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accident, epiphenomenon or contributory cause of the changing trends of mathematics. a sketch of development from the twelfth through the sixteenth century

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INTRODUCTORY OBSERVATIONS

The following is the result of an invitation to participate as a historian of mathematics in a symposium on Renaissance philosophy, dealing in particular with the rôle of the Aristotelian tradition in the genesis of the Early Modern era. I was, as anticipated by the organizers, somewhat amazed by the invitation--and all the more so because my immediate feeling was that Aristotelianism (as well as formal philosophy in general) and mathematics have *no* close connection during the Renaissance and the earliest Modern period.

This *was* only an immediate feeling, since my familiarity with the sources stopped by the fourteenth century, and since the question was in any event not one I had considered deeply before, not even in connection with the High Middle Ages.

What follows in then simply an investigation of the question *whether dominating philosophies, in as far as they are at all visible in the mathematical sources, have stamped (or eventually even determined) the ways in which mathematics developed from the twelfth through the sixteenth century, or they are purely epiphenomenal.*

The answer is of course partly determined by the level on which »philosophy« is understood. I shall restrict the use of the word to the level of *systematically organized thought*, and exclude the loose sense of »attitudes« even when the attitudes in question *could be* expressed in terms of some philosophical system--an artisan is not to be labeled an *Aristotelian* just because he prefers empirical methods for a-priorism; I shall, however, also discuss the influences of proto- and quasi-philosophical attitudes as well as the *relations* between philosophies and attitudes.

I shall *not* discuss the import of further socio-cultural factors. It will, however, be clear that major channels of influence for these are philosophical and (since they lack scholarly obligation to well-belaboured tradition and hence also internal rigidity) especially quasi-philosophical attitudes on the nature and purpose of mathematics and mathematical activity.

It will be convenient to discuss the problem in the grid constituted by conventional periodizations. A first period (»the twelfth century«, in reality with vaguer limits) is (when seen from the point of view of the history of mathematics) dominated by the enthusiasm for Euclid and the *Almagest*, and ends when Aristotle becomes all-dominating in the *artes* (and so becomes the all-dominating problem for ecclesiastical authorities nervous about university curricula). The second period (»the thirteenth century«) is that of assimilation of Aristotle. The third period (»the fourteenth century«) presents us with a wealth of creative developments of Aristotelian philosophy, including mathematical developments. During the fourth period (»Early Renaissance«, late fourteenth to mid-sixteenth century) mathematics and formalized philosophy live largely separately--and in the fifth period the foundation is laid *not least by developments related to mathematics* for the creation of the new philosophies of the seventeenth century.

Three problems of method should be mentioned in advance, one practical and two of principle. The first was already mentioned, *viz*. that before starting on the project I was only in possession of reasonable familiarity with sources from the twelfth through the fourteenth century (and even for this span of time of course only with a small part of the complete source material). Since then I have applied myself to cover at least essential sources representative of the most important Renaissance currents (but not of every major mathematician). Far from everything I wanted has, however, been accessible to me; nor have I had the time to go into reasonable depth with everything which deserved so. Finally may I of course have overlooked important characters and tendencies unintentionally or have assessed them wrongly, in which case I will ask for correction rather than indulgence.

The second and third problems both concern the reality of entities regarded. Specific philosophies may have some historical coherence over a certain span of time, though even that can be problematic. *Attitudes*, *tendencies* and *currents*, however, are elusive concepts though necessary if overall structures are to be distinguished; their demarcations will by necessity be blurred, at times they will overlap, and it will often be impossible to claim that a specific author belongs to one current and only there. It should be kept in mind that currents etc. may at times represent *poles with relation to which authors can be seen to orientate themselves* rather than *classifications*.

One entity plays a specific rôle: Mathematics (the third problem). Is it justified to think of mathematics as something well-defined and possessing continuous existence from (at least) 1100 to 1700, or is this an anachronism, a piece of *whig* *history?* Isn't such an idea in conflict with the obvious observation that the term covers something very different in the beginning and in the end of the period?

My answer to this Parmenidean point of view will be negative, which can be argued on several levels. I shall mention two: *Socially,* the actors themselves, those who generation for generation recreated the field, were convinced of both coherence and historical continuity; even a claim that previous generations had made *barbaric* or *adulterated* mathematics implied an acceptance that they made *mathematics*. *Metaphysically,* mathematics is characterized over the whole period by being an *abstraction* from sensible reality, dealing (as stated continuously from Augustine to Pascal) with a world created according to *measure, number and weight,* and susceptible of some sort of *argument* or *proof.* Over the centuries the substance covered by this global characteristic would vary; but the existence of the category itself was constant.

THE TWELFTH CENTURY

Medieval learning had inherited from Antiquity the scheme of the Seven Liberal Arts and, quite as decisive, the idea that these arts constituted the *apex* of *scientia humana*. This idea was promoted not least by Isidore of Seville (c. 560-636), who remained an important authority throughout the Middle Ages. If we concentrate interest on the *quadrivium* part of the scheme (arithmetic, geometry, music and astronomy), Isidore's attitude to the subject in his *Etymologies* is almost paradoxical: He is full of reverence for these important disciplines, but he knows next to nothing about them (if we define their contents according to the yardstick of the Alexandria school). All the same, the empty reverence proved important over the centuries: At every occasion where scholarly activity began burgeoning--be it Beda's (c. 673-735) Northumbria, Alcuin's (c. 735-804) Aachen, Hrabanus Maurus' (c. 776-856) Fulda or Gerbert's (c. 930-1003) school in Rheims, mathematical subjects were among those cultivated to the extent and in the sense allowed by current conditions[1].

Up to the end of the first millennium the interest in mathematical subjects is, it seems, mainly to be explained along these lines, as interest in *something in which the good scholar ought to be interested*, even though actual needs of ecclesiastical scholarly life made *computus* (Easter-reckoning) the only really living field from the seventh through ninth centuries[2]. The final result achieved was the re-establishment toward the end of the eleventh century of a complete *Latin*

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[1] I deal in somewhat more detail with this aspect of Early and Central Medieval history of mathematics and with the concepts of a »Latin« and a »Christian« quadrivium (see below) in my 1985a.

[2] »Not until forty years after Charle[magne]s's death, when diocesan schools began to expand and the manual of Martianus Capella began to influence the curriculum of some of them, was there a study other than *computus* which dealt with the mathematical sciences«, as stated emphatically by Ch. W. Jones (1963: 21)--maybe somewhat more emphatically than justified.

quadrivium, a cycle of mathematical disciplines considered to belong to and to round off the *scientia humana*-level of Latin scholarship. Its high point was Boethius' *Arithmetic*. Geometry was represented by compilations of (pseudo-)practical geometry combining the surviving fragments of Boethius' translation of Euclid with material drawn from Roman agrimensors; it included the use of the so-called Gerbert-abacus. Music was once again a mathematical theory of harmony (built on Boethius' translation of Nicomachos), after a dark interlude where it had dealt with actual song, and astronomy embraced the *computus*, basic description of the celestial sphere and the astrolabe, and a little (very little!) astrology taken over from the Islamic world. Besides, various general expositions of the aims and authorities of the quadrivial arts were at hand (Martianus Capella's *Marriage of Philology and Mercury*, Cassiodorus' *Introduction to Divine and Human Readings*, and a variety of Medieval compilations).

This was the foundation on which scholars had to build their understanding of mathematics, and with which the more ambitious became dissatisfied in the early twelfth century. To contemporary observers, this century was a bloom *par excellence* of the *artes*. Historians of philosophy would first of all think of its beginning as the period of Abaelard and the inception of dialectics' supremacy. Abaelard (1079-1142) himself, however, tells us indirectly that the *quadrivium* too was able to foster enthusiasm in the environment of young scholars, through the name he and Héloïse gave to their son: *Astralabius*[3].

More direct evidence is offered by the translators. A biography of Gherardo di Cremona (c. 1114-1187), the most prolific of all, tells that he was »educated from the cradle in the bosom of philosophy« and, dissatisfied with the limits of Latin studies, »set out for Toledo« to get hold of the *Almagest*. Having arrived he stayed there translating the Arabic treasures »until the end of life«[4]. Another, anonymous scholar pursued medical studies in Salerno when hearing that a Greek copy of the *Almagest* had arrived to Palermo; accordingly he left for Sicily, started preparing himself by translating some minor works from the Greek, and finally translated Ptolemy's *Great Composition*[5]. Adelard of Bath (fl. 1116-1142), finally, started writing in the tradition of the Latin quadrivium on the *Regulae abaci*. The treatise presents us with an overwhelmingly full discussion of this

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[3] Abaelard, Historia calamitatum, ed. Muckle 1950: 184f.

[4] The full fourteenth century biography will be found in Boncompagni 1851a: 387ff, from which I translate.

[5] See Haskins 1924: 159-162.

subject, referring to Boethius' *Arithmetic* and *Music*, to the traditional system of sub-units (the mutual multiplication of which gives occasion for many pages), to Gerbert, and of course to everything connected to the device itself[6]. Then he left home »to investigate the studies of the Arabs«[7], which resulted first in a metaphysical treatise *De eodem et diverso*[8], and then in a work on *Quaestiones naturales* built in part on what he had learned from the »studies of the Arabs«[9], and in a beautiful array of translations--including various astronomical treatises and at least one (probably two, possibly even three) translation of the *Elements*[10].

The first effect of the mathematical translations was the completion of what I have called the *»Christian quadrivium«*, that quadrivial syllabus which Christian Latin Europe (»Christianity« understood as an ethnic rather than a religious identity) considered its *legitimate heritage*, because it completed the range of authors, works and disciplines known (like Euclid, Ptolemy etc.) by name from Isidore, Martianus Capella and Cassiodorus; known (like optics) from Aristotle's works, of which most of those not translated before became available during the same century; or attributed by their titles to Ancient authors (as the »science of weights« was attributed to Euclid). Islamic authors continuing these same disciplines were accepted as legitimate and necessary (though morally clearly secondary) commentators and explanations of the same »Christian« quadrivium

Another effect was the completion of *the total range* of mathematical subjects, which came to include two obviously non-»Christian« disciplines, namely *algebra* (and more elementary commercial calculation) and *algorism*--the latter meaning

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[6] The treatise was edited by Boncompagni (1881). See the references to Boethius' translations p. 11119,22; the reference to Gerbert as »having given the technique back to us Gauls« p. 9123; and the Boethian reference to Pythagoras p. 917.

[7] As stated in his *Quaestiones naturales*, ed. Müller 1934:432.

[8] The »first« is hypothetical: According to the dedication the treatise is written during the seven year voyage to which Adelard refers in the beginning of the Quaestiones naturales, but the contents seems to belong to the intellectual luggage which made him set out, not to anything he had learned in Sicily or in the Near East. This combination fits the beginning of his stay in Syracuse best. See the edition and discussion in Willner 1903.

[9] Even though Boethius *De musica* remains an important source--cf. Müller 1934: 25^{13} and 27^{23f}.

[10] See Clagett, "Adelard of Bath", DSB I, 63.

calculation with Hindu numerals, which was soon accepted as a useful and neutral tool by the environment adopting the new mathematics and astronomy[11].

A total survey of the range of translations shows that mathematics played an important rôle (especially if astronomical and astrological works are counted as mathematics)--almost, perhaps fully on a par with medical subjects and the concomitant *philosophia naturalis.* Another measure of the importance of the mathematical imports is supplied by traditionalist polemics against the new learning. In a *Sermon to the Purification of the Blessed Mary* from the late twelfth century, Étienne de Tournai complained that many Christians (and even monks and canons) endangered their salvation by studying

> *poetical figments, philosophical opinions, the [grammatical] rules of Priscian, the Laws of Justinian, the doctrine of Galen, the speeches of the rhetors, the ambiguities of Aristotle, the theorems of Euclid, and the conjectures of Ptolemy. Indeed, the so-called Liberal Arts are valuable for sharpening the genius and for understanding the Scriptures; but together with the Philosopher they are to be saluted only from the doorstep.*[12]

The poets, Priscian's grammar, the rhetors, and even Aristotle's discussion of sophisms, belong to the traditional realm of the trivium; Justinian's Roman Law must also be understood as an extension of the study of dialectical Canon Law and theology, and hence as a traditional subject which aroused a sudden vigorous interest to the dismay of Bernard of Clairvaux and his companions-in-arms. The really *new* learning is represented, we see, by Euclid, Ptolemy, Galen, and possibly (but probably not) by the »philosophical opinions«. Broader interest in theoretical mathematics and in high-level astronomy (not necessarily followed

[11] An illustrative example is Gherardo di Cremona himself. In one of his translations from the Arabic, a *Liber mensurationum* edited by Busard (1968), Roman numerals, Hindu numerals and number words written in full are mixed up completely; even though the Arabic treatise is lost it is fairly certain that all its numbers were written as full words.

