

B: visual, construction.

8.2 THE SOCIAL

In this section I will discuss the importance of discussions, internalisation, guidance, and verbalisation as told by the pupils.

8.2.1 INTERNALISATION AND GUIDANCE

All the pupils say something in relation to this topic.

Pupil Z explains in her self-understanding (ID-Z6) that it is good to discuss with someone when one has problems learning. It is particularly helpful to discuss with someone who also does not know, because this way one gets some other angles, which could be the angle one misses. It is not always good to talk to people who have understood as they can be really

caught by that they have understood, and may bully. One can also learn from talking to oneself the same way as when one is in a group.

In connection with Theme 5, it seems that learning is individually but when one experience problems, one needs input from outside and one then works with these inputs individually. The reason why it is better to ask someone who has also not understood is not cognitive or pedagogical, but social, as the ones who knows may bully. She has to be introduced to mathematics and physics, otherwise they are closed country to her, and she does not understand it.

Besides the discussion above, the theoretical understanding can be argued to be that it is good to discuss with others, particularly pupils on the same level as others might bully. She therefore does not prefer the discussion with “equals” for pedagogical reason but for more “sociological” reason. The teacher is a consultant who guides. One can also discuss with oneself. Guidance plays a fundamental role in getting in touch with some parts of mathematics. This fits well with what she said in Theme 5 about that a teacher’s talk does not in itself make her learn mathematics but that one has to sit independently by oneself and work with the mathematics. Learning seems to be a duality between a social side of being introduced to something and using others when facing problems and then, on the other side, the individual engagement. It is on the individual side where the learning in itself takes place. She seems to describe a kind of ‘odd complementary’ relationship between the social and the individual, where the individual is the basics, and the social side assists the learning.

The self-understanding of Pupil *Æ* (ID-*Æ*6) is that learning does not only have to do with the teacher. It is a combination of that the teacher comes with some inputs which one then has to work with oneself, and then one can return to the teacher and get some new.

The theoretical understanding does therefore seem to be that the teacher guides but one also has to work oneself. It is a combination. At this point Pupil *Æ* does not seem to make any order of priority between the social and the individual side. In Theme 5 her explanation was quite similar to that of Pupil *Z*, and Pupil *Æ* does here argue in favour of individual self-activity. One might therefore argue that she is in line with Pupil *Z* in the sense that both seem to argue that the learning is a combination of the social and the individual side. A difference is, however, that Pupil *Æ* directly talks about a combination, whereas it can only be deduced

from Pupil Z. I will, however, put Pupil Z and Æ in the same group when the issue is if guidance important when introduced to something new or if one is stocked.

Pupil Ø explains in her self-understanding (ID-Ø6) that it is a self-perpetuating process if one sits and thinks about the mathematics oneself and gets stuck with oneself. One can only see the one direction where one began, as one has not understood the basics in the problem. If one gets some other angles this gives the overall understanding. It is necessary to know that one should not always follow one certain way, but realise that one cannot get far this way, and one has to think of something else, get some input.

The theoretical understanding does therefore seem to be that interaction is good as a way to be guided. Discussions are good when one is stuck, as it gives other angles and the overall understanding. In line with the discussion about Theme 5, we see here that learning has both a social and an individual side. It seems as if the value of the social side is mainly when one experiences problems learning oneself and that one after the input from the outside can move on “alone”. The various inputs may also give one a more overall understanding. Pupil Ø is thus in line with Pupil Z and Æ in terms of evaluating the relationship between the social and the individual side.

Pupil Å finds in the self-understanding (ID-Å6) that it is good to find someone to discuss with if there is something one cannot understand. It is better if one can discuss it on equal terms with someone who is in the same position as oneself.

The theoretical understanding can be interpreted to be that it is good to discuss with people on the same level as oneself. This seems to suggest that learning in itself is individual but that when one experiences problems; one needs input from the outside to be able to continue on one’s self-activity. Here she is in line with Pupil Z, Æ, and Ø. She says furthermore that it is *better* to discuss with equals, but she does not say *why* it is better or in *what way* it is better. She does not answer whether it is better for the learning or better socially, the latter was which Pupil Z states.

Pupil A finds in his self-understanding (IE1-A6) that it is good that he in his class has more opportunity to discuss everything. He feels he learns more as it is being worked more through. It makes him more confident in what he is doing and the smaller class makes him feel more

comfortable asking questions. If they do not understand the teacher, the class can all share ideas between them. He finds that reading aloud always helps as when one reads in one's head one tends to 'skim-read', not to read every word.

The theoretical understanding can be argued to be that guidance and interaction are important. Social interaction plays a fundamental role for their learning. The evidence suggests that the discussions have an effect not only for the *confidence* but also for the *learning*. He also says that he feels he learn *more* through discussions. Pupil A is thus different from Pupil Z; Æ, Ø, and Å, in the discussion about the relationship between the social and the individual side as he seems to prioritise the social side. This does, however, seem to contradict what Pupil A states in Theme 5. I will return to a discussion about this in Section 8.3.

Pupil C feels in his self-understanding (IE1-C6) that he learns more when they have discussions as it makes him more confident. He cannot learn from a book as this is not interactively, one cannot ask the book a question.

The theoretical understanding can therefore be interpreted to be that a way of learning is at the interpsychological level to negotiate the knowledge with an authority that guides. This seems to have an importance for the self-confidence. He furthermore says that the problem with a book is that one cannot ask it a question. Discussions might therefore also be useful for learning and not only confidence building. But he does not elaborate this much. In Section 8.1, when we discussed Theme 5, Pupil C is the pupil who by himself states that the learning is a two-way thing; it is about a person learning as well as one being taught properly. In Theme 5, Pupil C seems to give priority to the individual side, and states that the social side was useful when facing problems. This is much in line with what he states here. But here he also explains that another positive effect with discussions is that they make confidence grows. According to Theme 1, confidence is important, and thus the social side might indirectly through this facilitate learning. Pupil C is thus much in line with Pupil Z, Æ, Ø, and Å in relation to the discussion of the relationship between the individual and the social side.

Pupil D tells in her self-understanding (IE2-D6) that if they have a problem in class, they ask the teacher and her experience is that the teacher makes it quite clear. He goes through it on the board. Not the theory, but the problem. Doing it on the board means that one understands

why one is doing it, not just how to do it, and she thinks that this definitely helps. If one knows how to do it, that is one thing, but if one knows why one is doing it, then one can do any question really. Some teachers sometimes say that one does not need to know why one does certain things, which Pupil D finds to be good to a certain extent. But if one wants to work with difficult problems, one cannot work it out for oneself as easily. About the knot theory in the intersection, she thinks this is a kind of thing that it is very difficult to learn through a book - to represent a 3-dimensional object within a 2-dimensional way. It is where it would help to have a teacher explaining something and point at what is the vertex and what is an edge, and then draw little knots and say that this is so and so. At one place she says that she does not often learn things from the textbook, no matter how simple the language is. She normally has someone to explain it to her. Another place she says that she sometimes uses her notes to do the exercises. When she is revising for an exam, she does not think that she has ever used the notes. She always uses the book, as she knows that everything that is in there is relevant to the modules. Pupil D finds that it is better to do the mathematics in class as it is more interesting and one has other people to balance the result. It also helps to have friends who one can talk to if one is not sure about something. The approach they are used to is that one has the theory and then one has questions on it. It takes one in gently whereas the knot theory in the intersection goes straight in. When one first reads it, she feels that it is quite alienating.

The theoretical understanding is therefore argued to be that simple language is a way to keep the new mathematics within Vygotsky's ZPD. Discussion and guidance are important if one has problem learning and it is important to be able to really learn why. Pupil D is this quite similar to Pupil Z, Æ, Ø, Å, and C in her balancing of the social and the individual side. However, when she says that sometimes if there is something she does not understand, then she uses writing to get an understanding. Writing is naturally self-activity, but here the *tool* is the *language*, i.e. the written language functions as a thinking-tool, which is also in line with Vygotsky, and thus supports an Vygotsky-angle on the language-area. This is also supported by what Pupil D says in Theme 3 in Section 7.1. Here her selfunderstanding could be taken as meaning that language is an important thinking-tool for learning mathematics.

Pupil E's self-understanding (IE2-E6) is as follows: What they do if they meet some mathematics they do not understand depend on how difficult it is. Pupil E explains that he

goes and asks his classmates. And if it is a big problem he would ask the teacher to go through it. If a teacher teaches one how to do it, Pupil E thinks it becomes easier to approach things.

The theoretical understanding could therefore be interpreted to be that the language must be simple. This must mean that it is simple to the pupils. Interaction with knowledgeable people and a “right” presentation of the topic are also necessary. He says that if a teacher teaches a pupil, it becomes more approachable. It may seem a little odd compared to Theme 5 where he strongly argued in favour of the need for the learner to do different things and that things are organised in the head. What he said above in Theme 6 was said in connection with a discussion about what one does when one experiences problems learning *oneself*. Furthermore he does not say that the teacher makes it easier to *learn* but he uses the more “soft” word *approach*. One might therefore argue that this social side does “only” assist the individual learning.

His past experience of teaching was completely different from Pupil D’s. He had small books, a whole series, and there was an order in which one had to do them and all the pupils did the books by themselves and the teacher did not teach one anything, books told how to do it. The system worked for him but he did not think it was the right approach. He thinks that at the present school, when one does things as a class, it is better. It is good to do it in class as it is very difficult for an author to get an idea across in a book, it will be much easier and better explained by a trained teacher. Pupil E thinks that it would be easier if the author translated the difficult language instead of leaving the reader to do it. He finds it quite easy to learn from textbooks if it is written in a certain way and approaches the mathematics from a certain angles. Thus, he needs a certain angle or a certain way of being presented to the mathematics before *he* has a chance of learning/constructing in his way/himself. Pupil E is thus rather similar to Pupil Z, Æ, Ø, Å, A, and D regarding the balancing of Theme 5 and 6.

Pupil F explains in his self-understanding (IE3-F6) that the pupil thinks that if they were given more teaching time in class and less repetition of the same questions, he thinks they could go a lot faster. If Pupil F has problems he asks classmates. If he is on his own, he looks in the book, and if he does not understand the first section he gives up and goes through it with someone who is really good at mathematics. He has experienced that he can then do everything in the end, even though for instance his father who sometimes helps him knows

nothing about it in the beginning. However, it works best to talk to someone who knows the mathematics as it give some achievement whereas if one does not know anything beforehand, one does not know if it is right in the end. He prefers to discuss with some who are “unequal” to him as he then feels he knows that he will learn it.

The theoretical understanding does therefore seem to be that guidance and interaction are important particularly with someone who knows. Regarding the balancing of Theme 5 and 6 it becomes a bit difficult to place Pupil F as he did not say much about Theme 5. He did only seem to talk about visualisation as something that might make the learning easier. On the other hand, he does in the present section tell that when he faces problems learning, he seeks guidance. I have therefore put Pupil F in the same group as Pupil Z, Æ, Ø, Å, A, D, and E, but mentioned him with inverted commas to denote that this interpretation might not be unambiguous owing to the amount of data.

The self-understanding of Pupil B (IE3-B6) is that if he meets some mathematics that he cannot understand at first, he would ask the teacher, others, or go to the textbook. Pupil B finds that it certainly helps if one can discuss with someone else. Two brains are better than one as one person can have one idea which trigger another idea in the other person’s head which the first person would not have had, and then the second person having said that thing, then the one thing leads to another. It also helps if one person reads one piece, and someone reads another piece, and then explains it to each other, which saves time. He furthermore finds that it works best to talk to someone who knows the mathematics more than someone who also does not understand.

The theoretical understanding is therefore interpreted to be that interaction work, particularly with someone who knows. This suggests that when he experiences problems learning he expects guidance to be helpful. Pupil B seems to be even very positive of the use of discussions as he goes as far as to state that two brains are better than one and he explains why. Pupil B is thus quite similar to Pupil A in their balancing of Theme 5 and 6, namely that the social side plays a bigger role than for the other pupils.

The pupils therefore group in:

I: Discussion with equals/someone who also does not know is good; the reason is social

- and/or makes the pupil more confident {Z, A, C}
- II: Discussion is good, particularly with equals {Å}
- III: Discussion is good, particularly with someone who knows {B}
- IV: One can also learn from discussion with oneself {Z, Æ}
- V: Guidance important when introduced to something new, or stocked by oneself {Z, Æ, Ø, Å, C, D, E, "F"}
- VI: Learns more through discussion than sitting alone {A, B}
- VII: Reads aloud {A}.

8.2.2 VERBALISATION

Only Pupil Z, Æ, A, and C says something in this connection.

In her self-understanding Pupil Z (ID-Z6) says that her experience is that sometimes when she explains to someone what it is she has not understood then, while she is explaining, she suddenly realises herself what it is.

The theoretical understanding is therefore interpreted to be that through verbalisation, this audible speech brings ideas into consciousness. It seems to suggest that Pupil Z mostly is vocal as Pupil Z in Theme 5 herself states that she is not that visual in terms of pictures and diagrams, but she prefers to have things in writing, and then *see* that.

In her self-understanding Pupil Æ (ID-Æ6) says that when one tries to explain to someone what the mathematics is about then, while one is explaining it, one understands it oneself. Sometimes when one has not understood it completely, and one begins to explain it to another person, one also explains it to oneself.

The theoretical understanding could be argued to be that verbalisation makes it easier to understand. This seems to be quite the same as what Pupil Z says. In Theme 5 about visualisation, Pupil Æ explained that she does also have a visual way of learning, but that she can also understand the mathematics without visualisation. She explained that for instance in spatial geometry visualisation helps. In that sense Pupil Æ is different from Pupil Z as Pupil Z does not mention a positive effect of visualisation.

In connection with a discussion about the benefits of reading aloud, Pupil A says in his self-understanding (IE1-A6) that when one reads inside one's head, one tends to skim-read but when one reads aloud, one has to think about every word and sentence and what it means.

The theoretical understanding could be interpreted to be that audible speech and reading aloud brings ideas more clearly into consciousness. Perhaps one could argue that audible speech is a "bridge" between the unconsciousness and the consciousness. Verbal thinking, through either spoken or written word, is an example of a social activity. This type of social activity plays an important role for Pupil A. In that sense Pupil A is similar to Pupil Z and Æ as they also states that audible speech help the one saying it.

In his self-understanding, Pupil C (IE1-C6) tells that what he did to understand the knot theory in the intersection was to read the whole thing through and then remember the bits he did not understand and then he read these places aloud to himself. He found this approach good as he could hear it externally and he sometimes also thought of visual images. According to him, reading aloud always helps as if he just reads in his head, he reads without understanding. When he comes across a phrase that sounds awkward, he just reads it over but does not think about it, but if he reads it out loud, he thinks about it and visualises it.

The theoretical understanding could be interpreted as follows: Following Pupil Z, Æ, and A, also Pupil C talks about that audible speech helps the person saying it. The talk about reading aloud is also supported by the observations in the intersection where C reads aloud. Furthermore, Pupil C does also talk about that the audible speech helps the visualisation. Following the discussion in Section 8.1.2, Pupil C is more visual than Pupil A. Pupil C does therefore seem to use both approaches, with a stronger emphasis on the visualisation part.

The pupils therefore group in:

- I: Audible speech helps the person saying it {Z, Æ, A, C}
- II: Audible speech helps to clarify things for the consciousness {A}
- III: Audible speech helps the visualisation {C}
- IV: Say nothing {Ø, Å, D, E, F, B}.

8.2.3 DISCUSSION

Here I will summarise and discuss some of the main conclusions from Section 8.1.1. and 8.1.2.

Guidance and internalisation

Most of the pupils (Z, Æ, Ø, Å, C, D, E, and F) argue that guidance is important when introduced to something new, or when you are stuck in your individual work. According to Pupil Æ, the teacher guides, but one also has to work oneself. It is a combination. Pupil Z says almost the same, namely that the teacher is a consultant who guides. One can also discuss with oneself. Guidance plays a fundamental role in getting in touch with some parts of mathematics. One could perhaps argue that Pupil Æ has a slightly more “authoritarian” view of the teacher’s role, as she says that it is a combination, whereas Pupil Z talks about a teacher as being a consultant. To Pupil Å it seems that learning in itself is individual but that when one experiences problems, one needs input from the outside to be able to continue on one’s self-activity. Also Pupil C seems to argue in favour of that a way of learning is at the interpsychological level to negotiate the knowledge with an authority that guides. Similarly with Pupil D, E, and F. Some pupils (A and B) argue that they learn more through discussion than sitting alone working for oneself.

Pupil Z, A, and C state that discussions with equals/someone who also does not know, is best for social reasons or for reasons of creating confidence. Also Pupil Å seems to argue that discussion with equals is good, but she does not seem to argue that the reason is social or based on confidence. Pupil Z and Æ states that one can also learn from discussing with oneself. Pupil B says that discussion with someone who knows is best. According to Pupil Z, it is good to discuss with others, particularly pupils on the same level as others might bully. She therefore does not prefer the discussion with “equals” for pedagogical reason but for more “sociological” reason. Pupil Å also finds that it is good to discuss with someone where one is on equal terms. She says furthermore that it is *better* to discuss with equals, but she does not say *why* it is better or in *what way* it is better. She does not answer whether it is better for the learning or better socially, which Pupil Z did. Pupil Ø also finds that discussions

help one to learn as one gets other angles. She does not say anything about what is a best, discussion with “equals” or not.

Verbalisation

Only Pupil Z, Æ, A, and C says something about verbalisation. All four of them spoke in favour of audible speech being able to help the person speaking. A place where the four pupils differ from each other is for instance that Pupil Z does not at all mention a positive effect of visualisation whereas Pupil Æ is slightly more positive of the role of visualisation. Pupil C is still more positive of the role of visualisation and explains that the audible speech helps the visualisation. Pupil A explains that audible speech helps to clarify things for the consciousness. Pupil C talks about visual images as another way of learning and he is the most “visual” of these four pupils.

8.2.4 THE KEYWORDS FOR THEME 6

Z: discussion with people on the same level as others bully, guidance, verbalisation, discussion good when there are problems with the learning.

Æ: verbalisation, guidance in combination with self-activity.

Ø: interaction and discussion when one is stuck.

Å: discussion with people on the same level.

A: guidance, interaction particularly for the confidence, verbalisation.

C: interaction gives understanding and self-confidence, verbalisation.

D: ZPD, discussion, guidance.

E: interaction with knowledgeable person.

F: interaction with someone who knows, guidance.

B: interaction particularly with someone who knows, guidance.

8.3 THE THIRD PAIR OF THEMES SEEN TOGETHER

It seems that most of the pupils argue that learning has both a social and an individual side. The value of the social side is mainly when one experiences problems learning oneself. After the input from the outside one can move on “alone”. Learning is individually and the social side is more used when there are problems. One example is that audible speech helps either the visualisation or it helps to make things more clear. Particularly Pupil Z, Æ, Ø, Å, C, D, E, and F argue this. Pupil Æ and C are here particular as they by themselves use words such as “combination” and “two-way thing” to describe the relationship between Theme 5 and 6. However it also seems that Pupil A and B actually learns more through discussion than by self-activity and for these pupils the order may be reverse, i.e. social learning comes first, then the individual. I will now look a second time on Pupil A and B as Pupil A seemed to contradict himself and there were some unclear thing around Pupil B.

About self-activity and construction, Pupil A told that it is better to work things out for oneself from looking at examples. It is *better* to be able to see it for oneself when one has done something wrong instead of being told ‘oh that’s wrong, try again’. This could be interpreted as being supportive of a self-activity theory such as Piaget. It does therefore seem to contradict that Pupil A also says that one learns better in discussions. However, when Pupil A says that it is better to be able to see it for himself than being told, etc., he is actually choosing between the two alternatives: (1) working it out for himself from a number of examples or (2) being *told* the method in the beginning. Being *told* does not necessarily imply a *discussion*, perhaps rather the contrary. Pupil A is therefore not necessarily contradictory, but might instead be interpreted to prioritise as follows: 1. discussion, 2. working it out for himself, 3. being told. Therefore it might be that Pupil A is a “Vygotsky-pupil”.

Pupil B told that he needs to be able to see the reason behind the decisions; he is not satisfied if it seems arbitrary. This comment could be interpreted as meaning that he has to see the reasons *himself*; he is not satisfied with for instance being told it. Again, as said just above in the discussion of Pupil A, *being told* and *discussion* may not be the same. Pupil B may therefore also be a “Vygotsky-pupil”. And this shows that to learn through discussion, self-activity is also important.

Returning to the discussion in Chapter 4 about odd complementarity it seems that all ten pupils speak in favour of some kind of complementarity between the social and the

individual aspect of learning. Pupil A and B seems to argue that the main issue is the social side, whereas the eighth other pupils argue the opposite. This rather big difference in number does, however, not suggest such an 80-20 relationship for all pupils. The sample method does not allow such a conclusion.

In terms of another individual-social level, the visualisation and verbalisation, there are three groups about visualisation. The first group consist of Pupil Z who does not need to see pictures, but instead to see things in writing. A second group, (Pupil Æ, Å, A, D, E, F) is relatively positive of the role of pictures, as they state that pictures are sometimes helpful. A third group (Pupil Ø and C) finds visualisation to be very essential. About verbalisation, Pupil Z, Æ, A, and C all spoke in favour of audible speech being able to help the person speaking. Hence about the relationship between visualisation and verbalisation, one sees the following: I: Mainly verbal (Pupil Z). II: Seem to argue that it depends on circumstances, perhaps branch of mathematics (Pupil Æ, A, C). III: Relatively visual (Pupil Å, D, E, F). Thus it seems that various pupils divide themselves up into groups either mainly supporting an individual approach or a social approach, and some also seem to argue in favour of having both positions in a kind of complementarity relation.

*Of making many books there is no end,
and much study wears the body.
Now all has been heard;
here is the conclusion of the matter:
Fear God and keep his commandments,
for this is whole duty of man.
(King Solomon, The Bible, Ecclesiastes, Chapter 12: verses 12-13)*

9. DISCUSSIONS AND CONCLUSIONS

In this chapter I will first discuss the differences and similarities among the pupils as well as discuss what the pupils said which fell outside the six themes (Section 9.1). Then I will discuss the existence of types of pupils and to what extent these might be “cross-national” (Section 9.2). I will also discuss the CULTIS model for analysis (Section 9.3), some methodological issues (Section 9.4), and some perspectives for teacher training and education policy (Section 9.5). Section 9.6 is a summary of the conclusion on the research question.

9.1 THE PUPILS’ DIFFERENCES AND SIMILARITIES

In this section there will first be a matrix that displays the pupils’ utterances using the CULTIS model for analysis. Then there will be a discussion of what the pupils said that did not seem to belong to any of the theme in the CULTIS model for analysis. To recapitulate, the CULTIS model for analysis consists of six themes which various psychological learning theories seem focus on: The numbers 1-6 each stand for a theme; 1. Consciousness, 2. Unconsciousness, 3. Language, 4. Tacit, 5. Individual, and 6. Social.

9.1.1 MATRIX DISPLAY OF THE PUPILS’ NARRATIVES IN THE THEORETICAL UNDERSTANDING

Below is a matrix that is intended to create an overview over what the pupils said. It consists of the keywords from the previous three chapters. The matrix consists of six columns and ten

rows of data. Each row represents a pupil and each column a theme. The keywords are taken from the theoretical understanding.

	1 <i>Consciousness</i>	2 <i>Unconsciousness</i>	3 <i>Language</i>	4 <i>Tacit</i>	5 <i>Individual</i>	6 <i>Social</i>
Z	exercise, cognitive drive, affective drive, internal monitor, external pressure	preparatory work, incubation, illumination,	understand basics, systematic slow process, schematic learning from basics is best, language as thinking-tool, difficult to find meaning outside language, ping-pong between language and seeing it	language alone is not enough, some languages can hamper learning	self-activity, learning is individually, construction, you do not learn only through receiving, self-activity, knowledge is inside the head, visual in relation to writing/reading, own responsibility for learning	discussion with people on the same level as others bully, guidance, verbalisation, discussion good when there are problems with the learning
Æ	exercise, automatic manipulation, cognitive drive, force, no internal monitor, no plan	incubation, illumination	schematic understanding	cannot always explain what makes you learn	visual but learning can also happen without, work individually better than in groups, reconstruct, self-activity, visual but for more than reading books	verbalisation, guidance in combination with self-activity
Ø	exercise, self-confidence, no monitoring	preparatory work, incubation, illumination	language as thinking-tool, difficult to find meaning outside language, basics important, schematic learning	language can hamper learning	visual, more than auditory, individual ways of learning, self-activity	interaction and discussion when you are stuck
Å	exercise, cognitive drive	preparatory work, incubation, illumination	no rote-learning, basics	Nothing	self-activity, visualisation	discussion with people on the same level
A	exercise but not alone, reflection, automatic manipulation	preparatory work, incubation, illumination	schematic learning, no rote-learning, language as	Nothing	self-activity. knowledge is inside the head, the meaning/	guidance, interaction particularly for the confidence,

	but not alone, cognitive drive but not alone, encouragement/ self-confidence, plan based on past experience, needs to understand that it is true		thinking-tool but not alone		value of knowledge lies not in its function, visualisation	verbalisation
C	exercise, need to know it is right	preparatory work, incubation	assimilation, schematic learning, basics	Nothing	drawing, visual, construction but not alone, self-activity, individuality	interaction gives understanding and self-confidence, verbalisation
D	exercise, cognitive drive, force planning using past experienced methods	preparatory work, illumination	basics, rote-learning, schematic learning (better than rote-learning), language is a thinking-tool	language can sometimes obstruct thinking, sometimes learn mathematics without language	visual, self-activity, construction	ZPD, discussion, guidance
E	exercise but not alone, no plan, first big picture, cognitive drive, force, no automatic manipulation	preparatory work, incubation, illumination	accommodate, schematic learning, no rote-learning, language as thinking-tool but it depends on the kind of language	language can obstruct learning, sometimes you do not know what makes you learn	self-activity, construction, visual	interaction with knowledgeable person
F	exercise but one is enough, learn more than syllabus to see the application/ object process	Nothing	language as thinking-tool	cannot express, language can obstruct learning	visual	interaction with someone who knows, guidance
B	exercise	Nothing	language the major thinking-tool but it depends on the language, schematic understanding	language alone is not enough, some languages can hamper learning	visual, construction	interaction particularly with someone who knows, guidance

Matrix 9.1: Overview of pupil statements in the theoretical understanding.

I will use the matrix to get an overview over differences and similarities among the pupils.

9.1.2 THE SIX THEMES

The pupils' statements can mainly be interpreted using the CULTIS model for analysis. To a certain extent the pupils mention the same things, but there seems to be a difference between them on areas such as the role of practice, motivation, language, and individual and social issues. In summary, one can state the following:

In general, about Theme 1, some pupils do not talk about planning, others more openly denies it. But there are also some pupils who speak in favour of having a plan. Almost all the pupils talk about motivation, but there seems to be various views to the nature of motivations. The cognitive drive is mentioned as important. A more "outer" motivation is to be forced into it. Various pupils perceive this differently. Some pupils seem to take it as a positive challenge others as a negative. A more "inner" motivation is the motivation from being confused by something. Some does also explain that lack of motivation can be caused by lack of self-confidence, and that if one tells oneself that one can do it, it is the first step. There is also a show-off-effect/"social drive". In general it furthermore seems that the S-rationale is dominant among the pupils. All the pupils state that doing exercises is important for the learning process. There are however some variations. Not all pupils testify to the existence of an internal monitor, which one might have expected them to do, as it is one of the signs of high-achievers. In Theme 2, most pupils talked about what may be interpreted as all Hadamard's (1945) three phases.

About Theme 3 and 4, most pupils seemed to have a positive view of the role of language for learning, although there were variations. In general the pupils expressed that language have a dual nature, namely both as a thinking-tool and as an obstructer of meaning. Most of the pupils did also support the idea of schematic learning and that the basics are important. One pupil did also directly say that it was difficult to say what makes one learn.

About Theme 5, all pupils talk in favour of self-activity. Some of them mention this combined something which might be interpreted as construction, and some explicitly states that learning is both individual and social. There seem to be different, but few, views of the visual part. It goes from stating that one is not visual to others finding visualisation to be very

essential. Regarding Theme 6, most of the pupils argue that guidance is important when introduced to something new or when one is stuck. Only a few pupils said something about verbalisation, but they all spoke in favour of audible speech being able to help the person speaking. As discussed in the three previous chapters, the pupils seem to favour a discussion about complementarity.

It also seems that the usefulness of various approaches of learning depends on the branch of mathematics. For instance does the interpretations of the pupils' selfunderstanding suggest that language is not equally important for all branches of mathematics, but mainly useful for more algebraic expressions. For more visual problems it is probably better to have graphs or pictures in the beginning of the process of learning. However, these conclusions are cautious, as the pupils did not speak of this issue in an extensive manner.

The role of a coming examination

It might seem as though the future examination plays a role in what the pupils say. But the pupils do nevertheless seem to be able to see the difference, if there is one, in learning for an examination, and "real" learning. Pupil A and C clearly distinguish between what is means to learn for an examination and what learning means. For instance in IE1, 722-744, where it is seen that the pupils discuss what is helpful in terms of an examination, and what is generally helpful, which for Pupil C is not necessarily the same:

I: You don't think it learn, er, you learn anything by just doing example?

A: Yea, you do, you you learn how to apply the method. (C: You don't understand it) [some words are lost due to the interruption] helpful in terms of [C sighs] say what you gonna have to do in the exams (C: mmm) (I: mmm). It's being able to, you know, it helps you to work (C: mmm [very silent]) around slightly different forms of the problems being able to apply the method quickly without making any stupid mistakes, and so it is definitely useful in the form of exams, but

C: That's what I DO like, for revision (A: Yea) cause I understand (A: Yea yea, once you) if I if you know that basic integration for example, there is no point going through, for revision, you've got 2 weeks to the exam (I: mmm), there is no point in going through your notes on integration (I: mmm), cause you know how to integrate. What you need to do is have loads of loads of practice [interrupted]

A: Yea, you wanna get to understand it fully and understanding where the method comes from.

C: Mmm, not really, you don't wanna know where it comes from and prove it, you just wanna be able to [interrupted]

A: Well it CAN be helpful.

C: Right. For some people they they like.

Also Pupil D and E (IE2, 469-471) seem to be able to distinguish between what is helpful in terms of an examination, and what makes one learn in general:

D: It's a set of completion, isn't it (E: yea) It's like tightening up all the loose ends which you don't HAVE to do to pass the exam but it just, I don't know (E: Yea), it's more satisfying to you.

Later on in the same interview (IE2, 807-813):

I: But if you er were not suppose to learn this for an exam would you still turn to a book like this?

D: Yes, because I, cause you have to understand the basic and then you just want to know, you'll still know to how you would apply this knowledge and what [inaudible] it is (I: mmm)

E: I'd rather understand it [blows] and not care about what you're suppose to do with it.

The only pupil who seemed to feel very dependent on studying for an examination was Pupil F. But Pupil B separated, in line with the other pupils, what might be useful in terms of an examination to other learning, see the two interview quotes below. The first is from (IE3, 117-139):

B: I actually find it more useful when we ARE doing sets of exam questions

F: Exactly.

I: Why?

F: Er, cause they they sort of, [inaudible] the exam question, the whole topic area is condensed into one question, it has a bit of each sort of part. And er you get used to combining the whole lot together rather than just using one specific bit and repeating it over like you're doing in exercise.

I: Mmm, but do you need exam er questions to do this (B: Er) or is it more the type of questions?

B: It's getting use to the type of questions.

I: Mmm

F: Yea, it's the whole format cause one exam question can, if there is 6 chapters in a book you're studying er and each one is about a different subject er an exam question can test four of them all at once in one question and that's a lot more useful than doing each chapter and then forget the chapter and then you go to the next one, then you come up with an exam paper and you have to link them all together, that's the problem I think.

The second is from (IE3, 1046-1053):

I: What, you said, I think maybe it was you in the beginning that using the exam questions that was good for it kind of linked all different kinds of maths.

B: Yea, well, when when we say different kind of maths, the exam questions will be from [inaudible] mechanics modules linking all the maths (F: We get books) we are linking all the sort of chapters within the one book from that module (I: mmm) so it will all be within that topic area but it's separate, I mean each different chapter has a whole range of different techniques that you use for different things.

Thus, even though the role of the coming examination plays some role for how the pupils act it is also clear that the pupils are able to distinguish between what might be learning for an examination and what is “real” learning.

The pupils' abilities to talk about their learning process

As discussed in Chapter 1 and 2, the assumption behind this study is that the pupils are able to talk in a meaningful way about their learning processes. My interpretation of the pupils' selfunderstanding seen in the previous three chapters confirms that the pupils are actually able to have a meaningful conversation about their learning. The reason why I call it a meaningful conversation is that all four interviews were rather long and the pupils did in all of them very eagerly relate to what the other pupil(s) said and either validated this or entered a discussion about it. It is also clear that the pupils could recognise something in each other's explanations, and they did not talk past each other. Furthermore I found that most of the things the pupils said could be recognised in the various theories mentioned in the six themes of the CULTIS model for analysis. This was, naturally, an interpretation of the pupils' daily language and metaphors etc. However, the interview technique was very much non-leading. I will interpret this to imply that my recognition of the various theories within the pupils' narratives does actually show that the pupils had knowledge of their learning. It was not just their reply to leading questions.

9.1.3 OUTSIDE THE SIX THEMES

There are some utterances that did not fit into any of the themes and these utterances (placed in Theme 7) did also not seem to be covered by the theories that gave rise to the themes. All the pupils said something outside of the themes. What I find interesting is that one certain issue is mentioned by half the pupils: Pupil Z, A, D, F, and B. I will focus on this below.

How one is used to learn/being taught influence how one later on learns

Pupil Z explains (ID-Z7) that how they learn is influenced by the fact that they have been raised to having a visual cognition and therefore they learn most things through their eyes. Pupil A says (IE1-A7) something quite similar namely that the learning strategies one has, or the ideas one gets to overcome a problem learning something, are connected with the way one has been taught to do things. Pupil A also explains that when he was little, he was taught that if he cannot read a word he should try to read it through slowly and then he cannot help saying it out loud to oneself. The idea of reading aloud is therefore, according to him, something that one builds up through the way one has been taught to do things, right from the start. Also Pupil D says (IE2-D7) something like this when she explains that when she first came to this school, the mathematics was very difficult the first weeks as it, according to her, is hard to adapt to a different teaching style. Mathematics can thus be difficult if one cannot adapt to the (different) teaching style. After I asked Pupil B (IE3-B7) why he wanted to give his classmates examples in a presentation, he answered that it is because this is the way they have always been taught, through examples, and he therefore assumes that that is the way the people in the class will understand it easiest. Pupil F (IE3-F7) follows this up by supporting Pupil B's explanation and he then gives as example differential equations and complex numbers as branches where this would be the case.

It therefore appears that how they have been taught before, influences how they are able to learn now. This observation might also be supported by the fact that the only pupil (Pupil E; IE2-E5) who used the book extensively and actually rewrote it, was the only one who previously had experienced a school system where he was not taught, but had to teach himself everything from books.

The phenomena might be explained as that also the teaching methods must be part of, what I would term, a *zone of proximal teaching (ZPT)*. The talk of a ZPT is naturally inspired by Vygotsky's talk of a ZPD, zone of proximal development, which is the area between the tasks a pupil can do without assistance and the ones that require help. Similar for the ZPT, one could state that if a (new) teacher uses other teaching methods that are too "far away" from a teaching style the pupils are used to, the pupils might not learn. Furthermore a change of teaching style ought to be gradual.

One could then ask why this area does not seem to be covered by the theories in the six themes. One answer could be that the CULTIS model for analysis is focused on the cognitive side of learning, while the talk about ZPT might not be a cognitive factor. One might argue that this suggests that cognitive and non-cognitive factors could be complementary in terms of creating the opportunities for learning. In any case, a result of this study is that the pupils' previous experience learning mathematics, his learning history, to some extent influences how they later on are able to learn. This means that the single teacher's method of teaching is one factor that has consequences for the pupils' later learning successes.

9.2 TYPES AND NATIONALITY

As discussed above, the pupils are different. However as it also became clear in Chapter 6 to 8, the pupils could be put into various groups in each of the themes. It does therefore become important to investigate if some of the pupils tended to be in the same group within various themes or whether it was completely random from theme to theme. Below I will discuss different attempts to establish whether it is possible to talk about types of pupils.

9.2.1 WHICH PUPILS GROUPED TOGETHER

Below is a brief summary of the 23 groups formed in Chapter 6 until 8. Groups consisting of one pupil is not mentioned, neither are the "say nothing" groups.

1	Planning Motivation Practice Monitoring	A, D, E Z, Æ, Å, A, D, E Z, Æ, Å, A, C, D, E Z, Æ, Ø, Å, D, B A, E Æ, Ø
2	Unconsciousness	Z, Ø, Å, A, E C, D
3	Thinking-tool Schematic learning	Z, Ø, A D, E, F, B Z, Ø, C, D, E, B Å, A, C
4	Tacit	Z, D, B Ø, E, F
5	Self-activity Visualisation	Z, Æ, A, C, D, E Ø, Å, B Æ, Å, A, D, E, F Ø, C
6	Guidance Verbalisation	Z, A, C Z, Æ Z, Æ, Ø, Å, C, D, E, F A, B Z, Æ, A, C

Matrix 9.2: Summary of the 23 major groups formed in Chapter 6 until 8.

Below is a matrix that displays the pupils' differences and similarities within each theme. The purpose of the matrix is to use it to point at interesting cases for further discussion. Above the obliquely '*' line is mentioned the amount of times two pupils have been in the same group. Below this line is listed the number of times only one of the two pupils have belonged to a group. The number in brackets to the left denotes the particular theme where this was the case. In each box there is six numbers each referring to one of the themes. The total number is the sum of group-membership in all six themes. It is necessary to not only "count" the number of times two pupils are in the same group but also the number of times only one of the pupils are part of a group. One reason for this is that some pupils simply talk more than others. Looking at both numbers might be a way to eliminate the fact that if a pupil talks a lot, he may be put into many groups and a pupil who talk less might be found in a smaller amount of groups. Looking at the difference in the number above and below the obliquely line might eliminate the influence from the length of talk. It is also a way to avoid a situation as follows: a certain pupil is very often in the same group as a certain other pupil. It would then, at first, seem obvious to judge that these two pupils are quite similar. But to evaluate this, we also

need to know the number of times a group was formed but where only one of the two pupils were a member.

	Z	Æ	Ø	Å	A	C	D	E	F	B
Z (1) (2) (3) (4) (5) (6) Total	* 6	3 0 0 0 1 2 6	1 1 2 0 0 1 5	3 1 0 0 0 1 5	2 1 1 0 1 2 7	1 0 1 0 1 3 6	3 0 1 1 1 1 7	2 1 1 0 1 1 6	0 0 0 0 0 1 1	1 0 1 1 0 0 3
Æ (1) (2) (3) (4) (5) (6) Total	1 1 2 1 1 1 7	* 7	2 0 0 0 0 1 3	3 0 0 0 1 1 5	2 0 0 0 2 1 5	1 0 0 0 1 2 4	3 0 0 0 2 1 6	2 0 0 0 2 1 5	0 0 0 0 1 1 2	1 0 0 0 0 0 1
Ø (1) (2) (3) (4) (5) (6) Total	3 0 0 2 3 3 11	2 1 2 1 4 2 12	* 12	1 1 0 0 1 1 4	0 1 1 0 0 0 2	0 0 1 0 1 1 3	1 0 1 0 0 1 3	0 1 1 1 0 1 4	0 0 0 1 0 1 2	1 0 1 0 1 0 3
Å (1) (2) (3) (4) (5) (6) Total	0 0 3 1 3 3 10	1 1 1 0 2 2 7	3 0 3 1 2 0 9	* 9	2 1 1 0 1 0 5	1 0 1 0 0 1 3	3 0 0 0 1 1 5	2 1 0 0 1 1 5	0 0 0 0 1 1 2	1 0 0 0 1 0 2
A (1) (2) (3) (4) (5) (6) Total	3 0 2 1 1 3 10	4 1 2 0 0 4 11	6 0 2 1 4 4 17	3 0 1 0 2 4 10	* 10	1 0 1 0 1 2 5	3 0 0 0 2 0 5	4 1 0 0 2 0 7	0 0 0 0 1 0 1	0 0 0 0 0 1 1
C (1) (2) (3) (4) (5) (6) Total	2 2 2 1 1 1 9	3 1 2 0 2 2 10	3 2 2 1 2 2 12	2 2 1 0 4 2 11	3 2 2 0 2 2 11	* 11	1 1 1 0 1 1 5	1 0 1 0 1 1 4	0 0 0 0 0 1 1	0 0 1 0 0 0 1
D (1) (2) (3) (4) (5) (6) Total	1 2 2 0 1 3 9	2 1 2 1 0 2 8	4 2 2 2 4 0 14	1 2 3 1 2 0 9	2 2 4 1 0 4 13	3 0 2 1 2 2 10	* 10	3 0 2 0 2 1 8	0 0 1 0 1 1 3	1 0 2 1 0 0 4
E (1) (2) (3) (4) (5) (6) Total	3 0 2 2 1 3 11	4 1 2 1 0 2 10	6 0 2 0 4 0 12	3 0 3 1 2 0 9	0 0 4 1 0 4 9	3 2 2 1 2 2 12	2 2 0 2 0 0 6	* 6	0 0 1 1 1 1 4	0 0 2 0 0 0 2
F (1) (2) (3) (4) (5) (6) Total	3 1 3 2 2 3 14	4 0 1 1 1 2 9	1 1 3 0 3 0 8	3 1 2 1 1 0 8	4 1 3 1 1 4 14	1 1 3 1 3 2 11	4 1 1 2 1 0 9	4 1 1 0 1 0 7	* 7	0 0 1 0 0 0 1
B (1) (2) (3) (4) (5) (6) Total	2 1 2 0 2 5 12	3 0 2 1 3 4 13	1 1 2 2 1 2 9	2 1 3 1 1 2 10	5 1 4 1 3 2 16	2 1 2 1 3 4 13	3 1 0 0 3 2 9	5 1 0 2 3 2 13	1 0 1 2 2 2 8	* 8

Matrix 9.3: Matrix display of differences and similarities.

I do not want to use the numbers alone to determine who is the same type as whom, but the numbers create an overview. A thing to discuss in relation to determining types is when two pupils are little enough different to be able to be called a “type”.

The matrix below calculates the difference in how many times two pupils have been in the same group and from this subtract how many times one of them has been in a group where the other was not. For instance ‘+ 2’ means that two pupils have been in the same group two more times than one of them has belonged to a group where the other was not a member.

	Z	Æ	Ø	Å	A	C	D	E	F	B
Z	*	- 1	- 6	- 5	- 3	- 3	- 2	- 5	- 13	- 9
Æ	*	*	- 9	- 2	- 5	- 5	- 3	- 6	- 12	- 11
Ø	*	*	*	- 5	- 15	- 9	- 11	- 8	- 6	- 6
Å	*	*	*	*	- 5	- 8	- 4	- 4	- 6	- 8
A	*	*	*	*	*	- 6	- 8	- 2	- 13	- 15
C	*	*	*	*	*	*	- 5	- 8	- 10	- 12
D	*	*	*	*	*	*	*	+ 2	- 6	- 5
E	*	*	*	*	*	*	*	*	- 3	- 11
F	*	*	*	*	*	*	*	*	*	- 7
B	*	*	*	*	*	*	*	*	*	*

Matrix 9.4: Calculations of differences in the number of times two pupils are in the same group from the number of times they belong to different groups.

Particularly Pupil F and B have very low numbers. Perhaps this reflects that Pupil B and F from IE3 were the ones who seemed to be less able to express how their learning took place. However, this determination of types should not be taken too literally. A problem with this type of quantification is that it is difficult to quantify qualitative differences. For instance when some pupils were taken to belong to a group it was often decided after discussions of what exactly the pupils each had said and if the slightly different explanations were enough alike to justify putting the pupils in the same group. One could argue that it is difficult to determine if a pupil’s “membership” of various groups is equally “strong”. Furthermore one could argue that the differences between various groups within one theme might not be as big/small as the differences between various groups within another theme, and that this therefore makes it difficult to treat all these differences as if they were of the same size. It is also seen in the previous three chapters that some group-members were put in inverted commas to indicate that membership of this group is not unambiguous, but in the tables above this “semi-member” counted as full members. I agree with these reservations and it is partly

for this reason that I also counted the number of times groups existed but where only one of two pupils was mentioned. Another consequence is that I do not want to say that this quantifying investigation is final. It is a way to find an overview and it is a tool to point at connections that ought to be more carefully discussed. However, slight changes within some of the groups might only make a few numbers go up/down and the numbers will only change a bit. Therefore I will argue that the matrices give a useful picture in a more broad sense and that these more broad criteria can point to possible types. The matrices might, however, more rather clearly reveal who is *not* of the same type. A rule of thumb could here be to look at the pupil-pair where the difference among them is less than -10. The number ‘-10’ means that if two pupils, for instance, have been in the same group one time, there are eleven groups where only one of them is a group-member.

These pupil-relations, which had a number less than -10, are the following: F-Z, Æ-B, Æ-F, A-Ø, Ø-D, F-A, B-A, C-B, and E-B. It seems that particularly Pupil is different from many of the other pupils.

If we look for pupils who might be of the same type, a “plus” indicates quite clearly that two pupils might be of the same “type”. In this connection it seems that Pupil D and E are quite similar. At least they are more similar than they are different. The pupil-relations, who only have down to minus 2, as a rule of thumb, are: Z-Æ, Z-D, Æ-Å, and A-E. What might be interesting is that these four pupil-relations can actually be linked in a kind of domino-chain: Å-Æ, Æ-Z, Z-D, D-E, and E-A. One could then pose the hypothesis that Pupil Z, Æ, Å, A, D, and E are quite similar. The mutual relationship between Pupil A and D is -8 and some of the relationships between some of the others are down to -5 and -6. But on the other hand, it does not go beyond -8. Even though -8 is quite a low number there is some distance to -10, which was the number that I used above to denote that two pupils were not of the same type. One might argue that this could be an incident where some broad criteria are necessary.

These six pupils (henceforth the six-group) are from three of the four interviews. Pupil Ø and C were also group-members in two of these groups but they do not seem to be of the same kind as their group-mates. Three of the pupils are Danish and three are English. Four of the pupils are girls and two of them boys. This might suggest that what the six-group might have in common is something which is related to mathematics, not nationality, sex, or determined by something that happened in one particular group.

One can then conclude that if we look at all the six themes at once it might be possible to determine the existence of types of pupils, namely that the six-group consisting of Pupil Z, Æ, Å, A, D, and E are quite similar. Pupil B seems to be quite different from many of the other pupils. The remaining three pupils are Pupil Ø, C, and F. Below I will recapitulate some of the discussions from Chapter 6 to 8 where I discussed each pair by itself to see if this might give a clearer picture of the existence of types.

A qualitative description of the six-group could be the following: Only four of the ten pupils talked about the planning phase and all four of them are members of the six-group. The six pupils who talked in favour of a cognitive drive as an important motivation are exactly the same as the six-group. The six-group are also all part of the group of pupils who talk about that external forces are important motivations. The six-group do also all speak for either the need to do several exercises (not just one exercise) and/or that practice is necessary but not sufficient. Most of the six-group do also mention all three of Hadamard's phases for the importance of the unconsciousness. They do also all (except one) talk about both constructions and self-activity. About visualisation, the six-group stands for a relative positive view of visualisation but none of the six-group pupils are among the pupils who find visualisation to be *very* essential. The six-group is however divided on the question on the relationship between the individual and social side of learning.

9.2.2 THE EXISTENCE OF TYPES WITHIN EACH PAIR OF THEMES

In Chapter 6 to 8 the pupils were put into various groups in each theme. I will now discuss these groups.

In relation to the first pair of themes it was clear from the discussions in Chapter 3 that the consciousness and the unconsciousness could be seen as complementary. If we here look at the groups with a number of group-member, these groups consisted of following pupils: {Z, Æ, Å, A, D, E}; {Z, Æ, Å, A, C, D, E}; {Z, Æ, Ø, Å, D, B}; {Z, Ø, Å, A, E}. The members of these groups are almost identical with the six-group, which consisted of the following pupils {Z, Æ, Å, A, D, E}.

If we look at the second pair of themes, most of the pupils seemed to say, more or less directly, that language is the main thinking-tool but that it can also hamper thinking.

However, there are degrees of views: The pupils fall into basically two groups. Moderate positive of language: $\{Z, \emptyset, A\}$ and more positive: $\{D, E, F, B\}$. The six-group is here spread out on both these two groups and Pupil \AA and \AA of the six-group is not part of any of these two groups. Instead, the six-group members are now in group with other pupils. This reflects that the general partition of the ten pupils into the six-groups and the rest do not stand at more local levels in the specific themes. Here one sees other patterns emerging. In relation to the discussion about schematic learning, which is another issue within the second pair of theme, the pupils fall into various groups: $\{Z, \emptyset, C, D, E, B\}$ and $\{\text{\AA}, A, C\}$. Both these groups are a mix of the six-group and others.

In relation to the third pair of themes, it seems that most of the pupils argue that learning has both a social and an individual side. The value of the social side is mainly when one experiences problems learning oneself. After input from the outside one can move on “alone”. Learning is individually and the social side is more used when there are problems. Pupil $Z, \text{\AA}, \emptyset, \text{\AA}, C, D, E,$ and F argue this. This group consist of most of the six-group plus three other pupils. It seems that Pupil A (from the six-group) and Pupil B actually learns more through discussion than by self-activity and for these pupils the order may be reverse, i.e. social learning comes first, then the individual. In terms of the other individual-social level, namely the visualisation and verbalisation, there are basically two groups. One group seems to argue that it depends on circumstances, perhaps branch of mathematics: $\{\text{\AA}, A, C\}$ and the other is relatively visual: $\{\text{\AA}, D, E, F\}$. Again these groups are different from the six-group.

A result might be that concerning types, it might not be possible to say that if a specific pupil is a certain way in, for instance, Theme 6, then he is a certain way in Theme 1-5. I would need more pupils to determine this as I have only a few pupils here and individual variations might blur the picture. On the other hand, these ten pupils exist and the investigation shows a great variety of strategies of learning and that one pupil does not necessarily follow one theory. The pupils do not even agree which theory to support most. But, on the other hand, types of pupils definitely exist, both within each theme as well as on the overall level when all six themes are considered. The types are furthermore cross-national, which might suggest that the groups reflect something that is related to learning mathematics and not the school systems.

9.3 THE CULTIS MODEL FOR ANALYSIS

I created the themes as counter-poles, which was based on knowledge from the theories. For instance that the views of the importance of language for learning were very divided. But what do the pupils say? Pupil Z directly says that it is “ping-pong” (ID-Z4) which suggests that there is a complementary relation between Theme 3 and 4. This pattern can also be observed in relation to the individual-social relation. But the first pair of themes, the unconscious-conscious, does on theoretical level seem to acknowledge each other. Going back to Chapter 4 and the discussion about complementarity and the possibility of creating a grand theory, we could state that the pupils seem to argue in favour of some kind of complementarity. For instance in relation to individual-social it does seem as if most of the pupils’ explanations support the idea that learning is individual, but when these *individuals* experience problems learning, the *individuals* need inspiration from *others* to assist the *individual* learning. Two pupils do, on the other hand, seem to argue in favour of the opposite direction. This is also supported by what I referred Burton (1999) for saying in Chapter 1, namely that her investigation of mathematicians suggests that learning is neither wholly social nor wholly individual. The discussion about complementarity is however not the same as having made a grand theory. But it might suggest in which direction to go. But a conclusion to make from the discussion in Chapter 6 to 8 is that the content of a grand theory depends on the type of mathematics and the grand theory must include that the ways of learning is not the same for all individuals. Also based on the discussion in Chapter 4, the grand theory must accept realism on the level of ontology.

Regarding the relationship between theory and interview data one could again (as in Chapter 2) raise the question of whom to believe, the pupils or the theories of learning mathematics. Here I find it important to emphasise that most of the theories behind the CULTIS model for analysis arose from empirical study. This means that it basically arouses from what other pupils might have expressed at some other occasions. As stated in Section 1.1.3, most of these other pupils were in another age group than the pupils in the present study. This could suggest that if we want to get wiser on how high-achieving pupils learn, we, in this study, need to put priority to what the pupils here said. This does put us back to the discussion in Chapter 1 about the Diversity Thesis and the Similarity Thesis. The discrepancy between what the pupils say and what the theories say was not that big. This might suggest

that the theories seem to have some explanatory power but it does not necessarily mean that these theories have an equally good explanatory power for lower-achieving pupils. When most of the things the pupils said were actually reflected by these general theories, one might argue that the learning theories tell how the pupils who actually learn mathematics, have learnt it. This depends on whether the Diversity Thesis or the Similarity Thesis is correct. However, even among these ten high-achieving pupils there were differences between how they learn. One could argue that if the ten high-achieving pupils in this study are this different, this could point to the fact that if one looked at the whole range of pupils from low-achieving to high-achieving, the learning styles, or potential learning style, would be even more different. In this sense the Import Strategy mentioned in Chapter 1 might not be useful. This is unless there are things which connect a lower-achieving pupil to what characterises high-achievers, for instance the emotional attachment or some of the other things mentioned by Krutetskii in Chapter 1. Instead the Individualism Strategy seems more relevant.

Another thing discussed in Chapter 1 was that high-achievers cannot take care of themselves. The pupils did all, in various degrees, speak in favour of the social element in Theme 6. This supports the discussion in Theme 1 about the necessity of guidance, even for high-achievers.

The framework was necessary if I wanted to work with several learning theories at the same time. For instance would the discussion in Section 9.1 and 9.2 not have been possible had I not had the tool of the six themes to split the pupils' statements into modules which I could then shuffle around with and put together again. It would have been far too complex and "messy".

