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Some index formulae on the moduli space of stable parabolic vector bundles

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Review text:
Following [Int. Math. Res. Not. 2009, No. 4, 625–697 (2009; Zbl 1184.58008) and Commun. Math. Phys. 289, No. 2, 483–527 (2009; Zbl 1171.30017)], this is the third paper by the authors to perform concrete calculations of spectral invariants on punctured Riemann surfaces. These three investigations were all fuelled by the PhD dissertation [*Boris Vaillant*, 'Index- and spectral theory for manifolds with generalized fibred cusps', Bonn, 2001, accessible at arXiv:math/0102072 [math.DG]; MR1933455 (2003h:58034)], which combined Werner Müller's *number theoretical techniques* with Richard Melrose's and collaborators' *symbolic calculus* for efficient calculations of index and eta-forms on manifolds with genralized fibred cusps.
In the paper under review the authors extend Vaillant's method and their own previous results to the moduli space of stable parabolic vector bundles with vanishing parabolic degree on a Riemann surface with marked points. Parabolic vector bundles are holomorphic vector bundles together with a parabolic structure specified at each marked point by a set of weights and multiplicities. The notion of parabolic structures is very much related to that of *Hecke correspondences* appearing in [*M.S. Narasimhan* and *S. Ramanan*, C.P. Ramanujam - A tribute. Collect. Publ. of C.P. Ramanujam and Pap. in his Mem., Tata Inst. fundam. Res., Stud. Math. 8, 291–345 (1978; Zbl 0427.14002)] and elsewhere in algebraic number theory. It was worked out in [*V.B. Mehta* and *C.S. Seshadri*, Math. Ann. 248, 205–239 (1980; Zbl 0454.14006)]. As emphasized in [*H.U. Boden* and *K. Yokogawa*, J. Lond. Math. Soc., II. Ser. 59, No.2, 461–478 (1999; Zbl 1023.14025)], moduli spaces of parabolic bundles occur naturally as (i) unitary representation spaces of Fuchsian groups, (ii) moduli spaces of Yang-Mills connections on manifolds with an orbifold metric, and (iii), as in the present case, moduli spaces of certain semistable bundles over an elliptic surface.
The authors perform a computation of the eta forms (truly not an easy task) to express them in terms of explicit data coming from the parabolic structure leading to local families index formulae; they are also able to define the Quillen metric on the determinant line bundle via heat kernel techniques and determine its curvature; and they check the correctness of their results by re-proving a formula of [*L.A. Takhtajan* and *P. Zograf*, Math. Ann. 341, No. 1, 113–135 (2008; Zbl 1156.14028), Theorem 2]. The paper is difficult, but worth reading. Reading is not facilitated by the authors' decision to spread the arguments over three consecutive papers in three different journals demanding back- and forward browsing with some repititions in an unfortunate spaghetti structure. That said, the paper is highly inspiring in its Hirzebruch style aiming at concrete calculations derived from a thorough understanding of a generic case. This reviewer, e.g., could imagine consecutive explicit calculations of the eta forms and eventual zeta-regularized determinants on the underlying Riemann surfaces *with* boundary according to the program of [*I.M. Singer*, Mathematical aspects of string theory, Proc. Conf., San Diego/Calif. 1986, Adv. Ser. Math. Phys. 1, 239–258 (1987; Zbl 0669.58036)] and the encouraging results of [*R.G. Douglas* and *K.P. Wojciechowski*, Commun. Math. Phys. 142, 139–168 (1991; Zbl 0746.58074)], [*S.G. Scott* and *K.P. Wojciechowski*, Geom. Funct. Anal. 10, No.5, 1202-1236 (2000; Zbl 0973.58017)], and [*S. Klimek* and *K.P. Wojciechowski*, Differ. Geom. Appl. 3, No.2, 191-201 (1993; Zbl 0787.58045)] for the intricacies of this even-dimensional situation.

Reviewer: Bernhelm Booß-Bavnbek (Roskilde)

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