[12] *Non enim in figmentis poeticis, non in opinionibus phylosophicis, in regulis Prisciani, in legibus Iustiniani, in doctrina Galieni, in oribus rhetoricis, in perplexionibus Aristotelis, in teorematibus Euclidis, in conjecturis Tolomei, summan studiorum suorum ponere et tempus suum conterere debet christianus, multominus monachus et canonicus. Et quidem artes, quas liberales vocant ad acuendum ingenium et intelligentiam Scripturarum multum valent, sed, iuxta philosophum, salutande sunt a limine.* Quoted from Grabmann 1941: 61 (my translation).

by conspicuous competence) was evidently an important effect of the intellectual revolution in the twelfth century scholarly environment, and not just a queer preference of the translators. As we can imagine, below this high level a still broader interest in less requiring mathematical subjects thrived.

At the very turn of the century, a monumental expression of the interest in mathematics was created outside the environment of the schools: Leonardo Fibonacci's (b. c. 1170, d. after 1240) *Liber abaci*[13] (written 1202, and containing somewhere between 250 000 and 300 000 words). The work is in principle an enormously extended *algorism*, a guide to the use of Hindu numerals not only for computation but for commercial calculation and algebra in general. *If* the work represented more than its author it would be evidence that the interest in mathematics had penetrated not only the schools but also the commercial environment of Northern Italy. To some extent this is certainly true; during the thirteenth century a system of lay commercial education developed in Northern Italy. The system was centered upon commercial calculation, as evident already from the name: The *abacus school*.[14] In its immensity, however, Leonardo's work is a personal achievement of its author, and (apart from a number of copies of the manuscript) nothing similar was made for centuries. Leonardo's genius, impressive as it is for the historian of mathematics, tells little about the conditions and intellectual climate of his environment, neither during the twelfth nor the thirteenth century.

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[13] Ed. Boncompagni 1857 (from a manuscript of Leonardo's revision of the work in 1228).

[14] According to the chronicler Giovanni Villani, in 1339 about 1000-1200 Florentine boys (out of a total city population of 90000) went to one of the six schools where practical arithmetic was taught (C. T. Davis 1965: 415). See also Fanfani 1951 and Goldthwaite 1972 on Italian commercial teaching from the fourteenth through sixteenth centuries.

According to a document reproduced by Goldthwaite (pp. 421ff), the basic curriculum in a Florentine school »consisted in seven consecutive courses«: 1) arithmetical operations except division; 2)-4) division with one, two and more digits; 5) fractions; 6) the Rule of Three; 7) the Florentine monetary system. Higher subjects would be reserved for the few.

A more precise idea of the teaching can be acquired from the various »abacus treatises« which have been published. A fine specimen is found in Arrighi 1973. Obviously, *part* (but only part) of the inspiration from Leonardo was alive.

How are we, however, to explain those more modest but still revolutionary developments which *were* characteristic of the twelfth century?

Let us first return to Adelard. His *Regulae abaci* reflect his background in the traditional Latin Arts, and the *De eodem et diverso* and the *Quaestiones naturales* demonstrate that he belongs in full right to the current of »twelfth century Platonism«, with its inspiration from the *Timaios* and its interest in natural explanation[15]. This commitment connects directly to the empirical and naturalist aspect of his translations. There is, however, no direct link from this very atypical brand of Platonism to pure geometry or mathematics in general. Instead, we must see it as expressing an uncritical climate of intellectual hunger, where *anything important* in relation to the lost intellectual heritage (as understood in the schools of the early twelfth century, and hence understood not as antiquarianism but as comprehension of the universe) was to be seized upon--especially such fundamental works as the *Almagest* itself and the *Elements*. Already Isidore and Augustine had quoted the Bible to the effect that »YOU made everything in measure and number and weight«[16], from which Isidore concluded that

> *By number, we are not confounded but instructed. Take away number from everything, and everything perishes. Deprive the world of computation, and it will be seized by total blind ignorance, and will be indistinguishable from the other animals he who does not know how to calculate.*[17]

In the early twelfth century, Adelard and his fellows would see not only »computation« but the whole range of available mathematics as necessary if a world created in »measure and number and weight« were to be understood.

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[15] See also Jolivet 1974, on the conspicuous absence of Biblical explanations from the *Quaestiones naturales*.

Discussions of Adelard in the wider context of twelfth century naturalism will be found in Chenu 1966 and Stiefel 1977.

[16] *Omnia in mensura, et numero, et pondere fecisti*--*Wisd.* XI, 21; quoted in Augustine, *The City of God* XI, xxx (ed. McCracken et al 1966: III, 552), and Isidore, *Etymologies* III, iv, 1 (*PL* 82, 155). My translation.

[17] Per numerum siquidem, ne confundamur, instruimur. Tolle numerum rebus omnibus, et omnia pereunt. Adime saeculo computum, et cuncta ignorantia complectitur, nec differri potest a caeteris animalibus qui calculi nescit rationem--Etymologies III, iv, 3 (PL 82, 156). My translation.

A related but even more open attitude is expressed by Hugue de Saint-Victor (c. 1096-1141). In *Didascalicon* VI, iii he exhorts »learn everything, and afterwards you shall see that nothing is superfluous«[18]. The immediate context, to be sure, is »sacred history«, and strictly speaking we are only exhorted to learn everything from this subject--but the examples leading up to the conclusion show that acoustical, arithmetical, and geometrical experimentation and astronomical observation are no less praiseworthy.

This same all-devouring and undistinguishing appetite is also obvious if we look at the list of translations undertaken by the single translators[19]. The interest was (if we restrict the investigation to mathematics) directed to *anything mathematical* at hand; no higher point of view (philosophical or other) beyond availability and comprehensibility selected the material, which therefore turned out to constitute a rather eclectic heap by the early thirteenth century. Behind the total endeavor of translation lay, however, a philosophical formulation of the intellectual appetite: *The interest in the existing world*. As long as mathematics (and indeed *anything mathematical*) was understood as a necessary tool for this enterprise, eclecticism was in itself a consequence of the dominating philosophical point of view.

Referring to the title of my paper we may therefore claim that philosophy was only implicitly expressed through the new character of twelfth century mathematics but on the other hand an essential background to this character and hence *not epiphenomenal*. On the other hand, however, the »philosophy« in question was to a large extent an intellectual attitude rather than an explicitly formulated structure of thought, especially in its relation to mathematics. Philosophy *stricto sensu* was therefore neither essential nor epiphenomenal in relation to the twelfth century developments of mathematics: It was a *sleeping partner*. Moreover, the interest in mathematics was so unspecific, namely an interest in what might serve as description of the existing world and in systematic thought, that *mathematics itself* could be declared an epiphenomenon: Mathematics was chosen because *it was traditional*, because it *promised to fulfill urgent intellectual needs*, and because it *happened to be at hand*[20].

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[18] *Omnia disce, videbis postea nihil esse superfluum* (*PL* 176, 801). My translation.

[19] This is precisely the way the material is organized in Steinschneider 1904.

[20] Mathematics was not the only candidate at hand, unless we restrict the concept of »existing world« to that of »physical world«. For the worlds of metaphysics,

THE THIRTEENTH CENTURY

The thirteenth century is well known in the history of education and universities to be the century of Aristotelization. Of course, Aristotle could not displace *everything* else, and from critical sermons we know that not only Aristotle but also geometry and mathematical astronomy could keep students from the pure spring of theology[21]. Still, both ecclesiastical trials, surviving university curricula and the sources in general confirm that Aristotelian learning displaced every competitor to the position as main intellectual challenge, tool and stimulus.

What happened to mathematics and mathematical interests in the scholarly environment under these conditions? I shall try to approach this question from a variety of specific viewpoints before giving a synthetic answer.

First of all it should be emphasized that the general mathematical level among arts students was apparently raised from (say) 1180 to 1280. The enthusiasm of translators and their immediate followers should not make us believe that normal students (even when sharing the enthusiasm) had digested much of the meal of translations. During the thirteenth century, however, elementary introductions to the art of algorism and to elementary spherical geometry became widespread at the universities, and computus remained a living subject, treated several times by Robert Grosseteste (c. 1175-1253) and even by as fine a mathematician as Campanus de Novara (c. 1220-1296)[22]. Thanks especially to Alexandre de

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moral, Canon Law and the Scriptures the new dialectical method was the obvious choice, and it was certainly no less chosen than mathematics. Both, indeed, fulfilled the need for intellectual coherence growing out of the flourishing environment of schools and educated clerks.

[21] See the »combined sermon« compiled from a variety of real sermons by Haskins (1929: 46f).

[22] See Thorndike 1954 on the continuation of computistic creativity at least until the end of the thirteenth century. In fact, Cardano, Stifel and Clavius would still

Villedieu's (d. c. 1240) and Sacrobosco's (fl. 1220-1244) pedagogical successes these topics were certainly better mastered by many scholars by 1280 than a century before[23].

Interactions with philosophy are, however, not to be expected (nor, in fact, to be found) at the level of compendia and elementary treatises. At most they show us that the eclectic temper of the twelfth century had not vanished. What then about the *Elements*, probably the best occasion for metamathematical reflection that could be imagined?

One side of that question is the problem of diffusion: How much was generally taught? Hardly three or four propositions, if we are to believe Roger Bacon (c. 1219-c. 1292)[24]; 15 Books, according to a collection of *quaestiones* from Paris[25]. A commentary probably written by Albert the Great (c. 1200-1280), dealing (not always very correctly) in full with Book I and briefly with Books II-IV[26] is probably the best hint we can get of the range of normal teaching at the highest level. If the usual discrepancy between teaching and learning is taken into account we may safely assume that few scholars, and few active philosophers, were in possession of a knowledge of theoretical mathematics (or applied metamathematics) which could serious challenge their philosophical tranquillity.

The other way round, there is more reason to expect an influence, since everybody writing on mathematics in the thirteenth century university would be well versed in Aristotelian philosophy and in the traditional metamathematical theory of the Latin quadrivium, and presumably more disposed to accept his upbringing than those rebels who had left »the cradle of [Latin] philosophy« for Toledo or Sicily a hundred years earlier.

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write on the subject in the sixteenth century.

[23] Cf. Beaujouan 1954 and Evans 1977.

[24] *Opus tertium* VI, ed. Brewer 1859: 21. Given Bacon's polemical aim and irascible temper and his lack of deeper mathematical understanding there is of course no reason to take his testimony to the letter.

[25] Discussed in Grabmann 1934. The collection may date from the 1240es, the time when Bacon was in Paris.

[26] Discussed in Tummers 1980.

The obvious place to look is the Campanus edition of the *Elements* themselves[27]. Its character has been discussed on several occasions by John Murdoch[28], for which reason I shall be very brief. In a certain sense the above hypothesis is confirmed: Especially in Book V, we find references to Plato's *Timaios*, to Boethius' *De musica*, and in particular to Aristotle's conceptual gunnery, needed for the discussion of quantity versus number, degrees of abstraction, and of the necessity that all four quantities in a proportion be of the same nature in the *permutatim* mode.

--But only in a certain, restricted sense is Aristotelian philosophy an active moulding factor. It contributes no doubt to that greater stringency which distinguishes Campanus' work from mathematical writings from the tenth through twelfth centuries. Campanus would never regard the abacus a geometrical subject just because it makes use of a ruled board. But there is little specifically Aristotelian about the stringency, which is rather a stringent application of mathematical sources. This is revealed even in small details, e.g. that Campanus speaks of *communes animi conceptiones*, a traditional Latin translation of Euclid's *koinai ennoiai*, instead of using *dignitates*, the term used in current translations for Aristotle's *axioms*. What might look superficially as thorough orientation after Aristotelian modes of thought is rather a didactical dressing of the subject-matter, connecting it to familiar patterns of thought without really determining the choice of subject[29] or approach.