One could also argue that actually the pupils' explanations exemplifies the themes, for instance when the pupils mention certain branches of mathematics in relation to one theme. One could also state that the use of the knot theory in the intersection of the interview seemed to fill its purpose as a prompter of thoughts as most of the pupils could relate to this piece of mathematics. Sometimes the pupils also by themselves gave an example from some branch of mathematics. Some of the things they say are independent on which branch of mathematics, others are not.

In Chapter 1, Carlson (1999) argues in favour of the role of some non-cognitive factors. One of things mentioned was the importance of a non-imitating environment as well as the importance of a mentor. This is actually supported by for instance Pupil E in (IE1-E1) who

explained that when he got a “cool” teacher, he really started to learn mathematics. But this approach is also included by for instance Mason (1985) and one might therefore argue that this suggest that some non-cognitive factors play a role for allowing the cognitive processes to work. Thus there might also be a duality here.

9.4 METHODOLOGICAL ISSUES FROM THE STUDY

In this section I will discuss the experience of the methods chosen. The section will argue that the design and conducting of this study involved many discussions and a variety of compromises between different options. Sometimes the choice was the “least evil”. Considering these things in the future might (other things being equal) make it possible to get closer to truth and to get an even more accurate description of the pupils’ understanding of how they learn, hence, how they actually learn.

9.4.1 LACK OF CLASSROOM OBSERVATIONS

I did not observe the classroom teaching in advance, which I argued for in Chapter 2. However, during particularly the analysis I realised that even though the researcher has studied mathematics herself, using books in English, and also attempted to probe the pupils etc., the researcher did not know the particular discourse of their classroom. For instance, when the pupils talked about “doing examples”, what did it precisely mean? From the context, it was clear that it mainly meant exercises, but this does not rule out that the particular class and their teachers might have developed a certain kind of examples or a certain kind of discourse about this. This could have contributed to the validity of the study. Observation before the interviews could also have given the pupils an opportunity to get to know me, developed trust and feeling more comfortable with the research. However, their openness might suggest that the “warming-up” phase in the interview was enough to develop trust.

A negative side with having the observation before the interview is that I will then not be the “stranger” anymore, which would be damaging for the positive effects from being a stranger, see Chapter 2. Instead the observation could have taken place after the interviews.

Due to practical reasons this was unfortunately not possible as the pupils were all last-year pupils and I had furthermore interviewed them towards the end of the school-year. The reason for this late interview was that I had wanted the pupils who were high-achieving in all the mathematics branches at this level and I had therefore to wait with my investigation until the end of the year where they had learnt what they were also suppose to learn. I will argue that the negative effects of the lack of classroom observation are not serious and the loss of being a stranger might have jeopardized the validity more than the validity would have gained through the observations. Furthermore, classroom observation after the interviews would be observations of how they handle a process of revision and would therefore not necessarily give knowledge of how the classroom normally functions. Others might argue differently and it is a matter of difference of opinions and the need to make a choice between non-perfect options.

9.4.2 SAMPLING AND PAIRING OF THE PUPILS

The sampling of pupils was based on the teachers' knowledge of which pupils would "fit". One could ask if I would have had the same result at another school or if the pupils had been grouped differently. As discussed in Section 2.7, the aim in qualitative research is not on reliability in a quantitative sense but more on internal validity. As the researcher takes so much part in this type of data collection and construction, nominalist reliability becomes difficult unless it is detailed described how the research was conducted. A detailed description was an aim in this study. A different pairing would have affected the study. As feeling comfortable is vital when being interviewed, one could assume that their descriptions would not have been as extensive as they actually all were. One could conclude that the validity of pair-interview as a research instrument is fragile as it depends on the teachers' co-operation and their knowledge of the pupils. In the present case, it seemed to have worked. The interview where relatively little was said (IE3) was also the interview where the pair was not supposed to be interviewed together. However, the small sample of pupils does not allow us to make any generalisations, but it is nevertheless striking that this is how it was.

9.4.3 OTHER TYPES OF QUESTIONS

One can ask if it is a problem for the thesis that the pupils did not speak much about issues falling into Theme 4. Neither did many talk about the role of planning in Theme 1 or visualisation in Theme 5. One could pose the question what this “low” number means. Had they been pushed hard to make a statement, what might the remaining pupils had said? Is the reason why the pupils did not say it themselves that it is not an important aspect? Absence of evidence is, however, not evidence of absence. Rather, the low number of statements about planning reflects the interview technique and that some choices had to be made balancing “interviewer control” and “exploration of pupils’ opinion”. As I wanted to interpret the pupils’ explanations in the light of the theories behind the CULTIS model for analysis, the interview had to be very little controlled. Otherwise one could argue that the investigation was a self-fulfilling process. This gave less degrees of freedom for the interviewer but the alternative was to ask questions which arouse from the theories. The analysis would then “go in circles” and be leading as the same theories would be use to analyse the interviews.

The interview style here did actually lead to very elaborate explanations. However, if a later study wants to gain more knowledge of how various branches of mathematics functions in the various themes, it might be necessary to “push” the pupils more to give examples and not just wait and see when they give examples themselves. But in this study, this was not the research question, but something that came up owing to the pupils’ mentioning of it. Another option is that towards the end of the interview, the interviewer could “change strategy” and ask more directly about issues the pupils had not mentioned by themselves. This would not affect the validity of what they had already said as this had been said in an open explorative interview. On the other hand, this should be done very carefully as one could argue that what the pupils would say as a result of a more structured interview, would, as argued in Chapter 2, not be as valid as it could be said to be self-fulfilling prophesy. At least it requires carefulness in the wording of the questions and it requires a very experienced interviewer who is able to keep an overview of what has been said during the interview and which themes still need to be discussed.

9.5 PERSPECTIVES FOR EDUCATION POLICY AND TEACHER TRAINING

In Chapter 1 I discussed the concept of usefulness in terms of educational research. I quoted Niss (1999) for stating that the ultimate end of research in mathematics education is to improve pupils' learning of mathematics, directly or indirectly. One could therefore pose the question how the discussions and conclusions in this study could be used. I will also try to relate this study to some of the present political discussions about the education system.

9.5.1 IMPORTANT TO USE VARIOUS LEARNING THEORIES SIMULTANEOUSLY

I would argue that if a teacher favours one learning theory over the rest, he risks losing some pupils as it seems that the pupils learn differently and furthermore it seems that their learning process is a kind of "mix" of theories. The pupils tended to group in some types but these types do each cover different elements of the various theories. This does however not mean that even if one teaches having several learning theories in the back of one's head, that one can reach all pupils and that all pupils will then be able to learn mathematics at the highest level at high school. My study does not support such a conclusion, as it does not investigate it. In this connection Aagaard and Lindberg (2002, p. 6) quote the philosopher Nørretranders for saying that there is a need for more Darwinism and less Leninism in the education sector. The Darwinist principle is, according to them, that you create a big diversity in the environment and then there will always be someone with the right qualifications. The Leninist principle is that one can design everything and predict the future. This is said in a discussion about unifying the universities, but I would like to use the notions to say that owing to the Darwinist principle, the teacher could vary his teaching, and not be stuck on one teaching philosophy. This should be balanced with the ZPT as the pupils would not learn if the ways of teaching changes too rapidly from what they have been used to. In teacher education one could therefore argue that it is important that the students learn about all types of learning theories. It is also important that the school system should not be subject to various "fashions" of the kind which I quote Hansen (2002) for in Chapter 3.

9.5.2 GUIDANCE IS IMPORTANT

As stated in Chapter 1, in Denmark, pupils do not perform that well in international comparisons. Furthermore, children of parents with low education do on an average level perform poorly in the People's School compared to children in the other Nordic countries. According to Henriksen (cited in Internetavisen Jyllandsposten, 25 March 2002), one reason is that the People's School in the 1990s developed into that children should take greater responsibility for own learning, but this has an unexpected backside. Weak pupils cannot handle this, they cannot control their own learning, and they have no idea of what it is they are to learn and therefore experience failure. According to Henriksen, the result is that weak pupils need to be guided through the education. Responsibility for own learning also means that the pupils are responsible for their failure. If we compare this discussion with what the higher achieving pupils of this study say, it becomes clear that guidance and being introduced to something are important factors in becoming a high-achievers.

In this connection, Mellin-Olsen (1987, p. 33) argues that activity theory “embodies the individual and society as a unity: the individual acts on her society at the same time as she becomes socialised to it. And for the purpose of the educationist, Activity theory has another great advantage: its key concept, Activity, focuses right away on what our project is usually about: the initiation of learning in the context of the classroom”. Mellin-Olsen does here not talk about whether activity theory is *true* or not, he argues that it is useful as it addresses issues that is related to what happens in *classroom* learning. One could argue that he might say here that how you learn varies, but when you sit in a classroom, this is how it is done. What he argues is a kind of complementarity relation between pupil activity and pupils being consecrated into the topics, the existing body of knowledge. This is also what Winther-Jensen when he says that to be “consecrated” into a culture the learner must to some extent be brought in touch with all the symbols of the culture (Winther-Jensen, 1995, p. 187). This means perhaps that it is not enough with an experience-centred (constructivist) view on education. This is only one side of the coin. Equally important is the consecration (internalisation, guidance).

9.5.3 PSYCHOLOGY AND PEDAGOGY

As stated in above in for instance Chapter 1 and 4, this study is about psychology not pedagogy. According to Vejleskov (1998, p. 96), even if *psychology* should reach a complete explanation of how people learn or of how thinking is developed, which according to Vejleskov is highly unlikely, then it can never determine what school should teach. *Pedagogy* is not just a question of insight into man's psychology, but a question of what one thinks is best for man and for society.

One of the issues discussed by the pupils in this study is whether discussions with "equals" (who also do not know the mathematics) is better than discussions with someone who knows it. The answers varied. A similar question is whether effective schools/classes can be inclusive? According to Lunt and Norwich the schools with the highest examination results were not the one's with the highest concentration of pupils with special needs. But as is also argued: "inclusion is a very important value in education, but not the only value in education. Just as important is quality teaching that addresses individual needs. In accepting that there are multiple contrary values in education, we need to resolve dilemmas by finding optimal balances and trade-offs" (Lunt & Norwich, 1999, p. 84). Thus, I will argue, for psychological learning reasons, schools should not be inclusive, but for more sociological reasons, they might. This is a question for politicians.

9.5.4 THE PUPILS' LEARNING HISTORY IS IMPORTANT

As discussed above in this chapter, the pupils' learning history seems, to some extent, to determine the pupils' later learning styles. On consequence of this is that the teachers should teach within the ZPT. Another related issue is that the transition problems between various stages of the Danish education system, which was discussed in Section 1.1.2, might be smaller if the teachers not only took into consideration the knowledge necessary for the next stage but also the teaching methods which the pupils would meet. Thus if the teachers at for instance high schools to some extent introduced some of the teaching and study styles of universities, it might, all other things being equal, promote that the pupils get the optimum of

their future study. This does, however, not mean that other factors such as the actual understanding of various pieces of mathematics becomes unimportant.

9.5.5 A DEVELOPMENT OF THE PUPILS' METACOGNITION

As written in Chapter 2, Schoenfeld (1985, p. 138) argues that performance on many tasks is positively correlated with the degree of one's metaknowledge. He furthermore states that "Expert behavior, in which the appropriate resources are routinely accessed, is a result of the experts' possession of stable conceptual models. Conversely, many students' difficulties are due to the fact that their conceptual models are unstable" (Schoenfeld, 1985, p. 139). Schoenfeld thus argues that knowledge of own learning process results in higher performances. The pupils in this study do clearly show that they possess a metacognitive awareness and we know from their teachers that these pupils are also high-achievers. But we do not know, for these high school pupils, what caused what. But in any case, the evidence suggests that there is a correlation between the degree of metaknowledge and the performance.

Another result of this study is that the pupils are able to verbalise and reflect on their metaknowledge. One can therefore argue that this shows a potential for developing a learning potential as this might develop and improve the learning for the pupils. Furthermore, as discussed above, this study shows that the pupils' learning history plays a central role for how they later on learn mathematics. These things can be argued to signify the importance of making room in the teaching for developing the pupils' metaknowledge. To aid the pupils in this process it is however necessary that the teachers become equipped to be able to discuss the learning process with the pupils. This means that teachers need to have a rather good knowledge of various theories of learning otherwise they will not be able to recognise the theories in what the pupils express in their own words.

One can then discuss if this kind of approach will improve the learning for all kinds of pupils. If Schoenfeld's results are also applicable on high school pupils, one might expect that a development of a metaknowledge could help any pupil on any level. But this does also depend on whether it is possible to discuss metaknowledge with lower-achieving pupils. A problem for lower-achieving pupils is, according to Krutetskii (1976, p. 299; see Chapter 1)

that for instance the memory of lower-achievers and high-achievers is different. The high-achievers usually remember the general character of a problem-solving operation while low-achieving pupils usually only remember the problem's specific facts. One can argue that a requirement for developing a metaknowledge is that when one reflects on one's learning history, that one does not get lost in details but is able to remember the general character of what one did and abstract from this. This is a question for further study.

9.6 SUMMARISING CONCLUSION ON THE RESEARCH QUESTION

The research question is: How do the high-achieving pupils say they come to understand a mathematical concept that is new to them? How can these reports be informed by various psychological learning theories of mathematics?

This study investigates this question through a method of qualitative un-structured interviews with ten pupils and the pupils' explanations are interpreted using a range of psychological theories of learning mathematics. A specific CULTIS model for analysis is developed to aid this analysis. The pupils are very elaborate on this issue. And not only do each of the pupil say many words, the pupils' daily language and homemade descriptions of their learning are quite easily interpreted from the CULTIS model for analysis.

One can then ask the following question: "so, how do the pupils say they learn a mathematical concept that is new to them?" I do not wish to repeat all the details of the discussions of the whole study. But a more general answer is that there does not seem to be one way of learning for these ten pupils. The pupils in this study are different, but it is still possible to identify some similarities. This is particularly the case with the six pupils, which I have labelled the six-group. What characterises the six-group is that they have the same view of the planning phase as well as the cognitive drive and the external forces as important motivations. The six-group do seem to share similar views on exercises and the role of the unconsciousness and most of them talk about both constructions and self-activity. They have a relatively positive view of visualisation but they do not all share the same view on the relationship between the individual and social side of learning. Other groups, or types, of pupils emerge if one focuses on the level of each pair of themes. It might then be tempting to conclude that if there is no one way of learning, learning mathematics might happen an

infinite number of ways. The study does not support such a conclusion. The pupils do clearly vary, but they vary within a range of possible ways of learning. This range seems to be bounded by what the various and different theories describe. The ways the pupils learn are thus criss-crossing on the various theories. This conclusion is based on that very little of what the pupils said can not be interpreted from the CULTIS model for analysis. What falls outside the six themes of the CULTIS model for analysis is the concept of a ZPT (zone of proximal teaching) and that the pupils' learning history to some extent determines how the pupil later on is able to learn.

One can then ask another question: "so what can we use this knowledge for?" Again, I do not wish to repeat all the details of the discussions above. The study highlights the importance of not teaching according to only a few psychological theories of learning. Included here is the necessity of both an individual and a social aspect in learning and an awareness of both the possibilities of language as a thinking-tool and its limits. Also issues of conscious planning are important as well as to leave time for incubation. One can also conclude that the development of the pupils' metaknowledge might improve the learning for the pupils.

Apart from this, the discussion in particularly Chapter 4 about the creation/discovery of a grand theory of learning, as well as the pupils' explanations, which ran across various theories, seem to provide a ground for a development of such a grand theory. Hawking writes, for physics, that "if we do discover a complete theory, it should in time be understandable in broad principles by everyone, not just a few scientists. Then we shall all, philosophers, scientists, and just ordinary people, be able to take part in the discussion of the question of why it is that we and the universe exist" (Hawking, 1995). Following this line of thought, once a grand theory in learning mathematics is found, if it exists, this can, together with more pedagogical considerations, hopefully improve the discussion of the education system and learning even more. Not only among researchers, but everybody, and hopefully this will inform education policy with rational research based results.

C is the mark you should always have made.
 It's a simple and forthright and manly-type grade.
 For an "A" gives your peer group sad indication
 Of a social life lacking inspiration,
 While "B" is overreaching for most humankind,
 Yet displays lassitude in the genius mind,
 And "D" is the sign of mental defective,
 And "F" invites violent parental invective,
 "C" is the best. It shows moderation,
 The goal of philosophers in each age and nation.
 (O'Rourke, 1987, pp. 156-157)

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Dr. Josef Mengele experimented with about 3,000 twins in Auschwitz in World War II. Only about 200 survived. CANDLES stands for “Children of Auschwitz Nazi Deadly Laboratory Experiments Survivors”. Mentioned in Chapter 1.

http://www.eu2001.se/eu2001/news/news_read.asp?iInformationID=12545 & http://www.eu2001.se/eu2001/news/news_read.asp?iInformationID=12545 (30 August 2002). The EU focus on education, mentioned in Chapter 1. EU press releases.

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Appendix A: Matrix display of the pupils' processed narratives

A.1.1 The Danish interview

This interview was conducted the Thursday 18 March 1999. It lasted 80 minutes and the transcriptions fills 1680 lines.

	Pupil Z	Pupil Æ	Pupil Ø	Pupil Å
<p>1st The consciousness</p> <p><i>- practice</i> <i>- planning</i> <i>- reflection and monitoring</i> <i>- confidence and positive atmosphere</i> <i>- motivation</i></p>	<p>-I have sometimes experienced that I could solve some problems without really having understood completely the overall meaning. When I have worked with the problems and looked at them, perhaps drawn some pictures and imagined some things, for instance with spatial geometry and the distance formula, then I begin to understand it more overall (72-83).</p> <p>(When they were asked how they got to the point where they have understood, 87-88)</p> <p>-I often feel that when we have had something explained I do not understand a thing. But then when I've done all the problems, then I sort of have worked with them within myself (90-95).</p> <p>-I think that very few pupils, when they see a proof, get a lot of meaning out of this (120-121).</p>	<p>-now we have begun to revise some of the things we had a year ago and I can remember that at that time it was one big chaos, one did not understand a thing, now I think, was it not worse (127-132). (When asked what has happened in the meantime, 134)</p> <p>-one has been doing problems, calculating (136).</p> <p>-it is like when you begin to learn how to cycle, one does not understand that either, but tumbles over. There are some things which begins to become automatic, in the end you do not think about all these things, they are just in the head and one uses them automatically (150-154).</p> <p>-I agree with Ø [ID-Ø1, 394-395] in relation to that you do not know in</p>	<p>(In connection with ID-Å1, 106-108)</p> <p>-it is necessary (110).</p> <p>-you don't know in which direction to go when you don't understand anything (394-395).</p> <p>(In connection with ID-Z7, 1047-1065)</p> <p>-well, this is a barrier, when you do not think that you CAN do this, or something like it. Of course you can (1073-1074).</p>	<p>(After the pupils for some time have talked about doing exercises in a positive way, they were all asked if that meant that it was good to do the exercises, 106)</p> <p>-yes (many said yes at the same time) (108).</p> <p>(In connection with ID-Z1, 682-689)</p> <p>-I agree, you need to have an interest in it because if you just continues with 'what can I use it to, I cannot understand this, I do not want to understand this', then you are not going to learn it (691-693).</p> <p>(In connection with ID-Z1, 784-790)</p> <p>-also if the teacher simply says that this is easy, everybody can understand that, then you feel stupid if you can't (807-810).</p> <p>-also because it is</p>

(In connection with ID-Æ1, 127-136)

-we have been calculating all the problems we have been forced to do.

Then one has understood a lot of single cases and one has perhaps been able to imagine for instance a plane, a line and that $r_1 \times 2r$ is the normal vector. Then one knows what all these things are, one knows it 100%, the problems one has been calculating (138-148).

-and I also think that what Ø says about being thrown out into it, I think that is necessary to get something out of it. I think that one would not learn anything if one in 1.g just sat and gape and just said f is a function and A is a vertex, one would not learn anything through this (another pupil agrees). One is only challenged the moment it is exciting or such. The motivation is also important, I think there is a motivation in the confusion as one really WANT to learn it. And one needs to have this as it does not help just having to sit and learn the whole maths LANGUAGE first (272-281).

-for instance with differential equations, first I was able to learn it on the level of notation where you sit and then you multiply with dx on the other

which direction to go. Then you try different directions and perhaps you are lucky (401-407).

(In connection with ID-Z1, 706-711)

-yes it's a game, but at the same time it is also a challenge.

Calculating at the grocer's shop is not a challenge. This here is something fun. And the fact that there is something you cannot understand is in itself is a challenge (713-721).

-it's the game that is the fun part of it (763).

(In connection with ID-Z7, 1028-1035)

-yes, this is the one side, on the other side if I think that I cannot get through here I think, well I HAVE to and then I'll just HAVE to (1042-1045).

(In connection with ID-Z7, 1047-1066 & ID-Ø1, 1073-1074)

-well there is some maths which is not so easy to understand. I think you can accept that. I think I just accept that there are things that I do not understand at once (1093-1098).

In connection with ID-Z2, 1176-1183)

so old (someone else agrees) you always hear about the old scientists, so great [said in a positive way in connection with a discussion of motivation] (851-852).

-you can sometimes use it without understanding it (1600-1601).

side, even if we are allowed to do so (all laugh). You can sort of see it, the small numbers and such and then gradually as you have understood THAT then you can move on to say what it actually is that you are describing, what a differential equation actually is (308-319).

(In connection with ID-Z5, 505-520, when asked if based on what was just said, it is important with smaller pieces, 522)

-yes, I think that very few people can understand it all in one big bite right from the start. You sit with the problems to solve and these are kind of little bites. When you have solved an adequate amount then you can see the bigger picture (524-529).

-it is also very important for me to be motivated [this bit is also in ID-Z5] (641-658)

-it's a lot like a game. When maths is this crazy system we can build up and you have to have this motivation - you need to think that it is fun IN ITSELF because that even from 1.g the pupils sit and ask, can we not use this shit to anything. I think, maths perceived as a system is interesting, that you can do these things (682-689).

-well personally I really don't care what I can use maths for. I

-I more feel like there may be something which I do not understand completely, but after all I understand some of it, and I know what it is that I just accept. The I either I say I won't understand more than I already do or I say, no, I really want to understand it. I think it depends on how complicated the topic is (1185-1189).

learn maths as I think it is fun (697-701).

-I think it is fun, well, it's like playing with Barbie, it's the same, it's a kind of game (Æ agrees). The more formulas you have, the more clothes you have to Barbie, the happier you are. Oh dear, can I also do this, it's getting wilder and wilder (706-711).

-I think that solving maths problems is like doing criss-crosses. The motivation is the same because if you can work it out it is SO COOL right (723-731).

-it is more fun if you are allowed to explore the maths (736-741).

-it is also really cool to be good at maths, there is a kind of show-off-effect, we can go and scorn at the modern pupils as they do not know this (754-761).

-it is also the fascination that oh dear this is difficult, but I can do it, this is to a great extent the motivation. For instance a proof where our teacher says that this is very difficult, then you really get excited, you really want to learn it if you can (784-790).

(When asked if it is there an advantage that the teacher says that this is difficult (792-794).

-not in I.g, the first months (796-798).

-there is nothing worse than if the teacher introduces something

	<p>as trivial and then you cannot figure it out. I think that it is nice the way our teacher is, he says something like, of course this is difficult, but you can learn it and it is really interesting to learn and it is also good for you to learn it. Then you become more motivated (812-818).</p> <p>-people are different but I think that most people has a very clear feeling of when they have understood things (1163-1164).</p> <p>-I don't think that I have ever met someone who was capable of convincing themselves that they had understood a piece of maths which they hadn't understood 1224-1227).</p> <p>-I have often met a person in Danish who says they have understood a text but then when you discuss it you realise that this person thinks the text is romanticism, whereas it really is postmodernism, ahhh (1232-1235).</p> <p>-I don't think I have the right attitude, that maths is something you work with, it is not just something you sort of have to UNDERSTAND in chunks but it is also something where you work with your understanding of it (1566-1573).</p>			
2 nd The unconsciousness	(In connection with ID-Æ2, 6-11 & ID-Z5, 13-23 & ID-Z3, 13-23) -it is naturally an	(When asked what they do when they meet some new mathematics they	-when one re-reads something I really think one can get this aha experience which	-it's like if you sit with a problem you can't solve then perhaps if you

<p>- preparatory work - incubation - illumination</p>	<p>advantage if some time passes (13-23).</p> <p>-I think that sometimes the aha-experience is necessary to make it possible for you to really see it (83-85).</p> <p>-sometimes if it has been a while since we began on a new topic within maths, I read it through again, from the beginning, and THEN “the lady” [herself] understands what it’s all about. Cause then one has got the stupid basic things right, then one can see the point behind (98-101).</p> <p>(In connection with ID-Z4, 163-185 & ID-Æ4, 187-194, when asked if the teachers does not define what the notions alpha and beta usually stands for, 205-206) -it’s possible that the teacher SAYS it. But I just think that if the teacher has said it then this sentence is so much worked in inside the teacher that he cannot see that perhaps the pupil has difficulty understanding it, perhaps the pupil needs time to let it sink in (208-212).</p> <p>-I’ve talked to several people who have had this experience that through the year you are taught a lot of subjects and you keep hanging in and understand more or less what it is about and one can solve the</p>	<p>do not understand, 1-4) -first and foremost we try to keep a distance to it and then perhaps later return to it as one then sometimes can look at it with different eyes. If you have come home from school and thought about something different in the mean time then sometimes one understands it better (6-11).</p> <p>(In connection with ID-Z2, 339-350) -yes, suddenly there is something which falls in place. You have a lot of bricks in your head and all these bricks suddenly start to fit. It’s like there continues to be built on this, it’s like rings in the water, who spread more and more and suddenly more things are being put together, chain together (352-360).</p>	<p>Z talks about (110-111).</p> <p>(In connection with ID-Z2, 1135-1143) -but don’t you yourself determine whether it is an aha experience or not. I think it is quite relative (1145-1146).</p> <p>(In connection with ID-Z2, 1191-1203 & ID-Å2, 1205-1209) -but I still think that you can have degrees of understanding. If you haven’t understood all the things that are necessary to understand, you don’t get so far, but you can still have an aha experience (1211-1222).</p>	<p>are a bit away from it and then you suddenly notice something you did not see before (500-503).</p> <p>(In connection with ID-Z2, 1191-1203) -the teacher also often say that either you understand or you don’t (1205-1209).</p>
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	<p>problems. But then when for instance up to a mid-term test or something else where one sit down and read it, then it is that you really realise things (another pupil agrees). Because then one can really see the greater whole (266-272).</p> <p>(When asked if learning mathematics was like being on a deserted island and then going on exploring/discovery, 339-343) -I think it has more to do with that before understanding, there is nothing, there is no deserted island, but then one can begin to take things out of the nothing-ness. It really goes from not being there AT ALL to suddenly being there (345-350).</p> <p>(In connection with ID-Z5, 378-386) -it is not a thing that you have been able to explore before, it is brand new (386-388). (When asked if it was a kind of revelation, 390) -yes (392).</p> <p>-I do not get aha experiences in humanistic subjects. In maths things need to be in place before I feel that I have understood them. And I also often think that there must be an aha experience that is larger than the others I've had (1135-1143). (In connection with ID-Ø2, 1145-1146) -to me it is very well-</p>			
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	<p>defined. There is an extremely clear distinction between when I have understood something and when I haven't (1153-1154).</p> <p>-am I the only one who has this experience that you can sit one day and then feel that you have suddenly understood spatial geometry and then you read it through and SUDDENLY, yes, it's there, now I've understood, BANG, a light has been lit (1176-1183).</p> <p>-I just think that if I have only understood a little of it, then I haven't understood it. Not until the light has been lit and I do not think that there is a kind of semidarkness in between (1191-1203).</p> <p>In connection with ID-Å7, 1313-1315)</p> <p>-if I only understands half, I haven't understood it at all (1319-1322).</p>			
<p>3rd The language</p> <p><i>- language as basic thinking-tool</i></p> <p><i>- basics important</i></p> <p><i>- schematic understanding</i></p> <p><i>- rote-learning</i></p> <p><i>- assimilation and accommodation</i></p>	<p>(In connection with ID-Æ2, 6-11 & ID-Z5, 13-23)</p> <p>-if at some point in a presentation there is something which one does not understand, then one does not understand the rest of it, then one has to quit and start all over, take the whole way up again (13-23).</p> <p>(In connection with ID-Z5, 39-44, when asked what she meant by 'point 1')</p> <p>-if you for instance sit down and then say, all right, what is a</p>	<p>In connection with ID-Z2, 1176-1203 & ID-Å2, 1205-1209 & ID-Ø2, 1211-1222)</p> <p>-yes, there are several degrees of understanding (Ø says yes) (1254).</p>	<p>(In connection with ID-Z4, 163-185 & ID-Æ4, 187-194 & ID-Z2, 208-212 & ID-Ø4, 214-218)</p> <p>- But it is difficult to see how to avoid it [that language can hamper learning] as we have to be introduced to the new notation (218-220).</p> <p>(In connection with ID-Z4, 222-238 & ID-Z3, 240-246)</p> <p>-I also think it is difficult [to find the meaning without the language]. Then one would need a kind of</p>	<p>(In connection with ID-Z3, 1618-1619)</p> <p>-it bothers me a lot if there is something I do not understand but which I have to use to calculate with (1623-1624).</p> <p>(When asked if it bothers her to drive a car when she does not know exactly how the engine works, 1626-1627)</p> <p>-that's something else, as I do not have a wish to understand how such a thing works</p>

	<p>triangle, and then line up what kind of concepts we are working with, it is so very important to have the concepts lined up. What is the problem really about, and what kind of area are we in, and then you can begin to work with it. And I think that it is very important to have some terms and have them worked through (50-59).</p> <p>(In connection with ID-Z4, 222-238, when asked if one can find the meaning without the language, 240) -I think this is difficult, I think it is very difficult (246).</p> <p>-it is sometimes annoying to use a tool if one has really not understood where it comes from (1618-1619).</p> <p>-the pupils in the modern line are in the topic, nature-subject not ALLOWED to have the slow learning where you build the things up in the basic steps (1636-1643). -there is a platform at the high school. In the folkeskole I found it difficult to understand a lot of things, but it is a lot easier here as you are allowed to begin from the bottom instead of having to have these notions which are really not being explained (1650-1673).</p>		<p>flair as it is almost hopeless when you do not have the foundation to understand where it is you are going (248-251). -it is necessary with the foundation, but it is as if we just begins. If there are a series of bricks then we sort of begin halfway. I guess the aha experience is to have got the bricks underneath (255-262).</p>	(1629-1634).
4 th The tacit - words	-if the teacher first says, this is the plane alpha, then one thinks,	(In connection with ID-Z4, 163-185) -it's like there is a	(In connection with ID-Z4, 163-185 & ID-Æ4, 187-194 & ID-Z2,	

<p><i>obstruct thinking - cannot explain, but do</i></p>	<p>oh I cannot follow that but then after a while when one knows all the rules which maths teachers take for granted. We don't know that l, m, n are lines, but then if you know this, it is simply so logically, but it does not help you in the proofs as you do not know that in the beginning (163-171). (When asked if one cannot learn mathematics if one has not understood that l and m are lines and f is a function, 173-174). -yea, you can probably learn it, but it is difficult to learn it from someone where we simply begin on different levels. And then if this person says alpha, then we sit and think, well alpha can be anything. But then when you know this (another pupil say yea) is it easier as you learn it from someone who just presupposes that of course it is like that (176-185).</p> <p>-I am a pupil-teacher in mathematics and I have been sitting with a pupil who get so caught/stuck with what is f, and it is idiotically to sit and wonder about this, it is not what it is all about. I want to get to the point. But this is what is relevant when one does not understand what it is that is really relevant (222-229). (When I said that this sounded like she talked about bricks built on top of each other (someone said</p>	<p>barrier, but it is difficult to say what it takes to make you understand. Our teacher has been illustrating in spatial geometry book and formed then with the table to be able to see what it is that is asked. It was that which made you understand. It's such small things and it is difficult to say what it actually is that makes you understand (someone else says mmm) (187-194).</p>	<p>208-212 & ID-Ø3, 218-220) -I often think that when the teacher says, this is so SIMPLE, can't you SEE it, and then one does not understand a thing (another pupil says yes). And I think that this has to do with what we just talked about, that there are so many confusing things and one gets stuck in details which actually does not have any significance for understanding the difficult issue one talks about (214-218).</p>	
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	<p>yes yes) (231-232) -it's like the teacher speaks a different language than us. The teacher comes with this big mathematics language and then we become so stucked with what the things are called instead of what it means (234-238).</p> <p>(In connection with ID-Z5, 281-286) -And then one does not need all these notions if you can see it for yourself, then I think that it perhaps becomes possible to work a different way. You can also first notions - then examples (286-290). (When they were all asked if it was a kind of ping-pong between seeing it and the language, 292) -yes (294).</p>			
<p>5th The individual</p> <p>- construction - vision - self-activity</p>	<p>(In connection with ID-Æ2, 6-11) -one has to perhaps sit down all by oneself and look at the maths, independently of what is going on. One kind of has to take the learning over to oneself (13-23).</p> <p>-write it down, point by point, it is a kind of systematic process where one says, now I understand point 1 where I can work it through for myself. I take it myself and now I understand what it's all about (39-44).</p> <p>-I also think we should stick to what you [Ø, ID-Ø3, 248-251] say about a person having flair for maths - being</p>	<p>-sometimes if one has been taking notes in the classroom, then very often if one starts to read through these notes, then one does not really understand the notes, but then if one by oneself, with the book as starting point, begins from the very beginning and write down point by point, then one can use the notes. This is because one completely by oneself begins all over, it is very often a help to have it all worked through (31-37).</p>	<p>(In connection with ID-Æ5, 61-66) -that's right, what you said about getting it down in form of a picture, to understand where it is, for instance a tangent (68-70).</p> <p>(In connection with ID-Æ5, 428-433) -but then there are others who would like to se an example, and some problems and then through them understand, And others are more oriented towards the proofs and it is THEM you want to understand to be able to understand the whole thing (435-438). (When asked if there are people who</p>	<p>(In connection with ID-Æ5, 298-302) -it is the same way when you calculate problems (Æ says yes) when one for instance calculate a parable - when you can see it for yourself it is a lot easier to find the solution (Å says yes) (304-306).</p> <p>-when I sit and read in the book then you need to think more about it - write a bit read a bit - whereas when you sit in class and everything goes so fast (455-458).</p>

	<p>able to see this thing for himself another way. And I also think that this were the dividing line is - the line between starting to think YOURSELF and creatively, cause if you really have flair for it you can simply see the spatial geometry, you can just see it in your head as soon as you hear it (281-286).</p> <p>-if there is something which I haven't understood, then I simply can't see it (362-366).</p> <p>-you have to sit within yourself and then say all right now I have understood that this is how things are connected. And I think that this is where the dividing line is, if you are just receiving, then I don't think you have understood the things. The minute you get it inside your head and get the understanding inside your head then I feel that you can then pull this thing out and in (378-386).</p> <p>(When asked how they feel their way through the maths, if they had a specific technique, 409-410)</p> <p>-I actually think this is very individually (someone else says yes). When we sit and discuss there is often a difference in when the understanding comes to the individual. I think it is very much about finding the way that you personally can understand</p>	<p>-we have been working with spatial geometry and the first week it was one big something which we did not understand. But then if one first begin to make the drawings and when one makes a drawing, then very often one understands what it is one is working with (61-66).</p> <p>-when you sit and read in books, you also need to see it in your head. You can for instance see a proof in spatial geometry, but this is just on a flat piece of paper, you also need to be able to see in 3-dimensional, within the head, to be able to understand what it is all about (298-302).</p> <p>-to me it is very important that I can see it. I have very often made a drawing because I WANT to be able to see (428-433).</p> <p>-usually when we are told by the teacher to sit in groups and practice proofs then most people (S says 'very individually') sit by themselves, individually. What is obvious for one is not for someone else, and it is important that you</p>	<p>actually understands the maths through the proofs, 440)</p> <p>-yes. But I think it is very different how you understand. Sometimes I do not understand it when there is someone who explain it to me, I need to see it in writing to be able to understand (442-450). (When ask if she means that she actually has to tell it to herself, she says yes, 452-453).</p> <p>-visualisation can be a great advantage for instance a cosine equation to use to find a t. It is an advantage if we do not have something to which to use it, then it is on the philosophical level that mathematics should be understood (671-680).</p> <p>(In connection with ID-Æ5, 1471-1491, when Z says that if one cannot understand it completely one would be left with a feeling that something was missing, Ø interrupts)</p> <p>-I don't think so (1488).</p>	
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	<p>something, I do not think we can draw up a systematic which works for all. Somehow you have to find YOUR angle, and then suddenly you understand (412-426).</p> <p>-I do not think I have this visualisation-thing that much - but I more prefer, as Ø says, to have everything in writing - it has to be something I can see (462-464).</p> <p>-as a pupil-teacher you have to find new ways to explain a thing, and then you have to draw the problem and see it from different angles. I think that if you are allowed to rotate it and such, also because you simplify it by sitting and explaining it, you have it split up in some phases and then try to make the cognition less and in smaller bits, right (505-520).</p> <p>-I sometimes experience a resistance towards learning if I fear that it is too difficult. And I think that it is a threshold which is difficult to get kicked down. It is difficult to overcome that threshold if there is just someone who stands and talks and talks. Then you kind of keep a distance to it, but in the moment where the cognition has to be internalised in you, right, then you cannot keep it at a distance then you HAVE TO relate to it, you are forced into the</p>	<p>sit yourself and work it through so you KNOW all the little steps (579-586).</p> <p>-I'm not saying that I absolutely have to have a picture, only if it HELPS me to understand (1471-1475). (When Z asked her if she would be able to understand if she cannot have the representation, 1477)</p> <p>-no, I do not think that I would be able to understand it completely then (1483-1484). (Then Z ask if one would be left with the feeling that something is missing, 1486)</p> <p>-then I would just accept it, what I cannot understand, and then I would just understand the calculations and then accept that I can do this part of the theory (1488-1491).</p>		
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	<p>situation where you say that you cannot have this threshold (557-570).</p> <p>(In connection with ID-Æ5, 579-586) -and when you write it yourself you are better at locating the places where you do not understand, then when you take the step yourself then you can SEE, so THAT'S what I do not understand and then you can do something about it (588-598).</p> <p>-I don't think that what a teacher says brings me anywhere in relation to learning mathematics. I have to take the initiative myself to be able to learn. It is also very important for me to be motivated [this bit is also in ID-Z1] (another pupil agrees) I have to take the initiative to learn it myself. The role of the teacher is more like a consultant I can use when I have localised what it is that I cannot understand. BASICALLY I do really not think that the teacher can help me UNDERSTAND the stuff. The teacher can really not know if the pupils just sit like vegetable and do not understand (638-657).</p> <p>-I often think that in maths it irritates me that I do not understand the MEANING of it, right, as if I do not understand the connection within it,</p>			
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	<p>right (1331-1334) (When asked if she with 'meaning' meant the internal connection and not so much the use of it, 1336) -No, not what I can use it for, but there has to be some kind of MEANING with it right. Most maths is so beautiful, everything fits (1339-1343).</p>			
<p>6th The social -internalisation - guidance and interaction - ZPD - speaking aloud - hearing - discussing</p>	<p>-there is also another thing that is good for me, which is to talk to other people. It is extremely profitable. Also to talk with someone who also do not know, because this way you get some other angles to it which could be the angle one misses (469-475). (When asked what about talking to people who have understood the maths, 477) -this is not always good as they are really caught by that they have understood, and then they say oh that's so easy, ha ha (479-481). (When asked if you can sit and learn from talking to yourself the same way as when you are in groups, 572-574) -I REALLY think that you can do it yourself (Æ says yes) but you can also do it in groups (576-577). -it happens often that when you are about to explain someone what it is that you haven't understood then, while you are explaining it,</p>	<p>(In connection with ID-6Z, 469-481 & ID-6Å, 483-486) -very often if you try to explain to a person what it is all about then while you are explaining it, then you understand it yourself (1-2 persons agrees). It is really not certain if you have COMPLETELY understood what it is all about when you BEGIN to explain it to another person, but you really also explain it to yourself (488-493). (In connection with ID-Z5, 638-657) -well I do not think that it is just about the teacher, it is a combination of that the teacher comes with some inputs which you then have to work with yourself and then you can return to the teacher and get some new (659-663).</p>	<p>-it is a self-perpetuating process if you sit and think about it yourself and then it is like Z says, you can ONLY see the one direction where you began as you haven't understood the basics in the problem you are solving. Then it is better if you get some other angles. Gives the overall understanding (531-535). -you just sit and get stuck with yourself and think that THIS is the way to go (539-541). (When asked if that meant that one has to realise that one should not always just follow a certain way, you sometimes have to say, well, perhaps I can't get so far this way, I'll have to think of something else, 543-546). -yes, I guess this is necessary (548). -get some input (552).</p>	<p>In connection with ID-6Z, 469-481) -and then afterwards you sit, oh well, that's perhaps all right, but it is better if you can discuss it on equal terms with someone who is in the same position as you, then you discuss and can also reach it (483-486). -find someone to discuss it with or something like this (550). -talk with the others in class about problems nobody can figure out (554-555).</p>

	<p>you suddenly realise what it is (\AA says yes) (608-614).</p> <p>-I do not want to underestimate the humanistic subjects, but I feel that about maths and physics that if I am not introduced into these things they are closed country to me, I just don't understand it (888-906).</p>			
Outside themes	<p>-I think it also has something to do with that we have been raised to that our cognition is visual in some way or other. Many of the things we learn are learnt through our eyes (464-467).</p> <p>-I think that it is good for me to have a teacher who knows his material (638-639).</p> <p>(In connection with ID-Ø7, 1018-1023, when asked if maths really is this huge ghost or it is just bluff, 1025-1026)</p> <p>-I have this fear, which I do not have in humanistic subjects AT ALL, that there will be a ceiling to what I am able to understand. (1028-1035).</p> <p>(When Ø asks if this is the motivation, 1037)</p> <p>-no no I just feel that I might as well give up as I cannot understand this (1039-1040).</p> <p>-I just think that in maths, it's like I am a vessel to be filled and there is a limit in the vessel. I am convinced of such a point cause</p>	<p>In connection with ID-Z7, 1125-1126)</p> <p>-I also think that it is the maths (1130-1133).</p> <p>-there are some things which I'm confronted with that goes beyond my ability to understand, but I won't call it a wall (1307-1311).</p>	<p>-I find it difficult to believe that for instance Danish cannot be just as difficult as mathematics, but it is never being presented that way. It's like Danish is something everybody can understand, at least it is accessible to everyone whereas maths it uh a giant ghost of which you sometimes see a bit (1018-1023).</p> <p>-some say that when they can use it to solve a problem, then they have understood; others that they haven't understood before they can see the great picture (1170-1174).</p>	<p>-I do not see it as clearly defined as Z, well, it's like that at some point the maths gets so difficult that you only understands half of it (1313-1315).</p>

	<p>there has to be a reason why some people really cannot learn that maths no matter how many times they get it explained (1047-1065).</p> <p>-I have understood the topics we have been taught to far, but that does not mean that I'm good at maths (1100-1104).</p> <p>(In connection with a discussion about the difficulties learning maths they were asked if it was the teacher's way of formulation or it was of the maths (1125-1126) -I think it is the maths (1128).</p>			
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A.2.1 First English interview

This interview was conducted the Thursday 17 February 2000. It lasted 70 minutes and the transcriptions fills 880 lines.

	Pupil A	Pupil C
<p>1st The consciousness</p> <ul style="list-style-type: none"> - <i>practice</i> - <i>planning</i> - <i>reflection and monitoring</i> - <i>confidence and positive atmosphere</i> - <i>motivation</i> 	<p>(When asked how they know what they did wrong (99-100))</p> <p>-the teachers when they mark you, they usually write down how you should have done it (102-104). (When asked if it was every step or hints (106))</p> <p>-it depends on the teacher again. Mr. X tends to just write hints, prefer you to work it out for yourself with help if you need it. Mr. Y tends to write out just the answer of sometimes the whole method (108-111).</p> <p>-if you just got the answer written down, then you feel like you don't need to do it again even though you got it wrong. If you just got hints then you tend to feel that you have been hinted how to do it, you CAN work it out for yourself. So you are willing to try again (125-130). (After having interrupted C, IE1-C1, 219)</p> <p>-yea, you have to make it a leap. When you are told that this is how you differentiate something like $3x^2$ square differentiate $6x$, you are told that you have to accept that that's what you do before you can move on. (A little silence before he continues.) It's sometimes quite helpful to have a proof or something written out just to help you accept it, it is helpful to see how it's done cause then you know where the method comes from (237-241).</p> <p>-some people prefer when they have a really sort of puzzle-type-problem and not like this (the maths they got in the intersection). Some people find that the best way to approach it is to go through it really</p>	<p>(In connection with IE1-A1, when asked what works best (113))</p> <p>-the hints, because then you have a second go, cause there is only a limited number of questions on a topic (115-123).</p> <p>-if you just got the answer then you think, well I still don't understand how to do it. It is just the answer it does not mean anything. What you need is a written method of how to do it or a hint to show you the method (132-135).</p> <p>(A bit after having been asked to describe their learning process, 211-213)</p> <p>-First thing that happen in your mind it that it is hard to accept something new unless you know it is right (234-235), then interrupted by A.</p> <p>(When asked how to attack any specific problem (261-262))</p> <p>-think of the general solution, think of the general formula of equation you can put it into. Find similar examples and try to fit in to that, in a book or in the notes you have (268-273).</p> <p>(When asked if it is a coincidence that they find out the personal tricks or if someone told them (564-565))</p> <p>-it seems natural, nobody's told me (567).</p> <p>(In connection with IE1-A1, 658-661)</p> <p>-I find that really boring, I want to achieve with the minimum effort. But I'm not satisfied with minimum effort and bad results. I've got some tendency to just go for quick reasoning and quick approach</p>

systematically and write down the sort of problem and the techniques and then sort of trying to apply them systematically and work through, whereas other people and I just sort of look at it and not apply it systematically. It's difficult to describe exactly what actually goes on but I sort of look at it and sort of just make connections between something that you may have seen before. I tend to just sort of look at it. I suppose it's to do with the way you think when you come up with a new problem, you think about it systematically or laterally. I tend to think around it and tend to just see ways of doing things. It's hard to, I don't know exactly what actually goes on. Just sort of relax and sort of trying to think about the answer rather than start to work it out (593-631).

-Some people like to learn by doing, you know sort of finding general, they'll do loads and loads of example like E and sort of just being told how to do something and then just practice and practice all over again. Really boring (658-661).

(Said about one of their teachers) -he does have a bit of a tendency to sort of prove things. It can be quite useful to see where the methods come from, but sometimes he does get a little carried away. You may not understand, but it is interesting, cause if you are interested in something (interrupted by C) (688-698).

-some people just want to be told to do examples, find it helps them learn (717-718).

(When asked if he did not think one could learn from doing examples, 722)

-yes you do learn how to apply the method. (C (Interrupts): you don't understand it.) helpful in terms of exams. It helps you to be able to work around slightly different forms of the problems, being able to apply the method

instead of going through the whole thing all the way through. If a question is asking for a detail, I just go straight into it, instead of going all the way around and proving that the detail is valid. Some people think like that, if they see a question they wanna verify that the question is valid. Some people they think about the question and see is it a valid question, does it actually mean anything (663-680).

(In connection with IE1-A1, 688-698)

-if you find it boring (C: it makes it even worse). You're gonna do it anyway so you might as well do it and get it right and you feel good about it (700-708).

(In connection with a discussion of what is useful to do for an exam, IE1-A1, 724-742)

-if you know the basic integration, for example, there is no point going through it for revision. There is no point in going through your notes on integration, cause you know how to integrate. What you need to do is have loads of loads of practice (732-736).

(When asked if the situation in the intersection was different from the usual learning situation, 757-758) -this is individual, you don't get taught, this is not teaching, this is self-learning, and I can't teach myself something new. Well I can if I REALLY sit down and can be bothered, but it is really difficult to motivate yourself to something like this, it does not trick anything I know from before (760-764).

quickly without making any stupid mistake, and so it is definitely useful in terms of exams. (C: What you need to do is have loads of practice) yea, you wanna get to understand it fully and understand where the method comes from (C: Not really, you don't want to know where it comes from and prove it, you just wanna be able to) well it CAN be helpful (724-742).

(When asked what they would do if they had to present it to their classmates, 786-788)

-I would draw them out myself. I just take each paragraph or each set of ideas. Take each paragraph and draw out for myself, write it out for myself, try to work out how they fit into the argument I've got on page. I definitely find that helps me to understand the concept or idea. I need to see how it is applied and be taken through that applied or going through myself before I can get a proper understanding (790-800).

(In connection with IE1-A2, 710-713).

-for something like this I would find it useful if there were examples of how each thing work and not for me to do, but it working through an example like say, introduce some ideas like in the first paragraph. Went through some examples, I find that helps me to see how the method is applied. This gives you loads of new ideas and it is only when you get to the bottom or start to look at some other examples like knots in here (the knot book) you start to see how it works and sometimes you can miss out some early details if you do that, whereas if you got examples and you understand each bit before you move on. I think that's better. I definitely want to find examples of how each idea is used and what it relates to in terms of what you are actually using it for before I move up (766-783).