In so deeply a didactically determined tradition as that of scholasticism (down to the name, we observe!) superficial correlations with other parts of the curriculum should cause no amazement, and they can be found everywhere in the Medieval mathematical sources, which (like Medieval learning in general) were somewhat at odds with the Aristotelian compartmentalization of

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[27] Datable to the 1250es (see Benjamin & Toomer 1971: 4f). I used the second Basel edition from 1546 (*Euclidis Megarensis mathematici clarissimi Elementorum geometricorum libri xv*), which contains the Campanus edition in parallel with Bartolomeo Zamberti's translation from the Greek.

[28] 1968; and "Euclid: Transmission of the Elements", DSB IV, 446f.

[29] Even though we are dealing with a translation the choice of subjects is not fully fixed in advance. In fact, Campanus adds a number of extra propositions to Book V--cf. Busard 1972: 131ff.

knowledge[30]. A field like mathematics might from its internal epistemological impetus have a tendency to be governed by its own laws and rules, in agreement with Aristotelian ideals[31]; but scholars moving in their teaching from one *artes*-subject to another, teaching students who followed a broad range of courses, would rather act against the inherent tendencies of the subject than let their avowed philosophy strengthen it.

A characteristic instance of such purely external Aristotelization of a mathematical subject is found in Petrus Philomena de Dacia's (fl. 1290-1300) commentary (written 1291/2) to Sacrobosco's *Algorismus vulgaris*. Already Sacrobosco had quoted Boethius' *Arithmetica* to the effect that the art of number be a prerequisite for knowledge of anything, and given the Aristotelian epithets *materialiter* and *formaliter* to Boethius' two different definitions of *number*[32]. In his commentary to this, Petrus Philomena takes the opportunity to speak broadly about *the four Aristotelian causes* of the art of algorism[33]. Neither in Sacrobosco nor in Petrus Philomena's commentary is there, however, any influence of Aristotelization in what Petrus identifies as the *pars executiva*.

If we go back in time from Petrus Philomena and Campanus to Jordanus de Nemore (fl. somewhere between 1220 and 1250) we shall find an author more governed in his whole mathematical activity by a philosophical stance[34]. To some extent this stance was Aristotelian. Firstly, Jordanus appears to have taken the Aristotelian distinction between different sciences more in earnest than contemporary mathematicians. So, when writing mathematics (the only subject

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[30] Cf. the many instances of combination of Boethian arithmetic with themes from the trivium in Evans 1978.

[31] Ideals which had originally been inspired not least by the rise of theoretical geometry as an autonomous field of knowledge.

[32] Compare Sacrobosco's text in F. S. Pedersen 1983: 1741-4,16-18, with Boethius, Arithmetica I, ii and I, iii (ed. Friedlein 1867: 12-13).

[33] F. S. Pedersen 1983: 81-85. Concerning the »final cause« Petrus states that »according to the author the purpose of this art is the knowledge of everything; but *I* believe that its more immediate purpose is nothing but astronomy« (p. 82[35ff]; my translation).

[34] In the following discussion of Jordanus I draw heavily on my 1985a. It is to be observed that Jordanus's philosophical attitude must largely be read between the lines.

on which he wrote) he would not involve the usual array of didactical cross-references to other Liberal Arts, nor begin discussing the obvious astronomical applications of a theory of the stereographic projection. Secondly, one of his works, the geometrical *Liber philotegni* (which appears to have grown out of a series of university lectures, themselves known from a student's *reportatio*, the *Liber Jordani de triangulis*[35]) starts by a set of very Aristotelian definitions of *continuitas, punctus, continuitas simplex, duplex* and *triplex, continuitas recta* and *curva, angulus* and *figura*. Thirdly, Jordanus' *Arithmetica*[36] presents its initial axioms as *dignitates*, not as *communes animi conceptiones*. Fourthly and finally, most of Jordanus' works were labelled *demonstrationes* in their own time, and probably by the Master himself, i.e., they were understood as faithful to the ideals set forth in the *Analytica posteriora*, in contrast to the *experimenta* of algebra and algorism[37].

The second and third feature look like expressions of explicit philosophical commitment; they are, however, superficial and as irrelevant to the subject-matter as the Aristotelian causes to the *pars executiva* in the algorism (and in the proofs of the *Arithmetica* the *dignitates* are referred to as *conceptiones*). They can safely be seen either as didactically motivated philosophical lip-service or as joking flirt (indeed, the *reportatio* mentioned above suggests that Jordanus' lectures were full of jokes). On the other hand, the first and the fourth feature touch the very essence of the Jordanian *opus*, and are truly exceptional in the thirteenth century. Since Jordanus was obviously a pure mathematician by inclination, there is no

[35] Critical editions of both treatises in Clagett 1984. The characterization of the Liber de triangulis as a reportatio is my own conclusion from a close analysis of stylistic features of the text--see my 1985a.

[36] I used the edition in Lefèvre d'Étaples 1514, Book I of which is reprinted in my 1985. Barnabas B. Hughes is now preparing a critical edition of the work (personal communication).

[37] In the catalogue of his library (the *Biblionomia*, ed. Delisle 1874: 520-535), Richard de Fournival (b. 1201, d. before 1260) opposes the Jordanian genre *apodixis* (the Aristotelian term translated *demonstratio*) to precisely such *experimenta*. As discussed in my 1985a, Richard appears to have been personally acquainted with Jordanus and collected apparently all of his works. The characterization of the *genres* is hence probably faithful to Jordanus' own ideas and ideals.

reason to believe that he needed Aristotle's permission to be one[38];
on the other hand, his personal inclinations, obviously inspired by the Ancient pure mathematicians, made him agree much better with ideals formulated by Aristotle in the environment of these same authors than did those who tried to understand Aristotle on the conditions of the thirteenth century.

There is a certain parallel between Jordanus' Aristotelianism and that of the Averroists. In the *De eternitate mundi* Boethius de Dacia (fl. 1277) had distinguished *veritas naturalis,* the truth of natural philosophy, from *veritas christianae fidei et etiam veritas simpliciter*, »Christian, that is genuine, truth«[39]. As I read the treatise there is no doubt that Boethius was sincere in admitting the ultimate truth of Faith; still, being a philosopher by profession, by training and by inclination he claimed the right (and claimed it an obligation) to investigate that *natural truth* which was set into operation at God's creation. Boethius' position was only the extreme consequence of an otherwise accepted philosophy (and so indeed an appropriate expression of the inherent rationale of the Thomistic synthesis); but being extreme it revealed that the thirteenth century was not disposed to draw the full consequences of the Aristotelian division of the world into separate and semi-autonomous levels (nor that autonomy of single social groups which was its parallel), and Boethius was condemned in 1277.

Jordanus too was condemned--not by any bishop hostile to mathematics but by those closest to his enterprise. There seems indeed to have existed in Paris a whole Jordanian circle in or around the 1240es, embracing among others Campanus of Novara and in some way even Roger Bacon; but apart from the otherwise unknown Gérard de Bruxelles, author of a *Liber de motu* in Jordanian

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[38] There is even some positive evidence that he didn't care. Aristotle had once distinguished the *sophist*, who when discussing geometrical questions would use arguments which by nature were alien to geometry, from the *pseudographos*, who would use legitimately geometric though misleading arguments (see *De sophisticis elenchis* 171^b36ff and 171^b14ff, and *Topica* 132^a33). In thirteenth century mathematics both figures were identified with the *opponent* in a university disputation, and so they are in Jordanus' *Liber philotegni*, prop. 18: Jordanus apparently felt no need to support himself on Aristotle's strict distinction though obeying it himself.

The universitarian tradition continued the *quiproquo* for centuries: In a disputation from Leipzig in 1512, *falsigrafus* is used to designate precisely that argument which Aristotle uses to exemplify sophist ways (see Suter 1889: 19).

[39] Ed. Sajo 1964: 46. Similar formulations *passim*.

style, none of his disciples or associates cared to continue or defend the specific character of Jordanian pure mathematics. On the contrary: those who edited his treatise on the stereographic projection hurried to put in all those references to celestial circles, stars and astrolabe which Jordanus had discarded[40]. Jordanus, the only mathematician in the Latin thirteenth century doing mathematics in reasonable agreement with Aristotelian precepts, was *eo ipso* unacceptable to his contemporaries.

Essentially, this agrees with the evidence offered by Albert the Great and Thomas Aquinas (1225-1274). As mentioned above, Albert appears to have written a commentary on *Elements* I-IV. According to Tummers' analysis it is not very original, drawing heavily on al-Nayrîzî's commentary and other available material; the philosophical introduction is apparently »more in keeping with the Platonic-Pythagorean tradition than with the Aristotelian«[41]. In the Aristotelian paraphrases Albert is more philosophically stringent and clearly Aristotelian,- but he demonstrates no striking mathematical competence, nor were discussions of infinity and continuity mathematically productive (at most they were counterproductive, since Albert's basic point was to »maximize the gap between mathematics and the natural world«[42] and to concentrate interest on the latter). St. Thomas's treatment of metamathematical questions (in the Commentary to Boethius' *De trinitate*[43]) is philosophically more original and more interesting and much more positive in its evaluation of the relevance of mathematics for understanding the real world, but it floats miles above the level of actual mathematical work[44]. In its consequences, it will have been no more effective than Albert's more diffuse and more distrusting attitude.

The place to look for genuinely philosophical inspiration of mathematical activity is rather *outside* the most stringently Aristotelian circles, *viz.*--commonplace as it is--in the quarters of Neo-Platonic inspiration. Thomas himself

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[40] Both the original treatise and the different adaptions were edited critically in Thomson 1978.

[41] Tummers 1980: 483.

[42] Molland 1980: 472. The article contains many quotations from the paraphrases.

[43] See the translation of the relevant Questions V and VI in Maurer 1963, or the brief discussion in Weisheipl 1975: 134-136.

[44] The index in Weisheipl 1975 illustrates this beautifully by its »mathematics, *See* science and scientific method«.

is not fully a stranger to Neo-Platonic ideas, but dissociates himself from those, »for example, the Pythagoreans and the Platonists«, who asserted »that the objects of mathematics and universals exist separate from sensible things«[45].

This statement expresses Aristotle's interpretation of the Platonic view; closer to those inspired by Platonism in the thirteenth century would be a claim that mathematics was closer to *real*--divine--reality than are the sensible things. This point of view results (and resulted) easily when the twelfth century confidence in the descriptive power of mathematics (as described above) is taken to its philosophical consequence. In Roger Bacon's diffuse mind the two views are not easily separated. In other authors, a clearly Augustinian illuminationist stance is more obvious though rarely in sole and supreme reign.

A first name to be mentioned is William of Moerbeke (b. c. 1230, d. before 1286), who translated not only Aristotle but also Archimedes, Eutocios, Proclos, Alexander of Aphrodisias, Philoponos and others directly from the Greek[46]. His Neo-Platonic convictions are visible in his choice of authors to translate; in a *Geomantia* probably from his own hand; and especially through a dedicatory letter written by his friend Witelo in the latter's *Perspectiva*[47]. As it is made clear through the testimony offered by Albert and Thomas, a decision to translate the full Archimedes was far from inevitable even for a dedicated translator. The translation itself[48] gives no clues for Moerbeke's motives, but it is a fair guess that his Neo-Platonism played a major rôle, presumably together with an incipient philosophically supported friendship with Witelo the mathematician (Witelo arrived at the Papal court in Viterbo in 1268, at which occasion he met Moerbeke[49], and Moerbeke's mathematical translations are from 1269).

The connection to Witelo leads us to a whole cluster of names, *viz.* Grosseteste, Roger Bacon, Peckham (c. 1230-1292), Witelo (b. c. 1230, d. after c. 1275) and (as a partial contrast) Dietrich von Freiberg (c. 1250-c. 1310), and to *optics*, one of the two »new« mathematical disciplines of the Latin thirteenth century (Jordanus' statics being the other).

Grosseteste's rôle in this connection is mainly that of giving inspiration. Like

[45] *Commentary* ..., Question V, article 2, transl. Maurer 1963: 34.

[46] Detailed list in Minio-Paluello, “Moerbeke, William of”, *DSB* IX, 434-440.

[47] Ed. Risner 1572: II, 1-2; reproduced in Clagett 1976: 8f, note 30.

[48] Critical edition and translation in Clagett 1976.

[49] See Lindberg 1971: 72.

every philosopher of his century he was of course inspired by Aristotle,- but *The Philosopher* was only one of several authorities to Grosseteste, who was definitely no Aristotelian as regards his position on the relation of mathematics to other subjects. In a small and probably very early treatise *De artibus liberalibus*[50] he tells how informative these are for natural as well as moral philosophy. The discussion is not profound--astronomy, for instance, is important (in its astrological appearance) because it tells the right moment to act. Later works, however, show a fair acquaintance with astronomy, calendar construction and the fundamentals of optics[51]; when seen together with his philosophical works this mathematical competence makes his influence in his own and later centuries understandable. Important in the present context is of course his illuminationist coupling of optics with epistemology and with theologically tainted metaphysics (the theory of »multiplication of species«).

One of those to be impressed was Roger Bacon, whose grandiloquent confidence in his own mathematical competence has made later times accept it. So much truth is contained in the claim that Bacon was familiar with lots of mathematical authorities (and authorities of any scholarly discipline!), and that he was able to construct simple but relevant geometrical arguments pertinent to many optical observations and informal experiments. He combines sense for physical reality with a belief in the potency in mathematics which seems often more phantasmagoric than just Neo-Platonic. This would certainly have had more appeal a century or two later, but even in the thirteenth century it might have aroused an appreciable echo, had Bacon not been kept imprisoned and the circulation of his writings restricted for reasons which are only indirectly connected to his mathematical philosophy (if at all). His influence in broader circles was therefore modest, and passed mainly through whatever Baconian material was adopted by Peckham and Witelo[52]. Peckham himself appears to have belonged to the environment inspired by Grosseteste (in any case, he was a Franciscan and one of the founding fathers of neo-Augustinianism[53]) and wrote

[50] Ed. Baur 1912: 1-7.

[51] Also in Baur 1912. Cf. the discussion in Crombie, “Grosseteste, Robert”, *DSB* V, 548-554.

[52] See Lindberg 1971 on the connections from Bacon to Witelo and Peckham.

[53] Cf. van Steenberghen 1955: 98-104.

on mystical numerology in combination with Boethian arithmetic[54]; even in his *Perspectiva communis* he reveals himself as a Neo-Platonist of independent rather than Baconian inspiration--not only in his characterization of the Lord as *lux omnium* in the preface (which might be nothing but a poetical metaphor), but also e.g. in a discussion in prop. I.6[55] of Moses Maimonides' claim that there is an »influence of a particular star directed to each particular species« in this universe which is »like one organic body«. Probably, Peckham was therefore primarily interested in optics because of Grossetestian inspiration and Neo-Platonic inclinations; Bacon supplied him with factual material only.

Witelo seems to present us with a precise analogy to this. As he tells in the dedicatory letter mentioned above, Moerbeke made him commence the work as a means to know »how the influence of divine powers *(virtutes)* affects lower bodily things through higher bodily powers«[56]--and according to a remark in prop. X.42 his first interest in the matter had been aroused by observations of intriguing physical phenomena[57]. It appears that Bacon's optical manuscripts simply happened to be present and available (through Moerbeke's influence?) in Viterbo when they were needed.

It is hence probable that *all* important writers on optics in the thirteenth century were inspired from Neo-Platonic philosophy (or at least Neo-Platonically tainted philosophy), and that most of them sustained a eo-Platonic belief in the

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[54] See the description of the *Arithmetica mystica* in Lindberg, "Pecham", *DSB* X, 474.

[55] Ed. Lindberg 1970. The quotations from Moses Maimonides (which are *not* found as quotations but only hinted at in Peckham's work) are from the *Guide for the Perplexed* II,x (on »the influence of the Spheres upon the Earth«)--transl. Friedländer 1904: 164.

[56] Clagett 1976: 8 n. 30; or Risner 1572: II, 1.

[57] Risner 1572: II, 440. From the dedicatory letter it seems that Witelo was already engaged in Neo-Platonic reflections before meeting Moerbeke (and more specifically engaged in a work *De ordine entium* which he postponed). Moerbeke will then have explained to him the importance of *light* for understanding *that Divine light* which connects the different orders of entities--an idea which will have caught Witelo's interest because of his own physical observations. Witelo appears *not* to have brought any Baconian or Grossetestian inspiration with him to Viterbo: the terms in which the Neo-Platonic ontology is set forth differs from theirs, and is more orthodox.

explaining power of mathematics. The case of the first fourteenth-century writer on the subject, Dietrich of Freiberg, *may* be different. He rejected the Grossetestian »metaphysics of light« as well as the belief in the mathematical structure of Nature[58]. But he *was* interested in Neo-Platonic doctrines. All in all, he must probably be taken as evidence that a mechanical coupling between specific philosophical doctrines and the interest in optics was less important than more fundamental levels in the Neo-Platonic orientation, rather than as a witness of a mathematical autonomization of the subject which by c. 1300 would have cut it off from philosophical inspiration. The interest in optics is hence a link backwards to the twelfth century proto-philosophical enthusiasm for mathematics.

In this respect, the interest in optics is a close parallel to that in astronomy and astrology--and as we have just seen, the former is often interwoven with the latter. This rôle for astronomy is no marvel. If any field confirmed the twelfth century conception of mathematics as a way to true knowledge it was certainly astronomy, from *computus* to Ptolemean planetary theory--and astrology was then (as we have seen in Grosseteste) the way to make mathematics a way to knowledge of almost any kind. True, *music*, the mathematical theory of harmony, can be claimed to be equally well described through mathematical relationships. The theory of harmony, however, would only ask for the use of a fairly simple arithmetic of proportions and its relation to actual sound was limited. Planetary astronomy, on the other hand, dealt with the real celestial bodies, and could make use of almost any available level of mathematical sophistication. So, wherever you were on the level of mathematical learning you might see *your own* mathematics as an efficient tool.

Astrology was similarly accessible on many levels. It could be justified through sophisticated Neo-Platonic philosophy, as we have seen in Maimonides and the perspectivists;- but it could also be exerted as a complicated but aphilosophical technique of prediction and warning, and even be grasped as such by an illiterate public. No wonder, all in all, that the complex astronomy+astrology came to be regarded by many as the ultimate purpose of mathematics (its *final cause*, as stated by Petrus Philomena)[59]. This scale of values

[58] According to Wallace, “Dietrich von Freiberg”, *DSB* IV, 92.

[59] In reality, it had often been so already in the twelfth century. In this connection the importance of the *Almagest* for twelfth century translators will be remembered. As Lemay (1962) points out, astrological translations were also the first source for Aristotle’s natural philosophy.

was institutionally confirmed in the University of Padua, where quadrivial teaching was given in the common *artes* and medical faculty by physicians as a tool for astrologico-medical prognostication[60]. In other universities and centres of learning, where ecclesiastical, Thomistic or Albertian skepticism might be expected to have a greater influence, no institutional fixation occurred, but on the level of scholarly interests the difference was faint. The irony of history even led to the result that Albert's fame in later centuries connected him mainly with astrological and other occult subjects, on which the list of spurious Albertian works contain an impressing number[61]. Even though astronomy led to no significant development of *new* mathematical results or mathematical creativity[62] there is thus no doubt that the enthusiasm for astrology (and hence the quasi-philosophical attitudes giving rise to this enthusiasm) was the main incentive behind the spread of basic mathematical competence in the scholarly environment[63].

So, if we expected the century of Medieval Aristotelianism *par excellence* to

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Twelfth century scholars were also aware that astronomy was a main mobile for mathematical activity in the Islamic world. This appears from John of Salisbury's *Metalogicon* IV, vi (from 1159), where he tells that *»demonstration«*, i.e. the use of the principles expounded in Aristotle's *Posterior Analytics*, had by his times *»practically fallen into disuse. At present demonstration is employed by practically no one except mathematicians, and even among the latter has come to be almost exclusively reserved to geometricians. The study of geometry is, however, not well-known among us, although this science is perhaps in greater use in the region of Iberia and the confines of Africa. For the peoples of Iberia and Africa employ geometry more than do any others; they use it as a tool in astronomy. The like is true of the Egyptians, as well as some of the peoples of Arabia«* (transl. McGarry 1971: 212).

[60] See Siraisi 1973: 67f and *passim*. The situation was similar in Bologna (see Rashdall 1936: I, 242f, 248f). On Paris, see Lemay 1976.

[61] Albert had in fact written a survey of such subjects, in his *Speculum astronomiae, in quo de libris licitis et illicitis pertractatur* (in Albertus Magnus 1651: 656-666).

[62] In sharp contrast to what happened in the Islamic world--cf. discussion in my 1987.

[63] I leave the discussion of merchants' mathematics and of the gradually growing mathematical abilities in non-scholarly environments to my treatment of the fourteenth and fifteenth centuries.

be the century where Aristotelian philosophy shaped mathematics we will be disappointed. If we expect (e.g. because we know the wave of mathematical creativity which in the Islamic Middle Ages followed upon the translations) that theof shouldled

overwhelming mathematical activity during the thirteenth we will be equally How are we then to sum up the trend of scholarly mathematical development in the thirteenth century, and how are we to explain it?

Firstly there is absence an of creativity
what might look as a tendency to trivialization. Before considering this as enigmatic we should, however, compare the »trivial« level of thirteenth century scholars not with that of the translations but with that of twelfth century schoolmen,- in which case we shall see that the apparent trivialization is an illusion, due to wider dissemination of mathematical competence at an intermediate at that of normal
scholars (whose did always competence, observed
above). If we cannot speak of an blast of creativity we can at least compare the situation to a blaze of wildfire. Because of its own »epiphenomenal« character discussed above, the twelfth century enthusiasm for mathematics could hardly produced anything more.

Next there is the question of Aristotelian influence on the development of thirteenth century mathematics. As observed above it is generally superficial, one (in
educational) context of learning. Only as far as the progress in stringency and the relative regress of omnivorous eclecticism is concerned was *Aristotelianism egarded philosophy* of possible importance the of mathematics
and as we have seen, the agreement of Jordanus' mathematical style with strict Aristotelian principles was no success). What can be seen in the sources is the imprint of *Aristotelianism regarded as a way to organize teaching*. The »four causes« up in Petrus Philomena's Sacrobosco-commentary (and in numerous other not because of any relation to the *pars executiva* of the algorism but because *this the to speak in the Arts.* The *falsigrafus* (cf. note 38) is there not because he represents an important point in Aristotle but because a *word* was found in *Topica* which when sufficiently twisted could be identified with an opponent in a university disputation.

In the formulation of this I have grouped works like the Albert(?)- to the *Elements* with philosophy rather than mathematics. My reason doing so is that they were apparently not *mathematically productive*, i.e. that no further work in the field was inspired by them (at the moment). If they represent a mathematical *genre* then (in the thirteenth century) only a mathematical dead-end.

In the terms of my title, Aristotelian philosophy was then no real cause (not even a contributory cause) for the direction taken by the development of thirteenth century mathematics--at most an epiphenomenon and one of the forces holding back for a while that Neo-Platonic« mood which saw in mathematics the principal way to real insight. If any philosophy contributed to the development of mathematics in the thirteenth century it was, indeed, Neo-Platonism--or, better, the Neo-Platonic aspect of various eclectic philosophies. This was obvious above when the perspectivists' motivations were investigated; as a mood rather than an explicit philosophy it will also turn up if we try to formulate the attitude behind the interest in astrology. It is one aspect of a *medico-astrological naturalism* which had already been active in the first import of the doctrines of Aristotelian natural philosophy, and which in later centuries (thanks not least to the influence of Paduan Averroism) led to the observation that »where three physicians meet two atheists will be present«. In the thirteenth century things had not developed that far. Already then, however, when we should still speak of a tendency rather than a current, this tendency would often imply a bent toward heterodoxy, and be at variance with Thomistic and Albertian rationality (until it succeeded in turning Albert upside-down). In mathematics, it would care little for proof and demonstrative structure; Richard de Fournival, physician and »well versed in mathematics« according to his own words[64] and intimately familiar with Jordanus' works and ideals (cf. above, note 37), would himself prefer *experimenta* and astrology for mathematical *apodixeis*. He was no exception (and Roger Bacon, who shared in the same idiosyncrasies if only in stronger form, is hence only a caricature but no stranger to his century[65]). In the centuries to come this combination of a Neo-Platonic confidence in the

[64] *Biblionomia*, the introductory passage (ed. Delisle 1874: 520).