<p>2nd The unconsciousness</p> <ul style="list-style-type: none"> - preparatory work - incubation - illumination 	<p>-if you can't do something, and you spend AGES thinking it over and you start get frustrated and then you don't think clearly. If you go and do something totally different, you relax, sort of generally open yourself up to the problem and just sort of think about it when your are generally not trying hard to solve it. (837-845).</p>	<p>(When asked how they work out the personal tricks (554)) -It just happens, you happen to do it and it works. Different people different things (556-562).</p> <p>(When asked why it is good to read things again, IE1-C5, 813-814) -a word can have more than one meaning or a phrase can have more than one meaning and if you have the completely wrong meaning and you extend your ideas on that and you read back, then you see a clear difference, 816-822.</p>
<p>3rd The language</p> <ul style="list-style-type: none"> - language as basic thinking-tool - basics important - schematic understanding - rote-learning - assimilation and accommodation 	<p>(In connection with IE1-C5, 502-506) -if I have to memorize from notes, it doesn't happen so much in maths cause it is more understanding the method rather than facts you have to learn. If I am revising from my notes, I find that it helps if I actually write the notes out again, just copying them. Because if you sit down to revise, even if you read it aloud, you remember some things. But if you write it out, you sort of read it again and then write it again and it sort of reinforces it. And I definitely found that if there is something I need to memorize, sort of examples or methods, equations, how things works, I definitely find it easier if I write as an aid to memory (508-518). (In connection with IE1-C3, 525-536) -it's fairly easy to learn a general formula for loads of different things like trigonometry and things like that, you can just LEARN the general formula and every time you get a question you can just sit and use it and get the answer out. But then if you come up against a problem which is SLIGHTLY different, then if you've just memorized it and you don't understand how it got there, so you've got no change to work back and work it our for yourself, whereas if you understand it you can see where it comes from and where you need to change it to fit what you've got (540-547).</p>	<p>(When asked to describe their learning process, what actually goes on in your heads, while learning mathematics (211-213)) -I think of previous experience or maybe go back to what you know and current fit in the new thing to what you know. If you can do it, it's easy, cause it is just addition. If it is completely new, unless you are taught it, like so that it's very easy to understand. You can get quite, cause there is no way you can learn from a book (217-228).</p> <p>(In connection with IE1-A1 & IE1-C1 about only being able to accept something one knows is true) -the generalised form, like integration, if you can link it to something you know like $n+1$, then you can fit that into integration for example. Then it is easy, but if you just have an example, er $2x$ integrates to x square, then it is quite hard to think how did that happen (253-259).</p> <p>(When asked if they knew other tricks (520-521)) -remembering or learning (525)? memorizing something then you need to know the set number of points so you need to write them out and find some sort of sequence in order to remember them, But if you are just learning, then it's about understanding, you don't need to remember the detail, just need to know the overall concept, principals (532-536). -it's important to have good</p>

	<p>-think of how to do something for yourself and how something works, you put it into terms that you are happy with and you understand and then you find that you can apply that back to the more complicated explanation that you've first been given, then you know that you've understood (824-828).</p> <p>(When asked if it is important that the words come first and then you can see what the words are about, or opposite (863-865)) -I don't know really. It's difficult so I think it probably depends on the sort of problem. If it is a very sort of visual problem where you have to sort of think it through maybe in 3 dimensions, I think it is probably better to have the picture first and maybe dealing with graphs as well. But if a more of a sort of linear methodological process it might be better to have the words first and then the pictures to help you understand, cause it is the words you are trying to understand. It all depends on what it is you are trying to learn, I think (869-880).</p>	<p>grounding in maths, good basics. Otherwise it is really generally quite hard (589-591).</p>
<p>4th The tacit</p> <p>- words obstruct thinking - cannot explain, but do</p>		
<p>5th The individual</p> <p>- construction - vision - self-activity</p>	<p>(In connection with IE1-C5, 158-165) -it's better NOT to be told 'oh that's wrong, try again, it's better if you're told, well that's close but here you've got, you know, minus instead of plus, something like this. Cause when you can see for yourself what you've done wrong and how that's affected the answer. That helps you when you can wait and see what you've done wrong (169-174).</p> <p>(In connection with IE1, C7, 180-187, about if his answer had something to do with problem-solving, what about the mathematics BEHIND it, 189-190) -you may have understood it when you can skip ahead and see</p>	<p>(When asked how they are being help at the 'Clinics' (158)) -give you hints. Depends on the teacher again. Mr. X he gives you loads of hints, and encourage YOU to work through it, and YOU to achieve the answer. He won't say 'oh that's wrong' (162-165).</p> <p>(When asked if there were other techniques beside reading aloud, 502-503) -if you can put on paper then draw out, 505-506.</p> <p>(In connection with the discussion about learning and memorizing in IE1-C3, 525-547) -For basic principles you need to make up you own techniques which work to understand how it works,</p>

	<p>what the next step's gonna be. For instance in a proof. If you can SEE it, where it gonna be, and work it out for yourself before it's up on the board, you know that you can understand it for yourself. When there is somebody asking a question and YOU know the answer and you know how to explain it, you feel comfortable (201-209).</p> <p>(In connection with IE1-C1, 261-273)</p> <p>-I tend to find that I learn better if I look at loads of examples for myself and deduce for myself how it is done and how it works rather than being given at the beginning a general formula and fitting everything I get into that. If I can work it out for myself from seeing examples of bits of equations and things like that, then I find I can more easily deal with slightly different problems cause I think you have a better understanding of how something works if you worked out the method for yourself. I find it helps me to see. Seeing how something totally new like integration works in an example, just sort of looking at the way numbers move around in the equation. I find I can spot from looking at several different examples you can spot how it moves as you operate the method through it. And as you see more examples and just seeing what goes where, it just (snaps fingers) you know, make a connection between different parts (288-303).</p> <p>(When asked if the two hours they have in each lesson was enough to understand (305-308))</p> <p>-you can get the impression that you understand it and then you get back into doing your homework and come across a SLIGHTY, a different question – if it is something new you've only just learnt, you might not be able to see immediately how to do it. Find one of the teachers and get him to explain. I find that there usually is enough time if we've</p>	<p>how the problem is solved, how the actual detail in how the structure happen in between (549-552).</p> <p>(When asked if C did it the same way A, mentioned IE1-1A, 593-631)</p> <p>-more or less. I tend to learn more when visual. Try to make it more visual. I wouldn't just draw it out, I would think about it (635-642).</p> <p>-it all boils down to the teaching method and the teacher, not just about, it is a two-way thing, it's more about you learning as well as you being taught properly. If you are taught in a way you can fit in, then it is good (712-715).</p> <p>(When directly asked if he did the same as A explained in IE1-A1, 790-800, about what he would do if he had to present this to the class)</p> <p>-that's like a visual thing, that's how it works best. Draw it out. You cannot do many worked examples, it's just getting to know the theory so if it is like definitions, I'd write it out in my own words. Draw a diagram if that helps and trying to understand for yourself and read that thing again to see if it is the same and then present it and get at the key bits and then just talk about that and once you've got that frame then you can put the rest, all the details in afterwards (803-813).</p>
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	<p>been like I said working through it on the board and discuss it at class. But if we spend some of it doing examples I find it sometimes harder to understand and I need more help outside of the lessons. I think it is better to go over something, sort of completely and discussion and turn it around in your head and see how you understand that works and where the methods come from before you start trying to do too many practice questions (315-332).</p> <p>(In connection with IE1-C6, 453-465).</p> <p>-I find it much easier to understand I you just sort of visualise the whole thing. Cause some people aren't that good at sort of imagine things spatially; it's quite useful if you learn, especially with something like this where it is not easy to express the thing on a piece of paper in terms of a diagram or drawing or something like that; it's difficult to draw it. Then it's easier to visualise, if you can do that (467-475).</p> <p>-if you just read a sentence, if you just take that as the words. If you start to visualise it is difficult for you to see any other way it can work. If you start to visualise what it is talking about then you can SEE all the different ways in which it can happen, make you a broader definition a broader explanation than just what you are initially given (853-861).</p>	
<p>6th The social</p> <ul style="list-style-type: none"> - <i>internalisation</i> - <i>guidance and interaction</i> - <i>ZPD</i> - <i>speaking aloud</i> - <i>hearing</i> - <i>discussing</i> 	<p>-now there is more opportunity to discuss everything (55-56). (When asked (58) if they feel they learn more now that they have more discussions)</p> <p>-yea, I think it is much better to have a smaller class and do more discussions and so working through it. (when C says 'Confident' (65) A immediately continues) yea, it makes you more confident with what you are doing, you feel comfortable with asking questions if you don't understand a step that you have</p>	<p>(When asked (58) if they feel they learn more now that they have more discussions)</p> <p>-yea (60)</p> <p>(When asked why it is good with discussions (63))</p> <p>-confident (65)</p> <p>(When asked why C says that there is no way you can learn from a book (227-230))</p> <p>-it is not interactively, you cannot ask the book a question (232).</p> <p>(When asked just after the intersection what they did to</p>

	<p>gone through and other people in the class feel they can answer instead of the teachers, so if you understand it, we can all share ideas between us which makes you more comfortable in how you deal with it (60-72).</p> <p>(In connection with being asked if reading aloud always helps (480) -it is for true actually, cause when you read in your head, you tend to sort of, not necessarily skim-read, but not really read every word. But if you read it out loud, you HAVE to think about every word, every thing you are saying, so you think about what the sentence means and what the words means (484-489).</p>	<p>understand it (453-455)) -read it through, the whole thing, and then remember the bit which you don't understand and then you read that aloud to yourself. You can hear it, externally, and also sometimes if you think of visual images, cause this is mostly 3D, it's really hard to put on paper (462-465). (When asked if reading aloud always helps (480)) -it always helps (482). If you just read in your head, I'm just read it and don't understand. If you read it aloud and you come across like a phrase that sounds really awkward for example "in the plane whose interior intersects the diagram in one..." you just read it over, you read it, but you don't think about it, but if you read it out, you think about the disc, and then you visualise the disc, and then you visualise the plane (491-500).</p>
Outside themes	<p>-it's like when you are little, if you can't read a word, the way you are taught to read it, sound it out, like c-on-ven-tion. If you don't understand what it means, you read it through slowly and you sort of can't help saying it out for yourself. It is something you build up through the way you are taught to do things, right from the start (571-581).</p>	<p>(When asked how they know if they have understood a new piece of mathematics (180-182)) -when you get the right answer. At the end we are given that one question to do. Then if you do it and get it right then you know you've understood (184-187).</p>
Some utterances and observations during the intersection	<p>Observations: First they both read, C looks in (McLeay), A joins in and they begin to discuss the questions</p>	<p>- reads aloud during the intersection (406-409).</p>

A.2.2 Second English interview

This interview was conducted the Monday 21 February 2000. It lasted 65 minutes and the transcriptions fills 1239 lines.

	Pupil D	Pupil E
<p>1st The consciousness</p> <ul style="list-style-type: none"> - <i>practice</i> - <i>planning</i> - <i>reflection and monitoring</i> - <i>confidence and positive atmosphere</i> - <i>motivation</i> 	<p>(In connection with IE2-E7, 80-81, a comment to E)</p> <p>-you just go back and look at when we did an example in class, you read the notes and make sure that you understand it (83-84).</p> <p>(As a comment to what E says in IE2-E5, 109-115 about his use of notes)</p> <p>-I don't do that. I just read the book and make sure that I understand the book. I work through the exercises in the book and if I get it right, move on to next exercise. I very rarely write revision notes from the book (117-119).</p> <p>(In connection with IE2-E1, 199-200)</p> <p>-we take different approaches. The book has the answers in the back, so we work out when we got the right answer. Calculate backwards. Most of the times, there will just be one stupid mistake, that you actually understand it perfectly (202-207).</p> <p>(D interrupts E in IE2-E1, 356-370) in the discussion about how a teacher should be)</p> <p>-when you get to A-level your teachers are confident in that they can teach you well, but up until then in order to inspire an interest in maths, you need someone who is nice (interrupted by E) (372-374).</p> <p>-but it IS very difficult to be interested in maths (382).</p> <p>-yea, I used to like all those logic puzzles when I was younger (interrupted by E) (391-392).</p> <p>(When asked that they have to learn to calculate to apply the mathematics, 398-400, both said</p>	<p>(When asked how they work it out together, 193)</p> <p>-we just sit and talk about it and try different ways, trial and error sometimes (199-200).</p> <p>(About teachers)</p> <p>-they gonna be enthusiastic, I know teachers just have different approaches, I absolutely hated maths until my last year of GCSE (356-358).</p> <p>(When asked what made him change his mind, 360)</p> <p>-I had this really cool maths teacher, who was quite a good guy, he was really intelligent, he knew his stuff, but he was quite cool, he just made it fun, I actually learnt more with him in a year than I had done ever before cause he actually paid attention. Some teachers are better (interrupted by D) (362-370).</p> <p>(After the interruption by D, when discussing that there might be differences in how the teacher should behave depending on the pupils age, about being at A-level)</p> <p>-but by that time you want to do maths and you are interested in it and it's your choice to do it. I think a lot of people here are quite intelligent and didn't do well in maths when they were younger. I'll blame it on the teacher though. They are all lacy people (interrupted by D) (362-370).</p> <p>(When I ask them why it is difficult to be interested in maths, 384)</p> <p>-I don't know, I was interested in more applied mathematics, maths that had a purpose, or a problem that seemed impossible, like a brain-teaser (interrupted by D) (386-389).</p> <p>-I don't like basic addition and subtraction and a lot of rubbish, I like when it was used to solve</p>

yea, and then followed)
-but it's sort of boring stuff
(interrupted by E) (402-405).

(When asked to explain what she meant when she previously said that one is pushed harder now, and why this is good 449-450 & 454).

-you're pushed harder. It's good cause you're pushed to the edge of your ability where you just can't get away with just doing your minimum. They are pushing us to do more and that's good because it prepares you for going to university and study the maths more (452-460).

-it's like tightening up all the loose ends which you don't HAVE to do to pass the exam but it's more satisfying to you (interrupted by E) (469-471).

(In connection with IE2-E1, 727-730)

-I'd go through it all and get what I need and I'd take another book and understand the basic bits and then once I completely understand that I'd go on to the harder bit. I wouldn't go into each bit straight away (732-734).
-actually now when I come to think of it, I think when I went through it I DID try to make sure I understood each bit first (745-746).

-it's quite interesting that when you gave us this to do we didn't write anything, er, because I don't know why I didn't, but sometimes I find it helps to write notes, for example to write down things in bold, but I didn't this time for some reason (1199-1202).

interesting brain-teasers and things (394-396).

-I thought trigonometry was the most useless piece of rubbish until later o when I found out that you can solve other things with it, do loads of stuff with it. When it is just 3 graphs and that you are forced to memorise them, you can't cope with that (407-411).

(In connection with IE2-D1, 452-460)

-it is all those things that aren't in the textbook but those little extra's that some who studies maths further can appreciate and that you can then go and explore and that's a lot more interesting (interrupted by D) (464-467).

(In respond to IE2-D1, 469-471)

-that's what I mean by writing my notes. You've got all the little bits together and you have a nice picture. Everyone is satisfied when they know everything (473-477).

(When asked what they would do next to get to understand the maths in the intersection if they had to present it to the class, 723-725)

-I don't know. After you've read it a couple of times so you know what the whole thing is for, I'd go through each little bit in term to try to understand it bit by bit. Wouldn't do that the first time as there may be something later on that will help me (727-730).

(When I state to them that they actually agree that they would not go straight into trying to understand the details, I asked them why, 736-737)

-it's easier if you got a whole picture even if you don't understand it all. You are better off having a sort of vague whole picture before you start trying to attack each little bit at a time (741-743).

(As a respond to D when she confessed that she had actually tried to understand each bit first after all)
-yea, but only loosely, I wasn't bothered if I didn't understand something completely, I moved on, I went right on to the end. Sometimes you find something later on that will

		<p>make something that you're stuck on earlier clearer (748-751).</p> <p>(In connection with IE2-D5, 1044-1051) -yea (1053) -maybe some examples, not necessarily diagrams, but they say they're gonna denote this notation. They should give an example so that even if you think you understand it, at least you can look at the examples (1057-1062).</p> <p>(In connection with IE2-D1, 1199-1202) -I know, I normally do that. If you had a problem to solve, get a piece of scrap paper and scribble your ideas and notes and different things down (1204-1207).</p>
<p>2nd The unconsciousness</p> <p>- preparatory work - incubation - illumination</p>	<p>(In connection with IE2-E2, 216) -yea, but if you've been concentrated on it so hard, then sometimes you don't spot it (218).</p> <p>(When asked if there is anything we haven't talked about that is important in relation to learning mathematics, 1126-1128) -I don't know, cause a lot of this is hiding, it's subconscious, I'm not used to analysing why I learn something, so there might be things (1140-1141).</p> <p>(In connection with IE2-D4, 1209-1216 & IE2-E2, 1218-1219) -and then there is light (1219). -two wires, probably gathered and a spark comes (1227). (When asked if what they had been saying now had anything to do with the subconsciousness which D talked about a while ago, 1229-1230) [THE FOLLOWING ALSO BELONGS TO THEME 5] -probably, except that writing things down is much more of a conscious effort, trying to understand. When you understand subconsciously it's like an immediate shedding of light on the problem, when I write something down, that's much more consciously working it through in your mind. I</p>	<p>-if there is something wrong and you've just made a mistake, especially with long calculations or algebra, sometimes you can look at it for hours and not see the little mistake you've made and then someone else looks at it and see it straight away. See it with a fresh mind (209-212). (When asked if he can look at it with a fresh mind himself, 214) -maybe if you go away and come back (216).</p> <p>(In connection with IE2-D4, 1209-1216 & IE2-D2, 1219) -the feeling you get, you're frustrated and you don't understand it, suddenly you get it (1218-1219). (When asked where this light comes from, 1221) -a feeling of satisfaction, isn't it, having done something yourself, worked it out. It's like a link being made in the brain that wasn't really there before, two bits needed two (1223-1225).</p>

	<p>understand this bit and then linking all together on paper (1232-1235).</p>	
<p>3rd The language</p> <ul style="list-style-type: none"> - <i>language as basic thinking-tool</i> - <i>basics important</i> - <i>schematic understanding</i> - <i>rote-learning</i> - <i>assimilation and accommodation</i> 	<p>(When asked to explore the distinction between knowing how to do it or why, mentioned by herself, 273-275) -if you know how to do it, you've just learnt the method and you just change the numbers, but you basically recognise it because you've done the problem lots of times before. Understanding why you do it is more (interrupted by E) (277-280). (When asked if there is a difference between learning how to do something and learning why, 310) -yea, there is a big difference (316). -some of the answers may be easier if you go to a level that is more advanced than you actually need because then when you're in the exam it's easier, cause you can actually go beyond (330-332).</p> <p>(When asked why she had turned to the Puzzle Book during the intersection, 793-794) -I turned to the book because I was interested in what kind of questions you might ask from this. I'll look at it and then pick up the most important bit that you need from it. That gave me an idea of the questions you get about knots which isn't made clear from this and that's the way I work if I related it back to what I'll need for an exam (800-805). (When asked whether she would still do that if she was not suppose to learn this for an exam, 807-808) -yes, because you have to understand the basics and then you'll still want to know how you would apply this knowledge (810-811). (As a response to E in IE2-E3, 820) -I don't know. I thought reading this in fact helped me with understanding this. I just said it in a slightly different way and again it's a different approach and they sort of strengthen each</p>	<p>(When asked why it helps when the teachers explains using the blackboard, 251-252) -cause it is logical, going through things from the first principles, it is something you rarely do when you're solving a problem. Especially, you know for instance the method for taking an integral so you just write it out and take your integral, you don't actually go through the two pages of work to get it, cause you've got your rules and you've got your things in your head. Especially with differential equations, auxiliary equations and particular integrals, you've just got the method for doing it, so you just write it out and take your integral, you don't actually go through the two pages of work to get it, cause you've got your rules and you've got your things in your head (254-266).</p> <p>(In connection with IE2-D3, 277-280) -I think people in our class we get frustrated if we don't know why you do something, whereas a lot of maths students would be happy to know just how to solve the problem and get the right answer (282-286) -it's actually easier in an exam if you know why you do something rather than just know how to do it, cause if you are stuck on a question and you just knew how to do it, then you're completely stuck, whereas if you know the principles, right from the first principals and you CAN go straight from the beginning all the way through the end, it may take longer, but at least you can work through it logically and you know why you are doing it and eventually you will get the right answer or at least you can make some progress and get some marks (303-308). (When explaining the difference between learning how to do something and learning why, started by D, IE2-D3, 316) -it's like building a house. They can teach us how to build a house like it won't fall down, but they won't tell</p>

	<p>other and help you understand (822-824). (A respond to E in IE2-E3, 826-827) -[something quite unclear] I want to know what the point is [E agrees, says 'yea'] which isn't really clear from this (829-830).</p>	<p>you WHY you need pillars here and there to keep it standing up. SO that if you are stuck by yourself trying to build a house, if you don't know why you were doing particular things, you wouldn't be able to overcome the problem, you just know how to build a basic house (322-328). -it's like learning complex numbers and solving quadratic equations. It was quite simple cause the time when we first did it, we didn't need to learn, there were other ways of getting around it (336-339).</p> <p>(In connection with IE2-D3, 793-811 about the importance for learning of knowing how some knowledge is used for) -I'd rather understand it and not care about what you're suppose to do with it (813). (When I then said that he previously had said that he preferred if he knew when maths can be applied, 815-816) -yea, but you can't really apply it unless you understand the theory behind it (820). (As a reaction to D in IE2-D3, 822-824) -that would be a bit frustrating. You couldn't apply anything if you don't know what it's supposed to be. You don't understand the basics (826-827).</p> <p>(In connection with the discussion in IE2-D4, 937-940, about if it is possible to learn mathematics without the use of language) -no, but you can use different language, simple language to convey a point (942).</p> <p>(In connection with IE2-D6, 998) -it's better when you break it down (1004). (When asked what E meant by 'braking down', 1006) -sort of sentence at a time and understand it and then move on. Once you've read the whole thing and you got a basic idea of what it's about (1008-1010). (When asked by pupil D, if E thinks he understands it now, 1012) -more than I did when I first started</p>
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		<p>reading it (1016). -I don't have any problems understanding it, but I think if I was given a question, especially going straight into notation rather than theory, then I'll probably struggle first, probably need some help (1022-1025).</p>
<p>4th The tacit - words obstruct thinking - cannot explain, but do</p>	<p>(When asked why they had been laughing in the beginning of the interview, 755) -it was really complicated language, it's assuming that you understand what a mutually disjoint simple closed curve means (759-760). (As a comment to IE2-E4, 769) -I think (771). (As a comment to IE2-E4, 775-776). -if we open a textbook and got to have an exam over this then you are suppose to understand it, then it will be a bit definitely (interrupted by E) (778-779).</p> <p>(In connection with the discussion in IE2-D4, 853-855, when asked if the language always is an obstruction for learning) -it's always an obstruction if you don't understand it, it's frustrating cause you think that it's probably quite simple. When we discuss it with bits of our own words then it was fine, but this "mutually disjoint simple closed curves" (863-866).</p> <p>(When asked if mathematics has nothing to do with the language, or if one can learn mathematics without language, 937-938) -yea (940). (In connection with IE2-E3, 942, about if it is possible to learn mathematics without the use of language) -cause the mathematics in it is quite easy, what it is saying is what a knot is, what a link is. It took me a long time to work out what they were trying, whereas the fact as soon as I kind of translated it, I thought oh well, that's what a knot is, find that's easy (944-949). (When asked 'you translate it</p>	<p>(In connection with the question about having problems learning mathematics (167-168) -sometimes it is because the way the question is worded, more often than not, not because we cannot do the mathematics, but a lot of it is the notation and the way that you approach the question and if you learn in the textbook one way and you do it 50 or 60 times practice and then they approach it form a different angle or if you haven't learnt the actual theory really well. When you come from another angle, you are sometimes a bit stuck (186-191).</p> <p>(As a continuation of the discussion in IE2-D4, 759-760) -it's not written in the style we're used to studying from (762-763). (When asked if that would scare them off, 765) -it depends on what the work is for (769). (When asked what he meant by 'what it was for', 773) -depends on how serious it is (775-776). -if it was seriously important, run for help (781-782). -I think the English is worse than the maths in this (853). (When asked if the language is an obstruction for learning, 855) -yea (857). (When asked what they meant when they said that they did not like the notation, 878) -no I understood the notation. When I said I didn't like the notation I didn't mean it literally (887-894). (When asked if they did not like the way the notation was given or the fact that it was given, 901) -er, either way, I think I have more of a problem with what came just before they were given. You go and and you see the notation and you</p>

	<p>before you understand it, so, if you have understood, then you don't need to translate it, 955-956) -you have to do the two together, you have to translate while you're trying to understand (961-962).</p> <p>-much simpler language, it was pretty (1031). (When asked 'so the language is something that makes you work less?', 1033-1035) -but that's a good thing because what's important in maths is not knowing the most complicated way to explain something, it's knowing something (1037-1038).</p> <p>-sometimes if there is something you don't understand then I'll use the approach of writing down, writing down bits the same way, and so this was it, and then you write it down and work it through on the paper and then it suddenly clicks, and you understand it (1209-1212). (When asked what D meant my 'click', 1214) -you understand it, it just, I don't know (1216).</p>	<p>know they refer to this set and you don't know what the hell they are talking about (903-916).</p> <p>- don't use such big words they are aiming at people who don't understand. People would struggle with the language when they are suppose to be learning mathematics (929-935).</p> <p>(In connection with the discussion in IE2-D4, 1031-1038) -yea, it doesn't matter how you get there, once you get there, it doesn't matter how you'll explain it (1040-1042).</p>
<p>5th The individual</p> <p>- construction - vision - self-activity</p>	<p>(In connection with the discussion in IE2-E6, 978-979, about the use of graphics) -but don't you think they were in the wrong place, they should be telling you this is a knot there, not after the second paragraph (981-982). (When asked what D would like the first paragraph to be, 984) -in this paragraph it's telling you what a know is, but it doesn't actually show you a knot until a paragraph down (986-987).</p> <p>(When asked to describe how they get it in their head, 1044) -I looked at the diagrams (1051).</p> <p>(In connection with IE2-D2, 1126-1141 & IE2-E5, 1143-1147) -yea, but that's the whole point in learning why you do something rather than how because a lot of the times there are some things I do my own way, I don't necessarily follow the textbook</p>	<p>-I work of the notes and when there is coming up an exam, after I revised the notes and rewritten some more notes and then go back to the book and make sure everything in the book is actually in the notes. I generally work out a booklet (93-97). -I always write my own notes for exams. I generally use the book as a guideline but I write my own notes so that I understand it and if there is something in the book I do not understand I write it from other sources (109-115). -I have to have all my notes written out. I basically have to have the textbook rewritten in short for myself and I am not satisfied that I know everything till I have done it myself. I find that after I've sort of rewritten the textbook, I know it. And it's organised in my head because I learn what's on my notes in the order I make it, so when I am in an exam I don't think back to the book I think back to my notes. Rearrange the book into the way my</p>

	<p>example and that doesn't matter cause I understand what I'm doing, what I want to achieve by doing it and I can get the right answer by doing it a different way to the textbook, but, you know, it just works for me (1149-1153).</p>	<p>mind words (121-134). (When asked how his mind works, 136) -I don't know. It's going to be really logic. Everything's gonna be in the right order. I do work slowly because of it. I always have to have everything in logical order and everything perfect. I can't work with a textbook that jumps from here to there. I can jump back when I am using text but when it comes to revision I like everything sort of in a nice package and if it is not in a package I try to make it to a package, makes sure it is all nice (138-144).</p> <p>(In connection with IE2-E4, 929-935) -it depends on whom they are targeting it at (929).</p> <p>(In connection with IE2-D6, 958-959) about having to translate what the book say into a more simple language). - it depends on your audience. If you've got an audience who are used to this approach then it's fine (964-968).</p> <p>-actually the graphics are a big help. I was relieved when I got to this first graphic, it kind of showed what they were (978-979).</p> <p>(In connection with IE2-D2, 1126-1128) -I know people do differently; it's all very individual, even if you work on something together and you're both aiming to solve the problem. I mean you'll do it completely differently from someone else and quite often I find I don't like other people's styles, you know, you always get your own (1143-1147).</p> <p>-the people understand different things, it's different degrees. But if I can't solve a problem and I borrow someone else's work, it actually quite often doesn't help because the way they've written it out, you don't follow the same sort of logical thinking (1168-1173).</p> <p>-people have, for a start they</p>
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		<p>approach the problems differently. Some people will do the first two steps in their head and quite easily be able to go to the third step. And other people start right at the beginning, and when you write it out, for some people, of you miss out the logical step, that you need to see on paper for it to work, and yea, people do things in different orders and they've got different, and people have their own notation to things don't they (1191-1197).</p>
<p>6th The social</p> <ul style="list-style-type: none"> - <i>internalisation</i> - <i>guidance and interaction</i> - <i>ZPD</i> - <i>speaking aloud</i> - <i>hearing</i> - <i>discussing</i> 	<p>(As a comment to E in the discussion about notes in IE2-E5)</p> <p>-I sometimes use my notes. I use my notes to do the exercises when I am revising for an exam. I don't think I have ever used the notes, I always use the book. Just because we know that everything that is in there is relevant to the modules we are going to do (105-107).</p> <p>(In connection with IE2-E6, 172-177)</p> <p>-Yea. We all seem to have the same problems a lot of the time. It will be because we've gone through too quickly in class (183-184).</p> <p>-we'll ask the teacher, make it quite clear. He goes through it on the board. Not the theory, but the problem. Cause when we've done the theory most of the time (interrupted by E (234-242).</p> <p>-doing it on the board means that you understand why you are doing it, not just how to do it, and I think that definitely helps. If you now how to do it, that's one thing, but if you know why you are doing it, then you can do any question really (268-271).</p> <p>(In connection with IE2-E6, 344-350)</p> <p>-some of them can look out and say, this is all you need, you don't need to know why you do, you just go this and this, which is all right to a certain extent, but if you want to work with difficult problems, you can't work it out for yourself as easily (352-354).</p>	<p>(When asked what they do if they meet some mathematics they do not understand (167-168)</p> <p>-it depends on how difficult it is. I go and ask my friends who are doing the same work, classmates. And if there really is a big problem we ask the teacher to go through it (172-177).</p> <p>(In connection with IE2-D6, 242, where E interrupts D)</p> <p>-but he will relate it back to the theory, whereas when we do it we just try to solve the problem, but he'll do it step by step going through the theory and won't miss out any links or bits of theory he needs to solve the problem (244-246).</p> <p>(About being taught by teachers)</p> <p>-if they go ahead and teach you how to do it, I think you find it easier to approach things. It depends on who you teacher is (344-346).</p> <p>(When asked why it depends on the teacher, 348)</p> <p>-I just think (350, continued by D in IE2-D6, 352-354).</p> <p>(Said as a comment to D in IE2-D7, 418-422)</p> <p>-completely different from mine. We had these little books, a whole series, and there was an order you had to do them in and everyone just did the books by themselves and the teacher didn't teach you anything, books told you how to do it. Basically if you wanted to progress in maths there was a minimum that you had to do, which most people did, but if you took an interest and wanted to be any good at maths, it would be of your own back. I didn't actually like</p>

	<p>(When asked why it is better to do it in class, 437) -it's more interesting, you have other people to balance the result (E interrupts and says 'yea') (439-440). (Said in connection with IE2-E6, 442-444) -like E is saying, it also helps to have friends who you can talk to if you are not sure about something (446-447).</p> <p>(When asked what they could have done if they wanted to describe knot theory, 918) -I think this is the kind of thing where it is very difficult to about in a book and to represent 3-dimensional object within a 2-dimensional way, and it's where it would help to have a teacher explaining something and say pointing all this the vertex and this is an edge (920-923). (When asked what she meant by pointing, 925) -yea, drawing it or tiny little knots and say this is (927).</p> <p>(In connection with IE2-D6, 958-959 & IE2-E5, 964-968) about having to translate what the book say into a more simple language. -the approach we're used to where you have the theory and then you have questions on it, and it is taken you in gently whereas this straight in (974-976).</p> <p>(About the mathematics in the intersection) -when you first read it, it's quite alienating (998).</p> <p>-I don't often learn things from the textbook, no matter how simple the language is. I normally have some there explaining it to me (1078-1080).</p>	<p>that, I mean the system worked for me I think, but I didn't think it was the right approach, I think here when you do things as a class (424-433).</p> <p>(Said in connection with a discussion of why it is good to do it in class, IE2-D6, 439-440) -when people explain it to you it's just their way of learning, with the book it is very difficult for an author to get an idea across, it will be much easier and better explained by a trained teacher (442-444).</p> <p>(In relation to the discussion in IE2-E4, 759-782) -this is the sort of thing where you need a bit more basic knowledge, maybe someone else teaching it to you, rather than trying to understand it in written language. It's quite a task to make everyone understand something that's written (786-789).</p> <p>-I think it would be easier if the author translated rather than leaving the reader to do it (958-959).</p> <p>-I find it quite easy to learn from textbooks if it is written in a certain way, approaches from a certain angle (1082-1083).</p>
Outside themes	<p>-when I first came here, the first couples of weeks I found maths very difficult because it is hard to adapt to a different teaching style (413-414). -in the old school it was much more mixed ability, so it went a</p>	<p>-if you cannot do the questions you obviously did not understand the theory properly (80-81).</p>

	<p>lot slower and we did a lot more work in the lesson, and we worked from textbooks whereas here a lot of the times you don't even open the textbook during a lesson and sometimes I don't even bother bringing my textbook, I know all is done from the board (418-422).</p>	
<p>Some utterances and observations during the intersection</p>	<p>-reads a bit aloud (530).</p> <p>Observations: After reading it they look at (McLeay) (538).</p>	<p>- says he has more problems with the English (524).</p> <p>- says he thought a knot was something that held two things together (561).</p> <p>- says he reads it twice (685).</p> <p>Observation: Reads aloud.</p>

A.2.3 Third English interview

This interview was conducted the Tuesday 22 February 2000. It lasted 60 minutes and the transcriptions fills 1076 lines.

	Pupil F	Pupil B
<p>1st The consciousness</p> <ul style="list-style-type: none"> - <i>practice</i> - <i>planning</i> - <i>reflection and monitoring</i> - <i>confidence and positive atmosphere</i> - <i>motivation</i> 	<p>(When asked if they do not need all this exercising, prompted by the discussion in IE3-F6, 100-101)</p> <p>-you need an example of each one and you need to get it right and our books have answers in the back as well, you do need to make sure that they are not just looking in the back (105-106).</p> <p>(In connection with the discussion in IE3-B1, 108-111, just after that F (113-115) has said that some of the questions in the book are exam questions but most of them aren't)</p> <p>-I actually find it more useful when we ARE doing sets of exam questions (117).</p> <p>(When asked why, 121)</p> <p>-in the exam question, the whole topic area is condensed into one question, it has a bit of each sort of part. And you get used to combining the whole lot together rather than just using one specific bit and repeating it over like you're doing in exercise (123-126).</p> <p>(In connection with IE3-B1, 128-131)</p> <p>-yea, it's the whole format. An exam question can test four of them all at once in one question and that's a lot more useful than doing each chapter and then forget the chapter and then you go to the next one, then you come up with an exam paper and you have to link them all together, that's the problem I think (135-139).</p> <p>(In connection with IE3-B6, 155-157).</p> <p>-in the book we'll have worked examples, we always have examples, one example of the process and you just go through it (159-160).</p>	<p>(In connection with IE3-F1, 105-106)</p> <p>-I think it is not just one of every one, I mean you need a few just to get the sort of process into your head, but sometimes we get far too many, just repeating the same thing. And when you've done it for half an hour and you've learnt it and you have to carry on doing it hours of work on top of it, it's just the same thing again (108-111).</p> <p>(As a respond to IE3-F1, 117)</p> <p>-exactly (119).</p> <p>(When asked if they need exam question to do this or it is more the type of question, 128-129)</p> <p>-it's getting use to the type of questions (131).</p> <p>(In connection with IE3-F1, 159-160)</p> <p>-and you can find out how to do it from that (162).</p> <p>(When asked how they know if they have understood some mathematics, 197)</p> <p>-it's by looking at a question. If I can just look at it and see in my head how I am going to tackle and I'm not confused by any of this, asking me to do (199-201).</p> <p>(When asked to describe their learning process, 218)</p> <p>-for maths it is certainly different than other subjects cause it is learning how to use a technique rather than, like lists of meanings or whatever, and so I think the learning process is just practice, have it explained to you and then practice it to sort of get it over into your head so you can do it easily at a time (222-226).</p> <p>(In connection with the discussion in IE3-F1, 689-691)</p> <p>-well, yea, you couldn't go straight</p>

	<p>(When B was asked what he meant when he in line 200 said that he could see it in his head, 208) -it's something that I have learnt how to do, know how to do it without actually doing the question, you can look at it and say, ah I will use that technique to do it (210-212).</p> <p>(In connection with IE3-B1, 218-226) -we get taught through the principles, been given one example and just go practice it, that's the basic learning process I think (228-229).</p> <p>(When asked if they learn when the teacher tries to teach them a wider view of maths and therefore includes things that are not in the syllabus, 244) -yea, the process that we've been using that's taken one step further so you've get to see the sort of application, the object process (246-248).</p> <p>(When asked if they could learn it without examples, 689) -I don't think so (691).</p> <p>-you realise going through examples (712-714).</p>	<p>into an exam knowing all the techniques but not actually having worked through each one several times, and have to do it through, mechanically (693-694). -yea, I don't know exactly how they could really be explained without using some examples, just examples of numbers being put in, to see the technique (698-700).</p> <p>(When asked what about explaining the maths, 702) -for example like calculus, yea, differentiation you can explain how is the measure of the rate of change. I mean there is certain things that I just don't think could be done without these numbers (706-708). -I don't think you need numbers, they are still examples when you use a, b, and c, constants (716-717).</p> <p>(When asked what he got out of looking in the purple book [McLeay, (1994)], 772) -seeing the way they use (776-777).</p>
<p>2nd The unconsciousness</p> <p>- preparatory work - incubation - illumination</p>		
<p>3rd The language</p> <p>- language as basic thinking-tool - basics important - schematic understanding - rote-learning - assimilation and accommodation</p>		<p>-more notation instead of (740).</p> <p>(When asked what they had meant by saying that it is a good idea if things are simple, 801-802) -yea, certainly if you explain any of this simpler diagram, you understand it more quick (804-805).</p> <p>(Referring to what they had said previously they were asked if it was important to have examples or terminology first, 825-831) -we will be able to understand the examples they are given us without,</p>

		<p>cause in getting the examples they explain it using this terminology, but if we do not understand the terminology then we haven't hope of understanding the examples (833-835).</p> <p>(When I then said 'so the terminology is the most important?', 837)</p> <p>-I think, yea, it comes first (839).</p> <p>(When asked if there was anything they find important for knowing how they learn new mathematics, but which we haven't talked about, 1017-1018)</p> <p>-maybe the way that it is split to modules with different areas of maths in each different module, I think it helps keep one sort of general topic area you do all at once and you get examined on it straight away and you can put it behind you, gone (1026-1033)</p> <p>-keeps them separate so you don't get too confused (1041-1044).</p>
<p>4th The tacit</p> <p>- words obstruct thinking</p> <p>- cannot explain, but do</p>	<p>(In connection with the discussion in IE3-B4, 809-817)</p> <p>-I think it's here, language they sort of expect you to be with. And as A-level students we are not suppose to go and look up and make this sort of attempt to clear it before we fully understand what it's actually telling us (821-823).</p>	<p>(When asked what they had meant by talking about understandable language, 809-810)</p> <p>-the oriented link is an example which we understand in the English language, but I don't know how you see an orientation (815-817).</p> <p>-A-level is just sort of assuming, I mean it's all terminology, it is explained to us, you just expect to understand the terminology but here you can't cleanse all this these words, understand what they are referring to (845-848).</p>
<p>5th The individual</p> <p>- construction</p> <p>- vision</p> <p>- self-activity</p>		
<p>6th The social</p> <p>- internalisation</p> <p>- guidance and interaction</p> <p>- ZPD</p> <p>- speaking aloud</p> <p>- hearing</p> <p>- discussing</p>	<p>-if we are given more teaching time in class and fewer repetition of the same questions, then I think we can go a lot faster (100-101).</p> <p>(In connection with IE3-B6, 168-169)</p> <p>-or we go to, I've got three people in my class in my house, so I just go and ask them (171).</p>	<p>(When asked what they do if they meet some maths they cannot understand at first (147-148)</p> <p>-I would ask the teacher, if I was out of class time I guess I'd go to the textbook and look through it. Do you mean something on the topic area or something completely new (150-153). (When I said it could be both, 155)</p> <p>-if it's the whole topic area, I'll go to the book (157).</p>

	<p>(In relation to the discussion in IE3-B6, 899-918) -if I got a problem and I'm on my own I always get the book down and if I don't understand first section I just give up and I get through it with my dad or someone whose really good at maths, then I can do everything in the end, even though he knew nothing about it (920-923). (When asked what works best, to talk to someone who knows the maths or someone who also does not understand, 927-938) -someone who knows - give some achievement whereas if you do not know anything beforehand, you don't know if it is right in the end unless it is obvious it's right so you could be making a fool of yourself (940-944).</p>	<p>-if it is something we've never seen before and we don't have a textbook on it, leave it and ask the teacher (168-169). -yea, you can check with the others (173). (While referring to that they during the intersection had been discussing, they were asked why they discussed, 899-904) -it certainly helps if you can discuss it with someone else. Two brains are better than one (906-907). (When asked why is that, 909) -one person can have one idea which trigger another idea in the other person's head which the first person wouldn't have had, and then the second person having said that thing, then the one thing leads to another (911-913). (After Pupil F have mumbled something about read books, 915) -yea, it also helps if you read something of that one and I read some of this one and we sort of explain it to each other, that saves time (917-918). (In connection with IE3-F6, 927-944, when F said it works best to discuss with 'someone who knows') -yea (940 & 944). (When asked in what ways what they had been doing in the intersection was the same as what they normally do, 988-990) -that we've been discussing it together, help each other understand, we do quite often (992-993).</p>
<p>Outside themes</p>	<p>(As a respond to IE3-F6, 670-670) -for example if they are learning about differential equations or something and maybe taught any method for solving then, you go through all the proof, and complex numbers (680-682).</p>	<p>(When asked what they had meant, when they talked about presenting it for the class, they wanted to give their classmates examples, 670-672) -that because the way we have always been taught is using examples thoroughly to explain, so that's the way we think the people in our class will understand it easiest, explain through examples (674-676). (When asked why he had picked up the red book [Nelson & Wilson (1990)] after having been reading and thinking, 726-727)</p>

		-to see more of what sort of context it came from (729-730).
Some utterances and observations during the intersection	<p>-says: 'find the terminology' (349). (3)</p> <p>-says: 'useful to work out if there is some simple maths that would be quite next to it, without all this' (355-356). (4)</p> <p>-says: 'nice with an example there as well' (360). (1)</p> <p>-says: 'clear diagrams I suppose if making presentation to class' (381-382). (5)</p> <p>-says: 'the first thing we could do would be to draw some knots and then show then respected oriented diagrams' (496-498). (6)</p> <p>-says: 'explain by given examples and so' (599-600). (5)</p>	<p>-says: 'to get any further have to go and look up one of these terms mean' (337). (3)</p> <p>-says: 'more sort of simple ideas' (358). (3)</p> <p>-says: 'explain it slowly, more understandable language (F: Yea)' (377). (3)</p> <p>-says: 'look around for some more books, find some who do it more of understandable summary, maybe went through it a bit slower and sort of let you into it through more understandable terms' (392-395). (3)</p> <p>-says: 'Again we need to look it up' (420). (3)</p> <p>-says: 'we've got to define terminology and find something which will use basic math notation knot' (475-476). (3)</p> <p>-says: 'I can't see why that would be, seems to be an arbitrary decision to me' (564-566). (5)</p> <p>-says: 'I wanna know what the notation means' (586). (3/5?)</p> <p>says: 'if there are things a computer does for us, that's where we have to start by explaining to the class if doing a presentation to it ... and then we can go and talk about the ideas that are involved' (590-595). (6)</p> <p>-says: 'is this different from what we normally experience? Certainly, we are never asked to go and research things for ourselves, we just get explained the whole topic and then get explained how to do things and then get set practice questions on it. (662-666) (7)</p> <p>Observations: after both having read, B takes (Nelson & Wilson), then F talks to B, B takes (McLeay), F returns to paper, starts to write, then they discuss.</p>

Appendix B: Sheet with Knot Theory to the English pupils

Source: Nelson, R. & Wilson, R. J. (1990).

PLEASE SEE PAPER-VERSION OF THESIS

SORRY!

Appendix C: Sheet with questions to the English pupils

Was given to the pupils in the beginning of the intersection.

Working with Knot Theory

Take about 15 minutes to get some understanding of the new piece of maths.

Try to be aware of what it is *you do* to understand this new piece of maths.

After the 15 minutes some things will probably still not be completely clear to you, but think of what it is you would *do next* to fully understand this new piece of maths?

Is this learning situation different from what you normally experience? If yes, in what ways? If no, in what way is it the same?

Appendix D: Interview schedule, English interviews

Slightly edited.

1. Introduction.
 - i. Presentation of researcher. Give cookies and fuzzy drinks.
 - ii. Tell the pupils about the study and about what will happen this evening.
 - iii. Ask permission to tape and ask if they do have time for staying a whole hour.

2. How would you describe a normal mathematics lesson?
 - i. Same teachers?
 - ii. Same school before?
 - iii. Numbers of pupils in the class?
 - iv. Homework?

3. What would you *do* when you meet some mathematics you do not understand at first?

4. How do you know when you have understood?

5. Try to describe your learning process?

6. Introduction to intersection:
 - i. Look at some mathematics, try to understand as much as you can.
 - ii. Think about how you approach the mathematics.
 - iii. Possibility for using an extra room.
 - iv. Do what you think is necessary to understand this piece of new mathematics.
 - v. Give them pens, books, paper.

Appendix E: Interview schedule, Danish interview

Slightly edited.

1. Introduction.
 - i. Presentation of researcher. Give cookies and fuzzy drinks.
 - ii. Tell the pupils about the study and about what will happen this evening.
 - iii. Ask permission to tape and ask if they do have time for staying the one to
two hours.

2. What are your learning strategies meeting some mathematics you do not understand at first?

3. What are your own criteria for when you have learnt some mathematics?

4. Try to describe your learning process?

Appendix F:

Full transcription of the first English group interview

1 I: OK, first I want some, if you can describe a normal math lesson, how does this begin and end
2 and, how do you do it normally?

3
4 C: It depends on what teacher and which teaching. If we have [giggles] er say er Mr. X who
5 teaches us more pure maths (I: mmm) AS maths, then going, if it is something new, you can't,
6 we handed a prep and, and he wants to go through it, and he just goes through it and goes
7 through, and it takes a lot of questions if you (I: mmm) are doing the stats. And if it is
8 something new then he put more problems on board, ask us to work it out, if we can't, he gives
9 us hints all the way through and (I: mmm) you know proofs, the result at the end, and
10 generalises it and then give new examples (I: mmm). And er we have practices outside lessons
11 (I: mmm) - just little prep, go through it there. Then this really cover the whole course quite
12 quickly, but [A & C laughs]. If we have our other teacher er who teaches us statistics, er firstly
13 I'm not really enjoying it [laughs] (I: mmm) this course and also [laughs]. Well, not very good.

14
15 A: His teaching style is slightly different. We, er, we tend to write out more formal notes with
16 him, so we start [C coughs] starting a new topic, we tend to write up, he will write on the board
17 some maybe some basic form of notes er and then maybe an example [coughs, burps], sorry,
18 and then we will give, then we will do some practice question out from book, books that we've
19 got, like that, er same sort of style. And then sort of [inaudible] up [laughs a bit] the whole
20 lesson (C: mmm) because (C: [inaudible]) [C/A cough] yea (C: So [laughs]).

21
22 I: But, er, does it happen er, do you sit in groups sometimes or do work with the problems
23 yourselves or on the class or how does it [interrupted].

24
25 A: Generally (C: We) work as a whole class (I: mmm) in [1 sec silence] Mr. X we tend to sit
26 (C: Sit) we tend to sort of, open more of, a sort of open discussion (C: mmm) and sort of verbal
27 oral questions (I: mmm) and answers whereas Mr. Y's classes is more written practice er you
28 know style of questions (I: mmm). You know but generally, yea generally, mostly we tend to
29 do it as a whole class rather than smaller groups.

30
31 I: Do you have different teachers to different kinds of maths. (C & A: Yea) Ah yea, OK mmm

32
33 A: Yea, our course is split into six, ordinary A-level is split into modules (I: mmm) [cough] and
34 basically in three branches, mechanics, statistics, and pure maths. And, we, basically get taught
35 different modules, we do one module at the time, basically get taught different modules by
36 different teachers, so that's how it works.

37
38 I: Mmm OK, so does this, have you been in this school just this year or have you (C: 4 years)
39 (A: 5 years). OK (C/A: mmm) so, er, is this how the teaching normally is, or

40
41 C: No, this (A: er) is A-level, in GCSE there was, cause there were so many people (I: mmm) it
42 wasn't [A coughs] class discussion.

43
44 I: How many are you in the class.
45
46 [C/A talk at the same time 1-2 seconds]
47
48 A: 7 of us.
49
50 I: Ah OK.
51
52 C: But in GCSE we had like 24-25 (I: yea) so if we had an open discussion, with too many
53 people talking (I: mmm).
54
55 A: Yes, and it has been more taking notes than doing practice (I: mmm), plus now there are
56 more opportunity to discuss everything.
57
58 I: Do you feel you learn more (A: Yea) now, or.
59
60 A: Yea I think it is much better (C: Yea) have a smaller class and doing more discussions (C:
61 Yea) and so working through it (I: mmm) cause [1-2 seconds silence].
62
63 I: Why is it good with the discussions?
64
65 A: Er, (C: Cause) I don't know you felt (C: Confident) er er and you can, yea, it makes you
66 more confident with what you are doing cause if you are working say with an example from the
67 board, and you are following it down and working out for yourself as you get along, you fell
68 comfortable with asking questions if you don't understand a step that you have gone through (I:
69 mmm) and you know, and also or other people in the class feel that they can answer instead of
70 the teachers, so if you understand it (I: mmm), we can all share ideas between us (I: mmm)
71 which makes you more comfortable in how you deal with it (I: mmm) I think Which is, I just
72 quite (I: mmm)
73
74 [2 seconds of silence]
75
76 I: OK, What er about, I think about homework and things, er do you do a lot of your work at
77 home or do you do it mainly here or.
78
79 C: We do it at school (I: mmm) cause it is, yea, we do it from 7.30 to 9, er is a boarding school
80 (I: Everyday?) Yes it is a boarding school really so, [inaudible] between 7.30 and 9 every day.
81 So we can get, we can mostly get it done (I: mmm) [coughs]
82
83 I: Do you have assignments to submit every week or every month or.
84
85 A: We er
86
87 C: [inaudible] depends on what modules we are doing.
88

89 A: Yea we usually just gets at a few practice questions say after say maybe twice a week (I:
90 mmm) usually about that isn't it (C: mmm) twice a week. A set of questions to hand in, you
91 know the following lesson (I: mmm)
92
93 I: And then, are these marked or something?
94
95 A: Yea these are marked and handed back and sometimes if the has been a major, everyone in
96 the class has had a problem with it, we'll go through it (I: mmm) you know, as a class, we
97 discuss well, can discuss what we did wrong (I: mmm)
98
99 I: How er, so how do you then learn er what it is you did wrong or how do you learn how to do
100 [interrupted]
101
102 A: Well, when we, when the preps are marked [coughs] if you, you know, if there is a question
103 where you have done something wrong, then, the teachers when they mark you, they usually
104 write down how you should have done it.
105
106 I: Every step or hints or?
107
108 A: You, well, [1 sec silence] you, er Mr. X [C tries to break in] it depends on the teacher again.
109 Mr. X tends to just write hints, prefers you to work it out for yourself with help if you need it,
110 whereas Mr. Y [C tries to break in] [laughs] tends to write out [laughs a bit] maybe just the
111 answer or sometimes the whole method. (I: mmm) Er
112
113 I: Which er, what works best, would you say?
114
115 C: The hints.
116
117 I: Why?
118
119 C: Because its, because then you have a second go, cause there is only a limited number of
120 questions on a topic. So in chapter 4 [coughs], just picking a chapter, there is only so 10
121 questions at the back and if all 10 of them you've done (I: mmm), or 10 of them you've got
122 wrong, and if you just got the answer there is no way (I: mmm) but you got the method
123 [interrupted]
124
125 A: Yea, if you just got, if you, if you just got the answer, written down, then you feel like, you
126 don't have [C tries to brake in], you sort of don't need to do it again even though you got (I:
127 mmm) it wrong, you clearly er need to go through it, if you just got hints then you (C: mmm)
128 tend to feel that you, you know, you you could, that, now you've seen, you know, been hinted
129 how to do it, you CAN work it out for (I: mmm) yourself. So you are willing to try again (I:
130 mmm) and [interrupted]
131
132 C: And if you just got the answer, then you think, well I still don't understand how to do it (I:
133 mmm). It is just the answer it does not mean anything (I: No), what it means is, what's, er, what
134 you need is er, an er a written method of how to do it or a hint to show you the method (I:
135 mmm). You don't need, that's, that doesn't er
136

137 A: Yea, the books, our books got the answer in the back anyway (C: Yea you can check for
138 yourselves) (I: mmm) So that's.
139
140 I: So what, what do you do if, lets say you have some problem or, either a problem you have to
141 solve or some maths you have to learn, and you have difficulties learning it?
142
143 C: The clinics, the clinics.
144
145 A: Yea er.
146
147 I: What's that?
148
149 C: The maths clinics, they are just like, if you, in sessions, you want to come in [inaudible]
150 sessions, if you have any problem then (A: Yea), you go and see a teacher on duty.
151
152 A: The, the teachers they take in turns, once a day different teacher, they're in here from about
153 1.30 and (I: mmm) anybody, you know, any class any year, if if you've got problems, you know
154 something you don't understand, you just come and they go through it on a one to one basis or
155 maybe two people if there is [inaudible] (I: mmm) the same thing. [1 sec silence] And Mr. X
156 encourages us to go [C tries to break in] if we've got thing, and I've done that quite a lot.
157
158 I: How, how do they help you?
159
160 A: Well just sort [interrupted]
161
162 C: Give you hints. Depends on the teacher again (I: mmm). Mr. X er who acts as [inaudible] (I:
163 mmm) he er gives you loads of hints, and encourage YOU to work through it, and YOU to
164 achieve the answer. And he won't say, "oh that's wrong", he will, you know, say well
165 [interrupted]
166
167 A: Say what you've DONE [C & A talks at the same time a few seconds] rather than, rather
168 than (C: Yea) you know, if you, if you've tried to work something through and you got the
169 answer wrong [1 sec silence] it's, it's it's better NOT to be told, "oh that's wrong, try again"
170 it's, you know, it's better if you're told "yes well that's close but here you've got, you know,
171 minus instead of a plus so you haven't so you haven't multiplied this out right" or (I: mmm)
172 something like that. Cause then you can, you can see for yourself what you've done wrong and
173 how that's affected the answer. I think that helps you, you know, when, when you can wait and
174 see what you've done wrong (C: mmm). Its helps you to, you know [laughs] avoid doing that
175 again, see what I mean (I: mmm). Especially if a lot of the questions you do, are really similar
176 style, which, well er in our course they they are very very similar to each other, aren't they (C:
177 mmm) all questions in the books, and all the exam questions are, you know, basically very very
178 similar in style (I: mmm mmm)
179
180 I: But, but if you er, er, let's say if you have, you are er having some new maths and you er you
181 are solving a problem [C/A coughs], or lets say you you are learning some new maths, piece of
182 maths, er how do you know when you have understood it?
183

184 C: When you get the right answer. Cause usually in the lesson we learn, if we learn something
185 new it is usually in a double lesson (I: mmm), so we are not trying to go through anything and
186 er, and er at the end we are given that one question to do (I: mmm) or class question. Then if
187 you do it and get it right then you know you've understood it.

188

189 I: But that's the problem, you know how to solve the problem, but does, what about the maths
190 BEHIND it.

191

192 A: Well I think, when when we are holding class discussion and in Mr. X lessons as I say there
193 is only 7 people (I: mmm) and we are generally going through some [inaudible] new having a
194 having a, you know, having discussion at the BEGINNING when we are going to do some
195 brand-new, this is mainly the teacher just explain and go through the steps on the board but
196 after a while as everyone starts to understand it, everybody starts chipping in, and you may have
197 understood it when you can skip ahead and see what the next step's gonna be.

198

199 I: Next step, is that er in a proof?

200

201 A: Yea if you are doing, say working through a method of of of you know say, solving like the
202 differential equation which we're doing at the moment (I: mmm) say that you know the next
203 step in the method of doing it, or the next step in a proof like you say or something like that (I:
204 mmm) if you can SEE it, where it gonna be, and work it out for yourself before it's up on the
205 board, you know that you can understand it for yourself, and you know everyone, and and you
206 know when there is somebody ask a question and YOU know the answer and you know how to
207 explain it (I: mmm), you feel comfortably, you've understood the whole method of doing it, and
208 when the er you know, when everyone's understood, everyone is chipping in and you know (C:
209 mmm) the answers, it's quite good actually. (I: mmm) [A takes a drink].

210

211 I: Mmm, OK, can you er, if you should try to describe your learning processes, what actually
212 goes on in your head, while learning maths, what, how, can you er, what what do you think,
213 what comes into your mind when I ask this question?

214

215 A: Well [interrupted]

216

217 C: I think of previous experiences or maybe er you go back to what you know and current fit in
218 the new thing to what you know. (I: mmm) If you can do it, its easy, cause it is just er addition
219 (I: mmm) if it is completely new like when we first [interrupted]

220

221 [C & A talk at the same time about 1 sec]

222

223 A: Like differentiation, things like that. (C: Yea) For the first time. (C: Its something you know
224 it's not) (I: mmm) You have never seen anything like (I: No [laughs]) that before when you first
225 get to it, its totally new and different (I: mmm). Yea. Yes.

226

227 C: Unless you are taught it, like so that it's very simple to understand. You can get quite
228 [inaudible] (I: mmm) cause there is no way you can learn from the book.

229

230 I: Why, or or?

231

232 C: It is not interactively is it, you can't ask the book a question (I: mmm) unless your questions
233 are, cause the book's only got a couple of pages of the new things (I: mmm), the new chapter,
234 and it doesn't answer all your questions (I: mmm) and that's why er. First thing that happen in
235 your mind is that it is hard to accept something new unless you know its right so [interrupted]
236

237 A: Yea yea, you have to make it a leap, sometimes when, you you know, when you are told
238 something (C: Yea) and, you know, like take the example of differentiation, when you are told
239 that this is how you differentiate something like $3x^2$ differentiate $6x$, you are told that
240 that is what you and you have to accept that that's, that how you it, and what you do (I: mmm)
241 before you can move on (I: mmm).
242

243 [1-2 sec of silence, then both tries to begin a sentence].
244

245 A: It's it's [1 sec silence] sometimes quite helpful to have a a proof or something (C: Yea)
246 written out just to help you accept that that's, you know, how you do something, even if, you
247 know, even if you are not going to have to know how to use the proof or do the proof
248 yourselves at the exam, it is helpful to see how it's done cause then you know where the
249 method you got, you know where it comes from
250

251 [C & A speak at the same time 1 sec]
252

253 C: [inaudible] the generalised form, like integrating x [inaudible]. Once you've got THAT, and,
254 you know you are on a safe ground so you just put whatever it is you have in front of you (I:
255 mmm). So there is nothing much to understand there. [One coughs] If you can link it to
256 something you know like, $n+1$, you know, er learnt that when you're, er GCSE (I: mmm) then
257 er, you can, and you fit that into integration for example, then it is easy, but if you just have,
258 you know, an example er $2x$ integrates to x^2 , then it is quite hard to think how did that
259 happen, why and where er (I: mmm)
260

261 I: How would you, if you got this problem how would you attack this problem, if you were
262 asked to er work it out?
263

264 C: That specific problem?
265

266 I: Yea, or any other who is, which is [interrupted]
267

268 C: Think of the general solution (I: mmm). Think of the general formula or equation you can
269 put it into. If there is one, you just use it, but if there isn't one, then you've gotta er er [1 sec
270 silence], if this is a fresh topic you look back at the notes and find any similar examples (I:
271 mmm) and try and fit in to that. Well, integrating log for example (I: mmm) in a book er or in
272 the notes you just have, you know, log it so integrate but the question might be whenever
273 [inaudible] 2
274

275 A: I tend to find that I learn something better, say, you've got, like saying method like a general
276 formula for finding out that, come that's not a very good example cause it is pretty simple but
277 (C: mmm) er, you know, so finding out the integral of something, then [cough] I tend to find
278 that I learn it better if I look at loads of examples for myself, and deduce for myself how it is
279 done and how it works rather than being given at the beginning a general formula and fitting

280 everything I get into that. If I can work it out for myself from, you know, seeing examples of bit
281 of equations (I: mmm) and things like that, then I find I can, you know, more easily deal with,
282 you know, slightly, slightly different problems cause, I think you've got better understanding of
283 how something works, if you er liked worked out the method for yourself (I: mmm).

284

285 I: But how do you work out the method, you say, you look at at lot numbers lot a [one coughs],
286 many example, why is that helpful?

287

288 A: Well I I, it's just, I er, I find it helps me to to to see, I mean, if you, say, you may, take like
289 take integration you've got loads, if you're given, I mean, at the books we've got like one way
290 of explaining, and the way we're taught it, we tend to sometimes, you know, put up an example
291 and go through how to do it, and there'll be a couple of those in the books. And if you can it
292 anyway if I, if I [coughs] sort of see the examples and, you know, er, I mean seeing how
293 someone totally new like integration works in an example is just a, look, it's just sort of looking
294 at the way numbers move around in the equation, and you can spot, I find I can spot from
295 looking at several different examples, you can spot how it move, and you know how it shifts
296 itself around (I: mmm) as you operate the method through it. And as you see more examples
297 you can see, you know, how it works overalls (I: mmm) you get a broader picture of the whole
298 thing works in general, from looking at you know, from looking at different examples and just
299 seeing what goes where (I: mmm). It's just [snap his fingers], you know, making a connection
300 between, you know different parts of the, it's like spotting, you know, sequences where you are
301 given the first few terms and like we had a reasoning task where you are given the fist few
302 terms and you have to spot, you know work out what the answer is, that's the same kind of
303 thing really (I: mmm), I think.

304

305 I: OK. Is that also what you do about, I'm just thinking about, a while ago you said that er you
306 had these two hours lessons [C/A coughs] in the morning and then er, it it sounded like, it that
307 that was actually enough to understand the new math, but I was just thinking, maybe sometimes
308 it is not enough so [interrupted]

309

310 A: Yea, sometimes [1-2 sec silence], yea I mean [coughs]

311

312 C: It it depends on the er difficulty of the question er topic. If it is REALLY difficult then er [A
313 and C talks at the same time for 1-2 seconds]

314

315 A: You can get the IMPRESSION that you understand it, and you've understood what what
316 you've gone through in the lesson (excuse me) but then you might get into a, THIS COKE
317 [spills coke, a bit laughing around the table] and then you into the, get back into doing your
318 homework and come across a SLIGHTLY, you know, a a question you haven't seen any of
319 that, that you know that or that type before, then you might, you know you can't, if it is
320 something new you've only just learnt you might not be able to see (I: mmm) immediately how
321 to do it, so, sometimes it is not enough and I find it's [coughs] you know, have to go and have a
322 er just a chat at the maths (I: mmm) clinic so (I: Yea) find one of the teachers on their own (I:
323 mmm) just so that you can say exactly what it is you don't understand (I: mmm) and get them
324 to explain it (I: mmm). Cause sometimes, I mean [coughs] a a lot of the time I find it, it it IS
325 enough (C: mmm) if you spend the whole you know double period's is like an hour and a half
326 or so (C: mmm), more than an hour and a half, and so [clears his throat], I find that usually is
327 enough time if we've been, like I said working through it on the board and discuss it as a class

328 (I: mmm). But if we spend some of it doing examples [1-2 sec silence] I find it, sometimes find
329 it harder to, harder to understand, and I need more help outside of the lessons (I: mmm). I think
330 it is better to go over something, sort of completely and discussion and turn it around in your
331 head and see how you, you know, you understand that works and where the methods come from
332 (I: mmm) before you start trying to do too many questions, practice questions. [1-2 sec silence]

333

334 I: Yea, ok, good. I I'll try to er show you this maths er piece of math [paper-noise], which er I'll
335 like you to to spend maybe 15 minutes [C/A coughs] or something looking at. And it's about
336 something called Knot Theory, have you heard about that? (C/A: Er no.) [all three laugh]. Er,
337 it's sort of, have you been scouts or something? (C: Scouts?) You know, running around in the
338 bush with a tent and things. (C & A: No) (A: I'm quite proud actually [laughs]) [laughs] OK,
339 but, it's it's something to do with if you tie a knot and then, actually you can do a lot of maths
340 on this (C/A: [inaudible]) and it's er, well, I'll like you to look at this [paper-noise], for for this
341 question or whatever, this will give give you an impression of what it is about. You can just,
342 maybe have a look at this. And er er if about just and er then er have these er, where is, where
343 are they mmm, not not questions, but things that I'll like you to think about. It's er and you can
344 have a pen if you want. It's er, yea, have a look at what it says. Cause, what I want you to think
345 about while your are working through this is, what what it actually is you DO to TRY to
346 understand [C/A coughs] get some grasps of what's going on here. I don't expect you to have a
347 full completely picture about what's going on, after 15 minutes, but er but if you can while
348 working working through, think about what what am I doing and why am I doing this, and then
349 later tell me, what you want to do NEXT er to s to figure out the things that you don't (C: Yea)
350 now. (C: OK) So, I don't know, yea, you can sit here, you can, there is also the, the other room
351 is open if you like, I don't know, but you can sit here, it's fine (C & A: OK) and you can, er do
352 whatever you think is necessary, you can ask talk together, you can ask me, you can read, you
353 can sit and decide not to talk together, it's complete up to you, ha, (C/A: Yea) and you can have
354 some piece of paper if you (C/A: [inaudible]), and you can, and you can make notes on this
355 [questions] if you like and you can, yaa do what you like hmm. And this [C/A coughs] the the
356 copies from this book, and you can also have a look there if you want, but, yea. [inaudible]
357 Let's take this one [to herself]. There's I er, as you can see there is this black spot and this is
358 just because er it's something that is more necessary to if you want to find out what's on the
359 next pages, so that's why (C/A: Yea er [coughs]) I just erased it so [A/C coughs], it shouldn't
360 confuse you. Yea

361

362 [Silence: Can hear my scribbling in the beginning, some coughing.]

363

364 [In the next three sections it was very difficult to hear who said what; they talk at the same time.
365 The words are therefore just transcribes in one long sequence.]

366

367 [After silence in 1.10 min: Silent giggle

368 After silence in 2.20 min: Deep sigh

369 After silence in 25 sec: What's this [and some small talk]

370 After silence in 20 sec: Mumbling

371 After silence in 20 sec: Giggling

372 After silence in 40 sec: Excellent, laugh

373 After silence in 30 sec.]

374

375 That's not a knot. Talking to myself. That one, see this, that an, that's that not a knot, endless
376 sum, cause over the top, and from the top here, so you can put that bit, if you put that bit, a
377 loop, but that one doesn't. Yea [laugh] That's not a [inaudible], that's, [laugh] so it that, so is
378 that, mmm, hang on, yea yea yea, you're right, that no knot, that's a er, you can take that around
379 like that, that's. All these dark one are very good are they, no that one is not, yea that's a knot,
380 is that one a knot? Er, you cannot twist, yea, that's a knot, can put, you can out, that's
381 definitely, it is the same as that, well that's kind of [turn pages] think it's got the answers [turn
382 pages] I bet it does, yea, a, b, c, d, f, g, and h are not knots, yea, er, right er.

383

384 I: Just see if you can work thorough the theory in these er copy (C: Yea yea) and er (C/A:
385 [inaudible]) and through Reidemeister's Theorem (C/A: Sort of, yea) and the steps

386

387 A: Us, (C: Yea) let's just trying to work it out why.

388

389 C: So we can find the [inaudible] That goes to that. (A: Yea). It's when you pull, pull them
390 apart, the simplest [interrupted]

391

392 A: Simplest in the whole can be obtained from th that.

393

394 C: Yea.

395

396 [2 sec silence]

397

398 C: mumbles

399

400 A: I see.

401

402 [6 sec silence, then someone whispers something, sounds like reading aloud]

403

404 A: WHAT [laughs]

405

406 C: "Two diagrams are equivalent" [reads aloud from Reidemeister's Theorem at page 2 in the
407 copy]. Equivalent means [1 sec silence] "if and only if one" [silence 12 seconds, which include
408 a cough] "interior intersects the diagram" [silence with a bit of mumbling about 30 seconds].
409 All right.

410

411 C: There is a few different sort of things that you can do. That one is just (A: Is it?) [2 sec
412 silence] Sort of turning it upside down, er, swiping it over isn't it [mumbles together about 6
413 sec].

414

415 A: Nee, you are doing it wrong, it is sort of similar to it er it just

416

417 C: Just rotate, it's just rotated in fact isn't it. The bottom just crossed over isn't it.

418

419 A: Yea it's just rotated that's all.

420

421 C: Yea it's that's exactly the same.

422

423 A: You don't actually have to do anything to it [silence 1-2 sec] you just turn it around, or I am
424 not sure that what you're sup, that's what it means. [1 sec silence] How do you get that just by
425 [inaudible] isn't it.
426
427 [2 sec silence]
428
429 C: Mmm, that one [1 sec silence] put that one under there
430
431 [40 sec silence, the last 5 sec one can here pages being turned].
432
433 I: Should we discuss it now, or (C: Yea) are you not ready (A: Yea)
434
435 C & A: Yea.
436
437 I: OK, OK. What think you about this, first?
438
439 C: Different.
440
441 A: It's completely
442
443 C: [inaudible]
444
445 A: Yea.
446
447 I: It's completely what?
448
449 C: Er randomly (A: Laughs). (I: Er). You'll never think of anything like this (I: No) You
450 wouldn't think it's like maths (I: No).
451
452 I: I didn't know, I mean, I didn't know about Knot Theory until a week ago [laughs] (C: Yea)
453 So it was wow. But, you are not the only one (C/A: [inaudible]) who, so. But, how, how, what
454 did you, did you understand it or what didn't you understand, and what, how did you work it
455 through?
456
457 C: Read it through, the whole thing, and then remember the bit which you don't understand and
458 then you read that aloud to yourself. That sometimes helps.
459
460 I: Read aloud to, you mean?
461
462 C: [inaudible] up, yea just like, reading, (I: mmm) that's er, you can hear you hear it, er
463 externally, and also er, sometimes if you think of vi visual images (I: mmm) or, you know,
464 cause this is mostly 3D, it's really hard to put on paper (I: mmm) so if you think of piece of
465 string and er
466
467 A: Yea (C: Like like) I find it much easy to understand if you just sort of visualise the whole
468 thing (C: rotating) [a few words are not possible to hear a they speak at the same time] it just
469 depends on how you, you know, cause some people [1 sec silence] (C: Yea) er, you know,
470 aren't that good at sort of, you know, imaging things spatially in er, you know, that you can't

471 see, and how to change a diagram if you look things from [inaudible] like that (I: mmm) but I
472 find that's (C: Yes quite easy) it's quite er you know useful, if you learn, especially with
473 something like this, where, it is not easy to er express the thing [1 sec silence] on a piece of
474 paper in terms of a diagram or drawing or something like that or it's it's it's difficult to draw it
475 (I: mmm). Then it's easier to visualise you know, if you can do that.

476
477 C: Er [2 sec silence]

478
479 I: What did you do, I mean, you said you read aloud for yourself, er, the part bit part (C/A:
480 Cough] you didn't understand and that helped. Er, does that always help or, [interrupted].

481
482 C: Yea (I: Why, OK). Not just in maths, in my other subjects. (I: mmm) It always helps.

483
484 A: Yea, it is for true, actually, cause when you when you read in your head, you tend to sort of,
485 you know, (C: Swap) not necessarily skim-read but not really read every word. But if you read
486 out loud, you HAVE to think about every, you know, every word, every thing you are saying,
487 see, so you think about what sentence er means and what the words means rather than just, you
488 know, sort of ceiling through [A few words cannot be heard as C and A begin to talk at the
489 same time]

490
491 C: Yea (I: mmm), otherwise if you just read it in your head it like a just read it a feel [snaps his
492 fingers] I'm just read it and, I mean, don't understand. If you read it aloud, and you come across
493 like a phrase in which, sound really awkward, like, you know, the one here, where it says: "in
494 the plane whose interior intersects the diagram in one of the configurations involved". Like that
495 (I: mmm). You just read it over, you read it, but you don't think about it, but if you read it out,
496 then er, you think about you think about the disc, and then you visualise the disc, and then you
497 visualise the plane, (I: mmm) (A: Yea yea) One by one (I: mmm), like in er, my, I do biology, it
498 helps cause you've got to memorise a lot of sequences of event like, say neurotransmissions or
499 the seven steps, if you just read it, you just read it, er if you like, think about it, what is it, (I:
500 mmm) you know, you read it aloud and you think about what's [A: Laughs].

501
502 I: Are there other techniques like er reading aloud, are there other techniques or tricks
503 [interrupted]

504
505 C: Er, yea (I: you use?) if, if you can put on paper (I: mmm) then draw out er, [laughs] like (A:
506 Yea) in chemistry like drawing out certain equations.

507
508 A: If I have to memorize [1 sec silence] or not necessarily, er, if I, if I say memorizing from
509 notes, it doesn't happen so much in maths, cause it is more understanding of the method rather
510 than you know sort of facts you have to learn (I: mmm). Not the same thing in physics which is
511 really a combination of both (I: mmm). If, if I am revising from my notes, I find that helps if I
512 actually write the notes out again, just, just copying them, out out from you know [C coughs]
513 the previously. Because, if you just sort sit down to revise, you read through, even if you read it
514 aloud, you pick up some things, remember some things. But if you write it out, you sort of read
515 again and then write again (I: mmm) and it sort of reinforces it. And I definitely found er, if I, if
516 it is something where I have to memorize [1 sec silence], you know sort of examples, sort of
517 methods, equations, how things work, I definitely find it easier if I write, er, as an aid to
518 memory [2 sec silence] (I: mmm) er, [1-2 sec silence] Yea.

519
520 I: Do you know other tricks or do you know other people have other tricks, you know, you
521 know about?
522
523 [2-3 second where C and A talk at the same time].
524
525 C: Remembering or learning?
526
527 I: Er [tries not to rush into a question about this important distinction.] Why is? Is there a
528 difference between remembering and learning?
529
530 A: Yea, well [interrupted]
531
532 C: Memorizing something, then you need to know the [1 sec silence] set number of points (I:
533 mmm) so you need to write them out, and, you know, find some sort of sequence, in to remem
534 ber it. But if you are just learning, then it's about understanding, you don't need to
535 remember (A: Yea) the detail, just (I: mmm) need to know the overall, you know, concept (I:
536 mmm) [A tries to break in] you know, principals [interrupted]
537
538 A: Yea, it's important to understand how things (C: Before) how you GET the answer (C:
539 mmm). Because [2 sec silence] not [1 sec silence] as YEA, it is in math, it's going back to the
540 same thing, mean, it's, it's fairly easy to learn a general formula for loads of different things
541 like trigonometry and things like that, you can just LEARN the general formula (I: mmm), and
542 every time you get a question, you can just sit and use it and get the answer out. But then if you
543 come up against a problem which is SLIGHTLY different to the general [inaudible] you've
544 learnt, if you just memorized it and you don't understand how it's got there, so you've got no
545 change to work back, and work it out for yourself (I: mmm) whereas if you understand it [1 sec
546 silence], you can see where it's comes from and see where you need to change it to fit what
547 you've got. (I: mmm) Got to try and do. (I: mmm)
548
549 C: Small part er, small parts er, find go into difference [inaudible] [one coughs], basic
550 principles and deduce. For basic principles you need to make up your own techniques which
551 work to understand how it works, how, you know, the problem you've got, how its solved, not
552 the actual details in how the, structure happen in between (I: mmm).
553
554 I: But how do you work out these personal er tricks?
555
556 C: It just happens [1 sec silence] ss, you just [1 sec silence] you happen to do it and it works. It
557 is general for most people, I don't know (I: No) er some people like writing it out, everything,
558 the whole page [A coughs] read from that. Some people make notes from whatever, some
559 people just memorizing notes they get in class, some people er look at it and memorize [both
560 laugh]. Er different people different things (I: mmm). And er use diagrams as well but not in
561 maths, [inaudible] things, you can, different subjects [whispers]. [A mumbles something very
562 silently]
563
564 I: You say you happen to do it (A: Yes), but er, so is is is it a coincidence that you find out that
565 (C: Yea) reading aloud is a good idea or is, has have somebody told you or [interrupted]?
566

567 C: It seems natural, nobody's told me (I: mmm) [1 sec silence]. See [A coughs] cause er [1 sec
568 silence] well its, I've learnt it more when I was doing [interrupted by A, a few words cannot be
569 heard]

570
571 A: If you don't understand, it's like if you [C tries to interrupt], it's like when you are little, if
572 you can't read a word, the way you are taught to read it is to sound it out, you know come up
573 against and you and you sound like er you go c-on-ven-tion or something. It's like some of the
574 sentences, if you don't understand what it means, you read it through slowly and you sort of
575 can't help saying it out to yourself. If you if you sort of read it through slowly thinking about
576 each of every word like, "topological disk in the plane" you memorize, and you, sorry [burps],
577 visualize it [C laughs] as you [burps again] [C and I giggles] and you visualize it as you go
578 through thinking about [1 sec silence] thinking about what you are reading and how it how it
579 you know applies to what you are trying to do (I: mmm). It's just sort of er it is something you
580 build up through the way you are taught you know to do things, you know, different things right
581 from the, right from the start.

582
583 [1 sec silence]

584
585 C: I think it is important to have like good [inaudible], when you are small er

586
587 I: To have good, have good what?

588
589 C: Good grounding (I: mmm) er. In maths is is good basics [coughs] like, you know, you er
590 sound approach to ss any problem that comes to you. See, otherwise its really generally
591 [inaudible] quite hard [2 sec silence] [a bit mumbling]

592
593 A: Yea. Er. Some people [1 sec silence] prefer, when they have like a really general sort of
594 puzzle-type-problem and not one's like this [takes up the paper with Knot Theory], where it's,
595 things like this, there, you know, the sort of thing, again Puzzlebooks [Knot Puzzles, McLeay]
596 where it's just like er er er a written out puzzle, or the one's you gets like maths puzzle books,
597 sometimes you get, some sort of little problem. Some people find the best way to it, approach
598 upon like that, is to go through really systematically and write down [hear someone scribbling],
599 you know, write down what you know, and what you know about dealing with that sort of
600 problem (I: mmm) and the techniques you, and then sort of trying to apply them systematically
601 and work through, whereas other people, and I find I tend to just sort of, I don't know, just sort
602 of look at it [C laughs, coke fizz - some laughing]. Oh dear. [laughing]. I tend to sort of look at
603 it and [sniffles] sort of not not apply it, not apply it systematically, look at it [C laughs, coke
604 fizz again]. It's difficult, it's difficult to describe exactly what what actually goes on but I just
605 sort of look at it [2 sec silence] and sort of [C tries to break in, a few words are lost] sort of just,
606 you just, you just make connections, and I find [interrupted]

607
608 I: Make connections?

609
610 A: Yea between, and between something that you've, you may, something you've seen before
611 [1 sec silence, can hear I writing] ahh you know if [inaudible] right, this is, this is probably
612 when we got so in physics the other day, it's not related to physics in any way at all, (I: mmm).
613 But, you have, what you have is er [5 sec silence] If there is just two I can think of this, this is a
614 much more simple example, you have, mmm, you have, you have six matches (I: mmm) [A

615 draws six lines] and you have to arrange them so that each match is touching two others (I:
616 mmm) and when you come up with a problem like that, some people will solve, put down six
617 matches like this and start drawing out all the different ways you can put the matches (I: mmm)
618 you know, sort of like that [draws the second line] and trying to work out how you did it.
619 Whereas I tend to find it (C: [inaudible]) yea that's right, I tend to find it, I I just sort of look at
620 it and [1 sec silence]. I suppose it's, to do with the way you think when you come up with new
621 problem, you think about it systematically or laterally sort of just (I: mmm). I tend to sort of
622 think around it, and tend to just of see ways of doing things. It's hard to, I I don't know what
623 sort of actually goes on (I: mmm), whether you sort of, I said make connection with things
624 you've seen before, or it is just [3 sec silence] you know, being able to sort of se different ways
625 to approach a problem I suppose, er [1 sec silence] I find it more [2 sec silence] more effective
626 when I've got something totally new [2 sec silence] just to sort of, relax and sort of trying to
627 just think about think about the answer rather than start to work it out, if you see what I mean
628 (I: Mmm, yea). See the difference. Rather than, yea, that's, I suppose that's the best way to
629 describes it (I: Yea). Try to think about it, rather than try to work it out. Yes but it's just not, I
630 find it more effective not to try and work out the answer, but just think about the problems that
631 (I: mmm). It's a quite good way of describe it I suppose.

632

633 I: Mmm, do you [C] do the same, or?

634

635 C: Mmm, more or less, but [2 sec silence] I tend to learn more when visual or rather than just
636 [A coughs] look at it [A & C laughs, some words are missed]. Yea, I'll try to make it more
637 visual, like that (I: mmm). I wouldn't just draw it out, I would think about it like, 3 dimensions,
638 or, look at all the possible way, chain the possible ways, like you could do in a line, er you
639 could do it in 3D I think. Probably but mainly not in 3D works (I: mmm) [C laughs]. Think of
640 an example. And, that's probably because, to a certain extent I do it more visual. (I: mmm). I
641 just do in physics (I: mmm) which is more

642

643 I: You are biology?

644

645 C: Biology chemistry (I: Ah biology chemistry OK) So. I mean. You get taught the diagrams,
646 through diagrams mmm. You don't have these points er you think of them like diagrams (I:
647 mmm) or like a sequence of event, like I said before, you don't just see it like loose on a paper
648 (I: mmm) you think of, like actual visual appearance, moving pattern, like, if you've got
649 something [inaudible]

650

651 I: Er, what?

652

653 C: In biology, say if you've got, a diagram [A says a word so a few words cannot be heard], if
654 you've like a gate, or, [A laughs a bit] (I: What's that [laughs]). You say you've got

655

656 [Tape runs out]

657

658 A: Er, some people like to learn by doing [2 sec silence] you know sort of finding a general,
659 they'll do load and loads of examples like E [laughs] and and you know sort of just get been
660 told how to do something, and then just practicing and practicing and practicing over and over
661 again [C laughs silently]. Really bor [interrupted]

662

663 C: I find that really boring. I want to achieve with the minimum effort [I & A laugh].
664
665 I: That's honest.
666
667 A: Fair enough, yea.
668
669 I: I'm not going to tell anybody you said that [All laugh].
670
671 A: They probably realise [All laugh].
672
673 C: But er, the most effective way, not just like, to please yourself to go through with practice,
674 like, I am not satisfied with minimum effort and getting bad results (I: No). Er [One coughs],
675 I've got some tendency to, er just go for quick reasoning and quick you know approach, instead
676 of going through the whole thing all the way through [A coughs] if, if a question is asking for a
677 detail, I just go straight into it, instead of going all the way around and proving that the detail,
678 is, valid. Er you know that. Some people think like that, if they see a question [A laughs and
679 coughs] some people when they see a question, they wanna verify [smiles saying this] that the
680 question is valid (A: VALID, said at the same time [laughs]).
681
682 I: Well I didn't get that.
683
684 C: Some people, when you get a questions, some people they think about the questions and see
685 is it is that a valid question, does it actually mean anything (I: mmm), is it true, can't prove it.
686 [C & A laughs]
687
688 A: Mr. X has a, he is, I think he IS a good teacher and I like this sort of informal discussion
689 style, when we have in our lessons, he does have a bit of a tendency to sort of prove prove
690 things [A & C laugh]. It can be, it can be quite useful to see, like I said, to see where the
691 methods of comes from (I: mmm), but sometimes it does little get caried away [smiles] (I:
692 Mmm [Laughs a bit]). You say, a tendency to sort of prove things which might not become
693 relevant till say, like the THIRD [C giggles] year of our maths-degree or something like that (I:
694 mmm) which is what he went into the other day [C giggles] he tends to get a bit carried [laughs]
695 get a bit carried away, which CAN be interesting sometimes (C: Uh [giggles]) you know, just to
696 get into things which er er sort of, a complicated, and you may not understand it but they are
697 interesting [C giggles] (I: mmm) cause if you are interested in something, er you know
698 [interrupted]
699
700 C: If you find it boring (A: It makes it even worse) Yea (A: it's just sort of) it's like (A: Isn't it)
701 it's like saying for example, I am not going to mention any names or anything, say (I: No no no
702 [giggles]) you really like a subject, right, if the teacher is no good [A & C laughs, especially A]
703 then (A: If if the lessons are made boring) Yea (A: Then you won't to do the work and work (C:
704 mmm) things out for yourself, then you won't do it) (I: mmm) (A: but) It is not about force, it's
705 not like, you are forced to do these ten questions, it's more like [A tries to break in] you're
706 willing to do it anyway (A: understand how to do it) Yea, you're gonna to do it anyway, you
707 you might as well do it so you understand it and get it right, and you feel good about it and it
708 takes less time. (I: mmm) (A: Yea, some folks is good to sort of go off into branches that are
709 not necessarily) Yea (A: totally relevant) Yea, just to make it interesting (A: Yea, just to make
710 it interesting, yea) (I: mmm) Otherwise you do really dry boring stuff and you get a set of

711 questions that make you more depressed and then [A laughs] and you get bad marks at the tests
712 and the exams come and, you know, it all boils down to the teaching method and the teacher (I:
713 mmm) not just about, it's a two-way thing you see, it's more about you learning [1 sec silence]
714 you being able, no, you learning as well you being taught properly (I: mmm) if you are taught
715 in a way that you can fit in, you know, then it is good.

716

717 A: Yea, cause [C giggles] some people want the sort of, just be told, like I say just to be told to
718 do examples, find it helps them learn.

719

720 [1-2 sec silence]

721

722 I: You don't think it learn, er, you learn anything by just doing example?

723

724 A: Yea, you do, you you learn how to apply the method. (C: You don't understand it) [some
725 words are lost due to the interruption] helpful in terms of [C sighs] say what you gonna have to
726 do in the exams (C: mmm) (I: mmm). It's being able to, you know, it helps you to work (C:
727 mmm [very silent]) around slightly different forms of the problems being able to apply the
728 method quickly without making any stupid mistakes, and so it is definitely useful in the form of
729 exams, but

730

731 C: That's what I DO like, for revision (A: Yea) cause I understand (A: Yea yea, once you) if I if
732 you know that basic integration for example, there is no point going through, for revision,
733 you've got 2 weeks to the exam (I: mmm), there is no point in going through your notes on
734 integration (I: mmm), cause you know how to integrate. What you need to do is have loads of
735 loads of practice [interrupted]

736

737 A: Yea, you wanna get to understand it fully and understanding where the method comes from.

738

739 C: Mmm, not really, you don't wanna know where it comes from and prove it, you just wanna
740 be able to [interrupted]

741

742 A: Well it CAN be helpful.

743

744 C: Right. For some people they they like.

745

746 A: It's only [C giggles] pure in the in marking Pure, loads of the questions, the last part of all
747 the questions in the exams are like 2 marks and they are totally different, you know, there is
748 always something really to do with er [coke problems] something that is not necessarily [1 sec
749 silence] of the style you've done before, er if you know where that the (C: mmm) method of
750 sort of dealing with that sort of things comes from, you, sort of work backwards and (C: mmm)
751 (I: mmm) [2 sec silence] and find the answer. (I: OK)

752

753 [4 sec silence]

754

755 I: OK, I have one more question (mmm), er, if er, naa, you said, you were sitting and reading
756 this and [C/A coughs] er trying to work out what it was and reading aloud and these things, and
757 er, can you tell me in what way this learning situation was different or the same as what you
758 normally experience?

759

760 C: This is er individual, you don't get taught, this is not teaching, this is self-learning (I: mmm)
761 and I can't teach myself something new (I: mmm). Well I can if I REALLY sit down and can
762 be bothered [A coughs] but it's is really difficult to motivate yourself to something, like this (I:
763 mmm) it doesn't, like tricker anything I know from before (I: mmm) apart from you know
764 [interrupted]

765

766 A: I find it useful with something like this if there were examples of how each thing work and
767 not for me to do but it it working through an example, like say, in er like say, introduce some
768 ideas like in the first paragraph, if after that it then said you know, had a few pictures shown
769 like this, you know, this is a link, you know, this is what (I: mmm) a component you know, or
770 like things like that, and so it went through some example of of how it work and I find that
771 helps me to see [2 sec silence] to see how to see how the method is applied [2 sec silence] you
772 know, like apply to what you're trying to do (I: mmm) cause if you, like this, it gives you a
773 whole load of new ideas (I: mmm) and it is only when you get to the bottom or start to look at
774 some other sort of examples like knots in here [Knot Book, McLeay] you start to see how it
775 works and sometimes you can miss out some of the early details if you do that (I: mmm)
776 whereas if you get examples and you understand each bit before you move on [1 sec silence], I
777 think that's better. And [1 sec silence] if I was trying to teach myself to understand [1 sec
778 silence] properly you know completely everything that is in here [1 sec silence] I definitely
779 want to find [1 sec silence] examples of each bit each idea that given be, and try to work, you
780 know, sort of see how they see how they work and how each idea is used and what it relates to
781 in terms of what you are actually, you know, using it for, before I move on. [1 sec silence] So I
782 sort of take this and add to it with, trying to get some other examples from somewhere if I can't
783 put [2 sec silence] so [3 sec silence]

784

785 I: But what, what, would you do if you didn't have, let's say you only had these two pages, and
786 you only had to, let's say you had to present this to the class next Monday or something, how
787 would you work it through?

788

789 A: I would draw them out myself. I would try and understand it, I just take each paragraph or
790 each you know set of ideas like I said, [inaudible] ups I seem to have [he can't find the pen, 'I
791 helps and he finds it]. Er and then sort of take each set of paragraph and draw out for myself
792 you know some things and how they [1 sec silence] sort of how they link together (I: mmm) er
793 and then you know sort of like try to understand write it out for myself what different things are
794 and just draw out some sort of random, sort of knot and try to work out how they fit into the
795 arguments I've got on page [sniffles] because [3 sec silence] I definitely find that helps me to
796 understand the concepts an idea [1 sec silence], sort of method to doing something if I can see
797 how it's actually used (I: mmm) rather that just bah sort of method or whatever it, I need to be
798 able to see how it is applied [1 sec silence] and be taken through that applied or going through
799 myself as applied [1 sec silence] before I can get proper understanding. [1 sec silence]

800

801 I: What about you [C]?

802

803 C: [1 sec silence] er, that's like a visual thing, that's how it best work. [2 sec silence] Draw it
804 out [1 sec silence] you can't do many worked examples, it's just getting to know the er theory
805 so if it is like definitions or explanations er [1 sec silence] I'd write it out in my own words (I:
806 In your own?) own words (I: mmm) I'll just write it out cause some of, some of it, doesn't

807 really follow (I: mmm) er so you just write it out in your own ways and draw a diagram if that
808 helps and trying to understand for yourself and read that thing again to see if it is the same, and
809 just. And when I present it then er get at the key bits and then just talk about that (I: mmm) and
810 then un top, and once you've got that frame then you can put the rest. (I: Mmm) You can er put
811 all the details in afterwards.

812

813 I: Ah. You said, you would read it again, and then you [another] said "yes" does just right after,
814 why is it good to read it again?

815

816 C: [A & C say something at the same time, a few words cannot be heard] sure about your own
817 notes are correct notes, they are not just, they are not something completely wrong (I: mmm)
818 cause you could read a word and think of it in two ways. See, a word can have more than one
819 meaning or a phrase can have more than one meaning, and if you have the com, you know,
820 completely wrong meaning [1 sec silence] and you extend your ideas on that, and you read
821 back, then you [A coughs] see a clear difference (I: mmm) and er going back again, and do the
822 whole thing again, or not the whole thing but [interrupted a word is missing].

823

824 A: Think of, think of how to do something for yourself [1 sec silence] and how something
825 works, you put it into terms that you are happy with and you understand and then you find that
826 you can apply that back to the, you know, more complicated explanation or more complicated
827 example that you've first been given, then you know that you've understood it, or I'm just, yea,
828 I think (I: mmm) that's [interrupted]

829

830 C: And if you REALLY don't understand what's happening, you can go and ask someone (I:
831 mmm), if noone can help you, find another book, find another explanation (I: mmm) if you
832 can't do it, then just leave it (I: mmm). I'll do that cause (I: Nearly completely or) yea cause
833 there's, if you don't understand it, there is noone else to help you, no other book, or no other
834 resource to help you (I: mmm) then it's just going round and round and you're going
835 [interrupted]

836

837 A: I think, yea, if you, I mean if you get into a situation like that, it's like with some of these
838 puzzle, you're given, if you CAN'T do something (C: Yea) and you spend AGES thinking it
839 over and over and over and you start get frustrated and then you don't think clearly, if you go
840 out and do something totally different, you know relax sort of just generally open yourself up to
841 the problem and just [laughs] like I said, just sort of think about it when your are generally not
842 trying too hard to solve it (I: mmm) then I personally certainly, sometimes find that I can just
843 sometimes certainly see through the answer (I: mmm) you know, how I'm getting wound up in
844 how to how to get to it or what it should look like, or you can just sometimes see straight
845 through it.

846

847 I: Mmm. [1 sec silence]. You talked about words and their meaning and then about er er and
848 you also talked about er being able to SEE things. How is this er in relation to each other, the
849 words and the thing, if you understand, what I mean?

850

851 [2 sec silence]

852

853 A: Yea sometimes [coughs] I mean, from, if you just READ a sentence, and just take it as the
854 words, then, it's difficult to see er see sort of different ways in which it can work, if see what

855 mean (C/I: mmm) if you just read a sentence like the very first here “A link is a finite collection
856 of mutually disjoint disjointed simple closed curves” if you just take that as the words, then [1
857 sec silence] it is difficult for you to see, sorry [burps] (I: It’s all right) any other way that it can
858 work. If you start to visualize [2 sec silence] visualize what it is talking about, then you can SEE
859 all the different ways in wh in which it can happen (I: mmm) if you see what I mean (C: mmm).
860 Until you sort of [1 sec silence] (I: mmm) make you a, you know, a broader definition a broader
861 explanation than just what you are initially given.

862

863 I: So you’re saying there there is the words and then there is the, there is the er er the er you can
864 see it. And er so, does the, is it important that the words come first and then you can see what
865 the words are about, or do you want to see it first and then be explained and then get the words?

866

867 [1-2 sec silence]

868

869 A: Well. [1 sec silence] I don’t know really. It’s difficult, it’s difficult, so I think it probably
870 depend on the sort of problem [2 sec silence]. If it is a very sort of [1 sec silence] visual
871 problem where you have to sort of think it through maybe in 3 dimensions or something like
872 that, which something like this is, the I think it is probably better to have pictures first and
873 maybe dealing with graphs as well and something like that, and 3 dimensions graphs or or you
874 know, if you are doing vectors and you’ve got 3 dimensions it may be better to have the picture
875 first, and then the words explaining how it works, but if it more of a [1-2 sec silence] sort of
876 linear methodological process it might be better to have the words first and then pictures to help
877 you understand, cause it is the words you are trying to understand, but where there is something
878 visual like this or graphs or vectors [I coughs] it’s the it’s the it the pictures if you like, that you
879 are trying to understand (I: mmm) so the one supports the other depending on what what it is
880 you are trying to learn, I think (I: Mmm, Jaa, OK, Thank you very much.)

Appendix G: Full transcription of the second English group interview

- 1 I: First I want to hear, are are you in the same class?
2
3 E & D: Yea.
4
5 E: We're yea, everyone coming up is in the same class.
6
7 I: OK. Then how many are you in this class?
8
9 D: Seven.
10
11 E: Seven or eight (I: Seven). Not very many. (I: No)
12
13 D: It started up with a lot more people, and then (E: [inaudible]), it started up with a lot more,
14 about 14.
15
16 E: People dropped out, er they couldn't cope with the pace, cause it is quite a fast pace set (I:
17 mmm) er cause we're doing we are doing more than one A-level.
18
19 I: mmm, ah OK, you mean, so you are doing maths and for instance physics.
20
21 E: No, maths and further maths.
22
23 I: Oh.
24
25 D: We've finished the maths over in January (I: Oh) and then we spending from January to
26 June learning er hard and further maths.
27
28 I: Mmm. So you are very busy I guess? [E & D giggles] Yea Ok. So and you have different
29 teachers at different times during the year don't you? Or how is it.
30
31 D: Yea we have two teachers but we keep the same two teachers throughout the year (I: mmm)
32 but we er have [blows] rev certain number of lessons with one and then.
33
34 E: One of them is more a pure maths teacher (I: mmm), and the other one statistics-mechanics
35 tutor.
36
37 I: Mmm. OK. How long time have you been at the school.
38
39 E: Two years er, this is my second year (I: mmm) (D: The same)
40
41 I: Are you at, are you at are you er iv living are you living in town or.
42
43 [Begins talking at the same time]

44
45 D: No, living in the school.
46
47 E: I live in Burma. (I: Burma?) Next to er Thailand (I: Ah, OK, So you are far away from
48 home) Yea. (I: Ah OK, yea, so you have to stay at the school I guess). Yea.
49
50 I: Ah, OK, OK. Er. Can you describe to me how how a normal maths lesson takes place?
51
52 D: Er, it really depends on the teacher (E: Yes) [D & E laugh]. One teacher we don't actually
53 learn anything. (E: Yea er) we don't. Er, OK. Mr X
54
55 E: Let's take, OK, our pure maths teacher, we usually [inaudible] spend a full forty minutes or
56 an hour and 20 minutes doing theory. (D: mmm) And it is quite intense, and then er (D: Yea) as
57 he goes through it you can put up your hand and he'll sort of deviate and then, but he'll always
58 have sort of the main (D: He he goes) [inaudible]
59
60 D: He goes through it quite quickly and sometimes I find there are bits that I don't really
61 understand in class (I: mmm) so I have to read it up in the textbook later (I: mmm mmm) and
62 just make sure so, do I know it.
63
64 E: And if we've got any work to do he will set it to do outside class, he wont spend any time in
65 lessons doing works - its quite a fast pace set so - you do the theory in class, and do your work
66 outside, if you have any problems you gonna go back and see him (I: mmm) and go through in
67 class later. Er the other teacher er we do do work we do do work in class, don't we (D: Yea) we
68 got a bit slower er I don't know sometimes a bit too slow but I think we all
69
70 D: We all under yea, we all understand what's going on in the other lesson, but its gone through
71 much slower pace.
72
73 E: I think it depends, like, cause I think sometimes people get frustrated cause think, we go too
74 quickly, but at other times er you know, cause we all know what's going on er, we're all
75 suppose to be reasonable good at maths, you sort of spend two weeks going through something
76 which we could all learn in ten minutes. It's a bit frustrating, having still [inaudible] work.
77
78 I: Mmm, you say you have sometimes have to read it up afterwards at home or something?
79
80 E: Yea, er, when when we are doing er we do questions bases on the theory that we did in class,
81 so, if you can't do the questions you obviously didn't understand the theory properly, so, er
82
83 D: You just go back and look at, when we did an ex in class, you go back and you read the
84 notes and make sure that you do understand it.
85
86 I: You don't read the book, you read the notes?
87
88 D: Yea, you do both. Basically with er with our pure maths teacher he gives us all the
89 information in the notes, but sometimes to clarify things further then look up in the book as
90 well.
91

92 E: Our pure pure maths teacher been doing that for a long times, doing those [laughs] er what
93 we need to know and what we don't er, we get the full notes done and generally we I work of
94 the notes and then, when it's coming up to an exam, after I revised the notes and rewritten some
95 more notes and then go back to the book and make sure everything in the book is actually in the
96 notes, like to see that quickly (I: mmm) er, with the mechanics and statistics I generally work
97 out a booklet.
98
99 [2 sec silence]
100
101 I: You don't, so you don't use your notes in in in mechanics and this statistics
102
103 E: No I (D: Not much) we write our own notes really, don't we
104
105 D: Yea, I I use I sometimes use my notes, I use my notes to do the exercises when I am revising
106 for an exam, I don't think I have ever used the notes, I always use the book (I: Mmm) Just
107 because we know that everything that's in there is relevant to the modules we are going to do.
108
109 E: I always write my own notes for exams.
110
111 I: How do you write these notes, is it from the book or from how do you do it?
112
113 E: Generally from using the book as a guideline but I write my own notes so that I understand it
114 and if there is something in the book I do not understand I write it from other sources or other
115 notes.
116
117 D: I don't do that. Er differently I just I just read the book and make sure and to make sure I
118 understand the book I work through the exercises in the book, and if I get it right move
119 [inaudible] on to next exercise. I very rarely write revision notes from the book.
120
121 E: I am a bit angel/ament [inaudible] about things (D: Laughs) (I: Sorry) Angel, sort of er I have
122 to have [laughs] all my notes like written out, I basically have to have the textbook rewritten in
123 short form for myself and I am not satisfied that I know everything till I have done it myself, I
124 basically rewrite the textbooks [laughs]
125
126 I: Why does that work?
127
128 E: I don't know I find that after I've sort of rewritten it, I know it. And it's organised in my
129 head because I learn what's on my notes in the order I make it in my notes, so when I am in an
130 exam I don't think back to the book I think back to my notes.
131
132 I: So you rearrange the book actually?
133
134 E: Yea, into the own sort of way my mind works, so that if
135
136 D: How does your mind work?
137
138 E: I don't know, it's going to be really logic when everything's gonna (D: [inaudible]) be in the
139 right order [D laughs] looking for [inaudible]. Er I do work slowly because of it, Er cause I am

140 so er I always have to have everything in logical order and everything perfect I can't just sort of
141 er [1 sec silence] work at a textbook that jumps from here to there. (I: Mmm) I like things to
142 follow on. I can jump back when I am using text but, when it comes to revision I like
143 everything sort of, in a nice package and (? : [inaudible]) if it is not in a package I try I like to
144 make it to a package, I go and research it, makes sure it is all nice [laughs].

145

146 [1 sec silence]

147

148 I: OK, er, I mean, I just say, I am just interested in what it is you have to say, so, there is not
149 any right or wrong answer (D: Yes) it is just, I am just curious to know how you (D: The way
150 we work) yea, the way you work, exactly. (? : coughs] Mmm, you you, I wonder you said
151 something about er that that er that er when you work through a chapter you look at the
152 exercises and when you, if I understood you right, when you can solve these or you know what
153 they are about and can, then then you have, you think you have understood it, that (D: Yea) or
154 er, is that correct?

155

156 D: Yes, I I work through the exercises and I normally took the late questions cause they are
157 harder (I: mmm) and if I can get those right, you know, without having to think about it, but I
158 can just write down the answers and know what I have to do then I see my [inaudible] and go to
159 next exercise.

160

161 I: Mmm, does that mean that what makes you sure that you have learnt the mathematics is that
162 you can solve, you can solve the problems.

163

164 D: Mmm, because the problems in the book are often taken from exams (I: mmm), er from the
165 exam papers.

166

167 I: OK. [whispers: "great"] What do you do when you meet some maths that you can't really
168 understand, then you you take notes or ask other people or

169

170 D: Er, it depends on the situation.

171

172 E: Depends how difficult it is, I mean I usually spend quite a, you spend quite a long time
173 trying to solve it, but if, if you can't, the first thing I do is go and ask my friends who are doing
174 the same work. People in (I: The classmates) our class, yea. (I: OK) I wouldn't, yea, because I
175 am quite lucky cause out of our set, out top-set, 4 of us are all next door to each other. (I: Ah)
176 We all just happen to be in the same house. [Laughs] So if there is a problem, generally
177 between us we can work anything out really, some [inaudible] of it.

178

179 D: And we all at the same standard really aren't we (E: Yea) in that set.

180

181 E: And if there is a really big problem we ask the teacher to go through it.

182

183 D: Yea and we we all seem to have the same problems a lot of the time. (E: Yea) In some work
184 and then it will, then it will be because, if we've gone through too quickly in class (I: mmm)

185

186 E: Sometimes it is cause the way the questions worded though, more often than not, not cause
187 we can do the maths, but because we don't er a lot of it is sort of notation and the way that you

188 approach question and if you learn in the textbook one way and you do it 50 or 60 times
189 practise and then they approach it from a different angle er if you haven't learnt the the actual
190 theory really well you just can do the questions and when you come from another angle,
191 sometimes be a bit stock.
192
193 I: How do you work it out together then?
194
195 [1 sec silence]
196
197 E: Er (D: mmm) (I: This formu [inaudible] or what)
198
199 E: We just sit and talk about it and try different ways, trial and error sometimes. (D: Yea) We
200 get
201
202 D: You see, it's often, if [inaudible] take different approaches and then we work out, cause the
203 book has the answers in the back, and we work out when we've got the right answer. (I: You
204 calculate backwards?) Yea, sometimes, if we get really mixed up (E: Yea) So you look at the
205 answer and then you work out how they've got it and, and most of the times you probably
206 stupid because you know there will just be one stupid mistake (E: Yea) [laughs] you've made
207 (I: mmm) that you actually understand it perfectly.
208
209 E: Yea quite often if there is something wrong you've just made mistake, especially with long
210 calculations or algebra, you just show it to a friend and and, you know, sometimes you can just
211 look at it for an hour and not see the little mistake you've made and someone else looks at it
212 and see it straight away. (? : Mmm) see it with a fresh mind.
213
214 I: But can you, you can't, you, I mean, look at it with a fresh mind yourself?
215
216 E: Not if you've been, maybe if you go away and come back
217
218 D: Yea, but if you've been concentrating on it so hard, then sometimes you don't spot it.
219
220 E: Specially if it is due the next morning and its the last thing you do, and this is the last
221 question and you HAVE to get it done and you just sit there for 2 hours and you can't do it.
222 Maybe at, er a real bad thing in that test the other day (D: Mmm) didn't we, and we er we put
223 the rak-nought-rational for zero and not one
224
225 D: And we ALL thought it was zero, we work out, we thought, [inaudible] cause it will show
226 that something (E: Yea, big) we thought they got it wrong (E: Because) cause we all (E: Yea)
227 did it
228
229 E: Cause we were doing it together, we all just put not rational-zero and carried on, we got the
230 wrong answer in the end. It took AGES for some (D: [inaudible]), I think it was me [D & E
231 laugh] we went back and sort realised that naught-rational was one (D: [inaudible]), its things
232 like that (D: Yea).
233
234 D: And then we'll ask the teacher [inaudible] make it quite clear.
235

236 I: How does he help you or or why does it help to ask him?
237
238 D: He'll go through it on the board, er
239
240 I: The theory or er the problem?
241
242 D: No, the problem (E: But) because when we've done the theory most of the time
243
244 E: But he will relate it back to the theory, whereas when we do it we just try to solve the
245 problem, but he'll do it step by step going through the theory, and won't miss out any sort of er
246 links (D: Mmm) or bits of theory he needs to solve the problem.
247
248 D: And he'll ask you for the first principles. (? mmm) He'll say, well you know this results so
249 you just fits [inaudible] to here, and that helps.
250
251 I: Mmm, good, how, why does it help when when he does it on the blackboard er it's not a
252 blackboard it's a white board, but?
253
254 E: Er [1 sec silence] I think cause he's does it from first principles (D: Mmm) it's logical, going
255 doing the things from first principles er is something you rarely do when you're solving a
256 question.
257
258 D: Yea, specially with the later questions in the exercise because you look, [inaudible] you'll
259 like to have all the easy things, you've just be pushing them in, er and sometimes you forget
260 about how you got it or what it actually means.
261
262 E: Especially with sort of, well we're doing differential equations (D: Yea) at the moment and
263 doing auxiliary equations and particular integrals and whatever, you've just got, you know the
264 method er for doing it so you just write it out and take your integral, you don't actually go
265 through the two pages of work to get it, cause you've got your rules and you've got your things
266 in your head (I: mmm)
267
268 D: But doing it on the board means that you understand why you are doing it, not just how to do
269 it, and I think that definitely helps. (I: mmm) But again it's coming back to what E said about
270 how the questions are worded in a different way, if you know HOW to do it, I mean then that's
271 one thing, but if you know why you are doing it, then you can do any question really.
272
273 I: Can you explore that distinction between how to do it or why, er why is this different? [The
274 researcher here took a deep breath not to rush into a question, gained time to think about how to
275 probe]
276
277 D: Er, if you know how to do it, you've just learnt the method and you, you just change the
278 numbers er, but you basically recognise it because you've done the problem lot's and lot's of
279 times before, so understanding how to do it, er, understanding why you do it is more [2 sec
280 silence] er
281
282 E: I think there's a big distinction in our class because, I don't know, I think people in our class
283 we get frustrated if we don't know why you do something, whereas a lot of maths students

284 would be happy to know just how to solve the problem and get the right answer, er if there is a
285 particular thing you have to do to solve a, [1 sec silence] any sort of simple equation and you
286 don't understand WHY you do it, I mean you get a little bit frustrated.
287
288 D: And it's really frustrating when they say, well you don't need to know why you do it
289 [giggles] you just did so, so that (E: [inaudible]) [laughs]
290
291 E: If it's beyond er A-level er syllabus
292
293 D: But because a lot, most of our class would be studying something to do with maths when
294 they leave (I: mmm) then obviously they want to know (E: Yea) why you do it.
295
296 E: I mean it's a quite good thing in our class cause our teachers, well especially, our teacher for
297 pure maths knows that we'll we'll be going on and studying further so he take the time to show
298 why you do a particular (D: Mmm) thing, then in other classes you just skip through.
299
300 D: And (E: [inaudible]) then again it can be quite frustrating when he does something, he sit
301 there trying to prove it and he just get it, and then he just say ah [inaudible] [laughs]
302
303 E: It's actually easier er in an exam if you know why you do something rather than just know
304 how to do it, cause if you get stuck on a question and you just knew how to do it, then you're
305 completely stuck, whereas if you know the principals, right from first principals and you CAN
306 go straight from the beginning all the way through the end, it may take longer, but at least you
307 can work through it logically and you know why you are doing it and eventually you will get
308 out the right answer or at least you can make some progress (D: mmm) and get some marks.
309
310 I: Is there a difference between learning how to do it or learning why, why it is like this?
311
312 [Two next line came at the same time]
313
314 E: Yes.
315
316 D: Yea, there is a big difference. Er
317
318 E: I I can't think of a good example though. (D: [inaudible]) It's like er
319
320 D: It's like differentiation, I suppose. (E: [inaudible]) When you are integrating something.
321
322 E: Er it's like building a house. They can teach you how to build a house, that all, likes, it won't
323 fall down but they won't tell you WHY you need pillars here and there to keep it standing up,
324 er, [giggles] (D: No er I think that's, I I I like it really [I thinks D is trying here to support his
325 example by telling E that it is not silly]) that's [inaudible] you know what I mean, so that if you
326 were stuck by yourself trying to build a house, but you had a problem, if you didn't know why
327 you were doing particular things, you wouldn't be able to overcome the problem (? : mmm) you
328 just know how to build a basic house.
329

330 D: Some of the answers made be easier if you do go to a level that's more advanced than what
331 you actually need because then when you're in the exam it's easier, because you can actually go
332 beyond and you need that er we had that particularly with [1 sec silence] pure maths.

333
334 [2 sec silence]

335
336 E: Mmm, it's like er, learning complex numbers and things like that to solve er quadratic
337 equations and things instead of using other methods, it was quite simple cause the time, when
338 we first did it, we didn't need to learn, we didn't need it (D: Yea) there were other ways of
339 getting around it

340
341 D: And you just say, well, this is impossible to solve because. (E: Yea) And now we're
342 [inaudible] it's not impossible to solve.