[65] In the same connection (though with a change of emphasis from alchemy to Cabala) Ramon Lull (c. 1232-1616) could be mentioned, to the extent that his work are to be counted as mathematical. His squaring of the circle (critically edited with commentary in Hofmann 1942), at least, is a typical *experimentum*--and is, by the way, written as part of a skirmish with Paris scholasticism.

potency of mathematics« with emphasis on immediate practical insight and utility[66] would lead to continued reliance on the mathematical compendia written by thirteenth century scholars, which would thus form the mathematical culture of average scholars until well into the sixteenth century.

66 Cf. also Beaujouan 1957 on the tendency of scholasticism to require immediate utility of its science.

THE FOURTEENTH CENTURY

While the stereotype of the thirteenth century is that of classical Aristotelianism and Thomism, the stereotypical picture of the fourteenth century displays the *via moderna* and Ockhamism. And while the thirteenth century saw the transformation of the schools of Paris, Oxford etc. into »universities«, the fourteenth century brought the spread of the universitarian idea into German land and the creation of a multitude of new *studia generalia.*

The fourteenth century transformation of university learning can be discussed and described at many equally valid levels. On the level closest to the facts of daily teaching, the stabilization of the institution, of its teaching methods and of its curriculum, and the concomitant »professionalization« of the community of Masters of Arts[67] offer valuable explanations of the development. The social stabilization, the acquisition of a set of stable professional values and a teaching method based on intense and critical discussion allowed a systematic cumulation of scholarly insights--and since Aristotelian learning had been accepted as a common foundation, the cumulation resulted in cumulative development of Aristotelian doctrines in confrontation with new problems (development which at times brought them far away from Aristotle himself).

Some of these problems were of a mathematical nature, e.g. the »quantification of qualities«. And so, the fourteenth century produced that Aristotelian mathematics which had defaulted in the thirteenth. I shall not cover the current in depth nor mention all important authors or works[68], but only discuss a few select aspects pertinent to my subject.

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[67] » « because a modern *profession* is chosen more or less *for life*, while few Medieval scholars would remain Masters of Arts for decades. Still, many insights gained in modern sociology of professions are useful for understanding the Medieval Master of Arts. Cf. my 1980: 72ff.

[68] Excellent surveys are Murdoch 1961 and 1969.

Oxford appears to have been the place where the new philosophy was first developed[69]. It seems tempting to see this as a consequence of an old bent toward mathematics aroused by Grosseteste and Roger Bacon; the difference between the inspired Neo-Platonism of *their* mathematization of nature and the stringent intellectual style of fourteenth century scholars like Thomas Bradwardine (c. 1290-1349), Richard Swineshead (fl. c. 1340-1355) and William Heytesbury (fl. c. 1335) is, however, too great to make such a guess plausible. Furthermore, even though optical models were often used by the scholars of Merton College, the key abode of the new philosophy, the »metaphysics of light« appears to have played no rôle for them[70].

The problem which first comes to mind in relation with the Mertonians is the above-mentioned *quantification of qualities*, which has made many see the Merton school as a first step toward Galilei and Newton. A sense can of course be found which is vague enough to make such a conception plausible (especially if we content ourselves with the fact that Galilei was taught and learned from material going back to the quantifying schoolmen)--but in a strict sense it is definitely false: The Mertonian project was different, and had to be different, from that of the seventeenth century[71].

The main reason to see the Mertonians as proto-Galileans is that part of their work was concerned with the mathematical analysis of *motion*, in critical continuation of Aristotle's *Physica*. This, for instance, is the theme of Bradwardine's important *Tractatus proportionum seu de proportionibus velocitatum in motibus*[72]. But other qualities were also discussed from the quantitative point

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[69] See Weisheipl 1966.

[70] See Sylla 1971: 13-15. It should, however, be noticed that Bradwardine refers to Grosseteste with great veneration in his theological *De cause Dei* (see Baur 1912:108*).

[71] To state the difference pointedly: No fourteenth century user of the Aristotelian apparatus, however radically innovative his use, would claim with Galileo that »philosophy is written in this grand book, the universe, which stands continually open to our gaze» though written in the language of mathematics. On the contrary, in Mertonian hypothetical physics, as in »the *Iliad* or *Orlando furioso*, [...] the least important thing is whether what is written there is true«, the attitude with which Galileo charges his opponent (*Il saggiatore*, transl. Drake 1957: 237f).

[72] Ed., transl. Crosby 1955.

of view, both those of compound medicine[73] and those of the alchemical »primary qualities«[74], even though it must be admitted that the highest mathematical development was reached in connection with the treatment of motion. As it is evident from Bradwardine's title as quoted above, the mathematical theory of »proportions« (»ratios« in modern language) and proportionality was a central domain.

On the Continent, the most famous continuation of the Merton quantifications were Nicole Oresme's (c. 1320-1382) works. His *Tractatus de configurationibus qualitatum et motuum*[75], which is perhaps the most direct continuation of the Merton discussions, is famous for its introduction of a geometrical representation similar to a modern coordinate system of intensities of qualities subject to change[76], in which connection he also finds infinite sums geometrically (or rather, splits geometrical quantities into infinite sums)[77].

Other works from Oresme's hand are original not only in contents but also in aim. In the *Tractatus de commensurabilitate vel incommensurabilitate motuum celi*[78] (and elsewhere) he uses arguments from the theory of incommensurability against one of the pet tenets of astrology, *viz.* the existence of repeatable conjunctions. These arguments had been elaborated by himself in the *De proportionibus proportionum*[79]. This is one of two treatises where Oresme makes pure mathematics out of the problems of the theory of proportions as used to discuss

[73] See the summary of this question in Sylla 1971: 20ff.

[74] See Skabelund & Thomas 1969.

[75] Ed., transl. Clagett 1968.

[76] In fact, Oresme distinguishes such qualities from indivisible qualities and rejects the loose usage common among contemporary theologians applying concepts applicable only to quantifiable qualities e.g. to *caritas* (chapter I.ii, *ibid.* p. 170).

[77] Cf. discussion in Clagett 1968a: 206ff. Part of the discussion will be found in Oresme's *Quaestiones* to the *Elements* (ed. Busard 1961; cf. Murdoch 1964 and Zoubov 1968). This commentary is indeed quite different from those of earlier times, and raises questions concerned with Oresme's own theories to the Euclidean material, on which the *Quaestiones* have sometimes little bearing.

[78] Ed., transl. Grant 1971.

[79] Ed., transl. Grant 1966.

questions of motion. The other treatise is the *Algorismus proportionum*[80], which is much shorter but shares its main characteristics. As indicated by the name, it is an algorism, albeit a very special one. Like other algorisms it describes basic rules for computation--though not computation with hindu numerals nor at all with numbers but with ratios. The work is a beautiful piece of mathematical generalization, making use of the fact that the composition of ratios can be regarded as an addition--as the »addition« of a musical fifth and a musical major third gives an octavo, whence (3:2)“+”(4:3)=(2:1). This allows Oresme to define addition and subtraction of ratios, and to multiply and divide a ratio by a positive integer (whence even to multiply it by any rational number)[81]. Mutual multiplication and division of ratios cannot, however, be defined meaningfully, as observed by Oresme[82].

If we identify the ratio (a:b) with the real number a / b, »addition« of ratios becomes multiplication of real numbers, and the »multiplication« of a ratio (a:b) with the rational number p / q becomes the power (a / b)(p / q). It has therefore been customary to interpret Oresme’s theory as a theory for powers with rational exponents. In reality, however, Oresme’s scheme is much closer to the spirit of modern abstract group theory; in the language of abstract algebra his algorism is indeed, as he argues, a group allowing root extraction but not expandible into a ring or a field through techniques at Oresme’s disposal. (Similarly, the longer *De proportionibus proportionum* applies the structure abstracted from Euclidean arithmetic to the »addition« and »multiplication« of ratios and their inverse operations).

I have dealt at some length with this particular work because of its prototypical character for much of the new learning of the age. It continued a traditional subject (*in casu* one belonging to the rather elementary quadrivial level), but took it as nothing but a stepping stone for a discussion of pure

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[80] Ed. Curtze 1868. A partial translation accompanied by a not fully empathic commentary will be found in Grant 1965.

[81] It should be observed that the ratios in question need not be rational--the domain as defined explicitly by Oresme encompasses all ratios which can be written as »part or parts« of a rational ratio, in modern language all rational powers of rational ratios (ed. Curtze 1868: 12).

[82] I.xi, ed. Curtze 1868: 19. It *can* in fact be done, but only through introduction of the logarithmic function, and hence through an extension of the domain considered and the introduction of (e.g.) infinite »sums« of ratios.

principles (a second and third part then demonstrate »the very great usefulness« of these principles, *viz.* as structuring tools mainly in geometry, showing that abstraction was no goal *per se* to Oresme but rather the style of his whole intellectual context[83]). In the present case the result was a piece of pure structural mathematics, the underlying principles of which could not be understood before the twentieth century (and even then have normally been misunderstood by translators and commentators); in other cases the discussions led to semantic or logical theories with a similar fate. Generally, the *via moderna* led scholars into a highly sophisticated but also highly and narrowly specialized style of thought, particular results of which might at times be used in the following centuries, but whose outcome in the form of coherent structures was unable to find an echo in the Early Modern age.

Up to now the discussion was concentrated on the highest level of fourteenth century philosophical mathematics. As it is to be expected, this high forest was surrounded by a humbler underbrush of similar orientations. In order to get an impression of this broader environment we may look at a list of *quaestiones* from Paris, datable to c. 1330 (since the terminology is in many places too technical to allow meaningful translation without an extensive commentary I leave it untranslated):

1. *Utrum entia mathematica sint abstracta a sensibilibus qualitatibus.*
2. *Utrum mathematica abstrahant a motu.*
3. *Utrum mathematica sint coniuncta in esse cum qualitatibus sensibilibus.*
4. *Utrum mathematica sint priora qualitatibus sensibilibus.*
5. *Utrum de mathematicis sit scientia.*
6. *Utrum de substantiis possit esse scientia mathematica.*
7. *Utrum de qualitate sensibili possit esse scientia mathematica.*
8. *Utrum omnia entia mathematica et omnia sensibilia communia sint per se sensibilia.*
9. *Utrum scientie mathematice habeant aliquam communem materiam.*
10. *Utrum sint tantum quattuor mathematice.*

[83] We could even claim with Olaf Pedersen (1956: 99) that Oresme cannot be understood as a »mathematician« but only as a philosopher of nature.

11. Utrum geometria sit prior arithmetica.

12. Utrum mathematice scientie sint certissime.

13. Utrum scientie medie [like optics] *sint magis naturalis vel mathematice.*

14. Utrum entia mathematica diffiniatur per materiam intelligibilem.

15. Utrum quantitas sit per se divisibilis.

16. Utrum de numero sit scientia.

17. Utrum numerus sit ens reale extra animam.

18. Utrum unum et multa opponantur.

19. Utrum numerus componatur ex unitatibus.

20. Utrum numeri differant specie.

21. Utrum diffinitio numeri sit bene data.

22. Utrum continuum indivisum sit principium numeri.

23. Utrum subiectum in geometria sit magnitudo vel aliquid aliud.

24. Utrum quantitas precedat formam substantialem in materia.

25. Utrum sit dare dimensionem terminatam et indeterminatam.

26. Utrum dimensio terminata et indeterminata sint generales et corporales.

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27. Utrum punctus diffiniatur.

28. De diffinitione linee

29. Utrum linea componatur ex punctis.

30. Utrum punctus sit aliquid vel nihil.[84]

Many of these questions point back to discussions in St. Thomas and Albert (and the discussions refer indeed to both). Others remind of the introductory definitions from Jordanus' *Liber philotegni*; their real connection, however, is not to these (which, as we remember, had been completely external to Jordanus' *pars executiva*), but to active fourteenth century discussions of the nature of the

[84] Grabmann 1930: 77f (I corrected »intelligibidem« in question 14 to »intelligibilem«). The first 26 questions are due to Sebastian of Aragonia, the last four to Theobald of Anchora.

continuum and continuity, of the point, and of atomism; Clagett speaks of a general »current of “scholasticizing” geometry by techniques followed in natural philosophy«[85], and points out that it was felt even more widely: so in general theoretical geometry, as illustrated by Bradwardine’s *Geometria speculativa* [86]. While the Albertian commentary to the *Elements* had not been mathematically productive in its own century, a similar orientation *did* hence lead to the creation of an integration of mathematics, metamathematics and Aristotelian philosophy in the early fourteenth century.