343
344 E: As things like that, if they go ahead and teach you how to do it, I think you find it easier er to
345 approach things. [1 sec silence] It depends on who your teacher is I think (D: mmm). Probably,
346 all through maths.

347
348 I: Why does it depend on the teacher?

349
350 E: I just think

351
352 D: Some of them can look out and say, this is all you need, you don't need to know why you
353 do, you just do this this and this. (I: mmm) which is all right to a certain extent, er, but if you
354 want [inaudible] with the difficult problems you can't work it out for yourself as easily.

355
356 E: You're gonna make pl er they gonna be enthusiastic as well, I know teachers just have
357 different approaches, I absolutely hated maths until my last year at GCSE, and I was never
358 gonna do a maths A-level.

359
360 I: What makes you (E: [inaudible]) change your mind.

361
362 E: I had this really cool maths teacher, who who was quite a good guy but he, he was really
363 really intelligent [inaudible] like 3 maths A-levels or something, so he knew his stuff, and he,
364 but he was quite cool and he just made, he just made it fun, I actually learnt more with him in a
365 year than I had done ever before, cause he actually paid attention. I don't know what he did, I
366 mean it's just teach, some people have different methods and ways of approaching kids (?:
367 mmm) and sometimes er some are better [inaudible] (I: Some are better what?) Some are better
368 er sort of er what's the word (D: Er relating well) Yea (D: to, relating well to sixth form
369 students) I think, I don't know, once you get to A-level or university, standards are not so
370 important the the get on with the teachers approaches it. [inaudible]

371
372 D: Yea when you get, when you get to A-level you teachers are confident in that they can teach
373 you well, but up until then in order to inspire an interest in maths (E: Yea) its quite difficult,
374 you need someone who is nice

375
376 E: But by that time you want to do maths and you are interested in it and it's your choice to do
377 it (D: Yea). But I think people, I think a lot of people here are quite intelligent and don't do well

378 in maths when they were younger, and don't go on, its because of their, er, I'll blame it on the
379 teacher though, I don't know, they are all lacy people and all sorts (D: mmm) of different
380 reasons for that

381
382 D: But it IS very difficult to be interested in maths [inaudible]

383
384 I: Why?

385
386 E: I don't know, I've (D: [inaudible]) quite interested in maths, I was interested in like, I wasn't
387 interested in maths I was interested in like, little more applied maths, maths that had a (D: a
388 purpose) a purpose [D laughs] er er or a problem that that seemed impossible or, like a brain-
389 teaser in maths and er

390
391 D: Yea I used to like all those logic puzzles (E: Yea) when I was younger. (I: mmm) (E:
392 [inaudible])

393
394 E: I know some people don't like that sort of stuff [1 sec silence] I don't sort of basic addition
395 and subtraction and a lot of the rubbish, I like to I like when it was used to solve interesting
396 brain-teasers and things.

397
398 I: But you have to learn to calculate to to apply the maths?

399
400 E: Yea.

401
402 D: Yea, but it's sort of boring [E laughs] stuff, you has [inaudible] I remember actually when I
403 was taught Pythagoras the first time, I thought it was really clever, and I, it was the first time
404 that I had actually put [inaudible] the maths hat, I don't know, it was interesting, it was one of
405 those relationships that's just

406
407 E: I don't know, I thought I thought trigonometry was the most useless piece of rubbish until ye
408 later on when I found out you can solve other things with it er do loads of other stuff with it.
409 When it is just 3 graphs [E & D giggle] and that you are forced to memorise them and where
410 the er [inaudible] frequency, where it crossed the axe's and stuff, come (D: mmm) un, you can't
411 cope with that.

412
413 D: When I when I first came here, the first couples of weeks I found math very difficult because
414 it is kind of hard to adapt to a different teaching style.

415
416 I: How how was it taught where, at the old school?

417
418 D: Er, it was [1 sec silence] er, I'm trying to I'm trying to remember now, it was more, it was
419 much more mixed ability set, so it went a lot slower er and we did a lot more wok in the lesson.
420 (I: work?) and we worked from textbooks (I: mmm) whereas here a lot of the times you won't
421 even open the textbook during a lesson, and sometimes I don't even bother bringing my
422 textbook [inaudible] I know it is all done from the board. Er

423
424 E: Completely different from mine. We had these little books, there was a whole series of them,
425 maths in all the books and there was an order you had to do them in and everyone just did the

426 books by themselves and the teacher didn't teach you anything, books told you how to do it.
427 You could go to teachers there to help you obviously, the whole time they come around and
428 make sure you understood and stuff, but basically if you wanted to progress in maths there was
429 a minimum that you had to do, which most people did, but if you took an interest and wanted to
430 be any good at maths you have to, it would be of your own back, you know, you had to put the
431 work in and things, and I didn't actually like that, I mean [1 sec silence] the system worked for
432 me I think, but, although I didn't think it was the right approach, I think here, when when you
433 do things as a class

434
435 D: you are definitely pushed harder.

436
437 I: Why does it, why is it better if you do it in a class than, why is it good

438
439 D: It's not (E: [inaudible]), it's more interesting, you have other people to (E: Yea) balance the
440 result.

441
442 E: Yea and I think when people explain it to you it's just the their way of learning, I mean
443 sometimes if, you know with the book it is very difficult to, for an author, to er to get an idea
444 across, it will be much easier and better explained by a trained teacher.

445
446 D: Er, and like E is saying it also helps to have friends who you can talk to (I: mmm), if you are
447 not sure about something.

448
449 I: You said something about press, just a while ago, that it helped just or something, I was just
450 wondering what you meant.

451
452 D: Er, you're pushed harder.

453
454 I: Why is it, why does that work or why is it good?

455
456 D: Er, because you don't, you're pushed to er to the edge of your ability, where you can't just
457 get away with just doing your minimum, especially in the tops you know we could quite easily
458 do a maths A-level, not work very hard, in fact we could probably get away with not doing
459 much preparatory outside of lesson, and instead they are pushing us to do more er and that's
460 good because it prepares you for going to university and study the maths more.

461
462 E: I don't know, they press you harder because they they do a problem, that's in the textbook
463 and they give you the method, but them if someone else is teaching they'll go "Ah", what
464 happens if n [inaudible] equal to naught or you know, or not equal to something. And it is those
465 things that aren't in the textbook but, er those little extra's that a teacher of some of these
466 studied maths further can appreciate that you can then er go and explore that and that's a lot (D:
467 Mmm) more interesting. And you

468
469 D: It's a set of completion, isn't it (E: yea) It's like tightening up all the loose ends which you
470 don't HAVE to do to pass the exam but it just, I don't know (E: Yea), it's more satisfying to
471 you.

472

473 E: That's what I mean by writing my notes (I: mmm) you've got all the little bits sort of come
474 in together and you have a nice picture. And also of you do it as class, especially if people are
475 taken an interest, if er, usually before the teacher even finishing the main method there will be
476 people saying "Oh, what if this of that happens or this particular case" and then you know, go
477 and look into that (I: mmm) till everyone is satisfied with they know everything.

478

479 I: Does the teacher encourage you to to come with these suggestions and questions?

480

481 E: Yea, er, I don't think they have to anymore really, now that we are all working, people just
482 do (D: mmm) people want to know and into the habit of doing it, but I think when you are
483 younger, specially if you are a quite student, they always seem [D & E talk for a while at the
484 same time] If it was [inaudible] obvious you didn't understand er

485

486 D: We do, I mean [inaudible] [E & D talk at the same time] [inaudible] do that sometimes
487 [inaudible] occasions kind of write questions on the board, then then say adding, go around
488 class like that [inaudible]

489

490 E: X really does it like that, it's like a Monday morning and everyone is half a sleep. And make
491 sure you're on your toes. But when you're younger they always do that don't they, there's
492 always the guy who are staring out of the window and then [D laughs, says "Yea"].

493

494 I: Yea. OK. Well I have some maths I would I want to show you (D: OK, can we have one of
495 these? [cookies]) Yea, just take, it's for you, eat them now or take them with you, or do
496 whatever you like. [Some noise from paper] I have heard that you have never seen it before and
497 that's er the purpose. I want you to (E: together or separately) er, you can er, you can do it the
498 way you you prefer er, you have about 15 minutes to have have an understanding of what what
499 this is. You have, you don't have to break it to pieces (E: I only want a little bit) Ah OK [D
500 laughs] And then I have on this er sheet (D: [inaudible]) I have some some questions for you
501 and er. Well it's questions I am going to ask you afterwards.

502

503 D: So this explains it.

504

505 I: Yea, this copy is taken from this book about graph colouring and it is about something called
506 Knot Theory, I don't know if you have heard about it before (D: No) A knot it, I don't know if
507 you've been boy-scout or girl-scout, er things like this [paper noise from book] (E: mmm)
508 Actually you can do maths on this. I didn't know it until my supervisor told me [all laugh] it's
509 quite new for me as well, so I thought it was new for you too [all laugh] and and so if you, you
510 can look in these books if you want, you can discuss together you can have some pens, borrow
511 some paper, you can, do whatever you like to (E: Right) get this understanding (D: OK) and
512 then afterwards I want to ask you er how you approached it and what you did when you met
513 something you didn't understand, and, yea, (D: [inaudible]) and how this is different from a
514 normal learning situation, er er give you about 15 minutes and then (E: OK) er

515

516 [0-30 sec: Silence

517 30-35 sec. Laughs]

518

519 I: This black one is because it's some information you didn't know (E: Right) didn't need to
520 know.

521
522 D: [inaudible]
523
524 E: I don't know, I have more problems with the English.
525
526 D: [Laughs] "mutually disjoint"
527
528 E: [inaudible] know English [inaudible]
529
530 D: I do know English, I do know English [inaudible] [D reads a bit aloud]
531
532 [20 sec silence]
533
534 D: Ah I see.
535
536 [10 sec silence]
537
538 [paper noise from book]
539
540 [5 minutes and 10 sec silence]
541
542 D: Can we look at this [book]
543
544 I: Oh yea, please, oh yea
545
546 [paper noise from book], E hums
547
548 [5 sec silence]
549
550 E: [inaudible] crossing points
551
552 D: Er [4 sec silence] yea, I think, yea [inaudible] cross one connected to this [explains with
553 confidence] (E: OK)
554
555 [4 sec silence]
556
557 D: Is it a knot if you can trace it continuity? [inaudible]
558
559 [3 sec silence]
560
561 E: Is it? (D: [inaudible]) I thought a knot was something that held two things together.
562
563 D: Yea but you can [inaudible]
564
565 E: [inaudible]
566
567 D: But a link is [inaudible] than two knots together
568

569 E: Oh yeah
570
571 D: Like that, you can't continue, but that you can.
572
573 [paper noise from book]
574
575 [30 sec silence]
576
577 [paper noise from book]
578
579 [4 sec silence]
580
581 D: [inaudible], go and [inaudible] a un-knot
582
583 E: What's an un-knot
584
585 D: [inaudible]
586
587 E: All right, that is
588
589 D: Like that is
590
591 E: That is
592
593 [2-3 sec silence]
594
595 E: That is
596
597 [2 sec silence]
598
599 D: [inaudible]
600
601 [3 sec silence]
602
603 D & E: Yea
604
605 [1 sec silence]
606
607 E: That is
608
609 D: They are all knots [laughs]
610
611 E: That isn't
612
613 [1 sec silence]
614
615 E: That's [D laughs] is, I'm sure that isn't [laughs]
616

617 [2 sec silence]
618
619 D: Ah hang un [2 sec silence] ah I see. [inaudible] but [giggles]
620
621 [4 sec silence]
622
623 E: [inaudible]
624
625 D: No, that's it (E: interesting) that's it
626
627 E: You can pull it apart, you can see, pull it apart
628
629 D: You can pull that apart.
630
631 E: Yea, you can't plane [inaudible] to that
632
633 [4 sec silence]
634
635 D: You can't pull that apart.
636
637 E: That what I just said [D laughs]
638
639 D: OK
640
641 [5 sec silence]
642
643 E: It looks like the er
644
645 [3 sec silence]
646
647 D: The what?
648
649 E: [inaudible] set [1 sec silence] notation
650
651 D: He
652
653 [2 sec silence]
654
655 E: I think [paper noise from book]
656
657 [3 sec silence]
658
659 E: Can see what what sort of
660
661 D: [inaudible] kind of questions
662
663 E: Oh are these the questions
664

665 I: No, I mean er, if you want just to discuss it now, we can do that, I mean how you approach it
666 and things, but, er, it's not that you have to to solve the problems there (D: No) that [inaudible]
667 try to understand the theory (E: Yea) a bit of the theory behind.
668
669 E: He, Right
670
671 [2 sec silence]
672
673 D: [inaudible]
674
675 E: OK, yea
676
677 I: Yea
678
679 D: mm mm
680
681 E: We'll put, yea
682
683 [2 sec silence]
684
685 E: [inaudible] I read it twice, so [giggles] [inaudible] be much
686
687 [3 sec silence]
688
689 D: mmm
690
691 [5 sec silence]
692
693 D: [inaudible]
694
695 [5 sec silence]
696
697 E & D: [inaudible] [D laughs]
698
699 D: Like this, but you can [inaudible] couldn't you
700
701 [5 sec silence]
702
703 E: [deep sigh]
704
705 D: The naught
706
707 [3 sec silence]
708
709 E: [deep sigh]
710
711 D: Look
712

713 [6 sec silence]
714
715 E: I'm not expert [D laughs]
716
717 [3 sec silence]
718
719 E: Just, just discuss it
720
721 D: Yea
722
723 I: Yea, let's discuss. You said you read it twice (E: Yea), but will you do er er what will you do
724 next I mean if you want to let's say, let's say you had to present this for the class Thursday
725 morning or something. How would you er what would you do
726
727 E: [inaudible] er [1 sec silence], I don't know, after you've read it a couple times so you know
728 what the whole thing is for, you do, I'd go through each little bit in term try to understand it bit
729 by bit. Er. I wouldn't do that the first time I read through it cause there might be something later
730 on that will help me. Er (D: Er) Yea
731
732 D: What I would do, I'd go, I'd go through it all and get it [inaudible] what it needs and I'd take
733 some [inaudible] another book and understand the basic bits of it er and then once I completely
734 understood that then I go on to the harder bit. I wouldn't go into each bit straight away.
735
736 I: No, OK, actually you agree both of you you that you wouldn't go into the small bits straight
737 away. Why, why not?
738
739 [2 sec silence]
740
741 E: Well (D: mmm) because it's easier if you got a picture, a whole picture, even if you don't
742 understand it all er you're are better of having sort of a vague whole picture before you start
743 than trying to attack each little bit at a time
744
745 D: Actually (E: you don't really know what you are studying) now now when I come to think of
746 it, I think when I when through it I DID try to make sure I understood each bit first.
747
748 E: Yea, but only loosely, I wasn't bothered if I didn't understand something completely, I I
749 moved on, I didn't sit there and struggled with it. (I/D: mmm) I went on right to the end. Er
750 sometimes you find that something later on that you'll understand but understanding that will
751 make something that you're stuck on earlier clearer.
752
753 [4 sec silence]
754
755 I: In the beginning you've both laughing a bit, why? [D & E laugh]
756
757 E: Yea, it's the bit
758
759 D: It was it was really complicated language, I just, it was, It's assuming that you understand
760 what a mutually disjoint simple closed curve means [laughs]

761
762 E: Yea, I think, it's not written in the style we're used to studying [1 sec silence] from [1 sec
763 silence] I think, yea
764
765 I: Will that [inaudible] scare you off
766
767 S & D at the same time for a moment
768
769 E: It depends, I don't know it depends on what the work is for
770
771 D: I think
772
773 I: What do you mean, what is was for?
774
775 E: Well, depends how who how serious [D laughs] it is, if it was [talk at the same time for a
776 while]
777
778 D: If we open a textbook and [inaudible] got to have an exam over this (E: [inaudible]) then I,
779 you are suppose to understand it, then it will be a bit definitely
780
781 E: Yea er, yea cause if it was seriously seriously important, you might get a bit, but actually I
782 wouldn't ger put on fire, I mean, run for help though [laughs]
783
784 D: And also you s there is not much you can say about knots
785
786 E: This is the sort of thing where you need a er [2 sec silence] a bit more basic knowledge.
787 Maybe someone else teaching it to you, that understands it rather than trying to understand it, [1
788 sec silence] er in written language, it's quite a task to to make everyone understand something
789 that's written in writing.
790
791 [1 sec silence]
792
793 I: mmm, you turn to the book, the book with the knot er Puzzle Book, why and and did it help
794 you or
795
796 E: I didn't take the book
797
798 I: I know
799
800 D: I know, I turned the book because I was interested to see what kind of questions you might
801 ask from this, because that what they might do [inaudible], I'll look at it and then er pick out the
802 most important bit that you need from it. Er I'll say that gave me an idea of the sorts of
803 questions you get about knots which isn't made clear from this. (I: mmm) so and that was sort
804 of that's the way I work if I related it back to what I'll need for an exam [inaudible] [2 sec
805 silence] That's what
806
807 I: But if you er were not suppose to learn this for an exam would you still turn to a book like
808 this?

809
810 D: Yes, because I, cause you have to understand the basic and then you just want to know,
811 you'll still know to how you would apply this knowledge and what [inaudible] it is (I: mmm)
812
813 E: I'd rather understand it [blows] and not care about what you're suppose to do with it.
814
815 I: But you said (D: [inaudible]) previously that you preferred (E: something) to know when
816 math can be applied
817
818 D: [inaudible]
819
820 E: Yea, er, but you can't really apply it unless you understand [giggles] the the theory behind it.
821
822 D: I don't know (E: This is) I thought reading this, reading this in fact helped me with
823 understanding this (I: mmm) Just said it in a slightly different way and again it's a different
824 approach, and they, sort of strengthen each other and help you understand.
825
826 E: That would just be a bit frustrating, cause I wouldn't er understand it, you couldn't apply
827 anything if you don't know what it's supposed to be. You don't understand the basics.
828
829 D: Why sami [inaudible] understand the basics, cause I didn't, I want's to know what the point
830 (E: Yea) is. Which isn't really clear from this.
831
832 I: But how would you try and understand the basics?
833
834 E: May go back to to the chapter one, this is obviously chapter two [E & D laugh] read
835
836 D: Yea you find all the references, cause there are lot's of references to chapter 1 or 5 or things
837 like that (I: mmm)
838
839 E: I'd find out what a knot is first [2 sec silence] [D laughs] go right to the very basics [giggles]
840
841 D: What what is the basics, they are all basic concepts of knot theory, so this is the basics.
842
843 E: "Will be defined in an informal way. The reader interested in a rigorous mathematical
844 treatment can refer to [5,6]" (D: Yea, but this is) See, I'll refer to [5,6]
845
846 D: Yes this is the informal one but [E giggles] this is the easy one, so you you say, your saying
847 go straight to the difficult [inaudible].
848
849 E: No, but it will be er on a logic [E & D laughs]
850
851 D: This IS logical, (E: All set out nicely) it is logical. Actually it's not [inaudible]
852
853 E: I think the the English is worse than the maths in this (D: Yea), I'm not, I'm
854
855 I: Does this I mean, is that a a an obstruction for for the learning, the language.
856

857 E: Yea.
858
859 I: Is it always like this?
860
861 [2 sec silence]
862
863 D: It's always an obstruction if you don't understand it (I: mmm) (E: a) it's frustrating cause
864 you think that it's probably quite simple, and you would understand this in, and this is very
865 simple [laughs] knowing what a knot is, well, you know, when we discuss it with bits of our
866 own words then it was fine, but this "mutually disjoint simple closed curves" [laughs]
867
868 I: And in the third line.
869
870 E: You've quoted that about 12 times.
871
872 D: Yea I know [laughs] I don't like it
873
874 E: Look it up. [D laughs a while]
875
876 [5 sec silence]
877
878 I: You said something about that you, one of you, that you didn't like the notation.
879
880 E: Yea, well (D: I didn't mind [inaudible] it) It's almost like there, like where the noting set and
881 things er and I don't really understand before understand what they are defining as a set, and
882 they all these sort of er er yea didn't understand it.
883
884 D: I I didn't really have a problem, with that cause it, I think it's quite simously notations
885 because you are compared it to something [interrupted]
886
887 E: No I understood the notation, but I didn't I didn't [interrupt each other]
888
889 D: Why didn't you like it, I think it was quite logical? [interrupt each other]
890
891 E: No when I said I didn't like the notation I didn't mean literally [D & E laughs], the
892 [inaudible] the b's and the subsets, I meant I just didn't like the the they that [inaudible] kept
893 settings as notation and I didn't understand what it was form, and then sort of moving on, er,
894 from there, (? : mmm) I mean its nothing really really terrible difficult with the notation, so
895
896 D: So what did you [talk at the same time - inaudible]
897
898 E: I just didn't like that they were given us any [laughs], so I didn't understand what the sets
899 were, and the
900
901 I: Didn't you like the way they were given or the fact they were given?
902
903 E: Er
904

905 D: Oh way
906
907 E: Er either way (D: [inaudible]) I think I have more of a problem what came just before they
908 were (D: yea) given it
909
910 D: Like the “vertices correspond to crossings” [interrupted, inaudible]
911
912 E: And then they and say then they say “This set of vertices is denotes as s” but I (D:
913 [inaudible]) don’t quite understand [inaudible] before, so you get just notation and this is just,
914 this is the sort of thing that you be a bit daunting because you know and you go on and you see
915 this notation (D: [inaudible]) and you know they refer to this set and you don’t what the hell
916 they are talking about.
917
918 I: So, what could they have done, let’s say, if they wanted to describe knot theory.
919
920 D: I think this is, this is the kind of thing where it is very difficult to talk about in a book and to
921 represent 3-dimensional object within a 2-dimensional way, and it’s where it would help to
922 have a teacher explaining something and say pointing all this is the vertex and this is an edge
923 (E: [inaudible]) [inaudible]
924
925 I: Pointing at, I mean
926
927 D: Yea, drawing it or tiny little knots and say this is
928
929 E: Depends who they are targeting it at [1 sec silence], don’t be so [1 sec silence] so stuck up
930 [inaudible] [laughs] and so you get to a, I don’t know, don’t use such big words, they are
931 aiming to people who don’t understand it (I: mmm, well it’s part of er) and use basic, yea (I: so
932 I mean) no, I know, but I it would seem a bit sort of [1 sec silence] if it if it was in a GCSE and
933 A-level course (D: mmm) all this would have, language, it wouldn’t be right, it’s the the way
934 they approach it, the language, it’s just too, people would struggling with the language when
935 they are suppose to be learning the maths.
936
937 I: So is there a diff, I mean, er, so maths has nothing to do with the language? Or, can you learn
938 maths without language.
939
940 D: Yea.
941
942 E: No, but you can use different language, simple language to convey a point.
943
944 D: Cause the maths in it is quite easy, I think, well, it’s not (E: I’m sure it is [giggles]) [laughs]
945 What do you mean it is nothing really difficult what it is saying is this is what a knot is, this is
946 (E: Yea) what a link is, and, OK, that really really simplistic, it takes a long time [laughs
947 [inaudible]) it took me a long time to work out what they were trying (E: Yea what they were
948 explaining) whereas the fact as soon as I, kind of translated it, I thought oh well, that’s what a
949 knot is, find that’s easy.
950
951 I: What did you translated it [inaudible]
952

953 D: Into simple language [laughs] er, it er [inaudible]
954
955 I: You translate it before you understand it, er, so (D: [inaudible]) if you have understand, then,
956 it, you don't need to translate it.
957
958 E: I think it here would be easier if the author translated (D: Yea [laughs]) rather than er leaving
959 the reader to er [1 sec silence] to do it, I mean.
960
961 D: You have you have do the two together, you have to translate while you're trying to
962 understand
963
964 E: Depend, it depends on your audience though, er, I I mean, if you've got an audience who are
965 used to, this approach and don't understand then it's fine [1 sec silence] and I don't think many
966 people, and I think people, everyone will struggle with this, in our school in our (D: In our, yea)
967 set. Er, I mean some people would would take approach and probably solve it and er you know
968 understand it virtually
969
970 D: That would [inaudible] mean a lot simpler.
971
972 I: Why
973
974 D: Er, the language is much simpler (E: Yea, the pictures) yea, its an approach we're used to,
975 where you have there, you have the theory and then you have questions on it, and it is taken you
976 in gently, whereas this straight in
977
978 E: Actually the graphics are a big help, I know I was, I was (D: [inaudible]) relieved when I got
979 to this first graphic [S & D laugh], it showed, it kind of showed what they were
980
981 D: But don't you think they were in the wrong place, they should be telling you this is knot,
982 there, not [inaudible] after the second paragraph
983
984 I: Would what would you like the the first graph to be
985
986 D: Well, in [inaudible] paragraph it's telling you what a knot is (I: mmm) but it doesn't actually
987 show you a knot is, until a paragraph down
988
989 E: Yea, you need you need (D: more diagrams) more diagrams sort of a link
990
991 [Tape runs out]
992
993 D: It's just sort of funny that "an oriented link is a link" [laughs]
994
995 E: Yea, I don't know "each component of which has received an orientation", I suppose that
996 makes sense (D: Yea it does [Laughs])
997
998 D: It's just when you first (E: It's been talking about) read it, it's quite alienating [inaudible]
999
1000 [4 sec silence]

1001
1002 D: You think [inaudible]
1003
1004 E: It's all right it's better when you brake it down.
1005
1006 I: Brake it down?
1007
1008 E: Sort of sentences at a time and understand it and then move move on, once you've read the
1009 whole [D coughs] thing and you kind of got a basic idea of what it's about [2 sec silence] (I:
1010 mmm) that [inaudible]
1011
1012 D: But do you think you understand it now?
1013
1014 [2 sec silence]
1015
1016 E: More than I did the first, when I first started it, reading it
1017
1018 [2 sec silence]
1019
1020 D: I think I would understand in mathematical terms what is meant by a knot or a link.
1021
1022 E: I don't have any problems understanding it, I don't think, but I think if I was given a
1023 question [giggles], on er (D: Depending on the question) especially using just going straight
1024 into notation rather than theory er er [1 sec silence] then I'll probably struggle first, probably
1025 need some help (D: [inaudible])
1026
1027 D: Yea er
1028
1029 [Talking a bit about the cookies - noise from the paper].
1030
1031 D: Much simpler language, it was pretty [inaudible] (E: mmm)
1032
1033 I: But actually still you understood so it actually the language is something that makes you
1034 work less, I mean, makes you, er, how should I say it, er, you don't need to work as hard (D:
1035 mmm) if the language is easy, it's easier.
1036
1037 D: But that's a good thing (I: mmm) because what's important in maths is not knowing the
1038 most complicated way to explain something it's knowing something.
1039
1040 E: Yea, once, you need (D: [inaudible]) to get the theory, it doesn't matter how you get there,
1041 once you get there, it doesn't matter how you'll explain it (I: mmm) cause once you (D:
1042 [inaudible]) got the (D: in your head if you understand it) yea
1043
1044 I: How do you get it in your head, can you can you try to describe how actually how you get it
1045 in your head.
1046
1047 [3 sec silence]
1048

1049 E: Er
1050
1051 D: I looked at the diagrams
1052
1053 E: Yea, I I don't know people's diagrams definitely
1054
1055 D: Much easier to understand
1056
1057 E: Maybe some examples, not necessarily diagrams but, they say they're gonna denote this
1058 notation this set this notation, whatever, then they just move on, they should give an example so
1059 that, even if you think you understand it, at least you can look at the example and say, yea, I do
1060 understand it [2 sec silence] cause you you always sort of have that niggling thing in the back of
1061 your mind saying maybe I don't fully understand this, when you go into the next problem or the
1062 next complication
1063
1064 [3 sec silence]
1065
1066 I: Mmm
1067
1068 [3 sec silence]
1069
1070 I: But er, this er, how how is this different or the same from when you normally learn some
1071 maths.
1072
1073 D: Not having a teacher there to explain it to you, I think it's the main difference. [1 sec
1074 silence] You don't [giggles]
1075
1076 E: I think it's the language [1 sec silence] er, com
1077
1078 D: You know it's probably a combination of the two because I I don't often learn things from a
1079 textbook, no matter how simple the language is [1 sec silence] I normally have some there
1080 explaining it to me.
1081
1082 E: I don't neat to er, I don't know, I I find it quite easy to learn from textbook if it is written in a
1083 certain way, approaches from a certain angle [interrupted]
1084
1085 D: It's getting used to, it's getting used to it.
1086
1087 E: Yea
1088
1089 [3 sec silence]
1090
1091 I: In what way is it the same as what you normally do
1092
1093 D [Laughs]
1094
1095 E: Er
1096

1097 D: It's quite different.
1098
1099 [2 sec silence]
1100
1101 D: This one [Puzzle Book], this is much more similar (I: mmm) the way they explain it then
1102 they have puzzles and examples, er, this one just, explain.
1103
1104 [2 sec silence]
1105
1106 E: It doesn't seem very sort of [2 sec silence] I don't know it's very geometrical, and I don't
1107 really like [D laughs] I don't like that anyway, I just, I don't know, [D laughs and some words
1108 are inaudible] algebra I don't like to. [1 sec silence] I don't know they are they are sort of [1 sec
1109 silence] I know they are trying to denote it with bits of algebra (D: [inaudible]) so that I I
1110 suppose you you end up with er system of using sort of er mathematical language for it for er
1111 presenting it. Er, at the beginning it just doesn't interest you at all (D: mmm) It doesn't make
1112 you
1113
1114 D: You read it cause you have to, not because you're interested in it, whereas it contrasted
1115 [inaudible] the book if that looks quite interesting.
1116
1117 [3 sec silence, hear researcher write]
1118
1119 I: Mmm
1120
1121 D: [inaudible] relate it to an everyday situation [2 sec silence] so you start wondering what the
1122 whole point of it is.
1123
1124 [2 sec silence]
1125
1126 I: OK, I have a last question, well, it's not really a question it's about, is there anything we
1127 haven't talked about tonight which you think is important to mention in relation to learning
1128 maths or handling problems learning maths?
1129
1130 [3 sec silence]
1131
1132 E: Er
1133
1134 [7 sec silence]
1135
1136 D: [inaudible]
1137
1138 [3 sec silence]
1139
1140 D: I don't I don't think so, cause a lot of this is hiding it's subconscious, [inaudible] I'm not
1141 used to analysing why I learn something (I: mmm) so there might be things
1142
1143 E: I know people do er differently but it's all very individual [1 sec silence] even er even if you
1144 work on something together er [2 sec silence] and you're you're both aiming to solve the

1145 problem, I mean you'll do it completely differently from someone else (? : mmm) and quite
1146 oftenly I find I don't like other people's [E giggles a bit] styles, you know, you always get your
1147 own

1148
1149 D: Yea, but that's the whole point in learning why you do something rather than how because a
1150 lot of the times I will, there are some things I do my own little way, I don't necessarily follow
1151 the textbook example and it, that doesn't matter cause I understand what I'm doing (I: mmm)
1152 what I want to achieve by doing it and I I can get the right answer by doing it a different way to
1153 the textbook, but, you know, it just works for me.

1154
1155 I: What's your own way then?

1156
1157 D: I I well, this, I can't really think of anything specific, but it's [inaudible] just, I want to
1158 understand this and then I'll work out how to solve the problems involved in that, whereas I
1159 think that when you are at it alone, your GCSE level, then just stick very much to what's in the
1160 textbook, plugging in the numbers. (I: mmm) er, with me it's much more

1161
1162 I: It's (E: [inaudible]) much more?

1163
1164 D: more important to understand

1165
1166 I: Mmm

1167
1168 E: The people understand different things, it's different degrees, so, like I say, there're four of
1169 us, we're all doing the same thing we're all pretty much the same standard, but if I can't solve a
1170 problem and I borrow someone else's work, it actually quite often doesn't help because the way
1171 they've written it out [D & S laugh] (D: Yea that's true) you don't follow the same sort of
1172 logical thinking. You might start in the middle and then go back up to the beginning, on theirs,
1173 and and it sometimes takes longer and ah it gets you rapped up and nuts (I: mmm) in a way

1174
1175 D: [inaudible] very obvious, when you've copied their work [laughs] (E: Yea) you've seen it
1176 their way rather than you're way.

1177
1178 I: And the teachers notice?

1179
1180 E: Yea, they always notice [D laughs]

1181
1182 D: We all, we all work together on a test on Saturday [laughs]

1183
1184 E: We all got the, yea, [D laughs] cause we all did it together, and we all got 98% and we all
1185 got the same 2% wrong, so we all died, actually I think it was a bit tight I think we should have
1186 got 100% (D: Yea) as a nickling mark, but (I: [inaudible]) er

1187
1188 I: You said said something about a while ago that everybody has his own style or something.
1189 (E: Yea) Can you expand on this.

1190
1191 E: Well, people have [2 sec silence] well, for a start they approach the problems differently (I:
1192 In what) er, like, they'll they'll come in from a different, it's a basic method for solving

1193 something, some people will do the first two steps in their head and quite easily be able to go to
1194 the say the third step. And other people start right at the beginning, and when you write it out,
1195 for some people, if you miss out the logical step, that you need to see on paper for it to work,
1196 and, yea people do things in different orders and they've got different, and people have their
1197 own notation to things don't they
1198
1199 D: Yea, that's true, [inaudible] it's quite interesting that when you gave us this to do we didn't
1200 write anything, er, because I don't, I don't know why I didn't, but sometimes when I [inaudible]
1201 I find it helps to write notes, for example to write down things in bold, here, but I didn't this
1202 time for some reason.
1203
1204 E: I know, I normally do that, if you had a problem to solve [2 sec silence] get a piece of scrap
1205 paper and scribble your ideas and notes and different things down (I: mmm) I mean you finally
1206 get the right answer right (D: Yea) at the bit you need, and chuck the scrap paper away.
1207 [inaudible]
1208
1209 D: Yea but sometimes, if there is something you don't understand then, I'll use the approach of
1210 writing down, writing down bits the same way [inaudible] understand this, and so this was it,
1211 and then you write it down and work it through on the paper and then it suddenly clicks, and
1212 you understand it.
1213
1214 I: Clicks, what do you mean?
1215
1216 D: You understand, it just, I don't know.
1217
1218 E: [inaudible] the feeling you get, you're frustrated and you don't understand it, suddenly you
1219 get it (D: and then there is light) [D & E laughs]
1220
1221 I: Where does this light come from?
1222
1223 E: A feeling of satisfaction, isn't it. (D: yea) Having done something yourself, having (D:
1224 [inaudible]) worked it out. It's like a links being made in the brain, that wasn't really there,
1225 before, two bits needed two, you know [D laughs]
1226
1227 D: Two wires, probably gathered and a spark comes [laughs]
1228
1229 I: Does this have anything to do with that you talked about subconsciousness a while ago? [2
1230 sec silence] Or what did you mean by?
1231
1232 D: Er, probably except that writing things down is much more of a conscious effort, trying to
1233 understand, when you understand subconsciously it just, it's like an immediate shedding of
1234 light on the problem, when I write something down that's much more consciously working it
1235 through in your mind [inaudible] I understands this bit, and then linking all together on paper.
1236
1237 [2 sec silence]
1238
1239 I: OK, thank you.

Appendix H: Full transcription of the third English group interview

1 I: I was just talking to F before you came here. He was telling be about that you were one of the
2 four classes taking A-level in maths and this class takes one-and-a-half course, right.

3
4 B: Nine modules [inaudible].

5
6 I: Nine modules, mmm, OK great, OK. Can you can you describe to me how how a normal or
7 typical maths lessons is?

8
9 B: Er, it's usually er, we'll cut we'll start until like some news like a topic we've been doing
10 and take it one step further, er, the teacher explain us through (I: mmm) the methods an like,
11 using a [inaudible] for different sort of variations and how the question could come, and then
12 [snaps his fingers] we'll maybe do some practice, questions, or, maybe not. (I: mmm) Standard
13 format (I: Yea)

14
15 F: It depends who teaches us really, one of our teachers (B: Yea) tends just to set us exercise
16 questions and teach us for about 5-10 minutes in the beginning and, so there is exercise
17 questions for the rest of the lesson, and then the other one, er, [1 sec silence] teaches us entire
18 lesson and sets questions for prep (I: mmm)

19
20 I: These these questions do you solve them at home and then come back next lesson

21
22 B: Mmm, yea (I: mmm)

23
24 I: OK, what, does it does it only depend on the teacher or does it also depend on the er subject

25
26 B: Er, [1 sec silence] I think if it's er, sometimes yea if it's something we need a lot of practice
27 in.

28
29 I: What might that be?

30
31 B: Er [1 sec silence] I don't know well, I find personally a lot of this statistics very hard er then
32 if we can go through the questions in class where we have the teacher down and can ask them
33 how to go through [inaudible] (I: mmm) whereas if we do them at home we can't [inaudible] if
34 we get them wrong and check next lesson (I: mmm) so it is [inaudible] sometimes.

35
36 F: Also sometimes it is a bit waste of times if we're gonna go for something which some
37 understands if for example half the class understand and half of them don't er, it's a bit of a
38 waste of time spending half an hour going through going through the process if half of them
39 already know how to do it (I: mmm). We're encouraged to come, if you got a problem come to
40 him in the end so you don't waste class time.

41
42 [1 sec silence]

43
44 I: mmm. But how does he help you then, or

45
46 B: If you say if you say er what you're having trouble with and what you don't what you can't
47 understand, how er sort of step in reasoning, went on to another one and he'll explain it through
48 that, and explain why.
49
50 I: Is, you mean explain er why it is like that or
51
52 B: Yea, explain in more detail how he got from one stage to the next stage of the sort of
53 reasoning process.
54
55 I: Mmm, OK, so er how much homework do you have then? It depends on the (? [inaudible])
56 teacher?
57
58 B: Yea, it depends on the teacher and the subject and which part an what we are doing and
59 having a lot of trouble with and so on. Er, how much
60
61 F: It doesn't that really long, takes about 3 hours each week or something.
62
63 B: Yea about 3 and a half 3 hours a week.
64
65 I: Mmm
66
67 B: [inaudible]
68
69 F: It's not bad compared to some other subjects.
70
71 I: Do you work a lot with math compared to other subjects?
72
73 [1 sec silence]
74
75 B: I do, I do design, I do a lot a hell of a lot more work for design than I would normally, than I
76 would do for maths, er, physics is pretty about the same, this is my other subject.
77
78 I: So you take you take A-levels in Art er de
79
80 B: In maths, physics, design.
81
82 I: OK
83
84 F: I do languages er Latin and Greeks, [inaudible] and biology and for every single Latin and
85 Greek lesson, I could prepare for 45 minutes on the work and the translation we are gonna go
86 through so I spend a lot of time doing languages (I: mmm) and biology, that we get set very
87 very long preps each take five or six hours and and there's two teacher there so we get 12 hours
88 where we can work a week so we spend a lot of time doing that, so maths er kind of, on
89 borderline, for not really spending a lot of time there.
90
91 I: But do you er, do you feel you you spend er enough time to learn it or could you, I mean, use,
92 or are you just very efficient.

93
94 F: I don't [inaudible] think we could go through it faster, er, if we had more teaching time er, I
95 mean, once you're taught something, er all the questions in an exercise are going be repeating
96 that process.
97
98 B: It's the same thing with just different numbers basically
99
100 F: Yea, that's so, er, if we are given more teaching time in class er and fewer repetition of the
101 same question different guys [inaudible] then er I think we can go a lot faster. That's my view.
102
103 I: So you don't need all this exercising?
104
105 F: You need an example of each one and you need to get it right and not just er our books have
106 answers in the back as well, you do need to make sure that they are not just looking in the back.
107
108 B: I think it is not just one of every one, I mean you need a few just to get the sort of process
109 into your head when you don't [inaudible] but sometimes we get a set far too many just
110 repeating the same thing, and when you've done it for half an hour and you've learnt it and you
111 have to carry on doing it hours of work on top of it, it's just the same thing again (I: mmm)
112
113 F: It's sometimes crunching, er it's a bit boring and also I don't in er biology and Latin and
114 Greek we er get actual exam questions every week - you don't get set back in maths, really, I
115 mean some of the book questions are exam questions but most of them aren't.
116
117 B: I actually find it more useful when we ARE doing sets of exam questions
118
119 F: Exactly.
120
121 I: Why?
122
123 F: Er, cause they they sort of, [inaudible] the exam question, the whole topic area is condensed
124 into one question, it has a bit of each sort of part. And er you get used to combining the whole
125 lot together rather than just using one specific bit and repeating it over like you're doing in
126 exercise.
127
128 I: Mmm, but do you need exam er questions to do this (B: Er) or is it more the type of
129 questions?
130
131 B: It's getting use to the type of questions.
132
133 I: Mmm
134
135 F: Yea, it's the whole format cause one exam question can, if there is 6 chapters in a book
136 you're studying er and each one is about a different subject er an exam question can test four of
137 them all at once in one question and that's a lot more useful than doing each chapter and then
138 forget the chapter and then you go to the next one, then you come up with an exam paper and
139 you have to link them all together, that's the problem I think.
140

141 B: Yea (I: mmm) it's certainly be useful to do more exam questions (I: mmm)
142
143 F: Though we are given lot's in the end.
144
145 [2 sec silence]
146
147 I: OK, if, what what do you do with if you meet some maths you can't understand at first? [2
148 sec silence] (B: Er) You said you'd ask your teachers but
149
150 B: But it depends, it would depend, if we were in class time I would ask the teacher (I: mmm) if
151 we out of class time I guess go go to textbook look through, er, when you say that we don't
152 understand, do you mean something on the topic area or what, [inaudible] we just can't figure
153 out, or something completely new?
154
155 I: It could be both.
156
157 B: If it's on the topic area, I'll go to the book
158
159 F: And there we'll have worked examples, we always have of examples, one example of the
160 process and you just go through
161
162 B: and you can find out how to do it from that
163
164 I: Mmm
165
166 F: It's a question if you can't do [inaudible]
167
168 B: If it is something we've never seen before and we don't have a textbook on it and er (F:
169 Leave this) leave it and ask the teacher [laughs] (I: mmm)
170
171 F: Or we go to, I've got three people in my class in my house so I just go and ask them
172
173 B: Yea, you can check with the others
174
175 I: And from the same class?
176
177 F: Yea.
178
179 I: OK [1 sec silence] How do you help each other them, I mean, er the four of you?
180
181 [2 sec silence]
182
183 F: Then, usually one of us got the right answer, and they, and just explains it, [inaudible] we've
184 got the Russian G he he (I: Pardon?) G, he is from Russia and he is er his training in maths is a
185 lot more different er and so he approaches problems from different ways and he usually gets the
186 right answer, and so he has often helped me.
187
188 I: How is is his approach different from your?

189
190 F: He just uses er, I don't know, he's he looks at problems in a different way, I am not sure why
191 or how, it's just his way is a lot er more thorough er more vigorous than what we've been
192 taught, we just we just get taught the short-cuts and he has to go each section and er not miss
193 anything out.
194
195 [2-3 sec silence]
196
197 I: Mmm, so er, how do you know if you have understood maths?
198
199 B: Er, I think it's by looking at a question, I think if I can, when we get examples and, I can just
200 look at it and see [inaudible] in my head how I am going to tackle it and know how know that I
201 can [1 sec silence] [inaudible] I'm not confused by any of this, asking me to do [inaudible]
202
203 I: [inaudible]
204
205 F: Also we get tests every one, one two or three weeks (B: Yea) and if we fail there's,
206 miserable, then we know that we (B: Need to go back and do some more) mmm
207
208 I: What did you mean when you said, you could see it in your head?
209
210 F: Er, if it's er something that I have learnt how to do, know how to do it then, without actually
211 doing the question, you can look at it and say, ah I will use that technique to do it or, er, that
212 should work that way [1 sec silence]
213
214 I: Mmm, OK
215
216 [B's mobile phone rings - he turns it off, say sorry, I says OK]
217
218 I: Great, can, if I ask you to describe your learning process, what would you say?
219
220 [2 sec silence]
221
222 B: Learning process (F: We, nah) I think that for maths it is certainly different than subjects
223 cause when you learn it is learning how to use a technique rather than, like lists of meanings or
224 whatever, and er so I think the learning process is just practice, have it explained to you, and
225 then practice it to sort of get it, get it over [inaudible] in your head so you can do it easily at a
226 time.
227
228 F: It is er we get taught through the principles, been given one example and just go practice it,
229 that's the basic learning process I think.
230
231 I: But why does this I mean, what kind of practice?
232
233 F: Just (I: is it) just the pap the book, the questions in the book (B: The questions) (I: mmm, so
234 it's the problems, OK) Yea.
235
236 I: Mmm, great.

237
238 F: It's the does depend on the teacher, cause we've got one teacher and he has been teaching
239 maths for I don't know 50 years or something [B giggles] and er he doesn't like to be, he likes
240 to teach us a wider view of maths so he often goes on to things which aren't in the syllabus and
241 and he waffles on about that er, and though is not relevant when getting an exam it's quite
242 interesting.
243
244 I: Mmm, but do do you learn from it?
245
246 F: Yea yea, I mean it's it's like the process that we've been using that's taken one step further,
247 so you've get to see the sort of applications, the object process (I: mmm) and [inaudible]
248 situation
249
250 B: Yea the teachers [inaudible] (I: Pardon) the teachers is not clever enough to do that (F: Don't
251 let them hear this, [laughs])
252
253 I: OK, is it important that the teachers are are very clever in in the subject or is it more
254 important that they have this have a kind of pedagogical training.
255
256 [1 sec silence]
257
258 F: It's more important that they understand what they are teaching, you can't teach something
259 which you don't understand.
260
261 B: Certainly they gotta have a very deep understanding of it, so, much more than we would
262 need for the A-levels syllabus.
263
264 I: Mmm, [2 sec silence] OK great, OK. Now we've had this small talk about something in
265 general and then, I I want to give you this er math, and I want you to take about 15 minutes to
266 look at it and, you can have this sheet [paper noise], and er you can er going to give, you can
267 have these pens and I have some paper you can use, it is not, you don't HAVE to use the paper,
268 it's just, if you, if you think you could need it (B: Sure) and I have some [bag-noise] books
269 about it, it's about know theory, have you heard about that before (B: Er not really no) nah, it's
270 about when you tire a knot with some broken things, you can actually do some maths on that, I
271 I didn't know that myself until until recently. But in these books there are some theory about
272 that and this er this er these two pages I copied from this book so if you want you can use this,
273 have a look at, and then afterwards, I am going to ask you [inaudible] questions on this sheet
274 and if you want to have a look at the questions before, going to give you that, and you can do
275 whatever you think is necessary to try to understand these two pages. I don't expect you to to
276 get a complete understanding in 15 minutes, but er, so don't be, you know er, panic if you er
277 but er afterwards I would like to ask you about how how you actually approached this new
278 maths and what you want to do next in case you had to present this for the class in two days. (B:
279 I see) So if you have this in the back of your head while you're you're reading it and then
280 [paper noise], take notes if you want, some, look, talk together if that's what you normally do or
281 if that's what you want to do, so do whatever you think, and this, yea.
282
283 4 min 22 sec silence: [hear noise of bottle being opened and drunken from the first seconds]
284 [hear voices outside after 1 minute] [after 3 min and 45 sec hear drinking again]

285
286 A few seconds paper noise from book
287
288 F: Do you understand [inaudible] [whisper]
289
290 B: Ne [inaudible] information is [paper noise from book] 5, 6 [inaudible] [whisper]
291
292 [1 min 5 sec silence, paper noise from book]
293
294 B: When it says er, refer to [5,6] then it means chapter 5 and 6 (I: mmm [4 sec silence] I think it
295 refers to section five and six in the same chapter) All right (I: Otherwise it must be further) I got
296 to it, 5, 6 [paper noise from book]
297
298 [1 min 20 sec silence, paper noise from book]
299
300 F: [inaudible]
301
302 B: Er it's a lot of sort of pretty heavy equations which I actually don't understand.
303
304 [4 sec silence]
305
306 F: I think it is a bit important here, er, here, "To perform a move, one must find a topological
307 disc in the plane whose interior intersects the diagram in one of the configurations involved in
308 the move" [2 sec silence] so it means like, [inaudible] those two knots, and if by doing some of
309 these moves you can make that one into that one, they are the same, [inaudible], same diagram.
310 Which one of these knots can be represented this.
311
312 B: Yea yea I see.
313
314 F: Graph thing, that's [inaudible] numbers.
315
316 B: And if you just by certain moving like rearranging it, you can make one into the other.
317
318 F: Mmm
319
320 B: You see [inaudible] two separate pieces two separated knot, different, that's all one single
321 thing, that's two separate one isn't it.
322
323 F: Mmm, yea, it's just [inaudible]
324
325 B: That one into that one
326
327 [2 sec silence]
328
329 F: I don't know what that is
330
331 [6 sec silence]
332

333 B: [inaudible]
334
335 F: And why is it a diagram of 4-regular graph.
336
337 B: Certainly need, to get any further have to go and look up one of these terms mean
338
339 [4 sec silence]
340
341 F: Shall we just work one that sheet between us or should we work
342
343 I: You can do what you prefer.
344
345 F: [inaudible]
346
347 [13 sec silence]
348
349 F: Find the terminology
350
351 B: Mmm, which his certainly not in here [inaudible] syllabus [inaudible] we've come across.
352
353 I: It's not [giggles].
354
355 F: Think it will be useful to work out like if there is some simple maths that would be quite next
356 to it, without all this.
357
358 B: Yea a bit [inaudible] to more sort of simple ideas.
359
360 F: It would be nice if they had an example there as well.
361
362 B: Look up, find some more books, maybe and [2 sec silence] maybe some [inaudible]
363
364 F: This is [inaudible] interesting mathematical ideas, I would have er crossing numbers
365 [inaudible] and it's just how many times if you cross it like that, the crossing numbers of knot is
366 equal to [inaudible] and it's still cross over this [inaudible] put three [inaudible] and er this is,
367 the rest of this is just [inaudible].
368
369 B: That's 3-fold
370
371 [3 sec silence]
372
373 B/F: Exciting
374
375 [3 sec silence]
376
377 B: [inaudible] to explain it slowly [1 sec silence] more understandable language (F: Yea)
378
379 [4 sec silence]
380

381 F: Clear diagrams I suppose if making presentation to class [1 sec silence] we can use diagrams
382 out of this book [inaudible]
383
384 [4 sec silence]
385
386 F: We also really have to understand the maths too before we er [1 sec silence] make an
387 example or anything, it will be no good explaining it cause they'll just ask us what it's about
388 and we don't know, it's a mess.
389
390 [2 sec silence]
391
392 B: This sort of jumps in [1 sec silence] if we can find something like this, sort of look around
393 for some more books, find some who do it more sort of understandable summary, maybe went
394 through it a bit slower [2 sec silence] and sort of let you into it more, through more
395 understandable terms
396
397 [21 sec silence]
398
399 B: What else they're doing now. [inaudible]
400
401 [2 sec silence]
402
403 F: [inaudible] knots
404
405 [10 sec silence]
406
407 F: The [inaudible] seems to be about crossing numbers, and this isn't about crossing numbers
408 this is about picking a knot out of of a diagram.
409
410 B: [inaudible] mapping it to a graph [inaudible]
411
412 F: Yea
413
414 [2 sec silence]
415
416 F: But I don't understand what it's for, regular graph, whatever that means.
417
418 [2 sec silence]
419
420 B: Again we need to look it up.
421
422 [9 sec silence]
423
424 F: [inaudible]
425
426 B: A diagram like these seems to only show of some small part of the diagram, see
427
428 [2 sec silence]

429
430 B: Nah [coughs]
431
432 F: “A link is a finite collection of mutually disjoint simple closed curves”, which means pieces
433 of strength [inaudible]
434
435 [2 sec silence]
436
437 F: [inaudible] mmm, [inaudible] bit dudgy
438
439 B: [inaudible]
440
441 F: Yea
442
443 [13 sec silence]
444
445 F: Don’t understand what this is?
446
447 B: That’s just streaking out yea
448
449 [2 sec silence]
450
451 B: It’s slipping the knot over that way to that
452
453 F: Yea yea yea, (B: [inaudible]) it would happen if you pull those two like that, [inaudible] like
454 that, these all really simple, these all look very basic. Taken a [inaudible], this one takes this,
455 not
456
457 [2 sec silence]
458
459 F: [inaudible] colours to show which one to which [1 sec silence] numbers or something [2 sec
460 silence] that looks suspiciously like that 3-fold thing in this book, that mere image we’ve been
461 talking about.
462
463 B: It’s not an [inaudible] to the top down under.
464
465 F: Yea.
466
467 [13 sec silence]
468
469 F: What’s the introduction to this book, how far
470
471 [paper noise from book, 15 sec silence]
472
473 F?: It’s a bit odd,
474
475 B: So far we’ve got to define terminology, and find something which will use basic math
476 notation knot.

477
478 F: That will be chapter this chapter 2 part 2 of this chapter, that's, [inaudible] introduction
479
480 [14 sec silence]
481
482 F: [inaudible] representation to the class
483
484 B: Mmm [3 sec silence] and again it is pretty [inaudible] what comes out of every sentence I
485 don't understand in this.
486
487 [6 sec silence]
488
489 B: [inaudible] theory, probably with variance.
490
491 F: "No previous knowledge of knot theory is required, informal introduction", Section 2, that's
492 the informal introduction, knot
493
494 [9 sec silence]
495
496 F: The first thing to do would be to, draw some knots and then show the the respective oriented
497 diagrams whatever they are called, these graphs. But it hasn't given us [2 sec silence] examples
498 or
499
500 B: These two books, that one's just about knot basically not really, work, maths, doesn't have
501 any sort of theories [2 sec silence] doesn't relate it to any sort of figures or numbers (F:
502 Notation) seems to be heavily putting it into sort of graphical form, find another, [inaudible]
503 find something linking the two
504
505 [1 sec silence]
506
507 F: [inaudible] this is the vertices and this is where the crossing points are so it is there and then
508 there, and so that would be a
509
510 B: That would be $s(v) + - 1$
511
512 F: No, that's a minus 1 that's a a what, it depends on which on you are looking at
513
514 B: I don't really see what makes it a minus one and a plus one
515
516 F: Depends on which one you are looking at, don't say that in here.
517
518 B: You COULD say that [2 sec silence] you could say that this one is class 3 cause it's got three
519 that go over the top where you can say it's minus three cause it's got another 3 underneath.
520
521 [18 sec silence]
522
523 F: Perhaps you take one of the things and you do it in relation to [3 sec silence] in relation to
524 just one of the strings [inaudible] and it goes it goes under this one goes under over and this is

525 all round isn't it (B: It's all round yea) down this one, doesn't say that it doesn't say anything at
526 all
527
528 [1 sec silence]
529
530 B: It might have something to do with all of these [inaudible] all of them actually, under over
531 under over [inaudible] under twice
532
533 [2 sec silence]
534
535 F: [inaudible] change all that by doing these moves
536
537 [10 sec silence]
538
539 F: I don't understand this sign $s(v)$ at all
540
541 [7 sec silence]
542
543 F: Why, why are the arrows pointing this way and not that way
544
545 [1 sec silence]
546
547 B: Why [inaudible] what [inaudible] say in that one is an $s(v) + 1$ that one is an $s(v) - 1$
548
549 [12 sec silence]
550
551 F: [inaudible]
552
553 B: Is it something to do with the direction of the one that goes over the top or
554
555 F: [inaudible] both seem be coming into each other like that and going away from each other
556 like that and coming in and going away.
557
558 B/F: Pretty silly [very low voice]
559
560 [8 sec silence]
561
562 F: Doesn't give a clear it before, [inaudible]
563
564 B: The only thing I can think of this is taken the, so you take the one going from back from
565 right that goes underneath and seems to be positive, if it goes over the top it seems to be
566 negative (F: mmm) I can't see why that would be seems to be an arbitrary decision to me.
567
568 [4 sec silence]
569
570 F: Would it make any difference would it produce the same number in the end. Or or are we
571 suppose to sum them or not, might just be able to do the two, which go under and go over each
572 one.

573
574 [4 sec silence]
575
576 F: And there is one here it goes, over one
577
578 B: They both go over the same mount twice
579
580 [Suddenly someone enters the room, say sorry]
581
582 [3 sec silence]
583
584 F: Say, find out what we need to know [6 sec silence] what
585
586 B: I wanna know what the er notation means
587
588 [3 sec silence]
589
590 B: It there are a things computer does for us that's where we have to start by explaining to the
591 class if doing a presentation of it.
592
593 F: Mmm
594
595 B: And then we can go and talk about the ideas that are involved, and things
596
597 [3 sec silence]
598
599 F: [inaudible] explain by given examples and so, once we know this, we can make up a knot on
600 the board or whatever [giggles] and then show how it can be depicted by er er graph.
601
602 B: Yea
603
604 [5 sec silence]
605
606 F: It does say, [inaudible] convention so it could be something like left or right [inaudible]
607
608 [11 sec silence]
609
610 B: Keeps referring to [inaudible] binary, might have something to do with it, but I know binary
611 is er 1's and 0's computers use [1 sec silence] [inaudible] base 2 or whatever
612
613 F: Mmm
614
615 [4 sec silence]
616
617 B: [inaudible] I can't see quite how that would apply here but
618
619 [3 sec silence]
620

621 F: So you can get pieces of information from every single crossing point [2 sec silence]
622 and if by this you change a knot, by these Reideme, what this guy's theorem, you still get the
623 same piece of information out, it seems kind of logical.

624
625 B: Yea yea if you if you can make the one move [2 sec silence] the the same information from
626 that diagram [2 sec silence] then maybe not cause that's that's got a cross on it and that
627 [inaudible], $s(v)=1$ er, say that's it's [inaudible] across there.

628
629 F: Yea but they're also (B: $s(v)=1$ doesn't have to) it doesn't say
630

631 B: Straight line

632
633 F: If we've got something like this it's just one [inaudible] strain and we strengthen all out and
634 it's just the loop like this, it's just a circle, just turning into that one, I don't think that does but
635 if you've got things just turning into that one [inaudible] across this, drawing this physic work
636 figuration on the page, are we suppose to make it to the simplest it possible can, or are we
637 suppose to do the crossing overs as showing er [2 sec silence] on the page, [inaudible]

638
639 [6 sec silence, paper noise from book]

640
641 F: [inaudible] like that, you can unravel those so they won't have all those cross-overs [2 sec
642 silence]

643
644 B: That's that's the sort of underneath er crossing around isn't it.

645
646 F: Yea. [1 sec silence] seems that you can completely unravel that, yea er, crossing number of
647 knot [1 sec silence] So these crossing numbers may be the [inaudible] information here, oh yea,
648 it probably is, but it doesn't have minuses [2 sec silence] it has er if you put [inaudible] gonna
649 have three cross-over still, and that's gonna have four [2 sec silence] but it doesn't have
650 anything about minus ones.

651
652 [3 sec silence]

653
654 B: Yea

655
656 [2 sec silence]

657
658 F: Strange

659
660 [9 sec silence]

661
662 B: "Is this different from what we normally experience, learning situations?" Certainly I think
663 cause we are never asked to go and research things for ourselves (I: mmm) we just get
664 explained the whole topic er and then we get explained how to do things and then get set
665 practise questions on it, we'll never always never be told to go [inaudible] maths to go and
666 research something new, present it whatever.

667
668 [4 sec silence]

669
670 I: I notices when when you talked about presenting it to the class you wanted to give them
671 examples and you also mention while you were talking about that it would be nice, it would be
672 nice with examples. Why, I mean?
673
674 B: That's because we always, the way that we've always been taught is because this using
675 examples thoroughly to explain, so that's the way we think the people in our class will
676 understand it easiest, how explain through examples
677
678 I: mmm
679
680 F: For example if er they are learning about differential equations or something and maybe
681 taught any method for solving then, you go through all the proof how er [inaudible] complex
682 numbers
683
684 B: [inaudible] equations on the top of his head on the board (I: mmm) and then go through the
685 whole technique without [inaudible] using that equation.
686
687 F: Yea, afterwards [inaudible]
688
689 I: But could you learn it without examples?
690
691 F: I don't think so.
692
693 B: Well yea, you couldn't go straight into an exam er knowing all the techniques but not
694 actually having worked through each one several times, and have to do it through, mechanically
695
696 [2 sec silence]
697
698 B: Yea [inaudible] I don't know exactly how they could really be explained without using some
699 examples, just examples of numbers being put in, to se how the technique, [inaudible] see how
700 you could, explain the technique without the numbers being there
701
702 I: What about explaining the maths?
703
704 [2 sec silence]
705
706 B: Er, we talk about I mean for example like calculus, yea, differentiation you can explain how
707 is the measure of the rate of change of er [2 sec silence] I mean there is certain things that I just
708 don't think could be done without these numbers, but, er [inaudible]
709
710 [2 sec silence]
711
712 F: You realise going through examples you see er first of all, $6x$ er + $4 \frac{dv}{dx}$ er and then they
713 actually have numbers, it's [inaudible] a, b, and c, and so let him [inaudible] go to the proof and
714 show in terms of (B: Yea) these constants [interrupted]
715

716 B: Yea, you still you it, I don't think you need numbers they are still examples using [talk at the
717 same time for a while - inaudible] yea a, b, and c as you just, I don't know, (I: mmm) constant
718
719 F: That would be what we're doing here the kind of thing if we have to put [inaudible] and then
720 say if we have this many vertices how much [interrupted]
721
722 B: Have a amount of vertices and then [1 sec silence] b amount of edges viola.
723
724 [4 sec silence]
725
726 I: Mmm, [inaudible] when you were reading and thinking er you you after a while you took this
727 red book [Nelson & Wilson (1990)] [1 sec silence] why did you do that?
728
729 B: Er, because I this paper we've been reading come from there I thought it might that way
730 something more see more of what of the sort of context come out of a chapter (I: mmm) and er
731 saw it was number two part two of a chapter so I thought I would read it.
732
733 F: Also said that refer to 5 and 6 (B: Yea that was [inaudible] 5 and 6 but didn't understand
734 much of them)
735
736 [2 sec silence]
737
738 F: Er er thats another thing [inaudible] me
739
740 B: More notation [inaudible] instead of
741
742 [2 sec silence]
743
744 F: It's a bit odd
745
746 [2 sec silence]
747
748 I: Mmm
749
750 F: Yea, it's pretty unsatisfactory explanation of what to do and how, if you have a knot,
751 represented (B: Yes)
752
753 B: It's for the same thing L^+ that's the same er, that's the L^- , the same as minus one.
754
755 I: Mmm
756
757 [2 sec silence]
758
759 F: But if you're here, if you can pull them apart do you call it L-naught or can you manipulate
760 the knot?
761
762 [2 sec silence]
763

764 F: Suppose if you, mmm
765
766 [2 sec silence]
767
768 F: It's a bit odd
769
770 [3 sec silence]
771
772 I: Did you get anything out of this book, this purple one [McLeay][1 sec silence] was it helpful?
773
774 F: Er
775
776 B: Er seing the way they use [1 sec silence] these are not not good knots [inaudible] the strenght
777 crosses over and just [inaudible]
778
779 I: Actually this one is full of examples of knots
780
781 F: Er
782
783 B: Yea just knots but if you are talking about the crossing numbers
784
785 F: I understand what a crossing number is, (B: it's fine) yea, it's already er, it doesn't say talk
786 about [inaudible] plus 1 and minus 1 (B: Yea) [talk at the same time for a while - inaudible]
787
788 B: The number the the this whole thing, sort of area [inaudible] based around crossings (I:
789 mmm) the most important part I think worth focussing on
790
791 I: Mmm, [inaudible] talked about taking some copies from the purple book [McLeay, 1994].
792
793 B: Yea, I mean if we needed any examples [inaudible] the diagrams are much clearer in this,
794 whereas in this one they are just sort of simple lines and they are really small.
795
796 F: And they don't, not coloured or not even numbered here. [B: (inaudible)] You have to know
797 what the manipulation from there to there is about.
798
799 [4 sec silence]
800
801 I: You said er something about that it was more simple. Is is it a good idea if things are simple
802 or [interrupted]
803
804 B: Yea, certainly if you explain any of this simpler diagram what you understand it [inaudible]
805 (I: mmm) understand it more easily more quick.
806
807 [5 sec silence]
808
809 I: When you were discussion you also talked about understandable language (B/F: mmm) can
810 you elaborate it elaborate on that?
811

812 B: I love the terminology here the stof we just never come up with, can't comprehend before [1
813 sec silence] "mutually disjoint".

814

815 F: And the oriented link is that an example which we understand the English language but an
816 "oriented link is a link, each component of which have received an orientation", mean, I don't
817 know how you see an orientation.

818

819 [2 sec silence]

820

821 B: I think it's here, language they sort of they expect you to be fully com [inaudible] with. And
822 as A-level students we are not suppose to go and look it up and make this sort of attempt to
823 clear it before we fully understand what it's actually telling us.

824

825 I: Mmm [2 sec silence] so so now you're talking about that the terms should be clear, but before
826 you talked about having examples. Which is most important?

827

828 B: [inaudible]

829

830 I: You talked about that it was important to have examples before (B: [inaudible] examples)
831 now you are talking about er that er terminology [interrupted]

832

833 B: Will will we be able to understand the examples they are giving us without cause in in
834 getting the examples they they explain it using this terminology, but if we don't understand the
835 terminology then we haven't got hope of understanding the examples.

836

837 I: So the terminology is the most important?

838

839 B: I think, yea (F: [inaudible]), pretty, it comes first, yea.

840

841 I: Mmm

842

843 F: [inaudible]

844

845 B: A A A-level is just sort of [1 sec silence] assuming, I mean, it's all in terminology, we don't
846 for unless if it's a new thing, and if it is, it is explained to us so [inaudible] (I: mmm) so we
847 [inaudible] twice about it, you just expect to understand the terminology but here you can't
848 cleanse all this these words, understand what they are referring to.

849

850 I: mmm

851

852 B: That's why [inaudible]

853

854 I: That's why, pardon?

855

856 B: I know first when I was reading this article, it was [inaudible] very hard to understand what
857 it actually was trying to tell me.

858

859 I: Mmm.

860
861 ? : Mmm
862
863 [1 sec silence]
864
865 I: But even though you actually
866
867 [Tape runs out]
868
869 I: But from your conversation I could hear that you DID understand something?
870
871 [1 sec silence]
872
873 B: Mmm, yea, I we managed to get the er I don't know [F laughs a bit] er understand it's basic
874 ideas around crossing I said [laughs a bit] that's about as far as we understand, something to do
875 with the direction of the crossings [1 sec silence] and what you can and with er with the way
876 you can [1 sec silence] adjust that crossing here if it was a string the way you can [2 sec silence]
877 perform moves and this er.
878
879 F: For example it doesn't this is plain group plane crossing thing, it doesn't actually show a
880 plane crossing anywhere here at all, they are all 3-dimensional structure [inaudible] crossing but
881 er so what it a plane crossing, it doesn't say, I mean how would you have [1 sec silence] would
882 that be [interrupted]
883
884 B: Hang on it says a plane crossing there the two lines are actually going touch each other they
885 are on a plane with each other, so yea, they'll cross each other meet each other, but whereas
886 these two they [1 sec silence] they are not on the same plane, one is going above the other so,
887 [inaudible] same plane
888
889 F: So that't that's a piece of string with four different bits of it (B: [inaudible]) [inaudible]
890
891 B: Cause they they're joining the senses so therefore it must be on like [paper noise from book,
892 words inaudible] the table, and they cross, in a sense wouldn't it
893
894 F: But that that would be one of these one would be going over the other that must be a piece of
895 string which is actually in that shape [1 sec silence] (B: it must) one is going over the other and
896 it can't be in the same plane together (B: Mmm) cause one has to be higher up than the other
897 one (B: [inaudible])
898
899 I: But why did you actually begin talking together, I mean you have really been discussing this
900 and trying to, discuss the maths?
901
902 B: Er
903
904 I: Why did you [interrupted]
905
906 B: I think it certainly certainly helps if you can discuss it with someone else. Two brains are
907 better than one.

908

909 I: Why?

910

911 B: Er, one person can have one idea which should trigger another idea in the other person's
912 head which the first person wouldn't have had, and then the second person having said that
913 thing, and then one thing leads to another if you got two people to think er

914

915 F: Er [inaudible] if you read all that that book and then have to read that book as well.

916

917 B: Yea it also helps if you read some of that one and I read some of this one [1 sec silence] and
918 we sort of explain it to each other all that [1 sec silence] saves time.

919

920 F: [inaudible] if I got a problem and I'm on my own I always get [inaudible] bob/book down
921 and if I don't understand first section I just give up on it and er if I get through with my dad or
922 someone whose really good a maths then I can do it eventually in the end. [1 sec silence] even
923 though he know nothing about and I know nothing about it.

924

925 [2 sec silence]

926

927 I: What about is is one, if you ask the teacher, then he probably knows, so it's another situation,
928 from when two people don't know, in the beginning, do you understand what I mean?

929

930 B: Er, you're saying that, if what about if we go to the teacher and ask him?

931

932 I: Yea because then then it's another situation because he probably knows the maths.

933

934 B: Yea, I mean if, yea there's nothing we can do if we were asked to do a presentation we can
935 go and ask the teacher what exactly he knows about it and get him to explain it to us (I: mmm)
936 (F: [inaudible])

937

938 I: But what's best actually, what works best, to talk with someone [interrupted]

939

940 F: Someone who knows, yea (I: someone who knows?) (B: Yea [inaudible]) give some sort of
941 achievement whereas if you if you understand something (B: Through looking at it yourself)
942 looking at it yourself [inaudible] and not knowing anything beforehand, but you don't know if
943 it's right in the end unless it is really obvious it's right so you could be be making a fool of
944 yourself. (B: Yea) (I: mmm)

945

946 [3 sec silence]

947

948 F: How to be a genieous or so do something completely new.

949

950 [3 sec silence]

951

952 I: Someone had to learn it once, I mean, it had to start somewhere (? : mmm) mmm

953

954 [2 sec silence]

955

956 I: Great, er you did talk a little about how it was different.
957
958 B: Yea, instead, cause we never get asked to, certainly in maths, we never get asked to er go on
959 our own way with something completely new topic (I: mmm) and find out about it for
960 ourselves, we are always getting getting it explained to us [interrupted]
961
962 F: [inaudible] supposed to do is individual wises, to do the course, [inaudible] bits of modules
963 [interrupted]
964
965 B: Yea it's the coursework when there is er [interrupted]
966
967 F: Not allowed to teacher's help really, you are just given, yea [B and F talks at the same time
968 for a moment], the closest thing you are given are problem, and you you are to explain the sort
969 of technique you should use though, just [inaudible] you have to take data (I: Take?)
970 tapereadings of real situation and measure, for mechanics you measure lot falling down er
971 [inaudible] slope and how far it will er it will travel and then compare it with theory, how far it
972 should travel, and and then make sort of adjustment for the amount of friction that's [inaudible]
973 or something, and you do all that work yourself but you still, they are after you [inaudible] can
974 explain the theory of how to do it.
975
976 [3 sec silence]
977
978 F: We don't get [inaudible], we don't get say er go at look up how to use this theory in the
979 library [interrupted, talk a bit at the same time - difficult to hear]
980
981 B: Yea it's not [inaudible] give name of the theory and go and then you have to go and find out
982 about it.
983
984 F: Mmm
985
986 [3 sec silence]
987
988 I: Mmm, but in what ways is it the same, from what you normally, I mean, what you have been
989 doing tonight, in what way is that the same as what you normally do when you learn some new
990 maths, er [interrupted, a few words inaudible]
991
992 B: [inaudible] cross me that we've been discussing it together (I: mmm) er to help each other
993 understand er we do I mean quite often
994
995 F: [inaudible] (I: Pardon) we all laugh at each other when someone say something really stupid
996 [laughs a bit] [inaudible] really placed it wrong and [inaudible] often the teachers write up
997 something one line of an equation and says what will the next line be, and chooses someone, so
998 everyone in class is paying attention [3 sec silence] that's about [3 sec silence] er anything
999 that's similar, er we can talk to each other a little bit but not much.
1000
1001 [2 sec silence]
1002

1003 B: If we if we've worked through exercises like in class then if the teacher is talking to
1004 someone else about the problem that we've just discussed between us, see if we can get the
1005 answer without help from the teacher a bit from someone else [inaudible]
1006
1007 [3 sec silence]
1008
1009 F: [Mumbles a few words, inaudible]
1010
1011 [3 sec silence]
1012
1013 B: Yea
1014
1015 [4 sec silence]
1016
1017 I: OK, is there something we haven't discussed tonight which you think is quite important for
1018 knowing how you learn new math?
1019
1020 B/F: Mmm
1021
1022 [2 sec silence]
1023
1024 F: Oh oh what do you mean?
1025
1026 B: Maybe maybe the way that it is split [inaudible] to modules with different, different areas of
1027 maths in each different module er helps I think it helps keep one sort of general topic area, you
1028 do all at once and you get examined on it straight away and you can put it behind you, gone,
1029 [inaudible] next part, it's very helpful (I: mmm) er modules are clasted to three, three groups of
1030 six, there is mechanics, statistics and pure maths and if you want to you can do six modules in
1031 each one of them, but it counts as three whole A-levels er but most people, for the single A-
1032 level most people do er first 3 pure maths and then either 2 stats and 1 mechanics or two
1033 mechanics and one stats
1034
1035 [4 sec silence]
1036
1037 I: You can take it with you, you don't have to eat it now [about the cookies] [laughs]
1038
1039 [1 sec silence]
1040
1041 B: But, yea, I think certainly the sort of grouping in certain area and you have a different
1042 teacher (I: mmm) say while your [inaudible] mechanics modules and pure maths modules same
1043 time, you have one teacher for the mechanics and one for the pure (I: mmm) instead of, keeps
1044 them separate so you don't get too confused.
1045
1046 I: What, you said, I think maybe it was you in the beginning that using the exam questions that
1047 was good for it kind of linked all different kinds of maths.
1048
1049 B: Yea, well, when when we say different kind of maths, the exam questions will be from
1050 [inaudible] mechanics modules linking all the maths (F: We get books) we are linking all the

1051 sort of chapters within the one book from that module (I: mmm) so it will all be within that
1052 topic area but it's separate, I mean each different chapter has a whole range of different
1053 techniques that you use for different things.
1054
1055 [2 sec silence]
1056
1057 F: There is very little overlap between the the modules really, I mean (I: mmm), once you've
1058 done a module you can basically forget about it [interrupted]
1059
1060 B: Some of them some of them build on each other, er the first 3 pure one's certainly do (I: The
1061 first?) the first 3 pure maths ones (I: mmm) you can't do the second one without the first,
1062 without understand it, but then you can go on and do er [1 sec silence] in mechanics certainly
1063 you can go on to do er mechanics 4 one which what we are doing, without having done the third
1064 (B: Or the second really) yea yea, the fourth one is differential equations which, you really
1065 really need is the third pure maths to have any idea about it. [2 sec silence] Er, you go straight
1066 on do.
1067
1068 [3 sec silence]
1069
1070 I: OK do [2 sec silence] you don't, do you have anything to add to what B said?
1071
1072 [2 sec silence]
1073
1074 F: Er [1 sec silence], no I think that covers all the area.
1075
1076 I: Mmm. OK Well, Thank you for your time.

Appendix I:

Full transcription of the Danish group interview, in Danish

1 I: Jeg kunne godt tænke mig at høre hvad gør I, jeres reaktion på, altså, hvis I nu møder noget
2 matematik som læreren har gennemgået, noget, en matematikopgave, eller lad os sige læreren
3 er ved at gennemgået noget, og man ikke lige forstår, hvad snakker han om, hvad snakker hun
4 om, hvad gør I så?

5

6 Æ: Først og fremmest så prøver vi li'som at tage afstand til det (en anden elev: ja) og så
7 egentlig, og så måske vende tilbage til det, fordi så nogen gange så ser man på det med andre
8 øjne ikke, der er mange gange når læreren står ved tavlen og gennemgår noget, så bliver man
9 bombarderet med en masse ting ikke, og selvfølge så tager man det til sig, men så hvis man
10 kommer hjem, og man har tænkt noget andet imens, så forstår man det nogen gange bedre fordi
11 man sådan, altså ser på det [afbrydes af Z – det bliver lidt uforståeligt et par sekunder]

12

13 Z: ... et punkt hvor, hvis man får det gennemgået foran sig, hvor du så bare siger, dér, det kan
14 jeg bare ikke forstå, på den måde som det bliver gennemgået lige nu, og så står man så bare helt
15 af, (en anden: mmm) og så tror jeg mere, at man bliver nødt til sådan, måske selv at sætte sig til
16 at kigge på det, uafhængigt af hvad der foregår ikke? Så hvis man for eksempel har mistet det
17 første, så er det ret svært at komme ind i sådan et eller andet, en eller anden gennemgang på et
18 senere tidspunkt, så bliver man li'som nødt til [mumler] at tage den læring over på sig selv,
19 ikke. Så sige, nå ja så starter jeg forfra for at se det her [inaudible] hvad er det, det går ud på
20 ikke? Så er det selvfølge en fordel, hvis der er gået noget tid og læreren ikke står og snakker
21 samtidig. Ting der. Men jeg tror tit man kommer til at lægge afstand, hvis det er sådan. Hvis
22 man først begynder og ikke have fat i noget, så simpelthen ikke forstå resten af
23 sammenhængen, så står man bare af så, må man begynde forfra. Tage hele vejen op igen.

24

25 Å? En anden siger noget, selv læse igennem [inaudible]

26

27 Z: ... et eller andet man bliver kastet ud i, man skal regne en opgave i et eller andet, der lige er
28 blevet gennemgået på tavlen som man bare ikke har forstået, så må man lige sidde og kigge
29 igennem igen. Og så lige kig på opgaven en gang til.

30

31 Æ: Der er også mange gange hvis man sætter sig med sin bog og så, det kan godt være at man
32 har taget nogle notater til det, der er blevet gennemgået, men mange gange så hvis man
33 begynder at læse sine notater igennem så forstår man ikke rigtig nogen af de notater, men hvis
34 man så selv tager, med udgangspunkt i bogen ikke, og begynder helt forfra, og skriver punkt for
35 punkt ned ikke? så kan man godt bruge de notater man har lavet men, det at man simpelthen
36 selv helt starter fra en frisk, det, det er mange gange en hjælp til at man får det helt arbejdet
37 igennem.

38

39 Z: Men også det der med at skrive det punkt for punkt, det er meget sådan med, at det kan jo
40 godt være, det er sådan en systematisk proces ikke?, hvor man lige som siger nu forstår jeg
41 punkt 1 igås, så får jeg sådan arbejdet det igennem for mig selv, det er tit hvis det er meget
42 kompliceret at forstå når nu læreren står og bare kører derudaf, at det er en fordel hvis ... nu
43 tager jeg det selv og nu forstår jeg hvad egentlig er grunden til hvad er det egentlig det går ud
44 på ikke?

45

46 I: Du siger punkt 1? Hvad kunne det være for eksempel?

47

48 [Et par sekunders tavshed.]

49

50 Z: Nå ja men altså hvis man er ved at have noget trekantsregning ikke? Så hvis man sætter sig
51 ned og så siger, ok, hvad er en trekant ikke, og så ligesom får linet op hvad er det for nogen
52 overhovedet betegnelser vi arbejder med her ikke, og så kan man få dem skrevet op og tegnet
53 en figur og sådanne nogle ting, og så kan man lige så stille begynde at forstå nå, hvis den er så
54 og så lang igås det er virkelig, så kan man begynde at forstå. Men det er bare så vigtigt at få
55 linet op så. Mmm, hvad er det egentlig problemet går ud på, for det første ikke, og hvad er det
56 egentlig vi kigger på, hvad område er vi i. Og så kan man så begynde at arbejde med det. Og
57 jeg tror også meget det er vigtigt det med at få nogle betegnelser på, på en eller anden måde, og
58 få det sådan arbejdet igennem, ligesom Æ siger at skrive det punktvis op. [inaudible] [knipser
59 med fingrene]

60

61 Æ: Nu har vi haft, nu har vi arbejdet med noget rumgeometri, og den første uge [inaudible]
62 første da var det et stort, fatter vi ikke noget af, men hvis man så egentlig begynder, og man så
63 først får lavet de tegninger, og man også kan se jamen hvad er det egentlig, altså mange flere
64 betegnelser, det er måske beregninger eller sådan noget. Når man først får lavet en tegning, så
65 nogen gange så forstår man hvad det egentlig er man sidder og regner på. Det er i hvert fald
66 også vigtigt.

67

68 Ø: Det er rigtigt det der med at få det ned på billedeform. Altså, ja, enten også en kurve det
69 synes jeg det hjælper utrolig meget. Og forstå hvor det ligger, for eksempel med en tangent, øh
70 hvis du skal forstå hvad det var, og så få det tegnet, det synes jeg det [inaudible] bare.

71

72 Z: Altså jeg tror da også nogen gange jeg har oplevet det der med at, nå hvis jeg først, for
73 eksempel først ikke forstod rumgeometri, så var der nogle opgaver, som jeg bare skulle løse.
74 (en anden elev: Ja) Sådan foregik det i timen. Nu skal jeg bare sætte mig ned og løse de
75 opgaver. Og så kan man godt løse dem sådan lidt planløst, uden at forstå sådan egentlig, sådan
76 helt, ikke, sådan overordnet, kan man så, så når man li'som har fået løst nok af de små opgaver
77 som måske bare, sådan en eller anden distanceformel, vi lynhurtigt [inaudible] regne ud og så
78 bruger vi lige det og det. Og så på et tidspunkt når man så har fået gennemarbejdet de der
79 opgaver og kigget på det, tegnet nogle billeder, forestillet sig nogen ting, så begynder man lige
80 som at forstå sådan mere overordnet sådan mere [inaudible]. Og jeg tror måske også nogen
81 gange, da kan man forstå de der enkelte ting ikke, og så stadig have den der fornemmelse, nej
82 der er sgu et eller andet jeg ikke har fat på, fordi man ikke har forstået sådan overordnet, hvad
83 går det her rumgeometri egentlig ud på igås, hvad er pointen med det ikke li'som. Jeg tror
84 nogen gange det kræver den der aha oplevelse. Nå sådan hænger det heeele sammen, før man
85 sådan rigtig kan se det for sig.

86

87 I: Hvordan kommer man til det punkt hvor du siger sådan hov, nå det er sådan, den der,
88 jaaaahh, nu er man bare med? [afbrydes af Z].

89

90 Z: Ja, jeg har tit haft det sådan noget med, at vi har fået gennemgået et eller andet emne og det
91 forstår jeg ikke en wiut - det er skide træls. Og så, læreren kører løs, nu skal vi regne en masse
92 opgaver, og så, så er det som om, nå men så kan jeg sætte mig ned og så kan jeg regne de

93 opgaver. Jeg synes tit jeg godt kan regne nogle opgaver, selvom jeg egentlig ikke heelt er med
94 på hvad det er der foregår ikke altså. Når man så får regnet de der opgaver så er det som om
95 jamen så får man også arbejdet med dem inden i sig selv. Når man regner de der opgaver, og at
96 man skal prøve på at svare på de der spørgsmål eller et eller andet, ikke også. Selvom det måske
97 ikke lige er så [inaudible] så tror jeg nogen gange, jeg har haft det sådan, at det har været sådan,
98 nu har vi haft det her i lang tid, og det er lang tid siden vi startede på det. Altså ok nu læser jeg
99 det lige forfra, og SÅ, går det op for damen, hvad det egentlig er, det handler om. (Å: ja [sam
100 vil gerne sige noget mere]) Fordi så har man den der åndssvage basale ting på plads igås og så
101 kan man li'som så, nå ja det er det der, der er pointen, der ligger bagved.

102
103 Å: [inaudible] forstået noget meget teoretisk så lave nogle opgaver. Se hvad det er. I stedet for
104 bare bogen, de der tørre tal, hvordan man bruger det. [inaudible] tjekke det, lidt utydeligt.

105

106 I: Det vil sige, det er egentlig rart nok med de der slaveopgaver?

107

108 Flere i kor svarer: "ja".

109

110 Ø: Det er nødvendigt, det er det. Også for eksempel hvis man skal have en prøve og så man lige
111 læser op, så synes jeg nemlig man kan få den aha oplevelse (Z: Ja nemlig) som Z snakker om.
112 Altså det synes jeg, det er, det har egentlig rimelig stor værdi selvom man ikke gider det, men
113 alle de der prøver og sådan noget. Det kan godt hjælpe på forståelsen, synes jeg i hvert fald.
114 Netop fordi man er færdig med alt det der med at sidde med en masse opgaver og alt notationen
115 så man kan godt [Z afbryder, man hører hun siger ja, men de snakker et par sekunder i munden
116 på hinanden – ikke til at forstå].

117

118 Z: ... Det tror jeg nemlig også. Man ved hvad en retningsvektor er. (Flere elever: Ja) Og når
119 man så ved det, kan man godt se det for sig. Nå ja, det er jo smart, synes jeg. (En anden elev:
120 Ja) Jeg tror også tit det der med, at når de står og kører de der beviser af, da tror jeg altså, det er
121 de færreste der sådan bare tænker, nå ja, det er da masser af mening i, hvor er det sjovt det her.
122 Men altså sådan det der med at der [inaudible] de der opgaver som man SELV li'som skulle
123 sidde med. Og jeg tror det er opgaverne, der giver utrolig meget. (Flere: Ja; det er det også) Den
124 måde jeg har lært matematik på i gymnasiet (En anden elev: Det har jeg også) det er noget med,
125 at man har regnet opgaver. [Afbrydes af to andre Æ og ? på samme tid]

126

127 Æ: Men også det der med, at nu er vi begyndt at repetere noget af det vi egentlig fik
128 gennemgået for et år siden, i forbindelse med at vi snart er færdig, [en anden elev siger et eller
129 andet] ja, jeg kan huske sidste år, da vi fik det gennemgået, det var simpelthen, det var et stort
130 kaos, man fattede ikke noget, når man så får det gennemgået så tænker jeg jamen, var det
131 egentlig ikke værre [inaudible], altså man syntes bare dengang, da var det sss helt uforståeligt.
132 [inaudible].

133

134 I: Hvad, hvad er så det, der er sket i mellemtiden?

135

136 Æ: Det er at man har siddet og regnet, man har... [Afbrydes af Z, der fortsætter sætningen]

137

138 Z: ... har regnet alle de opgaver man har været tvunget til og, altså, man har forstået li'som (Æ:
139 Ja) en masse små enkelttilfælde tror jeg igås. Et enkelttilfælde der er beskrevet i en eller anden
140 opgave ikke, med en linie og et plan ikke, og det kan godt være at sådan, OK så har man måske

141 kunnet forestille sig konkret den der ene plan, den der ene [inaudible] det er man nød til (I
142 [måske]: mmm) men så når man kommer, når så det her bevis det kommer, som man bare ikke
143 forstod overhovedet der i 2.g, hvor det blev gennemgået. Så ved man hvad alle de her ting er,
144 man ved bare sådan, man er 100% sikker på hvad en normalvektor er [inaudible] de opgaver
145 man har regnet. [En prøver at bryde ind, og det er lidt utydeligt et par sekunder] Logik at ting
146 kører på den måde, selvfølgelig er r_1 krydset med r_2 det er bare det er normalvektoren (I:
147 .mmm), selvfølgelig er det sådan. Og før så har vi bare, Ahhh vi kunne ikke engang forstå, vi
148 stod af allerede dér. Og så [afbrydes af Æ]

149

150 Æ: Det er jo li'som alle ting igås, når man starter med at skulle lære at cykle, det fatter man jo
151 heller ikke, der vælter man jo også rundt ikke? Der er nogen ting, der begynder at køre på
152 rygmarven ikke altså, da, når man lærer, når man starter med et emne, jamen så er der så mange
153 nye ting, altså så til sidst, så, så tænker man slet ikke over dem, de, dem har man bare inde i
154 hovedet altså, dem bruger man automatisk uden.

155

156 I: Det sidder på rygmarven, li'som?

157

158 Æ: Ja, det tror jeg nok [Flere begynder at snakke i munden på hinanden et par sekunder, det er
159 ikke til at høre, hvad de siger]

160

161 ?: [inaudible] acceptere ting [inaudible] utydeligt.

162

163 Z: Jeg tror også det er vigtigt, det som [inaudible] siger, med notation. (En anden elev: Ja) Med
164 det der med, at at først så hvis læreren siger, nå men det her er så planen alfa, så tænker man
165 bare, nå nå plan ha ha. Det kan jeg ikke være med på. Men så efter et stykke tid så ved godt
166 bare de der regler som alle matematiklærere tager for givet igås, nå men alle ved at alfa og beta,
167 det er planer. (I: mmm) Det ved vi jo ikke, når vi sidder der. Vi ved ikke, at l og m og n
168 nødvendigvis er linier. (I: mmm) Men når man så ved dét, så er det bare så logisk, det hjælper
169 også een meget i de beviser, at hvis den hedder m, så er det selvfølgelig en linie ikke altså. Det
170 ved man jo ikke der i starten, så kan man sådan æææ [en lyd der vist betyder, at der er noget
171 man ikke kan finde ud af].

172

173 I: Kan man så ikke lære matematik hvis man ikke har, li'som har fattet at l og m det kan kun
174 være linier og og f det altid for det meste er en funktion og [afbrydes af Z].

175

176 Z: Jow man kan sikkert godt lærer det, men det er svært at lære det af én, der har den notation
177 og som bare siger, vi starter li'som på forskellige planer ikke (I: mmm), og så siger den person,
178 nå men alfa, og så kan man sidde sådan mmm, det kan jo være alt igås. (En: mmm) Og det er
179 li'som, det er svært, når der er det der misforhold mellem, at læreren tager det for givet ikke, og
180 eleven sidder aaahhhh igås. Og det er det samme med funktioner for eksempel, det der med at
181 når man kommer så ved man skal kunne [inaudible] $f(x)$, selvfølgelig er det bare en funktion,
182 sådan er det bare ikke. Men ok når man så først ved det, så bliver det altså (en anden elev: ja)
183 noget nemmere, fordi at, at man li'som skal lære det af en der bare forudsætter igås,
184 selvfølgelig er det sådan, og hvor det skriger i øjnene på den person som som har sat sig til at
185 skrive et eller andet p af [afbrydes af Æ].

186

187 Æ: Det er også li'som, der er jo li'som den der man har en eller anden måske bari., på en eller
188 anden måde en barriere ikke, så, og, og man kan blive, det er svært at sige, hvad det er der lige

189 skal til, før man siger, ov nu forstår jeg det, ikke. Vores matematiklærer har jo stået med, i
190 rumgeometri, med bøger simpelthen og, og formet dem med bordet ikke, for at kunne se at
191 hvad er det egentlig der bliver spurgt om. Måske så [er] det, så [er] det lige det han, han, han
192 gør der, som gør at man, så forstår man bare hvad det er, det drejer sig om, altså. Det, det [er]
193 sådan nogle små ting der pludselig skal til, og det er svært at sige hvad det egentlig er, der gør,
194 at man pludselig siger, aj nu forstår jeg. (En anden: mmm)

195
196 Z: Jeg tror også, det er mærkeligt med matematik, det der med at man skal altid lære det af en
197 person, som bare [griner] véd alle de der ting. Det er ikke li'som i dansk, hvor man sådan, hvor
198 det mere er en dialog mellem eleven og læreren for eksempel, og læreren ved ikke
199 nødvendigvis alt hvad der er at sige om den der tekst. Men i matematik der véd læreren bare, alt
200 hvad man nu skal lære [inaudible] i det konkrete eksempel vi står over for, ikke. Der er et
201 enormt misforhold der, og jeg tror også nogen gange det er svært, li'som at sætte sig ind i at
202 man som elev sidder sådan lidt puhh [puster], igås. Fordi der er så mange ting som læreren bare
203 forudsætter, selvfølgelig er det sådan. (En anden: mmm). Som man ikke selv, li'som.

204
205 I: Du siger læreren bare forudsætter, det, altså, det siger, læreren siger måske ikke at alfa og
206 beta det plejer at være, det plejer at være planer. [Afbrydes af Z].

207
208 Z: Det kan godt være læreren SIGER det. Men jeg tror bare, at hvis læreren først har sagt igås,
209 det her det er planen alfa, så er det li'som om at læreren har fået den her sætning, den, den er så
210 indarbejdet i læreren, så han slet ikke kan se, måske har eleven [inaudible] svært ved at forstå
211 det her, måske har eleven, skal det lige have tid til at synke ind igås. (En anden: mmm) Der er
212 det der misforhold igås.

213
214 Ø: Jow, det, jeg synes også tit at læreren siger, det er jo så SIMPELT kan I ikke (En anden elev:
215 Ja) SE det, og så, man sidder bare, og fatter ikke en disse. (En anden elev: Ja) Og det, det har
216 nok meget at gøre med det, netop det vi snakke[de] om, med at der er så mange forvirrende ting
217 som, og man sidder og hænger sig i nogle detaljer som egentlig ikke har betydning for
218 forståelsen af det problem man taler om. (En anden: mmm) Øhm, og, og det er måske nok lidt
219 svært at se, hvordan man skal undgå det, fordi vi bliver jo nødt til at blive introduceret i det, den
220 nye notation [mumler et sekund].

221
222 Z: Men øhh, selvom, det der som, som altså hjælpelærer i matematik, det der med at jeg har
223 også siddet og siger sådan, jamen f det er jo en funktion, selvfølgelig er det en funktion, og det
224 er li'som om at den der lille elev HÆNGER sig ENORMT meget i det her med, jamen hvad er
225 en f egentlig for noget, ikke. (En anden: mmm) Så man bare sådan, nå men det er jo idiotisk at
226 sidde og spekulere over det, det er jo slet ikke det, det handler om, ikke, se, nu skal vi til sagen
227 igås. [inaudible] eleven sidder sådan, ja, det er jo det, der er relevant, når man ikke forstår, hvad
228 det egentlig er, der er relevant. (En anden: mm) Jeg tror også tit det er det, man forstår ikke
229 hvad det er man skal frem til at forstå, hm.

230
231 I: Ja, ok, og det lyder også som om, det du beskriver det er en eller anden form for, sige
232 byggeklods, oven på hinanden, man bygger oven på hinanden. (En anden: ja ja) [Afbrydes af Z]

233
234 Z: Det er li'som læreren taler i et andet sprog, ikke. (En anden: mmm) Det er lidt li'som det der
235 med at hvis du skal lære dansk på italiensk, så ville vi også sidde og, og hænge os utrolig meget
236 i ordene som læreren sagde, i stedet for meningen, ikke. (I: mmm) Og, og læreren kommer med

237 det her store matematiske sprog, og så sidder man meget og hænger sig utrolig meget i hvad
238 alle tingene hedder, og er, i stedet egentlig og, og hænge sig i meningen?
239
240 I: Kan man så finde meningen udenom sproget?
241
242 Et par sekunders tavshed.
243
244 I: Det er måske lidt filosofisk? [siger det lidt undskyldende]
245
246 Z: Det tror jeg er svært. Det tror jeg er meget svært.
247
248 Ø: Det tror jeg også, det er. Det, det, så skal man have sådan et eller andet flair for det, eller en
249 eller anden fornemmelse af at, i hvilken retning det går. Fordi at det, det er jo nærmest håbløst
250 når du ikke har æhh fundamentet, og så forstå, hvad det er du skal frem til. (En anden: mmm)
251 Det, det.
252
253 I: Måske mere held end forstand, hvis man når frem til et eller andet?
254
255 Ø: Ja, det vil, er det jo nok. Men [en anden prøver at afbryde, og et par ord drukner herved] ikke
256 forståelse, man kan jo ikke nå frem til en forståelse via held (I: mmm), altså. Det, jo, måske
257 held i den retning ens tanker går. Men altså. Jeg tror det er nødvendigt med det fundament. Men
258 det er bare som om at vi starter, hvis det er sådan en række af byggeklodser, så starter vi altså
259 bare midt på, øhm, på en eller anden led ikke. [Der er samtidig lidt støj på grund af, at en
260 slikpose åbnes] Det, altså det er der, det vi sidder og siger er jo ikke at vi ikke får fundamentet
261 med fra starten, at det, det går hurtigt til næste trin ikke. Og så sidder man og lige skal have de
262 der [inaudible]. Aha oplevelsen det er nok at klodserne under med, ikke.
263
264 I: Aha.
265
266 Z: Jeg synes også det er sjovt, at det der med, jeg tror faktisk der er flere, jeg har også snakket
267 med flere der har haft den oplevelse af, at sådan i løbet af året, så får man bare gennemgået en
268 masse stof, og man hænger da på [En eller anden mumler et eller andet], og forstår sådan
269 nogenlunde hvad det går ud på, man kan godt regne opgaverne måske. Men når så for eksempel
270 en eller anden prøveeksamen eller terminsprøve eller et eller andet kommer, hvor man så sætter
271 sig ned, og så læser man det, og så er det, så er det altså virkeligt tingene de går op for én (En
272 anden elev: Det er det helt klart). Fordi så kan man altså virkeligt se sammenhængen. Og jeg
273 tror også nogen gange, det der som Ø siger, man bare bliver kastet ud midt i det, det tror jeg
274 altså også nogen gange, det er nødvendigt for overhovedet at få noget ud af det. (En anden:
275 mmm) Man [inaudible] vil i hvert fald ikke få noget ud af det i 1.g hvis man sad, hvis man og
276 måbøde og bare sagde, f er en funktion, og A er et punkt og så, og det vil man jo ikke få noget
277 ud af. (En anden elev: nej) Man bliver kun udfordret i det øjeblik, det er spændende og sådan.
278 Det er jo også vigtigt, altså den der motivation ikke, som jeg også tror der ligger i det, den der
279 forvirring, der ligger også en eller anden form for motivation ikke. Man VIL sgu gerne lære det.
280 (I: mmm) Og det bliver man så nød til at have, fordi at det hjælper ikke noget bare at skulle
281 sidde og lære hele det der matematikSPROG først vel. Jeg tror også det vi [inaudible] må holde
282 fast i det hvor du [Ø] sagde, at det du sagde, det kunne være en person der li'som havde flair for
283 det ikke, kunne se den her ting for sig på en anden måde. Og jeg tror måske også det, det er der
284 skellet ligger mellem og så begynde at tænke SELV og kreativt, på en eller anden måde ikke,

285 fordi hvis man virkelig har den der flair, [inaudible] man bare kan se det der rumgeometri, det
286 kan man bare se, inde i hovedet så snart man får det at vide. Og så behøver man ikke at have
287 alle de der betegnelser egentlig vel, hvis man kan se det for sig (En anden: mmm), og da tror
288 jeg måske også, da begynder man også at kunne arbejde på en anden måde med det. (En anden:
289 jaa – i betydningen jeg har forstået) [et par sekunders stilhed] Man kan også godt først
290 [inaudible] betegnelser og eksempler og [inaudible] [tøvende]
291

292 I: Synes I det måske er ping-pong mellem at man på en måde kan se det og så også sproget?
293

294 Z: Ja
295

296 [Pause et sekund, en anden hvisker vist også ja]
297

298 Æ: Det med altså [inaudible], det man egentlig sidder og læser i bøgerne. Det skal man også
299 mange gange ind, altså, altså se inde i hovedet altså. Man kan godt, man kan godt se et bevis
300 som for eksempel i rummet ikke, det er jo bare på et flat stykke papir ikke. (En anden: mmm)
301 Men man skal kunne se det så ind i 3-dimensionelt, inde i hovedet for li'som at kunne forstå
302 hvad det drejer sig om. Det.
303

304 Å; Det er på den samme måde når man skal regne opgaver [inaudible] (Æ: Ja) når man skal
305 regne parabel [inaudible] når man kan forestille sig hvordan den ser ud, så er det meget lettere
306 at komme frem til en løsning [inaudible] (Æ: Ja)
307

308 Z: Men også for eksempel differentiallyigninger ikke, der havde jeg bare sådan med først så
309 kunne jeg godt lære det, for det rent notationsmæssige plan hvor man sidder, nå ja så kan man
310 lige gange [inaudible] dx over på den anden side, det må vi fand'me ikke og sådan nogle ting
311 der [flere griner]. Man kan godt se, man kan godt se sådan, man kan godt se de der små tal og,
312 og de der små streger [inaudible] sådan og sådan og så efterhånden som man li'som har forstået
313 DET, så kan man gå videre til at sige, nå hvad er det egentlig det beskriver det her, hvad er det
314 egentlig en differentiallyigning egentlig er igås. (En anden: mmm) Og så har vi så været heldig
315 med at vi har haft det der fysik, fordi der måske også [inaudible] haft lettere [inaudible] (En
316 anden: ja [inaudible]) [inaudible] Ja, men først dér jeg egentlig har følt, nu forstår jeg at, at det
317 kan være et eller andet med varmeudvikling, eller et eller andet. Så forstår jeg hvad det her
318 egentlig kan bruges til, hvad det egentlig er det dækker over igås. (En anden: mmm) I stedet for
319 man bare får en eller anden opgave med y og x og små streger og sådan nogen ting igås.
320

321 I: Nu siger du, at nu forstår du, øh, sådan og sådan men (Z: Ja), men hvordan vil du sige, altså
322 hvornår er du li'som selv tilfreds, og li'som kan sige, hvad er det der gør at du li'som siger, OK
323 nu har jeg forstået (Z: Ja), hvad er din [afbrydes af Z]
324

325 Z: Det synes jeg også er svært, for jeg synes li'som tit jeg har det sådan med matematik, at jeg
326 kan gå og brokke mig over at jeg ikke har forstået en ting, som jeg sagtens kan lave opgaver i.
327 (I?: mmm) Fordi jeg synes der er stor forskel på og kunne sidde og, og bytte om på x'erne og
328 y'erne og, øh, følge nogle formler som der står i formelsamlingen og li'som bare bytte ud, og så
329 er det li'som altså [et sekunds pause] ja så er det li'som noget med bare at sætte ind i formler og
330 bytte lidt om og sådan nogen ting. Og jeg tror egentlig først selv jeg er helt tilfreds i det øjeblik
331 hvor jeg kan se, for mig selv, at, nå ja, det her er tiden og det her det er varmeudviklingen eller
332 et eller andet igås. For jeg egentlig kan se, hvad er det egentlig der sker når man bytter om på de

333 her tal, hvad er det egentlig det hele går ud på, fordi så kan man jo også meget nemmere se,
334 hvorfor man IKKE kan gøre noget. (En anden: mmm) Fordi i starten kan man godt få at vide,
335 ahm det kan du bare ikke gøre, det kan bare ikke lade sig gøre, vel. Men det kan man først
336 sådan rigtig forstå, når man li'som har set, nå ja det kan du altså ikke. [inaudible] (en anden:
337 mmm)

338
339 I: Det vil sige, kan man sige, det er et eller andet landskab man så skal, eller, man, man står på
340 en øde ø, fuldstændig, der har aldrig været en hvid mand, eller et menneske, der før. Og så må
341 man bare gå på opdagelse. Kan man beskrive det sådan? Så når man går rundt og rundt og rundt
342 og, og har faret vild nogen gange og så, så [inaudible] når man har gjort det tilstrækkelig mange
343 gange så pludselig så har man et billede over øen.

344
345 Z: Jeg tror mere, jeg har det sådan som om, at der ikke er noget, før at forståelsen er der. Sådan
346 igås. Så er det li'som bare blankt. Så er der sgu ikke nogen øde ø, så er der bare tomhed. Og så
347 kan man begynde at tage de her ting ud af tomheden, og det er jo også det, der er rigtig
348 underligt at forstå, ikke. Det er li'som om at man bliver nød til at gribe nogen ting ud af noget,
349 der ikke er der, igås. Sådan har jeg det mere, ikke. Altså sådan, at, at det går virkelig fra ikke at
350 være der OVERHOVEDET, til lige pludselig at være der.

351
352 Æ: Ja lige pludselig er der bare noget der falder på plads. Altså. [Klapper i hænderne] Jamen
353 man har, man har en, en, man har en masse netop byggesten inde i hovedet, ikke. Og pludselig
354 så begynder det bare at passe sammen. (En anden: mmm) Og så er det bare, jamen, så når man
355 ind sådan, det passer egentlig meget godt. Jamen, så, så har vi fysik og så pludselig en eller
356 anden byggesten som vi kan se den, nå ja den kunne vi egentlig også, den passer egentlig også
357 meget godt der. Det er li'som, der bliver hele tiden bygget videre på det. (En anden (I): mmm)
358 Og, og, det er lidt li'som, altså som ringe i vandet ikke, at det breder sig mere og mere og
359 pludselig så er der bare noget mere, der bliver sat sammen altså, som man kan kæde sammen
360 hele tiden.

361
362 Z: [inaudible] Jeg tror også, jeg har meget det der med, nu har du [I] det der med den øde ø. Og
363 det har jeg det lidt dårligt med for det er sådan meget med, udforsket eller ikke udforsket,
364 sådan, jeg kan godt se, at der ligger noget der men jeg, jeg har ikke kigget på det endnu, fordi
365 hvis det er sådan, at jeg ikke har forstået noget, så kan jeg simpelthen slet ikke se, at det er der
366 vel (I: mmm) altså. Men mere det der som Æ siger med, nå men så får man det her, så får man
367 den her formel, OK det er så li'som, nå men det kan så være en hylde, man kan ligge ting på,
368 ikke. (I: mmm) Og så kan man li'som få smækket nogen ting ind på den, på den hylde, ikke. Så
369 [inaudible] det er mere et eller andet system man kan gøre [inaudible], det er sgu ikke valgfrit,
370 nå nu vil jeg gå hen og kigge på den der, og så vil jeg (I: mmm) [inaudible] Det er mere sådan
371 noget, uh nu fik vi den ting, igås. (I: mmm) Sådan li'som trukket ud.

372
373 I: Altså, det var også, på et tidspunkt snakkede I også om, et eller andet med, at man blev
374 tvunget til at sætte sig, til at lave noget. (Z: Ja) Er det også lidt det du siger der, at, at så får, så
375 læreren giver dig bare et eller andet, altså, så, altså du kommer der bare udefra [afbrydes]

376
377 Z: Jeg tror ikke engang, at det er sådan rigtig at man, man kan få det. For læreren kan stå og
378 sige, hej hej, og sådan her hænger det sammen. Så bliver man li'som nød til og sidde inde i sig
379 selv og så sige sådan, ok nu har jeg fået at vide at det hænger sammen på den her måde ikke (En
380 anden: mmm). Og så må jeg kigge på kan jeg selv forstå, at det hænger sammen på den her

381 måde. Og så tror jeg det er li'som, det er det som skellet ligger mellem li'som bare at tage imod,
382 så tror jeg egentlig ikke man har forstået tingene. I hvert fald, ja ok det skal nok passe, ha ha
383 [inaudible] skrevet nogen ting oppe på tavlen. (I: mmm) Man bliver li'som nød til selv og sætte
384 sig ned og sige sådan, ja, en lineær funktion det ER det der, altså. I det øjeblik man li'som selv
385 får det med ind i hovedet og får erkendelsen med ind i hovedet ikke, så er det li'som om jeg
386 føler det sådan et eller andet med, så kan man trække den ting ud og ind, ikke (I: mmm). Det,
387 det bare en ting, den havde man bare ikke før. Og det er ikke sådan en ting, som man har kunne
388 gå og udforske tidligere. Det er virkelig sådan en ting, som man sådan får, hel ny igås.

389
390 I: En åbenbaring nærmest?

391
392 Z: Ja. [siger noget men afbrydes af Ø, uforståeligt]

393
394 Ø: Man har jo heller ikke, man har jo heller ikke nogen retning at gå, når du ikke forstår noget,
395 altså. Du ved jo ikke, hvor du skal gribe fat. Den æh kreativitet eller fantasi er der nok ikke
396 rigtig nogen der har, eller man iværksætter den i hvert fald ikke i så høj grad [afbrydes af at
397 rengøringsdamen pludselig åbner døren. Vi diskuterer, hvornår hun gerne vil komme, og
398 hvornår vi tror, vi er færdige. Hun spørger dog, om hun ikke nu må tage affaldsposerne med.
399 Hun larmer lidt med det, da Æ tager ordet, og man kan høre hende hele tiden mens Æ taler.]