It goes almost by itself that the tendency toward integration of mathematics and Aristotelian philosophy (be it forest or underbrush) did not dominate the landscape completely--and far from that. In universities many habits from the thirteenth century were continued (as revealed by the continuous use of the compendia from good old times); the trend toward astronomical preponderance was continued. Nothing in this situation calls for discussion beyond what was given above. Finally, however, it must be mentioned that some university scholars began taking active interest in the higher levels of practitioners’ mathematics. A first instance of this, *viz.* an anonymous treatise *De regulis generalibus algorismi ad solvendum omnes questiones propositas*[87] may even go back to the very late thirteenth century. It contains rules for solving problems of the first degree familiar from many older problem collections inspired by practitioners’ methods--e.g. to »find the length of a (broken) lance when a third of it is embedded in the bottom of a pond, a fourth is in the water, a fifth is lying on the water, and 26 feet remain outside the pond«[88]. The rules for solving the problems (rules given without any proof) are, however, *not* those current among the practitioners (in case the very flexible »single false position«) but derived from quadrivial arithmetic.

This short treatise is a first step toward integration between quadrivial and

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[85] Clagett 1964: 40. Works illustrating the field of interest are e.g. Bradwardine’s *Tractatus de continuo* (described in Stamm 1936) and Buridan’s (c. 1295-c. 1358) *Quaestio de puncto* (ed. Zoubov 1961). See also Zoubov 1959; Clagett 1962; and articles by Murdoch, Sylla and Normore in Kretzman 1982.

[86] Molland 1978 is a discussion of the work based on an unpublished critical edition. I used the 1530-edition.

[87] Critical edition and discussion in Hughes 1980.

[88] *Ibid.* p. 270 (Latin pp. 22186-2221).

»commercial« mathematics. Another, much longer step is taken by Jean de Murs (fl. 1317-134) in his *Quadripartitum numerorum* from 1343[89]. This treatise deals successively with the traditional quadrivial arithmetic (Boethian as well as Euclidean); with the art of algorism (concentrated on the treatment of fractions); with al-Khwârizmîan second-degree algebra; and with a richly varied material drawn from Leonardo Fibonacci's *Liber abaci* and *Flos super solutionibus*. The final part, on applications of arithmetic, presents *inter alia* Archimedean statics.

Another work written in part and brought to completion by Jean de Murs is the *De arte mensurandi*[90]. Thanks to Jean, a traditional mensuration treatise is integrated with much material drawn from Moerbeke's Archimedes and from *Elements* X; in many cases, proofs for the stated theorems are given, and in others it is told where material for a proof may be found (mainly in Euclid and Archimedes).

Two things should be observed concerning Jean de Murs and the two works just mentioned. First, Jean's approach to the material reminds of that of the better mathematicians of the following century, as we shall see below; and indeed, Regiomontanus planned to make an edition of the *Quadripartitum*[91] and possessed a copy of a treatise abbreviated from the *De arte mensurandi*.[92] Second, Jean differed completely from e.g. Oresme in his attitude to astrological prediction, and believed firmly in his own predictions[93]; in this respect he was no different from those of his contemporary mathematicians who like him fell outside the current of »philosophical mathematics«.

What was said up to this point on fourteenth century mathematics was

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[89] Brief, mutually supplementing descriptions in G. l'Huillier 1980: 194f, and Poulle, "John of Murs", *DSB* VII, 129; a number of excerpts were given by Karpinski (1912).

[90] Briefly discussed in Victor 1979: 49-51, and in Poulle, "John of Murs", DSB VII, 129. Busard 1974 contains a more thorough discussion of a single section, while Clagett 1978: 19-44 reproduces and discusses a number of Archimedean passages.

[91] See below. G. L'Huillier 1980 describes the annotations made in the manuscript by Regiomontanus.

[92] Busard 1974: 151.

[93] Firmly enough, in fact, to propose a crusade to the Pope, the success of which seemed guaranteed by a favourable conjunction--see Poulle, "John of Murs", *DSB* VII, 131.

concerned with mathematical currents associated with the university sphere. Universities, however, were not the only focus of mathematical development. Another one was the Italian *abacus school*, cf. above, note 14. Its early history is unclear, but around the end of the thirteenth century it seems to have reached maturity. The basic curriculum does not look particularly advanced, but it gave occasion for interest in more advanced extensions of the same fields. In this connection it is characteristic that only two of the manuscripts of the *Liber abaci* listed by Boncompagni[94] date from the thirteenth century (and both of these from the later part of the century); 4 were written in the fourteenth century, and 4 date from the fifteenth. From the fourteenth century a number of shorter abacus treatises are also known, dealing with the full curriculum as described in note 14 and containing a wide range of problems belonging to these fields together with some practical geometry and occasionally some second-degree algebra[95]. Problem collections of a similar sort, but concentrated on the »pure«, recreational outgrowth of practical arithmetic, are known from a number of fourteenth century monastic manuscripts[96]. None of all this contains the slightest hint of philosophical ideas or attitudes.

The principles and main trends of the fourteenth century development are then easily summed up. Around the creative Aristotelianism and the professionalization of the Master of Arts a highly sophisticated type of mathematics emerged, which in itself was also an important tool for the new philosophical style. Here, for the first time, Aristotelian philosophy was a contributing cause for the evolution of a new kind of mathematics, acting together with the social organization of knowledge.

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[94] 1851: 31-59.

[95] E. g. a *Libro d'abaco* from Lucca, published by Arrighi (1973), with second-degree algebra (pp.108-114) and geometrical calculations (pp. 114-121); the Florentine Paolo dell'Abbaco's (c. 1281-1374) *Trattato d'arithmetica* (ed. Arrighi 1964, with geometrical calculation pp. 104-138 and second-degree algebra pp. 143-148); and the mid-fourteenth century *Rascionei d'Algorsmo* from Cortona (ed. Vogel 1977, with geometrical calculation pp. 134-141 and 149f but no algebra).

[96] See Folkerts 1971. One short collection is dated to the thirteenth century and a manuscript containing a single problem to 1292.

I discuss the relation between practitioners' mathematics and its recreational outgrowth in my 1987 and 1987a.

Concomitantly, the traditional mathematical style was still in existence, bound up especially with astronomy and astrology. It was apparently not bound up philosophically, neither to Aristotelianism nor to Neo-Platonic currents[97], but continues thirteenth century medico-astrological naturalism. The utilitarian inclinations of this current reflects itself in Jean de Murs' integration of practitioners' mathematics with theoretical and even high-level mathematics.

In Italy, finally, the social basis for a genuine development of calculating mathematics arose around the turn of the century; for the time being, however, no philosophical (or merely proto-philosophical) formulations are found in this connection.

[97] Examples of such connections could perhaps be dug up at the cross-roads of Joachimism, alchemy, and astrology. Arnaldo di Villanova (c. 1240-1311) might be worthwhile investigating.

THE EARLY RENAISSANCE

To make »The Early Renaissance« follow upon »The Fourteenth Century« seems to presuppose a sharp but arbitrary boundary line at 1400 A.D. This is not quite the case. A closer look at the examples from the previous chapter will reveal that nothing significant took place after c. 1375. (Indeed, it seems that nobody entering a university for fifty years after the Plague contributed anything but the most faithful continuation of existing trends to the history of mathematics). Nor was anything important going to happen to the development of mathematics during the early decades of the following century. There is thus no reason to locate the break more precisely than »somewhere between 1375 and 1425«. During this half-century, however, a significant break *did* take place.

This is not to say that old habits and traditions were fully abandoned. To the contrary: until well into the sixteenth century most mathematics teaching, be it at the universities or in the merchant schools, was virtually unchanged--which not only tells something about what the students had to learn but is also informative about the knowledge and style of their teachers. This continuity holds on all levels. Elementary university curricula from the fifteenth and sixteenth century refer to subjects and compendia familiar from the thirteenth century (Book I of Witelo's *Perspectiva*, Campanus' *Theorica planetarum*, Sacrobosco's *De sphaera* and *Algorismus vulgaris*). But even the »philosophical mathematics« from the fourteenth century was transmitted and cultivated: Bradwardine's *Geometria speculativa* was read, and so was Albertus Saxonus' (c. 1316-1390) *Tractatus proportionum*, an introduction to the ideas of Bradwardine's *Tractatus proportionum seu de proportionibus velocitatum in motibus*. On the whole, the list of scientific *incunabula* is dominated by traditional works,--mainly by works written during the Middle Ages[98].

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[98] In Klebs's list (1938) of scientific books printed before 1500, Euclid is represented by 2 editions of the *Elements*. Bradwardine is represented by three editions (the *Geometria speculativa*, the *Arithmetica speculativa*, and the *Tractatus*

The teaching of practitioners' mathematics was grossly unaltered too. Until the mid-sixteenth century the only important change was geographical--*viz.* the spread of the Italian *practica* to Germany[99,100]. (On the boundary between university mathematics and practitioners' mathematics a brief treatment of the rule of three in a number of algorisms can be observed[101]). The novel trends in fifteenth century mathematics are hence not to be traced quantitatively; only future events were to demonstrate that these trends *were* indeed the future.

A modest illustration of the new tendencies is offered by Alberti's (1404-1472) *Ludi rerum mathematicarum*[102] from c. 1450. Its most striking novelty is perhaps found in a small phrase in the dedicatory letter, referring itself »all'umanità e facilità vostra«. This can be related, on one hand to the dedication of the Latin

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proportionum); Albertus Saxonus' *De proportionibus* was printed 9 times before 1500, and Richard Swineshead's *Calculationes* twice, on a par with the pseudo-Oresmian *De latitudinibus formarum* and Peckham's *Perspectiva communis;* Sacrobosco's *De sphaera* reached 31 editions.

[99] »Der ein kunst nit allein versteet und weist, sonder auch derselbigen kunst durch stete übung vorteiligen brauch überkomen hat, wirt von den lateinern Practicus genent. Dieweil nun die Wellisch rechnung nicht anders ist dann ein geschwinder außzug in die Regel de Tri gegründet, wirt sie auch derhalben practica gesprochen« (Rudolph 1540). The *Practica* is thus precisely the *genre* which we have already met as *Trattati d'abaco.*

[100] On the fringes of the teaching of commercial arithmetic, however, one remarkable development took place, *viz.* through its connection to pictorial art. Cf. below.

It could be mentioned that already Paolo dell'Abbaco's treatise (cf. note 95) contains a large number of drawings, demonstrating the artistic affinities of the environment.

[101] In the University Library of Copenhagen I stumbled upon an *Algorithmus linealis proiectilium* by one Magister Johannes Cusanus, printed by Hermann Busch in Vienna in 1514, containing 4 pages on the basic arithmetical operations (half of which discuss progressions) and 1½ page on commercial calculation; and upon a more extensive, anonymous *Algorismus novus de integris. De minutiis vulgaribus. De minutiis physicis. Addita regula proportionum tam de integris quam de fractis, quae vulgo mercatorum regula dicitur*, printed by Sigismund Grimm in Vienna in 1520.

[102] Ed. Rinaldi 1980. Also in Grayson 1973: 131-173.

version of Alberti's *De pictura*[103], which supposes that the Prince of Mantua will, on account of his *humanitas* and his interest in the *studia litterarum*, read, understand and relish the book in his leisure; and on the other to the dedication of the Italian version of the same work to Filippo Brunelleschi, which tells its purpose to be the resurrection of one of those *nobilissime e maravigliosi* arts of Antiquity *quasi in tutto perdute:* Painting, sculpture, architecture, music, geometry, rhetorics and augury[104]. As the title of the work suggests, the *Ludi* are meant recreationally; the various dedications show, however, that this recreation was meant as noble leisure, connected to the ideas of Humanism and to the resurrection of Ancient splendour. One almost starts wondering whether real history can fit the stereotypes of conventional periodizations so precisely. But since it does we may conclude that Alberti's *Mathematical Diversions* represent the mathematical version of archetypical Humanist ideals.

How does then Alberti's *Humanist mathematics* look? First of all, it contains no references at all to any philosophy or philosopher, be it Aristotle, Plato, Neo-Platonism or anything else. Mathematics is *in itself* a representative of Antiquity and humanity, and needs in Alberti's eyes no further philosophical justification.