400
401 Æ: Men jeg tror nok det er rigtig, som Ø siger, der er, det der med, man ved ikke hvilken
402 retning man skal gå, ikke. (I: mmm) Så, så det er lidt der, man prøver sig frem, og så er man
403 måske heldig eller pludselig kommer til at se det, eller, tager altså, man får forskellige
404 indgangsvinkler på det her emne som man slet ikke forstår [nogle ord drukner i
405 rengøringskonens larm] [inaudible] bare fandt den rigtige indgangsvinkel, så (Z(?): Det var
406 egentlig sådan, jeg mente det.) nu forstår man det altså. Det er lidt det der med, at når man
407 prøver sig frem, så pludselig, så er man bare heldig.

408
409 I: Hvad, hvad, hvordan, du siger, du prøver dig frem. Øh, gør du det på en bestemt måde, eller
410 er det mere sådan tilfældigt?

411
412 Z: Det tror jeg faktisk er meget individuelt fordi (En anden: Ja) det er tit sådan hvis, hvis vi for
413 eksempel skal sidde, hvis mig og Æ, vi har matematik sammen ikke, skal sidde og snakke lidt
414 om en opgave. Så er der tit forskel på, så følger vi li'som den samme erkendelse gennem at vi
415 sidder og snakker, men så er der li'som forskel på, hvornår erkendelsen kommer fra den enkelte
416 (Æ: Ja) til den enkelte. Fordi jeg tror meget det der med at finde den måde, som man helt selv
417 personligt kan forstå et eller andet på. (En anden: mmm) Og jeg tror ikke vi for eksempel sidde
418 og sætte sådan en systematik op sådan for alle. Først, så gør man sådan, og så gør man sådan.
419 Jeg tror mere det er sådan noget med at, at jeg har også nogen andre hylder og ligge tingene end
420 på end Æ har igås. (En anden: mmm) Det kan være, jeg har talt med M eller N [to andre elever]
421 om noget andet før ikke, der gør at [inaudible] – jo det kan jeg egentlig godt benytte her, hæii,
422 sætte ind på den her hylde som Æ måske ikke har, hun har en anden hylde, hvor hun kan lægge
423 det ind på (En anden: mmm). Og hvis man ikke har li'som det samme udgangspunkt så, så gør
424 vi [inaudible] så nogen forskellige ting, jeg har [inaudible] det er sådan en eller anden gnist (Æ:
425 Ja) der li'som tænder, igås. Så forstår man det bare. På en eller anden måde, man skal li'som
426 finde SIN indgangsvinkel til det igås. Og så lige pludselig så går det så op [inaudible]

427

428 Æ: Jeg har det bare meget sådan at, at det er vigtigt for mig at, at jeg, jeg kan se, hvad det er,
429 altså. Jeg har mange gange jeg li'som laver en tegning, fordi jeg, jeg VIL kunne se, altså. Det
430 der med at man bare har det, noget teori man sidder og læser om, altså. Det er vigtigt for mig
431 for at kunne forstå noget, at jeg har det på papir, jeg har en tegning af det. Altså det, det er i
432 hvert fald det første hvis jeg der er noget jeg ikke forstår, at jeg har det på tryk eller tegning.
433 [afbrydes af Ø]

434

435 Ø: Hvor der så er andre der gerne vil se et eksempel. (En anden: mmm) Og sådan, nogen
436 opgaver og så er det der igennem at (I?: mmm) det li'som kommer, ikke. Og hvor nogen de
437 sådan mere kører sådan i retning af beviserne og alt det der, det er DEM man vil forstå for at
438 forstå sammenhængen, ikke altså. [inaudible]

439

440 I: Det vil sige, der er også nogen der, der faktisk forstår matematik gennem beviserne?

441

442 Ø: Ja DET tror jeg da i hvert fald [inaudible]. Ja det kan man godt have nogen gange, at man
443 gør ikke. Ved nogle mere simple beviser, men måske igen fordi de ligner noget man har haft
444 før. (En anden: mmm) Eller at man kan finde nogen lighedspunkter i den der erkendelses- eller
445 forståelsesproces. Men det er nok meget forskelligt, hvordan man forstår, også [En prøver at
446 afbryde (Z?), men det mislykkes) om man er kreativ eller som det Æ siger, fordi det er jo
447 egentlig det, det handler om, ikke. At skulle se det visuelt. (En anden: mmm) Og en anden
448 måde det er også, at man gerne vil læse tingene i stedet for. Det sådan kan jeg godt have det
449 nogen gange, at jeg forstår simpelt ikke når der er [griner kort - hæ] en der står og siger tingene
450 til mig (En anden ?I: mmm) altså jeg skal se det på skrift for at forstå det.

451

452 I: Du skal li'som selv altså (En anden: Ja) (Ø: Ja) fortælle dig det, fortælle det for (Ø: Ja) dig
453 selv (Ø: Ja)

454

455 Å: [inaudible] Når sidder og læse i bogen, så er der nogen man li'som skal tænke mere over det
456 [En anden prøver at afbryde med: Jeg tror heller ikke] skrive lidt og læse lidt videre når man så
457 har forstået det første altså. I stedet for når man sidder i klassen og det hele går så stærkt.
458 [inaudible] Når der er noget man ikke forstår så kører læreren jo bare videre til det næste, ikke.

459

460 ?: Ja

461

462 Z: Jeg tror ikke helt, jeg har den der, den der visualiseringsting så meget. Men jeg tror meget,
463 jeg har sådan noget med, at li'som Ø siger med, at det skal helst være på skrift [inaudible] det
464 skal være noget jeg kan se, ikke. (En anden: mmm) Det er altså meget fordi, jeg tror måske
465 også meget, vi er opdraget til at vores erkendelse, den er visuel på en eller anden måde, ikke,
466 altså. (En anden: mmm) Mange af de ting vi skal lære, de foregår altså i, via, ved hjælp af vores
467 øjne. (En anden: Ja) Og der kan jeg, derfor synes jeg også det er vigtigt, det der med nogen
468 gange, når man, nu sætter jeg mig altså ned, så selv, så kigger jeg bare på det her igås. Så tror
469 jeg meget, det er der, hvor jeg har den der, så kommer punkt 1 punkt 2 punkt 3. Så tror jeg, også
470 en anden ting for mig som også er god, det er også det der med og, og kunne snakke med andre
471 folk, og det har vi sådan set, når vi regner vores opgaver. [de regner meget i grupper når de
472 regner opgaver] (En anden: [inaudible]) Og der er det, der er det utrolig fordelagtigt, også at
473 snakke med nogen som heller ikke kan finde ud af det, om det. (En anden: mmm) Fordi at man
474 på den måde også får nogle andre indgangsvinkler til det, som måske kan være den
475 indgangsvinkel, man selv manglede.

476
477 I: Hvad så med at snakke med andre, som kan finde ud af det, er det, det er ikke [afbrydes af Z]
478
479 Z: Det er ikke, det er ikke altid lige godt vel, fordi så får man tit den her forklaring, og de er
480 rigtig grebet af, at de har forstået det, og så kommer de aw-aw-aw det er da nemt [inaudible] ha-
481 ha-ha [inaudible] [afbrydes af Å]
482
483 Å: Og bagefter så sidder man bare, sådan lidt, ok [inaudible] det var selvfølgelig fint nok
484 [inaudible] Det er bedre, hvis man kan få diskuteret det, sådan på lige fod med en eller anden
485 (En anden: mmm) som har det li'som én selv. Så sige, synes du man kan gøre det sådan, nej
486 måske kan man gøre det sådan. Så når man også frem til [inaudible]
487
488 Æ: Men så også det, hvis man egentlig, æh, mange gange hvis man prøver at forklare en person
489 det, det handler om, så pludselig så når man selv er i gang med at forklare det, så forstår man
490 det. (En anden: 1-2 bryder vist bifaldende ind [inaudible]) så, så er det, det er egentlig ikke
491 sikkert man HELT har forstået hvad det er, det drejer sig om, når man BEGYNDER at forklare
492 den anden person, men, men så som man egentlig, man står egentlig også og forklare det for sig
493 selv (En anden: mmm) så forstår man det.
494
495 I: [inaudible] Hvad, hvordan kan det være? Øh, hvorfor, jeg kan godt, jeg kan godt genkende
496 det men hvordan tror du det kan være?
497
498 [Flere prøver at "komme til", men man "bliver enige" om, at Å svarer]
499
500 Å: li'som man sidder med en opgave, ikke forklare dig selv [inaudible] lavet før [inaudible]
501 mangler en eller anden ting, og så når man ser det igen, ikke [inaudible] det kan være
502 [inaudible] måske været gået lidt væk fra det og så noget [inaudible] et eller andet man
503 pludselig lægger mærke til, man slet ikke tænkte over før.
504
505 Z: Jeg tror også, (Å: ... man [inaudible] skal gå videre...) jeg har også haft det der med, at hvis
506 man forklarer det mange gange igås, også som hjælpelærer og så noget der, fordi så sidder man
507 så, og så skal man finde en ny måde at forklare den her ting på (En anden: Ja [inaudible]), det
508 bliver man nød til, for den der elev bliver ved med at kræve af dig, at du forklarer det. Hvis du
509 slet ikke kan forstå, [inaudible] forklare det på en anden måde, ikke. Så forklarer på en anden
510 måde igen og igen og igen, ikke. Og det er bare et krav, der er stillet, så (En anden: mmm),
511 indtil den elev forstår det. Og så bliver man li'som nød til at tegne problemet, ikke, og se det fra
512 andre vinkler. (En anden: mmm) Og så tror jeg nogen gange at, når man nu skal ud i situation
513 hvor man bliver nød til (En anden: mmm), hvordan fanden skal jeg forklare det her. Så tror jeg
514 nogen gange, så bliver [inaudible] nød til at [inaudible] for én. Fordi hvis man ikke har haft den
515 der aha oplevelse, så er det også rigtig rigtig svært at forklare tingen til en anden [inaudible]
516 ikke (En anden: Jaa) Så jeg tror nogen gange, det der med at man får lov til at dreje det, selv,
517 igås, og sådan, også fordi man forsimples det også et eller andet sted ved li'som at sidde (En
518 anden: Ja) og forklare det. (En anden: [inaudible]) Så får man det splittet op i nogle [inaudible]
519 faser og sådan nogen ting ikke. Og li'som sådan prøver at køre erkendelsen mindre og mindre
520 bidder ikke. (I: mmm) Og så [afbrydes af I]
521
522 I: Det er åbenbart vigtigt at det, det ikke er for store mundfulde (En anden: Ja) på én gang?
523

524 Z: Ja, jeg tror det er de færreste der li'som kan gribe en ting sådan "duk" (I: mmm) det store
525 hele fra begyndelsen ikke. Jeg tror også det er meget det der med, vi har snakket om før med at
526 man sidder og løser de her opgaver der li'som er små bitte bidder ikke (I: mmm) [der er lidt støj
527 idet nogle slikposer åbnes] og så når man har løst tilstrækkelig mange, så kan man se, så kan
528 man se det store, ikke (I: ja) fordi jeg tror, det skal fandme være let for at man li'som bare
529 li'som "klasker i hænderne" ahh [inaudible] det skal [inaudible]
530

531 Ø: Det kommer nemlig til at virke selvforstærkende, hvis man sidder der og tænker over det
532 selv og, og så er det nemlig som Z siger, man kan KUN se den ene retning, man startede på,
533 fordi man ikke har forstået det grundlæggende i det problem man prøver på at løse ikke. Altså
534 så er det li'som nå men så går det meget [inaudible], så forstår man det lidt bedre eller man får
535 nogle andre vinker på det (En anden: Ja), og det giver nok den der helhedsforståelse ikke.
536

537 I: Ja [Der er 3-4 sekunders stilhed]
538

539 Ø: Fordi man sidder bare og fastholder sig selv i at, at det er DEN vej du skal gå ikke, og det,
540 det er LIGE derhenne min forståelse den ligge ikke [griner lidt], men der når man bare ikke
541 hen. (I: mmm) [I + en elev vil sige noget på én gang]
542

543 I: Det vil sige man skal egentlig være klar over at man skal ikke altid bare følge æhh en bestemt
544 vej, man skal nogen gange sige ok jeg kommer måske ikke så langt den her vej, så må jeg finde
545 på noget andet. (Ø: Ja) Du skakker om på et tidspunkt, en eller anden, om og aktivere noget
546 fantasi, altså det skal man egentlig, det skal man så gøre på det tidspunkt? Eller [afbrydes af Ø]
547

548 Ø: Ja det er jo nok der, at det så er nødvendigt ikke. [inaudible]
549

550 Å: Finde nogen at diskutere med og sådan noget. [inaudible]
551

552 Ø: Ja og få nogen input. (En anden elev (Z?): ... rigtig godt...) (En anden: ... mmm ...)
553

554 Å: I klassen for eksempel når man skal lave afleveringer, og så snakke sammen om de opgaver
555 som der ikke er nogen, der kan finde ud af, men [afbrydes af Z]
556

557 Z: [inaudible] Jeg tror også tit for mig er det også meget vigtigt, altså sådan noget med, det ene
558 li'som følger det andet, se de her årsagskæder og sådanne nogen ting, ikke. Og så tror jeg en
559 anden ting som, som jeg nogen gange oplever både med mig selv og andre ikke, at der kommer
560 et eller andet punkt hvis, hvis stoffet har en karakter hvor man synes, aj det er altså bare for
561 svært det her. At så kommer den der, altså, så kommer den der modstand mod læring ikke altså,
562 så, så er det som om så modsætter man sig altså selv (I: mmm), at man skal lære det her stof,
563 ikke. Og det tror jeg det, det er også en tærskel som det er svært at få banket ned, og det tror jeg
564 nogen gange man kan hvis man også selv får lov til at forklare, både for sig selv og for andre,
565 for jeg tror tit det er svært at overvinde DEN tærskel, hvis der bare står en og bare fortæller og
566 fortæller og fortæller og fortæller, ikke. (I: mmm) Så er det li'som om du siger, så, så lægger du
567 li'som afstand til det ikke. Men i det øjeblik at det bliver li'som, at erkendelsen den skal
568 internaliseres i dig selv, ikke, så kan du ikke lægge afstand til det så bliver du li'som NØD (I:
569 mmm) til at forholde dig til det selv ikke. Så bliver du sådan tvunget ud i den situation, hvor du
570 siger, og så kan du ikke have den tærskel der vel.
571

572 I: Hvordan kan man så tilvejebringe, altså, kan man gøre det, altså for eksempel ved, ved at
573 man sidder og snakker, at læreren siger, ok nu arbejder I lige to og to sammen eller fire og fire
574 sammen. Eller kan man gøre det med sig selv?

575
576 Z: Jeg tror MEGET, man kan gøre det med sig selv (Æ: Ja), men man kan også godt gøre det i
577 grupper, men [afbrydes af Æ]

578
579 Æ: Jeg, jeg synes, det der er også det der, når, når vi nogen gange har fået at vide ahm, nu skal,
580 nu kan I lige sætte jer i grupper og øve på de her beviser, så, de fleste de sætter sig (Z: meget
581 individuelt) for sig selv (Z: Ja) individuelt og kigger på de beviser. Fordi det, det er jo igen det
582 der, noget af det som er indlysende for den ene er måske ikke indlysende for den anden. Altså,
583 og, og man skal bare selv, det er vigtigt man selv sidder og arbejder det igennem, ser det på
584 papir, det der man nogen gange ser oppe på tavlen, jamen det ser anderledes ud når man får det,
585 det bliver mere overskueligt på ens eget papir, fordi, man, man skriver det selv så man VED
586 hvordan man kommer, hvert, altså de der små skridt der. Altså [afbrydes af Z]

587
588 Z: Også når man skriver det selv, så kan man også bedre lokalisere de steder hvor man står af
589 (I: mmm), fordi det er tit når nu der står en anden og skriver det for én så, så er det svært at se,
590 hvor var [inaudible] jeg mistede grebet om det her igås (I: mmm). Men når man så selv sidder,
591 nå men så, når man ikke kan noget, men så stopper man jo bare naturligt [griner lidt] igås (I:
592 mmm), fordi så, jamen så går det jo bare ikke mere vel. (I: mmm) Så det der med når man selv
593 tager det i skridt, så kan man SE, det er DET her jeg ikke forstår, så kan man gøre noget ved det
594 ikke. Så kan man måske spørge om det og, og [En anden siger et eller andet uhørligt] fordi det
595 er først der, det handler om også li'som at lokalisere det, hvad er det egentlig der gør, at jeg
596 ikke forstår det her helt konkret (I: mmm) altså. Fordi, det tror jeg også kan være svært at se
597 nogen gange, hvad det egentlig er man ikke [En bryder ind og de næste par ord er uforståelige]
598 hvilket punkt eller hvilken lille sådan drejning [inaudible] (I: mmm).

599
600 [3-4 sekunders stilhed]

601
602 I: Man plejer også godt at kunne sige, i hvert fald som lærer, at jamen altså, hvis, hvis der
603 kommer en elev og kan sige hvad det er han eller hun IKKE forstår, så er eleven faktisk utrolig
604 tæt på at forstå (En anden: mmm) fordi man kan faktisk forklare, hvad det er man ikke (En
605 anden: Ja [griner også lidt]) forstår. Det er li'som bare så et, jeg er ikke med, så er man egentlig,
606 langt fra det. [inaudible]

607
608 Z: Det sker jo også tit når man, når man skal til at forklare en eller anden person, hvad det er
609 man ikke har forstået i en opgave, at man så, mens man står og forklarer det bare [inaudible],
610 går det bare (En anden elev: Ja) op for én ikke. (I: mmm) Det synes jeg UTROLIG tit sker når
611 [en anden råber meget højt et eller andet uforståeligt] hold kæft det kan jeg bare ikke finde ud
612 af, og så går man hen og så siger man så way, hjælpe med det her, nej igås [en anden råber et
613 eller andet op, der grines, lidt svært at høre hvad der egentlig sker og hvad de siger, men det
614 lader til at der er tale om en almen erfaring] så har man alligevel forstået (En anden: Ja)

615
616 Æ (evt. Ø): Det er faktisk rigtig nok [griner mens dette siges]

617
618 Z: Det tror jeg det er fordi man, man kommer meget tættere på det ved at se hvor er det jeg står
619 af henne, ikke. Sådan. (I: mmm) Så kan man li'så meget først [mumler] [inaudible]

620

621 I: Jeg tænkte på, alt det der, har det noget at gøre med den måde man måske underviser i
622 matematik på altså. Jeg tænkte på hvis nu for eksempel hvis nu man, altså nu, altså, hvis man nu
623 øh er tømrer, så har man også matematik bare på en anden måde (En anden: mmm). Men tror,
624 altså, tror I at det, det I beskriver der, eller hvad hedder det, det er selvfølgelig også lidt svært
625 fordi ingen af jer der er i tømrerlære, men altså, er det noget som har noget, er, er, hænger sådan
626 sammen fordi nu lærer man lige matematik på sådan som man lærer matematik på gymnasiet,
627 eller er det fordi matematikken er sådan som den er.

628

629 [2 begynder at snakke på samme tid]

630

631 Z: Jeg tror det er meget med lærer [inaudible]

632

633 Æ: Det ved jeg ikke. Jeg kan, jeg tror det, jeg tror det har meget at gøre med den enkelte lærer
634 hvordan vedkommende formidler stoffet, for der er mange gange, nogen de kører bare los oppe
635 på tavlen. (I: mmm) Andre igen de, de begynder netop det der at prøve og vise os med pegepind
636 og det hele, og prøve at gøre det sådan for at eleven kan forstå. Det ved jeg ikke.

637

638 Z: Jeg ved ikke om det er mig som individ, men jeg tror bare jeg har, jeg har den der med at, at
639 jeg tror egentlig helt basalt at det er godt for mig at have en lærer som kan stoffet. Men jeg tror
640 egentlig ikke at, at bare det at den lærer står og siger mig noget det bringer mig ingen steder i
641 forhold til at forstå matematik. (I: mmm) Der bliver jeg altså nød til li'som selv og tage
642 initiativet til at lære det. Og det tror jeg altså, altså det er meget vigtigt for mig det med, at det
643 skal helst være, jeg skal selv være motiveret for det, jeg skal selv (En anden: Helt klart) tage
644 initiativet til at lære det. (I + en anden: mmm). Og så er, er læreren li'som mere en konsulent
645 som jeg kan bruge, når jeg har fået lokaliseret, hvad det er jeg ikke kan forstå. (En anden:
646 mmm) Eller når jeg har begyndt at forstå noget at det, til at få bygget noget mere på, som kan
647 anvise over for mig, jamen hvad er det næste trin, når du har forstået det som du har forstået (I:
648 mmm) nu. Men HELT basalt så tror jeg faktisk ikke at læreren kan hjælpe mig til at FORSTÅ
649 tingene, ikke (I: mmm) og det er jo også tit [En anden forsøger at afbryde] man oplever også tit
650 i matematikken det der med, at læreren li'som vender tilbage og, tilbage og siger, er I med, har I
651 forstået det her, ikke. Og det gør de jo ikke særlig tit i de andre fag når man har lært et eller
652 andet, grammatisk regel, så siger læreren jo ikke sådan, nå har I forstået det her, har I set hvad
653 der var pointen, vel. Men i matematik er det utrolig meget sådan, har I forstået det, igås, fordi
654 det kan den jo heller ikke vide, altså læreren kan jo i det hele taget ikke vide, om, sidder
655 eleverne bare som grønsager nu og forstår ingenting, vel. Altså jeg tror det er utrolig vigtigt for
656 mig det der, og det tror jeg altså, det vil være det samme på tømrerskolen igås (I: mmm) altså.
657 Og så, ja, men det er selvfølgelig også li'som Æ siger igås, der er jo forskel på [afbrydes af Æ]

658

659 Æ: Jamen, jeg, jeg tror, jeg tror ikke kun at det drejer sig ikke kun om læreren, det er noget
660 [inaudible] en kombination af at læreren kommer med nogle inputs, som man selv så må
661 arbejde med, og så kan man egentlig vende tilbage til læreren og få nogle nye, ikke. Men den
662 der måde som de der input så kommer på, den tror jeg [inaudible], det, det spiller en rolle på,
663 altså, hvordan det kommer for at man videre kan forstå det, altså.

664

665 Ø: Jeg tror også det har meget at sige, at øh sådan, øh, for mange i hvert fald øh, hvad man skal
666 bruge det til, ikke, altså, det er (En anden: mmm) der tit nogen der spørger om. Hvad skal vi
667 bruge det til [Ø + et par andre griner]. Og det spørger man også selv om (I: mmm) når man

668 sidder der og bare ikke fatter, der er et eller andet man ikke fatter. Og der tror jeg der er stor
669 forskel på for eksempel at gå på øh, ja hvor er det, htx og (I: Ja) sådan, (I og Ø i kor): teknisk
670 skole og sådan noget hvor man skal have sådan lidt, det er jo mere matematik sådan til
671 husbehov eller hvad man nu skal sige (I: mmm) ikke, hvor man skal bruge det i en praksis. Og
672 derfor har det ikke så meget relevans alle de her filosofiske overvejelser over hvordan
673 matematikken den nu lige hænger sammen. Altså, det er bare sådan, her [klapper i hænderne]
674 har jeg en cosinusligning, den skal jeg bruge for at finde ud af, et eller andet øh [Z foreslår et
675 eller andet, og flere griner] ja sådan nogle ting ikke (I: mmm), og det er SLET ikke på det plan
676 vi bruger det (I: mm mm). Og derfor tror jeg nok, altså I [rettet mod de tre andre der går på
677 forsøgsholdet] har det så alligevel via fysikken ikke (Z: Jo), hvor I har koblet de to ting. Det kan
678 nok være en stor fordel som Æ også snakker meget om at visualisere det (I: mmm). Og det, det
679 tror jeg altså for os her på gymnasiet, hvis ikke man har noget udenom man lige bruger det til,
680 så er det meget på det filosofiske plan matematikken den skal forstås.

681
682 Z: Det er meget som, som sådan en leg, ikke. Eller [En anden prøver at bryde ind] som, som når
683 matematik er det her, det her sindssyge system, vi kan bygge op ikke, og man bliver nød til at
684 have den der motivation, man skal synes det er sjovt i SIG SELV, fordi at, at de der, som
685 [griner] allerede fra 1.g sidder og spørger, jamen vi kan ikke bruge det her lort til noget, ikke.
686 Altså, det, så kommer man ikke så meget videre hvis man ikke har den der med, nå man vil lære
687 det her [inaudible] fordi det er interessant i SIG SELV ikke (I: mmm) som (En anden: Ja)
688 matematik betragtet [en prøver at bryde ind] som system betragtet synes jeg det er spændende
689 at man kan gøre de her ting (I: mmm). Fordi eller så, så rykker man ikke nogen steder vel.

690
691 Å: Det tror jeg er rigtig, man skal li'som have en eller anden interesse for det før man
692 [inaudible] for hvis man bare bliver ved med, hvad skal jeg bruge det til, jeg kan ikke forstå det
693 her, jeg vil ikke forstå det, så, så kommer man altså heller ikke videre med det. (I: mmm)

694
695 [Et par sekunders stilhed]

696
697 Z: Altså personlig har jeg det meget sådan, jeg er, jeg er bedøvende ligeglad med hvad jeg kan
698 bruge det til (I: mmm) et eller andet sted ikke. Altså det er en fordel for mig, at jeg har det
699 fysik, men jeg lærer ikke matematik, fordi jeg regner med jeg kan gå ud i den store verden og
700 bruge det til noget. Jeg lærer matematik fordi jeg synes det er sjovt. Fordi (I: mmm) altså det
701 øh, kan godt være det lyder absurd [griner lidt]

702
703 I: Hvorfor er det sjovt? Altså jeg har [Z prøver at sige noget], jeg synes også, jeg elsker også
704 matematik, men [afbrydes af Z]

705
706 Z: Jeg synes det er sjovt fordi altså, jamen det er jo tilsvarende, det vil jo, svarer jo til at have en
707 eller anden [et sekunds pause], jamen altså hvorfor leger man med Barbie, ikke. Det er jo det
708 samme, det er jo sådan en eller anden leg (Æ: Det er en leg helt klart) altså jo flere formler du
709 får, jo mere sss, jo mere tøj har du til Barbie, jo gladere er du igås, jo mere tilbehør du har igås.
710 Har det bare, hold kæft man, man kan også gøre det ikke, det bliver jo vildere og vildere det her
711 igås [Z lyder vældig "høj"], og det [afbrydes af Æ]

712
713 Æ: Det er, det er da en, en, en leg hvor, hvor det samtidig også er en udfordring fordi man, det
714 er jo ikke bare sådan, at når man kommer øh, altså købmandsregning det er jo det man faktisk
715 bruger til dagligt, ikke (I: mmm). Det er der jo ikke nogen udfordring i. (Z (?): ... der er hele

716 tiden ...) Det her, det er noget sjovt noget ikke, det, netop det, altså. Og det man mange gange
717 ikke kan forstå noget, ikke, jamen det er jo også en udfordring i sig selv, ikke. (I: mmm) Og
718 netop, og så arbejde med det så man, så. På den måde så, så [En prøver at afbryde] er det jo et
719 spil, hvor nogen de kan lide at sidde og spille på computer, ikke, så nogen de kan lide og sidde
720 og lege med nogle (En anden: mmm) formler og sådan noget (Ø: Løse nogle problemer). Ja.
721 [afbrydes af Z]

722
723 Z: Jeg tror også, altså, jeg føler det bliver sådan rigtig [inaudible] grimt? for mig her i 3.g, for
724 jeg har det sådan, hold kæft man, løse matematikopgaver, det er næsten, det svarer næsten til
725 sådan nogen med at sidde og løse kryds og tværs igås. (I: mmm) Det [Æ siger et eller andet ord]
726 er li'som den samme motivation igås, fordi kan man finde ud af det her, det er jo DET, igås,
727 altså. (I: Et uforståeligt ord), når den går op så er det jo SÅ fedt, ikke. (I: mmm) Altså jeg tror
728 mere det er det som, som (En anden: Ja) der er pointen i det, ikke. Hvor i fysik der har vi jo den
729 anden indgangsvinkel på matematik, med nå [klapper i hænderne], det er bare noget vi skal
730 bruge til noget, og hvis vi ikke kan vores krydsprodukter, så kan vi ikke finde ud af hvordan de
731 her ting nu hænger sammen, ikke. Og det er en hel anden vinkel.

732
733 Å: Så når vi nu skal til og regne i de koordinatsystemer vi bruger, hvordan vi (En anden: Ja)
734 regner det ud [afbrydes af Z, mens Å + en anden griner lidt]

735
736 Z: Men altså jeg tror, jeg tror meget jeg har det sådan at, at jeg også synes det er sjovere det
737 andet der, hvor man får lov til mere at udforske matematik, at. At det tror jeg er en fordel at
738 have den indstilling til det, når nu man skal lære det på den her, på det her, på den her
739 gymnasie måde ikke. (I: mmm) Altså hvor jeg tror mere på tømrerskolen, der vil det måske ikke
740 være SÅ sjovt at sidde og have det her, hold kæft nu skal vi bare lege lidt (I: mmm) med de
741 [inaudible] kabelligningerne fordi der er det mere [afbrydes af Æ]

742
743 Æ: Jamen det sjove for dem, det er jo at komme til at hamre og sådan noget. Der, der er det jo
744 li'som en, det der matematik, der er derimellem ikke, det er jo bare noget der skal overstås, ikke
745 (I: mmm) så.

746
747 Ø: Det er et værktøj (Æ: Ja; I: mmm) mere end det, det sådan, det er DET, altså. Her er det jo
748 det egentlige, det er jo (I: mmm) selve matematikken.

749
750 I: Ja, matematikken både, egentlig for sin egen skyld, som (Ø: Ja) du sagde dér.

751
752 [Alle siger ja (det er)]

753
754 Z: Men det er også fordi, altså det er, det er enormt sådan, det er jo enormt VILDT at kunne
755 matematik ikke. Og, og også det der med, at der er jo også en eller anden blæreeffekt i det
756 sådan, nu ikke, igås [flere griner] det der med, at vi kan gå og pege fingre af de sproglige fordi
757 at de kan ikke finde ud af det her igås. Og det er der jo også, altså nu sagde du [I] [inaudible]
758 matematik da selv i dag med at, de store ånders område, det her, ha ha ha, nu er vi nok dygtige
759 og sådanne nogen ting, det er det der med [flere griner]. Jamen når man får tilføjelse [inaudible]
760 den viden ikke så (I: mmm), så kan man noget mere igås. Det er jo, det tror jeg, det er den
761 indgangsvinkel som, som mange gange af os som vi har valgt det på det her, synes det er sjovt.

762
763 Æ: Ja det er jo legen, der er det sjove ved det, altså. (Z: Ja)

764
765 I: Og så også lidt det blæret?
766
767 [Flere griner]
768
769 Z: Meget det blæret, altså vil jeg sige. For mig personlig i hvert fald, igås. Jeg har da set frem
770 til den dag, hvor jeg kunne sige, jeg kunne regne differentialligninger ikke, altså [flere griner
771 inkl. S]
772
773 I: Jeg kan godt følge dig lidt fordi, da, jeg blev student under den gamle reform, der hvor man
774 havde de der mat-fys grene eller naturfaglig gren og samfundsfaglig gren, musikfaglig gren, og
775 jeg var mat-fys'er. Og, altså, så var man noget (Z: Ja), hvis man var mat-fys'er, ikke og så, så
776 var der nogen der hed mat-kem'er som havde matematik på højt niveau og kemi på højt niveau.
777 Og, og dem hånede vi fordi de havde fysik sammen med de billedfaglige [flere griner eller
778 kommer med "kvælelyde" [inaudible]]. Det er jo ikke rigtig fysik og de var ikke rigtig kloge.
779 Og man var jo sådan, man var jo lidt mere (En anden: Ja ja) sådan.
780
781 Å: Sådan er det også lidt nu, nu er (En anden: Ja) de fleste [inaudible] HF og sproglige sådan,
782 der kommer nogen kommentarer ind imellem. (En anden: Ja) Sådan tror jeg altid det er.
783
784 Z: Men det er den der fascination af, at hold kæft det er svært det her, men jeg kan godt finde
785 ud af det igås (I: mmm) altså. Der ligger [En anden griner] også helt klart et. Det er også meget,
786 det der er motivationen ikke, altså, når man går ind og så ser man en eller anden, et eller andet
787 bevis som vores lærer, hold kæft det er svært det her, det er bare uhh det er hard-core [det er
788 læreren der åbenbart siger dette til eleverne], ikke. Og så er man jo helt vild og blodig, fordi nu
789 skal man bare lære det her, fordi hvis man kan [En anden griner] DET, ikke, så er man jo
790 virkelig (Å(?): Når så det går op for én, griner). (En anden: Ja)
791
792 I: Det vil sige, det er faktisk en fordel hvis han står og siger, hun står og siger, at det her er
793 faktisk ret svært, altså [afbrydes af Z, der er et par ord begge siger på én gang der drukner] Det
794 er egentlig bedre end hvis han siger, jamen det [afbrydes af Z]
795
796 Z: Jeg tror ikke det er i 1.g, vel [Alle (vist) griner]. Altså, de første par måneder der [Der er en
797 masse grinen som vist indikere at ting er meget svært og forvirrende i den første tid i 1.g] nu
798 skal vi bare.
799
800 Æ: Det, det er sjovt, det der, det er sjovt det der, med, når man sidder, nu, hvor man, når man
801 sidder og tænker tilbage på noget som, hold op det var godt nok svært en gang og, [inaudible]
802 sss, ja det er det ikke mere. Den, det er altså. Det der er sjovt altså, de beviser man synes der var
803 svære for et år siden, og så hører man, nå de [altså elever der er på klassetrinet under] er nok
804 ved det der, ja bare vent, altså, det bliver værre ikke [flere griner], altså. Det er da lidt den der
805 [alle (vist) griner og afbryder her faktisk]
806
807 Å: Jeg tror altså også, hvis nu læreren bare siger, det er jo let, altså, det kan alle jo finde ud af.
808 Så føler man sig også sådan lidt dum, ikke, hvis nu man så ikke kan (Z: De KAN ikke forstå
809 det, så) det er ret svært, fordi så kan man finde noget andet, så har man li'som gjort noget (Z:
810 Ja) (I: mmm) Så bliver man li'som motiveret for at få noget mere (I: mmm).
811

812 Z: Der er ikke noget værre end når læreren li'som har introduceret et eller andet som trivielt, og
813 [En griner] man så ikke kan finde ud af sådan en ting. Iyap. [Flere griner] Men, altså, det er jo
814 enormt træls. Jeg tror også det er rart nok, hvis læreren har den der indstilling som vores lærer
815 meget har med, det der med, nå men selvfølgelig er det svært, men det kan læres igås. (I: mmm)
816 Og det er virkelig interessant at lære det og, og det er godt for jer hvis I lærer det ikke altså. Så
817 bliver man også sådan lidt tændt på det ikke, altså, nå men ok, det lyder det her. (I: mmmm) [Et
818 par sekunders stilhed] [inaudible] var det også helt sikkert.

819

820 [Et par sekunders stilhed]

821

822 I: Jeg tænkte på med det, man kan også sige, at, ok, så er I så eksperter i matematik og alle de
823 andre de, ja ja [En griner]. Men, men dem der så er sproglige, de er jo så endnu mere ekspert i
824 engelsk og sådan noget. [Z prøver at afbryde] Og aijd de [altså matematikere] er sådan nogen,
825 de kan ikke snakke [En griner] med udlændinge, de forstår ikke, de kan ikke sige tre ord på
826 engelsk uden det er forkert grammatisk, ååhhååhhø [en lyd der betyder: ih hvor er de dumme]
827 Så tror [afbrydes af Z]

828

829 Z: Men stadigvæk så har jeg det bare sådan igås, og det tror jeg også er en generelt sådan
830 udbredt holdning, og det er altså, enhver idiot kan lære mange af de her humanistiske ting, men
831 det kræver altså noget at lære [En griner] det her matematik og fysik, ikke [Alle (vist) griner og
832 der er også en der siger en lyd – århhh – der vist betyder, at Z lige skal styre sig]. Altså, seriøst,
833 seriøst, og det tror da jeg også, det er da også en holdning til det, jamen det tror jeg sgu også det
834 er en holdning som ligger sådan [En griner, Å(?)] i befolkningen som sådan og i vores
835 kulturelle arv som sådan. Fordi det er meget sådan noget med, at [En griner, Å(?)], at hvis man
836 virkelig skal tænke på noget, en, en person der er fagidiot (I: mmm), fordi personen skal bruge
837 enorm lang tid på at sætte sig ind i et givet emne, fordi så stort og svært og besværligt er det her
838 emne altså. [Nogen prøver at sige noget] Så ER det fysikken og det er en, en matematikken og.
839 Der er fandme ikke noget der hedder en psykologi-nørd eller en filosofi-nørd, vel. Altså sådan,
840 de findes ikke. De er ikke i den (I: mmm) samme kategori, vel (I: mmm). Fordi det er li'som
841 om at de her fag, de har sådan en eller anden STATUS, ikke. Og det er da også, altså, jeg tror
842 da også, at det er noget af det der gør sådan, at folk ikke vælger det så meget ikke, men altså.

843

844 I: Men de er bange for at, altså, eller, de ikke tror eller [afbrydes af Z].

845

846 Z: Men jeg tror da også, det har sådan et eller andet, altså, det har da sådan en eller anden, æh,
847 ære at, at det her, det er bare kompliceret, ikke. (En anden: mmm) Og det her det er altså noget
848 som man kun kan forstå, hvis man gider og bruge de næste fem år på at sidde og, og regne
849 opgaver, ikke.

850

851 Å: Også fordi det er så gammelt (En anden: Ja). Man hører altid om de der gamle (En anden:
852 Ja) videnskabsmænd. Så, så store

853

854 Æ: Men jeg tror da generelt, så er der da et eller andet med, lige meget hvad for en linie man så
855 har valgt ikke, så dem der har valgt en anden linie, altså [der grines] det er jo, de skulle bare
856 vide hvad de [hun siger egentlig "vi", men hun må mene "de"] går glip af.

857

858 Z: Jeg tror også forskellen den er altså også, at matematik og fysik det er altså lukket land for
859 dem som ikke har lært noget. Og det ER sådan noget som engelsk og tysk og de der mere, mere
860 humanistiske [Afbrydes af Æ]
861
862 Æ: Jamen det er det altså ikke.
863
864 Ø: Der er også nogen, der har, der er nogen der har flair for grammatik og (Z: Ja, ja, det er klart,
865 det er jo klart) (Æ: Det er klart) som så ikke kan noget matematisk, men dem der sådan, mat
866 virkelig matematiker jamen (Z: Men) der er grammatik jo ikke [afbrydes af Z, og et par ord
867 drukner, [inaudible]]
868
869 Z: Forskellen er, forskellen er, forskellen er, hvis nu, hvis du nu skulle have sådan en
870 blærekonkurrence mellem to matematikere og to sproglige. Så kunne de to sproglige, de kunne
871 sætte sig til at snakke, snakke engelsk for eksempel igås. Og du kunne stadigvæk godt følge
872 med selvom du slet ikke havde haft noget engelsk, for du kunne følge lidt med ikke, altså,
873 sådan. [Ø og Z snakker lidt i munden på hinanden og ordene drukner]
874
875 Ø: De har jo også sådan nogen ting, altså latin for eksempel ikke, hvis vi mere fordi altså det,
876 det fatter man jo ikke en brik af, det [afbrydes af Z]
877
878 Z: Jamen jeg synes bare. (Ø: det der) [afbrydes af I]
879
880 I: Det er faktisk også rimelig matematisk, altså med, med altså meget [afbrydes af Z]
881
882 Ø: Jamen det er tysk jo også (I: mmm) for eksempel også, ja (En anden: Ja, En anden: Ja). Vi
883 har også snakket om, men altså, så, så det med, nej, aij, jeg tror ikke der er sådan.
884
885 Æ: Altså jeg tror det er noget, det er en udfordring, altså på hver sin måde (Ø: JA, det tror jeg
886 også). Det tror jeg.
887
888 Z: Nå ja men altså, men det er jo heller ikke fordi at jeg overhovedet vil nedvurdere de
889 humanistiske fag, slet ikke, fordi jeg er selv meget humanistisk øhh, er egentlig ikke så stor
890 [inaudible] matematikperson, men, men jeg tror bare meget det der med, at jeg har det også
891 sådan, at, matematik og fysik, hvis jeg ikke bliver sat ind i de ting ikke, sådan ahh, så er det
892 lukket land for mig, så forstår jeg det bare ikke vel. Vi kan sidde og tale hen over hovedet på
893 enhver person der aldrig nogen sinde har haft integralregning om de der, om de der altså, folk
894 forstår ingen gang hvad det ER, vel. (I: mmm) Altså, det tror jeg alligevel det er det, der er
895 forskellen igås. Hvis vi sidder og snakker differentiallyigninger i bussen på vejen til Nytorv
896 [Centrum i Aalborg, der holder alle bybusserne], folk de forstår ikke hvad det er vi snakker om,
897 vel. (En anden: mmm) Men, men vi vil godt forstå, at de sad og snakkede om, om, om et eller
898 andet som lå li'som inden for, for humaniora sådan. Også tit, at nogen ting som vi har oplevelse
899 med personer, som man møder i sin dagligdag, fordi forskel er at på det der højniveau
900 matematik ikke, at det møder du, det bliver du ikke konfronteret med i lige så høj grad, ved bare
901 li'som at VÆRE i din dagligdag jo ikke. Så det er li'som også meget et valg, du siger, jamen
902 jeg vil beskæftige mig med matematik ikke, du kan, du kan sagtens undgå og have matematik
903 på A-niveau, du kan sagtens undgå at støde ind i de problemer ikke (I: mmm). Men det er
904 svært, og når du går ude på gaden og i det hele taget ER, at undgå at støde ind i nogle af de der

905 mere sådan humanistiske problemstillinger ikke. (I: mmm) Altså du ved jo næsten hvad det er
906 igås. (I: mmm) Men det er li'som om, det her matematik og, synes det er sådan pui langt væk.

907

908 I: Ja, og måske også på det niveau fordi man, man altså, så almindelig geometri,
909 trekantsberegninger, det, det støder man også på, at hvis man har en flagstang og så ser en
910 skygge. (En anden: Ja [+ noget mere der ikke kan høres]) Det er jo gange almindeligt, altså, det
911 er en retvinklet trekant.

912

913 Å: Mmm, men man behøver ikke tænke over [inaudible] hvordan man skal beregne vinklen i
914 det så. Tænker ikke over hvis man ser en flagstang om sommeren...

915

916 [Båndet løber ud, vendes, lidt af samtalen går derved tabt]

917

918 Æ: Men jeg kan, jeg kan godt følge den der i, at ja altså, man, man kan på en eller anden måde
919 sige et eller andet, altså, på et andet sprog, altså, det, det er jo rigtig nok, men altså.

920

921 Z: Men jeg tror måske også, det kan også godt være [afbrydes af Æ]

922

923 Æ: Måske ikke lige så meget lukket land, som matematik er, det ved jeg ikke.

924

925 Z: Det kan også godt være noget med min tankegang, ikke. Fordi jeg har det bare sådan, nå ja
926 men det studium, som jeg skal have til næste, som jeg skal begynde næste år, det er
927 humanistisk, selvfølgelig er det det. Og der tror jeg meget jeg vil have det sådan, at hvis jeg nu
928 ikke læser nogen af de andre humanistiske studier, så vil jeg ikke føle mig sådan, sådan meget
929 uvidende når jeg møder en, en eller anden psykologistuderende for eksempel, eller en eller
930 anden danskstuderende. Fordi jeg for eksempel vælger kunsthistorie, så vil jeg ikke føle mig
931 meget uvidende [inaudible] situation?, men jeg tror hvis der kommer sådan en eller anden
932 professor i matematik ikke, så kan den der altså nogen ting, som er sådan fuldstændig, det kan
933 jeg aldrig nogen sinde nå i mit liv vel, altså. På den måde ikke. (I: mmm) Og det synes jeg har
934 sådan en eller anden fascinationskraft [inaudible]. Men også den måde det bliver fremstillet i
935 medierne på, nu så vi lige det der, det der, så den der udsendelse med ham fyren, der ledte efter
936 Fermats Sidste Sætning, altså igås. Der er sådan en eller andet (I: mmm) hrruuuhh, igås. Det er
937 sådan lidt.

938

939 I: Han var lidt småskør. [Flere griner]

940

941 Z: Der er sådan et eller andet næsten overnaturligt i de der folk som gider at beskæftige sig så
942 meget med noget der er så teoretisk og så langt væk fra, hvad vi egentlig vil kalde sådan
943 almindelig menneskelig sådan relationer (En anden: Hverdage, agtige [inaudible] ting).

944

945 I: Men det bliver jo alligevel brugt, altså, i alt muligt, altså, i, i, for eksempel nu har I, nogen af
946 jer har det med L [læreren for forsøgsklassen] i fysik, men altså, hvor, hvor det faktisk bliver
947 brugt. Altså, også til, altså, biler, der er jo rigtig matematik i hvordan en motor fungerer, altså
948 [førsøgt afbrydes af flere]

949

950 Z: Der er jo heller ikke nogen mennesker, der forstår det. En, en motor ikke vel, altså, så kan
951 man jo ikke li'som, vøøø [sound of a car driving].

952

953 Å: Når man tager kørekort, så lærer du, ahh nu trykker du på speederen og så kører bilen. Der er
954 ingen, der [flere griner, og et par ord drukner]

955

956 Z: Jeg synes også det er meget fascinerende det der med at, at det er den her store verden, som
957 man ikke kan få indblik i, men, eller, medmindre man virkelig vælger at gå ind og beskæftige
958 sig med den. (En anden: mmm) Aktivt li'som gå ind i det, det her.

959

960 Æ: Den er ikke så tilgængelig som altså, filosofi det kan man, man kan, enhver kan sidde og
961 være filosofisk, sådan lidt (Z: [inaudible]) altså.

962

963 I: Jeg tænkte på, der, der er noget som, fordi, da jeg havde en elev sidste år, så var der, så havde
964 jeg en oplevelse som jeg egentlig godt kunne tænke mig, og, altså det er ikke nogen, en nogen
965 elev som I kender. Men, hvad hedder det, sådan lige hører jeres reaktion på. Det, det var en som
966 jeg havde og vedkommende, X, kalder vi vedkommende X, øh. Jeg syntes vedkommende X øh
967 virkelig havde fært af matematik. Og, men hele vejen gennem året blev vedkommende ved
968 med at insistere på, at vedkommende ikke kunne finde ud af matematik. Og så skete der så det
969 til eksamen, at vedkommende fik 11 til eksamen, og så sagde jeg så til X, kan du ikke se at jeg
970 havde ret, altså du, du kan finde ud af matematik. Og så siger X så, nej, jeg kan ikke finde ud af
971 matematik.

972

973 Z: Jeg har også, tror jeg da også måske nogen gange godt, at, at jeg kan lidt også kan se mig
974 selv i den der rolle der igås. For det er sådan noget med, at, at jeg kan godt have nogle lærere,
975 der siger, nåå men du kan godt finde ud af det her, og at man kan godt, klarer sig godt til
976 prøverne og sådan noget, og så stadigvæk føle, hvis man ikke virkelig føler, eller sådan, jeg tror
977 også der er en eller anden forskel på det der med, at i de, i de humanistiske fag, der kan jeg
978 li'som føle, at, at der har jeg altså begrebet om hvad det egentlig er, det hele det handler om, vel
979 (En anden: mmm) Men tit så kan jeg sådan føle i matematik, at der må være en eller anden
980 hemmelighed, eller sådan et eller andet [Z + et par andre griner lidt], som de andre går og ved,
981 som jeg bare ikke [1-2 griner lidt] har fattet, igås. Fordi (En anden: mmm) hvis, hvis det her det
982 skal være så smart og logisk så MÅ der altså være sådan et eller andet, som jeg bare ikke har
983 forstået, som jeg bare aldrig kommer til at forstå igås. Det, jeg synes tit jeg kan have den der
984 følelse af, at der er sådan et eller andet.

985

986 Å: Det synes jeg altså også godt man kan se [inaudible] Nogen gange siddet, når der er nogen
987 lærere, der [inaudible] Hvordan når de frem til de konklusioner der. (En anden: Ja ja) Hvad,
988 hvad tænker de for at nå frem til [inaudible] endnu svært [Æ prøver at afbryde, og et par ord er
989 lidt utydelige]

990

991 Æ: Hvis man synes man, man kunne sige, man kan finde ud af matematik, jamen så, så vil man
992 også kunne, så ville man virkelig kunne forstå det og, og, altså [afbrydes af Z]

993

994 Z: Så ville man jo sidde og udnytte de der beviser selv ikke og (Æ: Ja og) alt

995

996 [Z og Æ snakker lidt i munden på hinanden, Z griner lidt]

997

998 Æ: Jeg tror det er det, at, at det kan da godt være, at jamen [inaudible] kunne sagtens finde ud af
999 ikke, men man sidder bare, nej, man fatter ikke noget altså, altså så, så på den måde, da er det er

1000 nok mere den der forståelse for det som man (En anden: mmm), man mangler, som man så
1001 siger, man kan ikke finde ud af det, fordi man ikke har noget.
1002
1003 Z: Jeg tror også det er fordi, vi alle sammen har den der oplevelse af, i matematik er [inaudible]
1004 hvis man ikke forstår. (I: mmm) Sådan, hvis du får at vide af læreren igås, du er godtnok en
1005 dygtig barn. Og så bliver man undervist i differentiallyigninger, man sidder bare hrrruu, det går
1006 bare hen over hovedet på én. Og så føler man sig fandme ikke som noget dygtig barn, vel. (I:
1007 mmm) Men i dansk, der kan du godt gå videre. Du er god til dansk, og det sker ikke særlig tit,
1008 at, at man li'som har noget i dansk, hvor man bare, hold kæft, jeg forstod ingenting af det her
1009 vel, altså sådan. (En anden: Ja) Jeg tror hele det der med, at man hele tiden bliver konfronteret
1010 med (En anden: Ja) nyt stof, og jeg forstår det ikke, men har ekstremt tit, synes jeg, i matematik
1011 den oplevelse, jeg forstår ikke det her uhhh ikke, og det er slemt og hvad skal jeg gøre og
1012 [afbrydes af Å]
1013
1014 Å: [inaudible] haft nu [inaudible] skal igås, [inaudible] så nævner læreren specielt, altså hvis I
1015 nu kan se det, så vil I jo også høre om noget der var meget værre igås [En eller andet siger
1016 noget (en sætning), der er meget utydeligt]
1017
1018 Ø: Måske så er problemet også, at man, man, at matematik bliver fremstillet som sådan noget.
1019 Altså, jeg har lidt svært ved at, at tro at, at for eksempel dansk eller et eller andet, det ikke kan
1020 være li'så svært. Men det er jo sådan, bliver det aldrig fremstillet vel. Altså det er sådan, det,
1021 det kan [klapper i hænderne] enhver forstå. Det er i hvert fald tilgængeligt for enhver. (I: mmm)
1022 Og det der med at matematik det er sådan uuuhhh et kæmpe spøgelse, som man sådan nogen
1023 gange får set det lidt af [afbrydes af I]
1024
1025 I: Jamen er det et stort spøgelse eller, eller altså (Ø: Jamen det er dét) Er det et blufnummer at
1026 matematik bare [afbrydes af Z]
1027
1028 Z: Jeg tror, jeg tror meget jeg har sådan en eller andet angst for at, at, som jeg ikke har i nogen
1029 humanistisk fag OVERHOVEDET, at der skal komme et loft for hvad jeg kan forstå. Og jeg
1030 har hele tiden den der ting med, det næste emne vi skal gennemgå, det kan jeg garanteret ikke
1031 forstå. Der må komme et eller andet punkt, ikke. Jeg har det meget med i matematik, der må
1032 fandme komme et eller andet punkt, hvor jeg ikke kan forstå mere (I: mmm). Og så har jeg tit
1033 sådan noget med at når, når jeg så lige bliver introduceret til et eller andet nyt emne, så tænker
1034 jeg bare [banker i bordet]: Nu er det kommet. Nu kan jeg ikke forstå mere igås. Det var
1035 ærgerligt, det [flere griner] var så det, igås.
1036
1037 Ø: Jamen er det så ikke en motivation, altså.
1038
1039 Z: Nej, nej så har jeg bare den der, wow, nu kan jeg li'så godt stå helt af, fordi jeg kan ikke
1040 forstå det her [afbrydes af Æ]
1041
1042 Æ: Ja det er den ene side, og på den anden side tingen, så, jeg kan også følge det med at, nu kan
1043 jeg bare ikke mere. (Z: Nej) Det, det kommer ikke igennem, jeg kommer ikke igennem det her.
1044 Og nå så tænker jeg bare, jamen, man SKAL jo igennem. [En prøver at bryde ind] Og så er det
1045 man får den der med, så MÅ jeg bare [afbrydes af Z]
1046

1047 Z: Jeg tror bare, der tror jeg meget jeg har tendens til at tænke på i matematik, at tænke på, på
1048 mig selv som sådan, altså som det der kar, der skal fyldes op, og så er der bare ikke nogen
1049 grænse hvor, hvor jeg så ikke kan nå videre [hun må mene, at der er en grænse i karet] igås. (I:
1050 mmm) Og hele tiden så har, så går jeg fandme og er sådan lidt halvbage for, at det næste der
1051 bliver introduceret det kan jeg ikke forstå. Og jeg har også haft, fået et meget mere afslappet
1052 forhold til det efter vi er blevet færdig med pensum, ikke, [flere griner] fordi, nå ja, der kommer
1053 ikke mere i år, som jeg ikke kan forstå, som der, som der er mulighed for at jeg ikke skal kunne
1054 forstå igås. (I(?): mmm) Og samtidig også den der spas? [inaudible], wagh, forstod jeg egentlig
1055 det der helt, eller var der noget [inaudible] som gik forbi min næse, ikke, men meget det der
1056 med, at matematik, der er jeg fandeme bange for, at der skal komme den der grænse. Og det er
1057 ikke sådan noget med, nå men det kan jeg ikke forstå lige umiddelbart så jeg må arbejde med
1058 det. Det er sådan noget med, at jeg er bange for at der skal komme den der hvor, hvor guderne
1059 siger stop [flere griner]. Hvor min hjerne li'som siger stop. Jeg har sådan, jeg er også overbevist
1060 om, at der er sådan et eller andet punkt, hvor man ikke KAN lære mere igås. Fordi der må være
1061 en eller anden årsag til, at der er nogen folk som ikke, fuldstændig, KAN lære det matematik,
1062 uanset hvor mange gange de får det forklaret, ikke. Altså, så, så jeg føler mig rimelig overbevist
1063 om, at jamen der er jo [griner lidt] nok en eller anden grænse for, hvor meget jeg kan lære, og
1064 jeg er bange for at den grænse skal jeg snart støde ind i, ikke. [Flere griner] Jeg føler næsten, at
1065 jeg hele tiden [afbrydes af Ø]

1066
1067 Ø: Men er det så ikke sådan en barriere, som vi også snakkede om, sådan et eller andet (Z: NEJ)
1068 psykologisk.

1069
1070 Z: Jeg opfatter det ikke som en barriere, jeg opfatter det meget som, mm, jamen, altså, sss, mere
1071 kan min hjerne ikke, altså. Og det er ikke sådan [afbrydes af Ø]

1072
1073 Ø: Og det er DA også, det er da også en barriere, det er da også sådan, nej det KAN jeg nok
1074 ikke, eller sådan, altså. Gu' kan man det ikke.

1075
1076 Z: Jamen jeg tror ikke så meget jeg opfatter det som sådan altså, jeg oplever det [afbrydes af Ø]

1077
1078 Ø: Det er heller ikke det, jeg mener også mere, ER det ikke det, altså, er det ikke [afbrydes af Z]

1079
1080 Z: Jo. Om der er det. Men det tror jeg altså, sådan. Den forestilling som jeg har om hvordan det
1081 skal være, og jeg tror det er den forestilling, som begrænser mig og som får mig til at sige (Ø:
1082 Ja) jeg kan ikke finde ud af matematik. Men den forestilling, som jeg har om hvordan det skal
1083 være det er, at en eller anden dag så kommer min lærer og lægger en eller anden ting foran mig,
1084 som jeg aldrig nogen sinde i hele mit liv vil blive i stand til at forstå, uanset hvor mange gange
1085 han prøvede på at forklare det for mig, ikke. Og det har jeg det bare ikke i dansk og i de der
1086 andre fag vel. Der er jeg sikker på igås, at uanset hvad så vil jeg altid kunne forstå igås. Altså,
1087 jeg vil altid kunne forstå, jeg vil altid forstå meningen med en eller anden tekst, ikke, jeg kunne
1088 godt, fordi det er også meget sådan, jeg vil altid kunne forstå min personlige mening (I: mmm)
1089 ind i det her ikke. Min personlige relation i forhold til det her, men matematik det er vist meget,
1090 nej, jeg er sikker på det igås. Og jeg føler det sådan lidt, jeg kommer løbende efter den her
1091 grænse, hold kæft mand, en dag ikke, så gå klappen ned ikke.

1092
1093 Æ: Jeg synes da også, at allerede nu, jamen der har vi da nogen gange oplevet sådan, jamen der
1094 er noget matematik som ikke lige sådan er til at forstå, og sss. Det tror jeg også, man må da

1095 godt acceptere, at der er noget matematik, som, altså, der er der jo også i fysik, altså, i det hele
1096 taget, der er nogen ting også med, med verdensrummet, på en eller anden måde, fysik også
1097 fungerer på. Der er nogen ting som man ikke lige sådan bare kan forstå, og, det ved jeg ikke, så
1098 accepterer jeg nok bare, at dem kan jeg ikke forstå, altså.

1099
1100 Z: Men så det får mig på en eller anden måde til at føle, at nå men, jeg, jeg har faktisk ikke helt
1101 greb om det her, vel. Altså så, jeg kan ikke sige, jamen selvfølgelig er jeg da god til matematik
1102 fordi jeg vil ikke føle altid, jamen, at jeg vil være god til alting i matematik. Jeg kan godt sige
1103 (I: mmm) jeg har forstået de emner som der er blevet gennemgået for mig nu, og det betyder
1104 ikke, at jeg er god til matematik, vel.

1105
1106 I: Men du er god til det som du har mødt indtil nu.

1107
1108 Z: Ja, ja. Altså jeg, jeg kan sagtens finde ud af alt det som der er blevet gennemgået indtil nu,
1109 men det betyder ikke nødvendigvis at jeg er god til matematik (I: mmm), altså som hvis du kan
1110 forstå det skævt [inaudible]

1111
1112 I: Ja, ok men det vil sige det med at være god til matematik, det, det er et meget meget abstrakt
1113 begreb og meget, altså meget altomfavnende.

1114
1115 Z: Det er li'som om man har tal., man li'som har evne, evne til at uanset hvad der vil blive lagt
1116 end foran én, så vil bare kunne forstå det igås. Man har sådan et indtryk af at der er nogen folk i
1117 ens klasse der har det sådan [flere griner] nogen gange, [inaudible] hold kæft mand. Men, men
1118 også bare det der med at øhm jeg måske ikke sådan føler, at jeg har sådan, den der meget meget
1119 naturlige flair for det vel, altså sådan. (I: mmm) At, jeg tror også, det er også de færreste jeg har
1120 mødt egentlig, som bare lige kunne sige det der med igås, hvis noget med, hvis læreren
1121 fortæller et eller andet om, så man forstår det bare, og sådan har jeg det meget i fag som dansk
1122 igås, hvis læreren siger, nå men han er sur ham der manden, ja selvfølgelig er han det igås.
1123 [afbrydes af I]

1124
1125 I: Men er det så læreren, er det, undskyld jeg afbryder, er det så lærerens øh måde at for.. li'som
1126 formulere sig på der gør, eller er det matematikken der gør det.

1127
1128 Z: Det tror jeg er matematikken.

1129
1130 Æ: Det tror jeg også er matematikken. (Z: Altså det er) (Å (?) [inaudible] forskel i dag
1131 [inaudible]) (Z: Det er, det er) Der er nogen begreber i matematik, altså, der er nogen begreber
1132 også i fysik som altså det, dem, man lærer nogen formler ikke, men (I(?): mmm) der ér nogen
1133 ting som man ikke vil kunne forstå baggrunden for altså.

1134
1135 Z: Men jeg tror også der er utrolig meget, men det ved jeg så ikke om det er, for mig personligt,
1136 fordi jeg er så meget humanistisk i mit hoved. Men jeg, jeg får ikke a-ha oplevelser i et
1137 humanistisk fag, det gør jeg altså ikke. (I: mmm) Der ER tingene bare sådan som de er igås.
1138 Der er ikke noget man kan sige, man har en sætning på, selvfølgelig er der en sætning. Men i
1139 matematik der skal man li'som have den der fornemmelse af at man har forstået det, fået sådan
1140 et eller anden dybere greb ikke altså (En anden (I?): mmm) sådan. At, at tingene li'som er faldet
1141 på plads før man føler man sådan har fod på det ikke. Og da tror jeg tit, at jeg føler sådan lidt,

1142 der må være en a-ha oplevelse der er større end den jeg lige har haft, som de andre har haft,
 1143 men som jeg ikke har og derfor er jeg ikke sådan helt god til matematik, ikke.
 1144

1145 Ø: Men afgører man ikke selv om man har den der a-ha oplevelse eller ej. Altså, jeg tror den er
 1146 jo, den er vel relativ.
 1147

1148 Z: Jo sikkert. Men jeg synes bare at for mig der føles den som sådan meget, altså meget
 1149 veldefineret, sådan.
 1150

1151 Ø: Ja, ok, ja. Men det kan jeg godt se. [Afbrydes af Z]
 1152

1153 Z: [De første par ord kan ikke høres idet de drukner i Ø's sidste ord] Der er et utroligt klart skel
 1154 mellem hvornår jeg har forstået ting og hvornår jeg ikke har. Sådan utroligt.
 1155

1156 Ø: Men altså, jeg synes nogen gange jeg kan opleve det sådan, hvis nu man er træt ikke, og så
 1157 tænker man sådan, NU er den der, ikke. Og er det så ikke sådan lidt, sådan, fordi så har man
 1158 nået, så er man tilfreds. Altså. Det der med at blive tilfreds, det er vel også individuelt hvornår
 1159 man er det, altså, æhm, hvor en anden måske vil sige ahhr nu, nu har jeg dæleme forstået det
 1160 igås, så (Z: Jo) en anden vil sige, hov hov hov (Z: Jeg tror også der er forskel) [de taler på
 1161 samme tid og nogle ord som Ø siger kan ikke tydes] jeg er slet ikke nået til målet vel.
 1162

1163 Z: Jeg tror også der er forskel fra person til person, men jeg tror de fleste har det sådan meget,
 1164 en meget klar fornemmelse af hvornår de selv har forstået tingene ikke. Sådan jeg har det bare
 1165 sådan forstået eller ikke forstået. Der er to kategorier når det har at gøre med matematik og
 1166 fysik igås. Og det er virkelig sådan, det har jeg fod på eller det har jeg overhovedet ikke fattet.
 1167 Og så, og i dansk er det mere sådan noget, nå ja igås, det kører bare ind på li'som altså på, på
 1168 [afbrydes af Ø]
 1169

1170 Ø: Men der er flere grader af det, altså [En anden prøver at sige et par ord, og derfor drukner et
 1171 par af Ø's ord også] der er jo nogen der kan sige, at når, når jeg kan bruge det til at løse en
 1172 opgave, SÅ har jeg forstået det. (En anden: mmm) Andre der vil sige, det er først når jeg har
 1173 forstået den der store (En anden: Ja) øh sammenhæng at jeg har forstået det igås. Og den, den
 1174 forskel ligger der nok.
 1175

1176 Z: Men er jeg den eneste der sidder med den her oplevelse sådan, sådan af at man kan sidde en
 1177 dag og så, man kan føle man har ikke forstået rumgeometri, og så læser man det igennem og så
 1178 [banker i bordet] så føler man LIGE PLUDSELIG, yes, nu er den der, nu har jeg forstået det.
 1179 Fordi sådan har jeg det meget sådan. Det er ikke sådan noget, måske har jeg næsten forstået det
 1180 nu, og nu kan jeg løse opgaven, så kan jeg mmm, har jeg forstået, ja det kan jeg godt sige. Det
 1181 er meget sådan noget [inaudible] SLAM, der, der, det er li'som om, så går, så bliver der bare
 1182 tændt et lys. Det er li'som om at, der er sådan et værelse inde i hovedet ikke, og så går man lige
 1183 pludselig hen og så lige pludselig så bliver lyset bare tændt, og så er den der bare igås.
 1184

1185 Æ: Jeg har det mere på den li'som, der er måske noget jeg ikke lige forstår helt, men jeg forstår
 1186 trods alt lidt af det. (Z: Jeg har bare det der) Jeg ved godt, hvad jeg accepterer, nå ja, men jeg
 1187 kommer ikke til at forstå det mere end lige det jeg gør, eller også siger jeg, nej jeg vil dæleme
 1188 forstå det. Altså (En anden: Ja). Og, og det er nok hvor, hvor kompliceret området er, så siger
 1189 jeg nå ja, bare jeg sådan kan forstå lidt af det ikke, så er det udmærket.

1190
1191 Z: Ja men der tror jeg bare, jeg har det der med, at hvis jeg kun har forstået lidt af det, så har jeg
1192 ikke forstået det, altså sådan, hvis jeg kun føler det der med, at, at, at jeg har forstået sådan en
1193 lille smule igås, sådan, li'som, nu gennemgår vi lige lidt af det der [mumler et par ord sammen
1194 med nogle fortælle-lyde] og sådan det der, så kan man lige [en griner] forstå det lidt, man kan
1195 måske også sådan lige [klapper], nå ja, men det er sådan noget med nogle broer og noget ikke.
1196 Aahhh fint nok. Men, men, men jeg har først selv den der med, nu bliver lyset tændt, når det er
1197 sådan at jeg føler, at, for eksempel en retvinklet trekant, og nu kunne der komme en person, der
1198 kunne stille uanset hvilket spørgsmål det skulle være om den der retvinklet trekant, jeg ville
1199 bare kunne svare på det, igås. Sådan høhø emnet det er bare udtømt (I: mmm). Nu har jeg bare
1200 forstået det igås. Og jeg har endda forstået hvorfor det ER sådan ikke. Hvorfor forholder det sig
1201 sådan ikke. Og det er først der lyset bliver tændt, og der synes jeg ikke rigtig der er sådan noget
1202 halvmørke indimellem de der ting, men det føler jeg mere sådan i de der andre sådan sprogfag
1203 og sådan noget, der er sådan bwaøøøhhh jaaa, det skal (Å: Det tror jeg også) nok passe, den der.
1204
1205 Å: Læreren i matematik siger også tit, at enten så forstår I det her eller også så forstår I det ikke.
1206 Og karaktererne i matematik det er også tit en masse der får helt vildt lave karakterer, fordi de
1207 ikke har forstået (I: mmm) og de høje de har. Ikke sådan, der er ikke noget, ikke særlig mange
1208 indimellem som forstår (Z: Der forstår til et otte-tal) sådan forstår lidt, men noget af det går. Jeg
1209 tror også tit så enten har man fået fat i et eller andet emne eller så [afbrydes af Ø]
1210
1211 Ø: Men altså jeg tror bare stadigvæk godt at man kan have de der flere grader af forståelse,
1212 fordi at det også handler om, altså, ja det ved jeg så ikke hvad det er [Ø lyder lidt irriteret], om
1213 det er fantasi eller, æhm et eller andet sammenskrumpning inde i hjernen [griner selv lidt + en
1214 anden] der gør at, hvor langt man når i forståelsen ikke, fordi at det som, som, som Z øhh så
1215 siger ikke, altså det kan godt være en der har samme oplevelse men bare ikke når lige så langt i
1216 forståelsen som S, det kan vi af gode grunde ikke rigtig sige [Ø griner lidt] noget om fordi
1217 forståelsen det er et vidt begreb (I: mmm) men altså, at, at man blot kan have det på et, et andet
1218 plan end hvor æhm, fordi man ikke har forstået, hvad det er man ikke forstår, og det er jo i
1219 grunden det som du [I] også sagde dengang, du sagde at, at når eleven kan sige hvad det er (I:
1220 mmm) han ikke forstår, så er de i virkeligheden tæt på ikke. (I: mmm) Men hvis nu man ikke
1221 har opfattet alle de der ting som er nødvendige at forstå (I: mmm) altså så, så når man ikke så
1222 langt vel, men man kan alligevel godt have en a-ha oplevelse.
1223
1224 Z: Jeg tror faktisk jeg kender nogen, som, eller jeg nogen sinde har mødt nogen, hvor det
1225 lykkedes for dem at bilde sig selv ind, at de har forstået et eller andet i matematik som de ikke
1226 har forstået. Sådan altså, det er jo imod [jeg tror hun i starten må mene, at hun IKKE kender
1227 nogen, som ...] [afbrydes af Ø]
1228
1229 Ø: Det er jo heller ikke indbildning, ikke indbildning, fordi at det er ikke indbildning, i og med
1230 du selv tror det, altså du er selv overbevist om det. (Z: Ja ja, men) du KAN ikke [afbrydes af Z]
1231
1232 Z: Men jeg tror, jeg tror bare aldrig jeg har mødt en person for eksempel, jeg har tit mødt en
1233 person i dansk som siger, nå jeg har forstået den her tekst, og så begynder vi at snakke om den,
1234 og så kan man bare mærke håååå den tror det er romantik og det er postmoderne igås [nogen
1235 griner] og sådan er det jo ikke vel. Og så, men i matematik der har jeg aldrig mødt en person
1236 der siger, sådan nå men jeg har forstået retvinklede trekanter, og så bare ikke kan løse en
1237 opgave rigtigt. Det har jeg aldrig mødt. [afbrydes af Æ]

1238
1239 Æ: Jamen, det behøver ikke, altså jeg kan godt, altså det ved jeg ikke, jeg kan godt tro jeg, nej
1240 nu har jeg forstået, og så finder jeg ud af, det har jeg, pludselig finder jeg ud af, det var egentlig
1241 slet ikke sådan som man faktisk troede det var, altså. Og der, der HAR man også gået med den
1242 der, nu har jeg, nu har jeg forstået altså. Pludselig så går det (Ø: JA) op, så går det op [Ø siger et
1243 eller andet, snakker med, “frem og tilbage”] for én man bare ikke har forstået alligevel [Æ +
1244 sandsynligvis Ø griner højt]. [afbrydes af Z]
1245
1246 Z: Jeg har så tit oplevet, fordi jeg har netop den der med igås, hold kæft man, enten så fatter jeg
1247 det bare ikke, eller også gør jeg. Altså der er bare (Ø: Det er det jo ikke) Jeg, jeg, jeg, jeg tror jo
1248 også, jeg fatter det lige til [Z griner] det går op for (En anden: Ja) én, at man ikke (En anden: Ja.
1249 en eller flere prøver at snakke med) gør det alligevel, altså. [afbrydes af Ø]
1250
1251 Ø: Og det kan måske netop godt være, hænge sammen med det der med, at der er en
1252 sammenhæng man ikke LIGE har fået med ind i sit [afbrydes af Æ]
1253
1254 Æ: Ja, og der ER flere grader af forstå.. (Ø: Ja) Ja (I: mmm) [afbrydes af Ø]
1255
1256 Ø: At man så, så er der en der trykker på en knap, som man ikke lige (Æ: og så falder det hele
1257 tviinggg, hov, [Æ griner]) inddraget i en stor (Æ: så er det bare slet ikke sådan alligevel) Ja.
1258
1259 [Et par sekunders stilhed, lidt småmumlen]
1260
1261 Z: Men føler man sig ikke, så synes jeg bare, hvis man har den der [afbrydes af Æ]
1262
1263 Æ: Hvor..., hvornår kan du være SIKKER på, at du så HAR forstået?
1264
1265 Z: Jamen det ved jeg ikke, men det er sådan [En griner “lydløst”], det er for mig meget
1266 veldefineret, sådan inde i mit hoved igås, det der med, jamen det har jeg forstået nu igås. Det er
1267 UTROLIGT veldefineret. Fordi jeg tror altid, hvis jeg har den der, sådan jamen jeg har forstået
1268 det her igås sådan [tonefald etc. indikerer at der her er tale om at hun mener at man på
1269 overfladen forstår], så vil jeg altid have den der tvivl i baghovedet, hvis det er sådan jeg ikke
1270 har haft den a-ha oplevelse. Og SÅ vil det være i DE tilfælde, hvor jeg har gået sådan lidt,
1271 jaaaahh det har jeg forstået, det arrhh igås. Og så vil jeg komme hen, og så min lærer [En
1272 “grynter”] siger, nå ja så skal vi lige se på det, og så wow igås. Men der vil jeg også have haft
1273 den der fornemmelse af, jeg har sgu ikke helt greb om det her, fordi det er først når jeg sådan
1274 virkelig, sådan, jeg kan KUN have den der sådan virkelig a-ha oplevelse når det er sådan at jeg
1275 altså også virkelig har forstået [Z griner her kort] det. (En anden: Ja) Sådan virker det meget,
1276 men jeg tror altså også måske som du siger igås, der kan godt være forskel på hvor man så
1277 sætter grænsen, ikke, og der tror jeg måske også, hvor..., hvornår man har forstået det, og der
1278 tror jeg måske også, at vi kan få den der elev, som du taler om ind ikke fordi, at hvis man sætter
1279 den der grænse så højt, at man li’som siger, at man har set sådan en eller anden indre
1280 sammenhæng i det, der virkelig gør, at man er urokelig i sin tro på, at man har forstået det. Og
1281 jeg tror også meget det er sådan noget med at man har forstået sådan alle sammenhænge, alle de
1282 der små trin ikke. Hvis man så føler, man har forstået det, ikke, hvis man så FØRST DER føler
1283 man har forstået det (I: mmm), så tror jeg man kan den der, jeg er ikke god til matematik uanset
1284 om man så får 13 eller hvad man gør (En anden: mmm) igås. Men det der med, at man, måske
1285 mere, hvis man, hvis man har det sådan, jamen jeg kan regne opgaverne og jeg sådan, jamen jeg

1286 følger med igås, og jeg har forstået de der ting og sådan noget, og man så føler man har forstået
1287 det, så tror jeg godt man kan have det på den måde, jamen jeg er ok til matematik ikke, og så.
1288 Men jeg tror sådan lidt der er forskel ikke, fordi jeg har også stadigvæk den der følelse af, at jeg
1289 har ikke forstået DET HELE og derfor vil jeg heller aldrig sige om mig selv, at jeg var skide
1290 god til matematik, fordi jeg har stadigvæk den her følelse af at DET HELT STORE LYS i
1291 matematik det er ikke gået op [En griner kort, vist nærmest bifaldende] for mig endnu. Den
1292 HELT STORE A-HA oplevelse, hvor jeg sådan siger, “rumgeometri og differential.., ligninger,
1293 det hele det passer bare saaaammeeeen” [synger nærmest det sidste ord; det i “ siges meget
1294 hurtigt] igås (Andre: mmm [kort grin]) Den har, den har jeg altså ikke haft endnu, og derfor tror
1295 jeg aldrig nogen sinde få mig til at sige, hold kæft jeg er GOD til matematik, jeg skal læse det.
1296 Altså for eksempel. Jeg tror også det er sådan noget, der holder mig tilbage fra at sige, jeg skal
1297 læse det her fag, fordi, fordi, at, at jeg føler ikke jeg vil kunne gå ind og læse matematik nu, for
1298 jeg føler stadigvæk den der med, hold kæft, der kommer garanteret på andet år, der kommer der
1299 noget som jeg ikke kan forstå igås. Det ved jeg bare sådan ikke, altså sådan. Sådan har jeg det
1300 lidt.

1301
1302 I: Den der mur der, Z snakker om, er, er det sådan generelt, altså har I også en eller anden idé
1303 om, at på et eller andet tidspunkt så klapper den i, så er det bare go’nat. [En griner]

1304
1305 [Et par sekunders stilhed]

1306
1307 Æ: Jeg tror da, at der er det. Der er da nogen ting, som man bliver stillet over for som jeg vil
1308 sige, det der, det går ud over MIN evne til at forstå det. (I: mmm) Men jeg vil, altså, jeg ved så
1309 ikke om man kan kalde det en mur [Z prøver at bryde ind, og et par ord drukner] ... jeg kan nok
1310 mere acceptere, jamen, det, det der, det kan jeg altså ikke lige forstå, det der. Men så heller ikke
1311 [afbrydes af Å]

1312
1313 Å: [inaudible] Jeg ser det i alt fald ikke nær så klart defineret som S. (I(?): Nej) (Z: At alt hvad
1314 der hedder rumgeometri, det kan jeg bare ikke.) Altså det er måske sådan lidt, på et eller anden
1315 tidspunkt så bliver det så svært, at man kun får fat i halvdelen af det. (I: mmm)

1316
1317 [2-3 snakker lidt i munden på hinanden og ordene drukner]

1318
1319 Z: ... så kan du [Å] netop sige sådan igås, nå men hvis jeg så fik fat i halvdelen af det, jamen
1320 det er jo så egentlig så også, så vil der ikke være tale om en mur, men jeg har det med, at hvis
1321 jeg kun har fået fat i halvdelen, i halvdelen af det, så har jeg ikke forstået noget som helst igås,
1322 altså. NEJ, så er der ikke nogen pointe i det her vel. Og jeg havde det også sådan, altså, vi
1323 havde regner rumgeometri i et halvt år, og jeg følte stadigvæk, jeg havde forstået det, jeg kunne
1324 sagtens regne alle opgaverne, men det [griner selv kort] er først når man sætter sig ned, og man
1325 læser det hele igennem, og jeg forstår alle beviserne og jeg ser meningen med det hele igås, det
1326 er først der, hvor jeg vil sige, jamen jeg har forstået rumgeometri igås. Sådan. Og, og jeg vil
1327 aldrig nogen sinde kunne sige der er et punkt, hvor jeg var nået sådan halvvejs. (En anden:
1328 mmm) Og derfor så er jeg også utrolig bange [griner kort selv] for at der skal komme den der,
1329 det kan godt være, at sådan, når, når min mur den kommer, så kan det godt være, jeg vil sådan
1330 sige, nå men jeg kan måske godt forstå lidt af det igås. Men, men det vil bare ikke hjælpe mig
1331 fordi, at jeg VIL sådan kunne forstå det hele og meningen og sådan nogen ting ikke. Og jeg tror
1332 også tit eller, jeg sådan synes tit, at det, der [inaudible] irriterer mig i matematik er at jeg forstår

1333 ikke MENINGEN med det igås altså sådan. Li'som jeg ikke forstår, og [inaudible]
1334 sammenligning i det her igås.

1335

1336 I: Meningen, altså forstået som den indre sammenhæng, og ikke så meget med, hvad det kan
1337 bruges til?

1338

1339 Z: Nej ikke hvad det kan bruges til, men li'som om, at der må være en eller anden MENING
1340 med det her igås. Der må være et eller andet med [1-2 bryder ind og ordene drukner,
1341 abstraktionsniveau" [inaudible] bliver nævnt] det meste matematik, der er det jo så smukt op
1342 det hele går op, og man har det hvor man li'som kan se, hold kæft det er smukt, og det går op
1343 der igås, hvis jeg ikke kan se det, så, så er det bare ærgerligt [griner].

1344

1345 Æ: Altså jeg vil sige, så noget som, nu har vi arbejdet med idræt, hvis man kan tage det, og det
1346 er nok nogen som jeg ALDRIG vil sige jeg vil kunne forstå vel, men altså, og, der er der ikke,
1347 altså jeg vil ikke sige, der er da ikke den der mur der, fordi der er nok, på et eller andet,
1348 NOGET af det man forstår, ikke, men jeg vil ikke kunne få [afbrydes af Å]

1349

1350 Å: Fordi vi har det i tre uger eller sådan noget ikke, så har man jo ikke forstået ALT hvad der er
1351 at forstå om komplekse tal vel. Men hvis nu det er et eller andet man har arbejdet med, det kan
1352 man jo godt [inaudible], og så lige det bevis her og, og hvorfor siger man kan gøre det her
1353 [inaudible]

1354

1355 Z: Jeg må indrømme det der irrationelle [hun må mene imaginære tal] tal, det står som vi har
1356 haft det, som vi har snakket om, nu har [inaudible] vi haft det samme forløb der, det står bare
1357 som sådan ikke-forstået kategori ikke [En griner lavt, sukker nærmest] sådan ikke, sådan
1358 virkelig. Ikke fordi jeg, jeg var da, der var det der med de små i'er og noget [flere griner] sådan
1359 nogen ting, det kan jeg godt se, jeg kan også godt sætte tingene ind i formel, og sådan, fordi jeg
1360 ikke forstod, men jeg kan ikke forestille mig hvad, det der irrelevante, det der lille bitte i det går
1361 ud på sådan.

1362

1363 [2-3 snakker et øjeblik på samme tid]

1364

1365 I: Forestil dig et tal som et punkt. Man, man altså, man, man laver bare en ekstra dimension på
1366 sin tallinie, og så li'som et tal er ikke bare noget på en linie det, det er bare et punkt.

1367

1368 [Æ begynder at sige noget, men afbrydes af Z]

1369

1370 Z: Før jeg har den der, igås, så kan jeg sgu ikke, før jeg kan li'som kan SE, hvad er meningen
1371 med det der lille i ikke. Det kan godt være at jeg kan, jeg kan sådan regne tingene og se på
1372 formlen men jeg fatter ikke [afbrydes af Æ]

1373

1374 Æ: Der er da også li'som, man kan da heller ikke, man kan heller ikke, det der med at der er
1375 nogen, der er nogen tal eller der er nogen brøker der er uforkortelige eller nogen decimaltal
1376 ikke, som bare fortsætter, og det kan man jo heller ikke forstå at der er et tal der kan fortsætte i
1377 det (I: mmm) uendelige vel. Og det er også det med de uendelighedsbegreber man arbejder
1378 med, der er jo ikke nogen [1-2 andre (Z + ?) begynder også at sige noget her, så et par af Æ's
1379 ord drukner] man kan ikke, man kan ikke, man kan ikke helt forstå det der uendelig vel, men,

1380 men det accepterer man også bare, at man ikke kan forstå, fordi (I: mmm) det går på en eller
1381 anden måde så bare ind i, li'som i (En anden: fysik det) [afbrydes så af Z]
1382
1383 Z: Der tror jeg måske godt, jeg kan acceptere at det er så uforståeligt fordi, fordi nå ja men ved
1384 godt, at, at måske det ligger ikke lige sådan i den menneskelige natur at kunne forestille sig
1385 uendelighed, ikke. Det kan jeg godt sige til mig selv. (I: mmm) Jeg kan ikke forestille mig
1386 uendelighed, jeg har ikke NOGEN som helst basis for nogen sinde i hele mit liv at komme til at
1387 [afbrydes af Æ]
1388
1389 Æ: Men så går den klap vel heller ikke ned for dig.
1390
1391 Z: Nej DER går den klap ikke ned for mig, fordi der kan jeg godt sige DET ER MIT VILKÅR
1392 SOM MENNESKE, li'som jeg kan ikke forstå hvad der er meningen med mig selv, vel. (Æ:
1393 Jamen det) Det kan jeg ikke forstå. Men jeg kan godt, jeg kan se på L for eksempel, som er
1394 matematiklærer, hvis jeg kan se på ham, han har forstået de der irrationelle tal, det har han bare
1395 FORSTÅET, og det har jeg bare ikke. [Flere griner] SÅ er klappen der altså igås. Det er meget
1396 det der med at [afbrydes af Æ]
1397
1398 Æ: Der accepterer jeg altså, lisom jeg accepterer det der med, [en anden snakker med, men det
1399 er uforståeligt] med de, med, med uendelighedsbegrebet (Z: Ja) ikke, det forstår jeg ikke
1400 [afbrydes af S]
1401
1402 Z: Der ved jeg så, jeg VED at det er menneskeligt muligt at forstå de der irrationelle tal, jeg
1403 VED [banker i bordet] der er nogen derude på universiteterne [banker i bordet] der går rundt
1404 [banker i bordet] og siger ha ha jeg har forstået de irrationelle tal [et par stykker griner et stykke
1405 tid] og det KAN jeg ikke. Og det irriterer mig bare SÅ MEGET ikke, altså sådan. [En griner]
1406
1407 Å: Så sige for eksempel i fysik, hvor vi havde de der artikler om bølger, den opgave hvor man
1408 har en partikel der går igennem to huller, så sidder man også bare sådan der, wow det kan den
1409 ikke (En anden: Ja ja), det kan ikke lade sig gøre igås, men alligevel der kan man godt så sige,
1410 nå men jeg kan godt regne opgaver i det, jeg kan også godt forstå hvorfor man skal gøre sådan
1411 og sådan, der er bare et eller andet der går galt [inaudible] [afbrydes af Z]
1412
1413 Z: Men der er også det punkt, hvor det sådan mere [flere bryder ind, og det er svært at skelne
1414 den ene fra den anden samt overhovedet høre hvad der siges] (Å: bare accepterer, at, jamen,
1415 sådan er det, slut.) (En anden: mmm) Men, men der tror jeg også, der er den der, eller det der
1416 med at, at jeg kan godt acceptere, at der er nogle dimensioner inden for matematik som nærmest
1417 er ren filosofi (I: mmm), og jeg kan også godt acceptere, at, at der er jo nogen, altså matematik
1418 ER jo ikke NOGET som sådan vel, det er jo bare, nu tager vi nogen aksiomer og så bygger vi løs
1419 på det, og det er skide sjovt ikke, og det kan jeg godt acceptere. Og jeg, jeg kan godt acceptere
1420 det der med at, også i fysikken der, som Å siger, med at jeg kan godt sådan acceptere, at der er
1421 nogen af de her ting, de er altså for, for langt ude til at, at, at det li'som ligger LIGE for i
1422 erkendelsen. Men hvis jeg har den der meget klare fornemmelse af, at hvis jeg bare fik en a-ha
1423 oplevelse, så ville jeg kunne forstå det her, hvis jeg bare, og det ved jeg at jeg kan igås. Jeg ved,
1424 jeg ved at der er nogen der kan, jeg ved at den, den er li'som lad-sig-gørelig for den
1425 menneskelige igås erkendelse. Hvis jeg ved at den er en potentiel ikke (I: mmm), så vil det
1426 irritere mig ufattelig meget hvis jeg ikke har fået (I: mmm) sådan virkelig, så vil jeg ikke føle

1427 jeg har forstået det. [Et par sekunders pause] Men jeg kan godt se det når I siger [mumler noget
1428 der ikke er til at høre] [griner lidt selv]
1429
1430 I: Altså man, man, jeg tror også nogen gange, det har jeg i alt fald selv oplevet, at nogen ting
1431 har jeg bare været nød til at acceptere, for eksempel på universitetet, når man har sådan noget
1432 der hedder matematisk analyse eller abstrakt algebra og altså, det er virkeligt meget meget
1433 abstrakt, men altså. For eksempel det der matematisk analyse der, der opererer man med, med
1434 kugler af n dimensioner, hvor n det kan være 10.000, ingen problem, åkk, og så tegner man lige
1435 på tavlen, så tegner man så lige sådan et eller andet øhh øhh [nogle griner lavt] sådan kugle der,
1436 det er n dimensioner og så regner vi lidt på det og, og kuglen er simpelthen, altså at radius den
1437 er simpelthen så tæt på 0 som, ufattelig tæt på 0, og så, så kuglen er bare af, af, af n
1438 dimensioner. Og så sidder man der, nåååhh [afbrydes af Å]
1439
1440 Å: n dimensioner, det kan man slet ikke se for sig [flere griner].
1441
1442 I: men øhh, så er man bare nød, jeg plejer altid at have et billede af en 3-dimensionel kugle inde
1443 i hovedet, for ellers så kan jeg ikke finde ud af det (En anden: Ja) altså, så det er man nød [En
1444 prøver at bryde ind] til altså, det kunne, det kunne jeg [afbrydes af Å]
1445
1446 Å: Det er tit hvis, det er nogen ting man virkelig ikke kan se for sig, så, så er det tit, mange der
1447 [et ord kan ikke høres på grund af at en laver noget støj, måske med en taske] uendelig, det kan
1448 man ikke sådan se sådan for sig (En anden: mmm), og man kan ikke se n dimensioner eller en
1449 partikel der li'som går igennem to steder på én gang, sådan nogen ting, det kan man simpelt.. (I:
1450 mmm) det kan man ikke [afbrydes af Z]
1451
1452 Z: Men det er da også [mumler et eller andet] når vi snakker om de her spørgsmål, om det så
1453 ikke også mere er filosofi igås, man har at gøre med, når det når til det punkt der.
1454
1455 I: [mumler et eller andet] altså til forskel fra filosofi så kan matematik faktisk bruges i
1456 rumvidenskab og sådan noget. (Z: Ja ja) Så det, så, så et eller andet sted er det [afbrydes af Z]
1457
1458 Z: Der, der synes jeg så også, at nåå der er det selvfølgelig det fascinerende ved, ved
1459 matematikken, det er jo også, at, at jamen på et eller andet punkt der er det jo filosofi, men det
1460 er bare en filosofi der er, der er på en eller anden måde er mere anvendelig ikke (I: mmm), eller
1461 sådan kan nå til nogle erkendelser, som man ikke kan nå, hvis man bare sidder og snakker, fordi
1462 det har nogle andre muligheder ikke. (I: mmm) Og så, og så, men stadigvæk ER en form for
1463 filosofi ikke, altså. Hvor det så, altså også noget af det som jeg synes der er mest fascinerende
1464 ved det ikke, at, at den li'som har de to dimensioner. Men jeg synes også, jeg kan heller ikke
1465 forstå, hvis nu for eksempel I har den der eksempel, hvor du [I] siger den her, den her kugle i n
1466 dimensioner ikke. Hvad skal man så gøre hvis, hvis, hvad skal I så gøre, når nu I SKAL have
1467 det der billede inde i hovedet for at forstå tingene, li'som for at forstå rumgeometri, man skal
1468 have det her billede af planen og linien for sig selv inde i hovedet, ikke. Hvad gør I, hvad gør
1469 [afbrydes af Æ]
1470
1471 Æ: Jeg siger nok [hun må i stedet for "nok" mene "ikke", hvilket fremgår af det følgende] at jeg
1472 absolut skal have et, et billede af, jeg siger [en anden siger et par uforståelige ord] bare det
1473 HJÆLPER til forståelsen (En anden: Ja), og jeg vil sige, hvis jeg kunne forestille mig den kugle

1474 her kugle med n dimensioner inde i hovedet, så vil det nok også kunne hjælpe mig til at kunne
1475 forstå det, altså.

1476

1477 Z: Men vil du, vil du kunne forstå det, hvis du ikke kan, kan have forestillingen tror du.

1478

1479 [5 sekunders stilhed]

1480

1481 Z: Kan du [afbrydes af Æ]

1482

1483 Æ: Nej jeg vil nok ikke, jeg vil nok ikke kunne forstå det fuldt ud, jeg vil nok altså [afbrydes af
1484 Z]

1485

1486 Z: Ja så vil man have den der følelse af at (Æ: fordi) der mangler li'som noget, ikke.

1487

1488 Æ: Ja. (Ø: Det tror jeg ikke) Så vil jeg bare, det, jeg vil bare acceptere at det, det vil jeg ikke
1489 kunne forstå, og så vil jeg bare [en anden mumler noget] forstå de beregninger som man så (Z:
1490 Ja ja) udfører, og så acceptere at den del af det [en bryder ind] teorien, kan jeg godt sss
1491 [afbrydes af Z]

1492

1493 Z: Det er jo stadig skide irriterende.

1494

1495 Å: Men man kan så sige, der er ikke ret mange der forstår hvordan en kugle med n dimensioner
1496 ser ud, så hvorfor skulle jeg kunne forstå det ikke, altså.

1497

1498 Æ: Altså der går det jo ov., ud over (En anden: mmm) den der, ud over ens fatteevne (En
1499 anden: mmm) nogen steder, og det [afbrydes af Z]

1500

1501 Z: Det er også li'som en skattejagt igås, hvad, hvad, hvad man også selv indser for, anser for at
1502 være INDEN for, for ens egen sådan (En anden: Ja) forståelsesevne (I: mmm) igås. Og det er
1503 ikke li'så ubehageligt ikke at kunne forstå noget som man, som man siger, ahm det, det ER der
1504 jo ikke nogen der kan forstå, det her, der er jo ikke (En anden: Næ [hun er altså enig]) nogen
1505 der kan begribe noget der er i 10.000 dimensioner, det har vi ikke nogen menneskelig mulighed
1506 for at gøre vel. Fordi, jamen altså, det har vi slet ikke erkendelsesapparatet til vel. Det er meget
1507 mere irriterende det der med, at hvis man ikke kan forestille sig en kugle igås, altså fordi man
1508 VED li'som at man burde [griner lidt] kunne forestille sig, altså. (I: mmm) Altså sådan, der
1509 synes jeg, der, der er det mere irriterende på en eller anden måde, der går det én mere på, ikke.

1510

1511 Æ: Det er nok rigtig nok, det kan godt, det kan også godt irritere mig, hvis der er nogen der har
1512 forstået noget, som jeg ikke har forstået, men sss (Z: Det er li'som om så føler jeg ikke jeg kan
1513 trænge ordentligt ind i det) nej, men så tror jeg på nogle punkter så accepterer jeg, nå ja, men
1514 det kan jeg ikke forstå. Og så pludselig så kan det egentlig godt være, at der går nogle uger og
1515 så forstår jeg det endeligt. Men altså (Z: Jeg er også sikker på det er mere) der går ikke den (Z:
1516 indstilling) der blok ned, fordi så accepterer jeg bare, nå ja men der ER altså lige den der del af
1517 (Z: Ja), den forstår jeg altså ikke lige. Men så er der de andre dele, som jeg forstår.

1518

1519 Z: Ja, men altså jeg, jeg tror da også, jeg tror heller ikke, det er en hensigtsmæssig indstilling at
1520 have til det. Fordi at mit problem vil jo også være, at hvis jeg nu KOM til på det der
1521 længerevarende naturvidenskabelige studium, at hvis der så kom noget som jeg virkelig virkelig

1522 ville opfatte som uoverstigeligt svært, så ville det jo være meget bedre at have Æ's indstilling,
1523 hvor man (I: mmm) så siger, jamen så arbejder jeg med det, og så forstår jeg sådan dele af det
1524 igås og (I: mmm) så lader jeg mig måske også ubevidst arbejde med det igås, og jeg prøver på
1525 li'som at [1-2 prøver at afbryde] acceptere at nu kan jeg, jeg kan forstå NOGET af det igås. Og
1526 så kan man måske på ét eller andet tidspunkt komme til en eller anden forståelse af det. Men jeg
1527 har bare utrolig svært ved li'som at finde min i den der, det der, altså som om jeg synes man
1528 hænger sådan (I: Lidt perfektionistisk) Jamen jeg ved ikke om det er perfektionisme, men jeg
1529 føler sådan man hænger i riuw, fordi hvis man ikke har forstået (I: Man hænger i hvad for
1530 noget?) man hænger i sådan et eller andet tomrum igås (I: Nåååh.) hvor man ikke har forstået en
1531 eller anden ting sådan helt igås. Så, så er det li'som om, at det, det synes jeg ikke sådan er helt
1532 tilfredsstillende, på en eller anden måde, eller sådan, så det.
1533
1534 I: Jeg skal lige høre, skal du [Æ] læse noget ingeniør noget bagefter, eller sådan noget.
1535
1536 Æ: Mmm, ja, det har jeg planer om. Mmm farmaceut har jeg tænkt på, det skal ikke, det skal
1537 ikke kun være, det der med universitetet, hvor du læser tre år for eksempel matematik og så,
1538 altså (I: Nej) hovedfag. Det, jeg ved ikke helt hvad det hedder.
1539
1540 I: Ja, hva..., hvi..., hvis man læser 3½ år så har man hovedfag (Æ: Ja), men du kan kun læse
1541 matematik.
1542
1543 Æ: Ja, men det skal det ikke være, det skal være en kombination af, af det hele [griner lidt].
1544 Også altså med lidt humanistisk må det også godt.
1545
1546 I: Hvad skal I to andre læse.
1547
1548 Å: Det ved jeg ikke.
1549
1550 Ø: Øhh, jeg regner med medicin, men det ikke er ikke noget jeg ved. (I: mmm) Så det har ikke
1551 så vildt meget med matematik at gøre.
1552
1553 [4 sekunders stilhed]
1554
1555 I: Det er lidt skægt at I, at det altså, at jeres [Æ og Z] to syn, at nu er det mest jer der har
1556 snakket lige til sidst, og så at, altså at, så du [Æ] vælger noget naturvidenskab og har DET syn,
1557 og du [Z] vælger ikke noget naturvidenskab og har DET syn. (En anden: mmm) Mærkeligt, det
1558 er lidt skægt.
1559
1560 Å: Jamen altså, jeg overvejer også helt klart noget der har noget med naturvidenskab at gøre. (I:
1561 mmm) Ikke noget humanistisk, det er [mumler noget uhørligt]
1562
1563 Z: Men jeg tror også det er fordi at det der med at hvis man skal arbejde med det, så bliver man,
1564 altså fordi jeg, jeg, jeg synes at, at det, det er sådan spændende og fascinerende på det der, på
1565 det der interesseplan, ikke. Øhm, men det ville irritere mig utroligt meget hvis det var noget jeg
1566 skulle arbejde, at jeg ikke følte jeg kunne trænge ordentligt ind i det, ikke. Og nok også fordi at
1567 jeg ikke har den der rigtige indstilling, med at matematik også er noget man arbejder med, det
1568 er ikke bare noget man sådan, skal li'som FORSTÅ i sådan nogen blokke, men det også er
1569 noget man, man li'som arbejder med en eller anden erkendelse af ikke. Og dér, der tror jeg

1570 heller ikke jeg ville, heller, heller aldrig nogen sinde kunne komme til det punkt hvor jeg sådan
1571 kunne, kunne tilføre noget nyt til det jeg havde lært. (I: Altså selv finde på noget helt nyt,
1572 andre) Det tror jeg ikke, fordi der tror jeg man mere skal have den indstilling med, ahm det er
1573 noget man kan arbejde med det her igås. Altså sådan.
1574
1575 Ø: Så det at finde sådan en ny æhh bevisrække eller sådan noget, det opfatter du ikke som noget
1576 nyt, altså det er en del af denne (Z: Jo, men det er dét) normal [inaudible] del, det er egentlig
1577 bare ikke udforsket endnu.
1578
1579 Z: Det er det som jeg ikke tror, at, at jeg, jeg kan komme til fordi da, da tror jeg nemlig at, at
1580 hvis du skal til at gøre sådan nogen ting, som at finde på et eller andet nyt ikke, så skal du mere
1581 have den der indstilling med, at det er noget du kan sådan arbejde med ikke, og det er noget du
1582 sådan kan, kan, jamen altså sætte ind i nye former igås, altså, og måske forstå LIDT af og så
1583 prøve igen og igen og igen og sådan noget der igås. Og det ikke lige nødvendigvis skal være
1584 hele systemer der går op sådan på én gang. Og det tror jeg altså, men den indstilling tror jeg
1585 måske også er lidt nødvendig hvis man SKAL på de der naturvidenskabelige studier ikke.
1586
1587 I: Jeg tror det kommer lidt an på om man skal være ingeniør eller man skal læse hard-core
1588 matematik. (Z: Ja det kan godt være) Der, der tror jeg, der er en forskel. Fordi mange af dem
1589 der læser ingeniør, øhh altså de, de har ikke beviserne. De får at vide, altså hvis du har det og
1590 det problem så har vi, så har du en formel, som virker på den og den måde (En anden: mmm)
1591 og, og så bruger du de og de redskaber, her har vi en computer, der lige kan beregner noget, så
1592 man approximere nogen ting, for det er sjældent en ingeniør regner eksakt. Det er sådan ok, så
1593 er det sådan, så ligger vi lige lidt ekstra til for at være [flere griner] sikker. [et par stykker
1594 snakker lidt i munden på hinanden, uforståeligt nogle sekunder]
1595
1596 Å: ... matematiker og sproglige, der er det tit sådan, så læreren skriver en formel op på tavlen,
1597 og siger, med den der kan I regner noget [inaudible] ud, og så giver han os nogen ark, og så
1598 sidder vi sådan og regner lidt på det. Hvis så man så li'som vil forstå det, så må man bare sådan
1599 ty til det man har så har lært i de andre fag, ikke. Det er sådan mere sådan, nu skal vi regne ud
1600 hvor langt der er ud til den her stjerne. (I: mmm) På den måde så kan man jo også godt bruge
1601 det, uden at forstå det, nogen gange. (I: mmm)
1602
1603 Z: Men så er det et værktøj, li'som, vi snakkede også om før med den der tømrer, eller et eller
1604 andet, at så er det et værktøj.
1605
1606 Å: Altså nu er jeg, nu er det så ikke så galt nu når vi så har, altså jeg har haft matematik og
1607 fysik ikke, i så lang tid, så mange af de ting dem har vi set før grafisk, og forstået noget af det,
1608 og. Jeg kan forestille mig HF og nogen af de sproglige, de sidder sådan lidt, sådan, formel, så
1609 må de bare bruge den, som et værktøj, altså tage den ind, og så sige, nå men så er der så og så
1610 langt. (En anden: mmm)
1611
1612 Ø: Det øhm altså meget med, at de sproglige de bakker helt ud altså, når der kommer noget med
1613 nogle formler (En anden: mmm) eller nogen [inaudible] eller sådan noget og det, det kan godt
1614 være det hænger sammen med det som Z sagde for noget tid siden med, at, at der er sådan en
1615 opfattelse af, at de naturvidenskabelige fag som sådan noget ret særligt og (I: mmm)
1616 utilnærmeligt på en eller anden måde.
1617

1618 Z: Jeg tror også nogen gange det er træls at bruge et værktøj som man egentlig ikke har (En
1619 anden: mmm) forstået hvor kommer fra (En anden: mmm) altså sådan [afbrydes af Ø]
1620

1621 Ø: Men jeg tror ikke det er dét, altså [afbrydes af Å]
1622

1623 Å: Det genererer mig rigtig meget, hvis der er noget som jeg ikke forstår og så jeg bare skal
1624 regne med det.
1625

1626 I: Det, det generer dig jo ikke at sætte dig ind i en bil, og nu, så sætter du dig og drejer på
1627 nøglen, og så kører motoren.
1628

1629 Å: Det, det er nok noget andet, fordi jeg ikke har noget ønske om at vide, hvordan sådan en
1630 [flere griner højt] fungerer. Sådan intern [denne bemærkning skal vist forstås som at det med
1631 ikke at ønske at vide hvordan en motor fungerer er en slags intern joke eller hemmelighed].
1632 Hvis man sidder i [inaudible] og man regner noget, og så man ikke LIGE helt har forstået
1633 hvorfor det nu er, så har man sådan lyst til sådan at finde ud, hvorfor det nu er man lige kan
1634 gøre det på den måde.
1635

1636 Z: [Her snakker hun meget lavt, så nogle ord er ikke til at høre] Men det er rigtig nok, jeg har
1637 sådan en opfattelse af at filosofi [inaudible] det er sådan noget med det at det afhænger af
1638 sproglige [inaudible] det er bare sådan noget [inaudible] nå, men så kan det også være lige
1639 meget ikke. (En anden: Ja) Men det har også noget at gøre med at deres, altså, det fag som de
1640 har, naturfag, ikke, at jeg tror meget af det der med, der får man ikke LOV til at have den der
1641 sådan, langsomme erkendelse (En anden: Nej), hvor man bygger tingene op i de der grundtrin,
1642 der skal man li'som have sådan viuw det her, fuijj, store (En anden: Ja) overblik, og derfor er
1643 der ikke rigtig tid til at [griner kort] starte fra bunden ikke.
1644

1645 Ø: Det er meget rodet, det virker, ja, ufatteligt rodet, synes jeg. Altså, det, det er noget og så er
1646 det alligevel ikke noget, vel altså. (I: mmm) Det synes jeg, det [afbrydes af Å]
1647

1648 Å: Sådan lidt af hvert (Ø: Ja) Å: ... har mødt, sådan virker det (I: mmm) (Ø: Det er det)
1649

1650 Z: Jeg tror også det, noget af det der, har været så irriterende for mig i folkeskolen, for
1651 eksempel i et fag som fysik, det var, det der med, nå men hvis vi for eksempel skal have om
1652 potentiel og kinetisk energi, så siger vi bare, der er noget der hedder potentiel energi og det er
1653 hvis man tager og gør sådan her, og så løfter en ting (I: mmm) igås. Og så kommer man, så
1654 kommer man i gymnasiet og man tror bare at man ved alt om det her potentiel og kinetisk
1655 energi, og så er det som om så begynder de bare, to meter [griner kort] længere nede igås, og så
1656 er det som om, nå, men hov hovsa, der var altså en platform her igås, li'som. (I: mmm) Og der,
1657 der synes jeg da i folkeskolen, der havde jeg utroligt svært ved at forstå mange af de ting, som
1658 jeg synes var, var, var meget meget nemmere i gymnasiet EGENTLIG, fordi man fik lov til og
1659 li'som starte fra bunden af, i stedet for [griner lidt] bare at skulle have de der begreber, som
1660 man egentlig ikke havde fået forklaret. Altså sådan. (I: mmm) Jamen hvad er det egentlig for
1661 noget det her igås, hvad er det egentlig det går ud på sådan en, altså sådan. For eksempel meget
1662 med strøm og sådan noget, strøm og spændingsforskel, jeg kan huske det irriterede mig utrolig
1663 meget i folkeskolen, jeg vidste overhovedet ikke hvad det var vel, altså sådan, og der tror jeg
1664 det hjalp utrolig meget, da jeg kom i gymnasiet og [inaudible] fik at vide, nå ja, der kommer de
1665 små elektroner og, og de (Å: og kører rundt på en bane [En griner] og alle de der forklaringer)

1666 Og der kommer måske også den der Æ og Ø's sådan visualiseringsting ind ikke, med, nå man
1667 så kan man pludselig se, nå men det ER de små elektroner der kommer kørende [flere griner]
1668 ikke, altså det er jo (En anden: mmm) altså. Før var det bare sådan, strøm, ikke, der var det bare
1669 sådan, ord. (I: Noget magi) (En anden: Ja ja) Nå men som om at, at begrebet li'som er nok, og
1670 det er det jo ikke. (I: Ikke kun meningen, altså) Nej, altså navnet er jo ikke nok vel (I: mmm)
1671 altså sådan. Det er li'som, det er li'som som, sådan altså, funktion, det er jo ikke nok vel altså,
1672 hvis man ikke li'som har forstået, hvad, hvad, hvad er det egentligt, ikke. Da tror jeg det er
1673 meget vigtigt det der med at lære tingene sådan fra, fra li'som nul og så opefter.
1674
1675 Å: Man har det også meget bedre med at bruge det til noget, når man.
1676
1677 [Der kører et tog forbi, og i 9-10 sekunder kan man ikke høre, hvad der bliver sagt; ordene
1678 'li'som' og 'forstået' kan dog lige anes.]
1679
1680 I: Ja, men, jeg tror jeg vil sige mange tak, jeg synes det har været rigtig spændende.