The mathematical *contents* of the treatise marks no watershed in the history of mathematics. Much space is occupied by practical geometry, especially the measurement of heights and distances, which is of course in touch with Alberti's conception of vision and hence related to his interest in the theory of the central perspective but also fully traditional. Besides, practical geometry is represented by area measurement (referring in particular to Columella and Savasorda (!) among the ancients, an to Leonardo Fibonacci among the moderns[105]), involving triangulation and the use of lunules. Finally, the treatise contains some statics, describes »Hero's bottle« and his hodometer, and explains how to find by systematic trial and error the elevation and the correct quantity of gunpowder to use for a bombardment.

According to this Albertian treatise, mathematics is hence *applied mathematics*. That does not change if one goes to Alberti's works on perspective, which are concerned precisely with a novel and ingenious application of (fairly unsophisticated) mathematics. In another respect, however, we should be careful not to judge Alberti's mathematical ideals on the basis of the *Ludi* alone. In one work

[103] Ed. Grayson 1973: 9.

[104] *Ibid.* p. 7.

[105] Ed. Rinaldi 1980: 50.

he shows that his pretensions go beyond those of Vitruvius and Columella: The *Elementa picturae* constitute indeed an attempt to combine the practical aim with Euclidean systematics and structure, but in a way which is adapted to the subject and not expressed in the words of Euclid[106].

Similar orientations are found in the works of some famous teachers of applied arithmetic. I think of Piero della Francesca (c. 1410-1492) and Luca Pacioli (c. 1445-1517)[107]. In Piero's *Trattato d'abaco* as well as in their works on the Golden Section and on regular polyhedra, they bring together methods of algebra and practical geometry with interest in art and references to Antiquity[108]. Luca Pacioli's treatise is particularly interesting because of its copious extra-mathematical observations and commentaries, from which a totally eclectic conception of Ancient philosophy is obvious. Mathematics is primarily an priority for mathematics over philosophy. His work is indeed *necessary* to »everybody wanting to study philosophy, perspective, painting, sculpture, architecture, music, and other most pleasant, subtle and admirable doctrines«, and he concludes from his discussions that

> *the mathematical sciences of which I speak are the fundament for and the ladder by which one arrives at knowledge of any other science, because they possess the first degree of certitude, as the philosopher says when claiming that »the mathematical sciences are in the first degree of certitude, and the natural sciences follow next to them«. As stated, the mathematical sciences and disciplines are in the first degree of certitude, and all the natural sciences follow from them. And without knowing* them *is it impossible to understand any other well. In* Solomon's Wisdom *it is also written that »everything consists in number, weight and measure«, that is, everything which is found in the inferior or the superior universe is by this necessity submitted to number, weight and measure. And Aurelius Augustine says in* De civitate Dei *that the supreme artisan should be supremely praised because »in them he made exist that which was not«*[109].

It will be observed that a quotation from Aristotle (called by his Medieval pseudonym *philosopho*) is twisted to bring home a point of view which is anything but Aristotelian (*follow* being understood as »follow by logical derivation and in rank«, not merely as coming next in exactness), and that the familiar words from *Wisd.* XI, 21 (see above, note 16) are still quoted. New and still fermentative

[109] Conciosia che dicte mathematici sienno fondamento e scala de peruenire a la notitia de ciascun altra scientia per esser loro nel primo grado de la certeza affermondolo il philosopho cosi dicendo. mathematice enim scientie sunt in primo gradu certitudinis et naturales sequuntur eas. Sonno commo edicto le scientie e mathematici discipline nel primo grado de la certezza loro sequitano tutte le naturali. E senza lor notitia fia impossibile alcunaltra bene intendere e nella sapientia ancora e scripto. quod omnia consistunt in numero pondere et mensura cioe che tutto cio che per lo vniuerso inferiore e superiore si squaterna quello de necessita al numero peso e mensura fia soctoposto. E in queste tre cose laurelio Augustino in de ciuitate dei dici el summo opefici summamente esser laudato perche in ella fecit stare ea que non erant (my translation from Winterberg (ed.) 1896: 36). Cf. also chapter II in general, pp. 35-40. The description of Archimedes as ingegnoso geometra e dignissimo architetto is found on p. 36; the claim that Luca's book is necessary for »ciascun studioso di Philosophia, Perspectiua, Pictura, Sculptura, Architectura, Musica e altre Mathematiche suauissima sottile e admirabile doctrina« is taken from the title page, p. 18.

wine in disparate second-hand bottles.

In his classification of mathematical disciplines, Luca presents us with similar eclectic innovations. His personal preference seems to be a scheme of three genuinely mathematical disciplines: *Arithmetic, geometry* and *proportion*. Then he runs into the traditional quadrivial scheme, based on distinguished authorities like Plato, Aristotle, Isidore and Boethius, but opposes to these eminent philosophers his own judgment (though *imbecille e basso*) that if *music* is included *perspective* must be so too, leaving us either with a different set of three or with a set of five disciplines[110].

In their way to bring together different ideas these men, from Alberti to Luca Pacioli, represent something original. Their ideas *inside* and *on* mathematics are, however, not very original except when it comes to the classification of disciplines. Who looks for real mathematical originality in Alberti's time should go to Nicolaus Cusanus (c. 1401-1464).

This originality has several sides. First there are the ideas which he brings to mathematics. In as far as they go beyond the limits of his own background (practical geometry and Bradwardine's *Geometria speculativa*) they are indeed so original that they have neither precursors nor followers, for the simple reason that the arguments are wrong--in a strictly mathematical sense often trivially wrong, because Cusanus takes his philosophical axioms for mathematical facts. Only thanks to Cusanus' philosophical importance and political rôle are his many rectifications of the circle available in modern print, and even in German translation with competent commentary[111].

Another aspect of his originality lies in his *confidence in his own originality*. While Alberti, and so many others with him, believed that all there was to do was to resurrect the forgotten knowledge of Classical Antiquity (another stereotype, but confirmed in the above quotation), Cusanus knew better in the introduction to *De geometricis transmutationibus:*

> *Wohl haben die alten, mit starkem Forschergeist begabt, in unermüdlichen Fleiß versucht, viel damals Verborgenes für sich und die Nachwelt ans Licht zu bringen; wohl haben sie in den meisten hohen und schönen Künsten mit Erfolg gearbeitet, aber in einigen der höheren Wissenszweige haben sie nicht alles Erstrebte erreicht. Der beste*

[110] *Arithmetic, geometry and proportion* is the claim of chapter 2 (ed. Winterberg 1896: 35-40), the modified quadrivial scheme that of chapter 3 (*ibid.* pp. 40-42).

[111] Hofmann 1952.

Erhalter aller Dinge hat es nämlich so bestimmt, damit die göttliche Kraft des Erkennens in uns nicht erlahme, sondern durch immer lebhafteres Interesse auf das noch Verborgene, aber der Erkenntnis zugängliche gelenkt werde [...]. Unter den Aufgaben, die bisher den geometrischen Spekulationen hindernd im Wege standen, blieb vornehmlich eine auch von allen denen ungelöst, deren Geisteskraft uns die überkommenen Bücher gewissenhaft wiedergeben, nämlich: Zwischen einer geraden und einer gekrümmten Linie Gleichheit herzustellen oder eine Verwandlung ineinander zu leisten. So kam es, daß es vielen, ja fast allen, die sich dieser Untersuchung widmeten, nach unermeßlichen mühen schien, der Weg zur Einsicht in diesen Sachverhalt sei uns entrückt, und zwar wegen der Unmöglichkeit des Unterfangens, da die Natur der Koinzidenz einer solchen Gegensätzlichkeit widerstrebe. Ich aber glaube, die Schwierigkeit dieses Unternehmens liegt vielmehr in einem zu geringen Verständnis, in mangelnder Sorgfalt und im Fehlen der äußersten Aufmerksamkeit, wie sie eine völlig ungelöste Aufgabe erfordert [...][112]

In one sense, of course, this belief in ones own progress over all predecessors is by necessity common to all circle-squarers. It is, however, coupled to a general belief in the possibility of a continued cognitive progress guaranteed by the Lord. This brings us to the third aspect of Cusanus' originality, his metaphorical use of mathematics as a guide in philosophy.

Metaphorical use of mathematics in philosophy sounds somehow like Neo-Platonism. If Neo-Platonism is used as an easy catch-word, Neo-Platonic« inclinations can easily be read into Cusanus' writings. Catchwords, however, are of little use if one wants to understand Cusanus' unique use of mathematics. True, the *De docta ignorantia* II, xiii quotes the invariable *in numero pondere et mensura*, and explains how the Creator made good use of all four quadrivial disciplines for his creation, »whence it comes that the machine of the world cannot perish«[113]. This, of course, is fairly traditional (apart from the »machine of the world«), and not very different from the »conventional Neo-Platonic« spirit of various Medieval writings. But already his metaphors are different from the uncommitted imagery of earlier times[114], and a very definite epistemological

[112] Transl. Hofmann 1952: 3f.

[113] *... ex quo evenit mundi machinam perire non posse*. My translation from Wilpert 1967: I, 67f (ed. Argent. I, 50).

[114] One thing is to explain Trinity numerologically, quite another is to prove (through analogy with the basic rôle of triangulation in surveying coupled with the identity of maximal and minimal entities) that Quaternity or further

rôle is assigned to them, the investigation through symbols *quasi in speculo et in enigmate* of that which cannot be reached through rational discourse[115]. In this use of symbols, Cusanus' thought is related to the Joachimite, alchemical and Cabalistic tendencies of surrounding centuries--but even this comparison doesn't do him justice: None of these currents went beyond numerology in their use of mathematics; Cusanus went astray, but so precisely because he went far, into what we might call a »dynamic approach to infinity«[116]. The strict discipline of fourteenth century Oxford scholasticism might have transformed this mixture of mathematical and theologico-philosophical inspiration into something more rigorous; a Renaissance politician-philosopher and mathematical amateur submitted only to discreet critical questions from friends did not possess these opportunities, even when he listened to the questions[117].

We may then contrast Cusanus to the academic mathematicians of the day.

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extensions of the Divine are *impossible*, as done in *De docta ignorantia* I, xx, (ed. Wilpert 1967: I, 59f [ed. Argent. I, 20-22]).

[115] *De docta ignorantia* I, xi, ed. Wilpert 1967: I, 13 (ed. Argent. I, 11). Cf. Volkmann-Schluck 1968: 25-35. This position is, if one wants to put a label on it, closer to Plato than to Neo-Platonism--and the same chapter *does* represent it as Platonic.

[116] In this respect, then, his approach foreshadows the whole *analysis infinitorum* of the Early Modern age which, in its disrespect for Archimedean rigour, created something effective and *potentially rigorous*. The mathematicians from Cavalieri to Newton and Leibniz (to name but a few) could do this because they dealt carefully with the infinitely great and the infinitely small. They might well have accepted Cusanus' claim that a circular arc approaches gradually to a straight line as its radius approaches infinity--but they would have parted company with him when discovering that he referred not to an arc of fixed, limited length but to a quarter of the full circle (e.g. *Aurea propositio in mathematicis*, transl. Hofmann 1952, 180; cf. *De docta ignorantia* I, xiii (ed. Wilpert 1967: I, 15f [ed. Argent. I, 13-14]).

Cusanus' failure can be explained through a lack of trained mathematical intuition, or as the result of an interest directed by his philosophical aims and convictions rather that by those practical norms which are acquired by the working mathematician. Conversely, the success of the seventeenth-eighteenth century mathematicians of the infinite can be ascribed to a well-trained intuition, and to dominance of the outspoken and tacit standards of the discipline over those imposed externally by philosophical principles.

[117] See Toscanelli's letter and the commentary in Hofmann 1952: 128-135, 233-235.

Most of these, it is true, were already discussed anonymously above, as carrying on fourteenth century traditions into the sixteenth century. They lectured on the basis of texts which had once been related to one or the other philosophy or proto-philosophical attitude. But they did so indiscriminately, and hence apparently from institutional inertia rather than through any living philosophical commitment. They may have been Aristotelians when asked questions on epistemology or natural philosophy; but when asked about the importance of mathematics they would quote *Wisd.* XI, 21 (*in measure, number, and weight*) and Boethius' *Arithmetica* (everybody did, as we have seen); under astrological examination the immense majority would have committed themselves to »medico-astrological naturalism«. In German, one might speak of this mixture as *Gewohnheitsaristotelismus*--which has almost as much to do with serious philosophy as *Gewohnheitserotik* has to do with the passions of love[118].

We shall meet an extreme form of this superficial »Aristotelianism by habit and convention« below, but for the moment remember that some fifteenth-century academic mathematicians were more than mere transmitters. I think first of all of Peurbach (1423-1461) and Regiomontanus (1436-1476), who, like Alberti etc., though a small minority represented the incipient transformations of mathematics.

Both also represent a new, autonomous professionalization of scholarly mathematics. In Vienna, their common home university, this had already begun in the early fifteenth century when Johann von Gmunden (ca. 1380-1442) became the first specialized professor in mathematics and astronomy; Peurbach, by becoming a court astrologer, represents another aspect of the new professionalization. In Johann, the new specialization had coexisted with a fairly traditional attitude to the contents of the subject[119], and is mostly to be seen in prolific work, interest in instruments, and care for the tools (bibliographic as well as intrumental) of the discipline; a skeptical opinion on astrology as more than a way to earn a living *may* also depend on that intimate familiarity with

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[118] The two algorisms mentioned in note 101 are excellent examples of this Aristotelianism by facile habit and convention. The first quotes the *Topica* to explain the ratio 10 between successive lines on the line abacus; the second introduces the subject in a beautiful mix-up of references to Boethius, Augustine and Aristotle (in descending order of importance).

[119] See Vogel, "John of Gmunden", *DSB* VII, 117-122; cf. also Benjamin's demonstration (1954) of Johann's very close dependency on Campanus' *Theorica planetarum*.

astronomical technicalities which resulted from specialization[120].

Through writings and disciples, Johann was the indirect teacher of both Peurbach and the latter's student Regiomontanus. In their generation, specialization coupled to relations with the Italian Humanist environment came to fruition, and for the first time Latin mathematics began catching up with its Ancient and Islamic precursors *in one of their own fields* (the philosophical mathematics of the fourteenth century being original not only in contents but also as a field of study). That field was (of course, one might say) *mathematical astronomy* and *the mathematics of astronomy* (which includes trigonometry). Peurbach, who was intimately familiar with the *Almagest* wrote a *Theoricae novae planetarum* intended to replace Gherardo di Sabbioneta's unsatisfactory but still popular thirteenth century compendium and began working on an abridged version of the *Almagest* itself; Regiomontanus finished the latter after Peurbach's death, and himself wrote a devastating critique of Gherardo's compendium[121]. The term »devastating« is to be read literally, in the sense that Regiomontanus oft-printed attack undermined its popularity[122]. Both scholars were hence actively engaged in the onslaught upon what could be seen as mathematically sloppy scholastic astronomy on behalf of Ancient (i.e. Ptolemean) standards, and hence in full right as part of a Humanist spring cleaning in the discipline. This interpretation of their common endeavour is corroborated by their biographies, showing close personal relations to Italian Humanism (not least to Cardinal Bessarion); Peurbach, furthermore, used his university chair to propagate Humanistic classical studies[123], and Regiomontanus' literary style is clearly that of the Humanistic scholar (be it the dedicatory letters, whole works in dialogue form, or even the terse definitions of the *De triangulis*).

It seems, however, that both were »Humanism's good servants--but astronomy's first« (to paraphrase Thomas More on King and God). This becomes obvious in the plans which Regiomontanus' had for the use of his printing establishment

[120] In the sense, at least, that intimate technical knowledge of astronomy would almost inevitably entail skepticism toward all sorts of market-place astrology--be it even the well-paying market-place of the court.

[121] All these works are reprinted (together with Regiomontanus' *De triangulis* etc.) in Schmeidler 1972.

[122] See Schmeidler 1972: xix.

[123] Lecturing *inter alia* on the *Aeneid* and on Juvenal--see Vogel, "Peurbach", *DSB* XV, 474.

in Nürnberg[124]; besides his own works and the great works of Antiquity (Ptolemy, Euclid, Theon, Proclos, Firmicus Maternus, Archimedes, Menelaos, Theodosios, Apollonios, Hero and Hyginus), the list includes Witelo's *Perspectiva* (designated »an enormous and noble work«), Jordanus' *Arithmetica* and *Data* (a work on theoretical algebra) and Jean de Murs' *Quadripartitum numerorum* (»a work gushing with subtleties«). Regiomontanus was neither an enthusiastic amateur nor a mere ideologue, and would never claim like Alberti that the mathematical arts had been »almost lost« since Antiquity; his own quality as a mathematician made him recognize sophisticated mathematics even in scholastic garb.

One type of sophisticated work, however, is lacking from his circular: Bradwardine, Swineshead and Oresme are all conspicuously absent. What they had made fell outside the canon defining Regiomontanus' enterprise. It cannot be because it was not astronomical--for Oresme *had* written on astronomical problems, and Jordanus' two treatises are on the other hand definitely non-astronomical; nor can it be because their works had been forgotten--as we have already seen, more traditionally minded printing houses *did* print them. It will rather be their primarily philosophical involvement and the purely hypothetical nature of their investigations *secundum imaginationem* which made them uninteresting--that same characteristic which had separated them from the astronomical naturalism of a Jean de Murs. Regiomontanus' interest in mathematics was indeed, though on a high theoretical level, an interest in a scientific *tool* to be used in the description of nature.

This is seen very clearly in the *De triangulis*. Not only is the whole work written to procure a mathematical underpinning for Ptolemean astronomy; this aim reflects itself even in the initial definitions, which build on actual *measurement*[125]

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[124] The advertising circular is reprinted in Schmeidler 1972: 532.

[125] The difference is highlighted by a comparison between the first definitions of Euclid's Data and Regiomontanus' De triangulis, which run, respectively:

»Surfaces, lines and angles to which we can procure equals are said to be given in magnitude«--a question of theoretically possible construction (ed. Menge 1896: 2; my translation and emphasis), and

»A quantity is called known when it is measured, either by a well-known or by an arbitrarily fixed measure, according to a known number« (ed. Schmeidler 1972: 283; my translation and emphasis). Obviously, the intricacies of irrational ratios are of less concern than the actual process of constructing a table of numerical values.

while being metatheoretically problematic. The quasi-philosophical attitude expressed (directly as well as indirectly) by Regiomontanus combines the naturalism of previous centuries (and even medico-astrological naturalism, although he was somewhat less sanguinary on the subject of astrology than e.g. Jean de Murs[126]) with the conviction already pointed out in Luca Pacioli, that mathematics was *in itself* a way to Ancient splendour as good as any philosophy[127]. No explicitly philosophical claims or professions of philosophical faith are to be found, only general expressions of reverence for e.g. Plato, Aristotle, Plotinos, Anaxagoras, Democritos, John Scotus and Thomas.

A final mathematician from this period to discuss at some depth is Cardano (1501-1576). His *Ars magna* from 1545 belongs (with Copernicus' *De revolutionibus* from 1543, Vesalius' *De humani corporis fabrica*, likewise from 1543, and a few other works) to the milestones demarcating the transition to the mature scientific Renaissance. Here, however, I shall concentrate on two of his earlier works.

Cardano was, like Peurbach and Regiomontanus, a professional scholar; but his profession was that of a physician, besides which he was by inclination a naturalist philosopher, maintaining that everything in the universe was living and animated. Mathematics was, as stated by his pupil Ludovico Ferrari, a subject

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[126] The advertizing circular contains, besides the classical astrological works of Ptolemy and Firmicus Maternus, only one astrological author with possibility to include fragments of another and eventually still other authors of predictions »if they are seen to be worthy«. However, if the prefatory letters to a horoscope of the later Emperor Maximilian I are to be believed (which could of course be problematic, since the addressees were the Emperor and the Empress), Regiomontanus believed precise astrological prediction possible but difficult in practice, as great knowledge was required (extensive quotations in Zinner 1968: 51f). More unambiguous evidence is constituted by numerous quite private notes, aiming e.g. at building up an astrological meteorology (*ibid.* pp. 54f, and *passim*). In the Padua lecture (Regiomontanus 1537: 4^r-5^r; cf. below, note 127) it is also stated once more that astrological prediction is possible but requires much more knowledge than what is acquired from Sacrobosco's *Sphere* and similar compendia.

[127] This is made explicit in the introduction to a lecture (Regiomontanus 1537: 3^v) held in Padua. Precisely as Luca, Regiomontanus claims for mathematics the rôle of a *first philosophy*, that of a foundation on which philosophy can build.

An extensive account of the lecture is found in Zinner 1968: 111-114, with subsequent discussion; similarly in Cantor 1900: 260-262 and onwards.

he exerted »for enjoyment, to seize for himself some recreation and solace«[128], but also to gain an income and to win fame[129]. His *Practica arithmeticae generalis* (1539) and the *De numerorum proprietatibus*[130] extending one of its chapters are thus products of a semi-professional mathematician of genius, addressing subjects which could be expected to have (and indeed had) resonance in his times.

The *Practica arithmeticae generalis* is, according to its title, a generalization of the *Trattato d'abaco* (cf. above, note 99), and it contains what *should* be contained in such a treatise: The numeration system, the basic arithmetical operations, the rule of three and related commercial arithmetic, basic algebra and basic mensuration. Numeration and arithmetical operations are, however, discussed not only for integers and fractions, but also for *surds* and for *powers of the algebraic unknown* (which involves automatically some classification of irrationals). Also contained in the introductory chapter is a proposal for an (explicitly generalizable) designation of the first 10 algebraic powers. Further on, sexagesimal arithmetic, Boethian and Euclidean theoretical arithmetic, *computus* and astronomical regularities, magic squares coupled to »their« planets, Biblical numerology, the conceptual instrumentarium from *Elements* X, the »rule of six« used in spherical geometry (with a reference to Regiomontanus and older astronomers), a chapter on games and one on the principle of *Data* (referring amply to Regiomontanus), and various geometric constructions (including Philon's extraction by moving geometry of a cube root[131]).--Truly an overwhelming work, and truly far from any standard of mathematical normality.

The *De numerorum proprietatibus* is an extended but purged version of the chapter "De proprietatibus numerorum mirificis" of the previous work. The astronomical regularities, the numerology and the astrologically connected magic squares have disappeared; what remains is an account (with heuristic proofs) of the main concepts and results from the arithmetical books VII-IX of the *Elements*, critically correlated with the Boethian theory of figurate numbers, and augmented with a number of observations on the proprieties of numbers (including the casting out of nines) and with an arithmetical translation of select

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[128] *... a guisa di giuoco, per pigliarse alcuna ricreatione & solazzo*. My translation from Ferrari's *Primo cartello*, ed. Masotti 1974: 5.

[129] Cf. Ore 1953: 10f.

[130] Both in Cardano 1663.

[131] The history of this and other equivalent constructions from Leonardo Fibonacci's *Practica geometriae* onwards is presented in H. l'Huillier 1979: 54-56.

results from *Elements* II and XIII. Here, as in the *Practica*, free use is made of current algebraic abbreviations for *plus*, *minus* and *radix*.

In the first work Cardano behaves as the prototype of a »universal Renaissance genius«, speaking of everything and respecting no customary disciplinary boundaries; in the second he shows that he *could* restrict himself to mathematics, but that this subject was on the other hand an entirety. Like Regiomontanus, though in a very different manner, Cardano brings together the practitioners' and the theoreticians' aspect on mathematics, no practical question being too humble for theoretical elaboration, and no theory too elevated for application. Each in his own way achieved what had been foreshadowed by Jean de Murs, and what had been the implicit project of twelfth century naturalism--impossible that early for lack of adequate practice, for insufficient understanding of theory, and for lack of adequate social structures able to carry on the project.

In neither of the two works does one find references to major philosophical systems. If we go to Cardano's encyclopedic *De subtilitate*[132] (which can be said to represent Cardano's own philosophy), we shall find that Book XVI "De scientiis" is in fact mainly concerned with mathematics (some music and meteorology and a little medicine being included). The concluding list of key workers is headed supremely by Archimedes. Then come, in order of succession: Ptolemy; Aristotle (the naturalist); Euclid; Scotus (no further identification is given); Swineshead; Apollonios; Archytas; Eutocios; al-Khwârizmî; al-Kindî; »Heber Hispanus«[133]; Galen; and Vitruvius. The philosophical context can be (briefly!) characterized as naturalism bent toward occultism and influenced by Neo-Platonism; but the attitude toward mathematics seems to be a very open-minded »Archimedism«--Archimedes being both the ingenious geometer, the calculator of the apparently incalculable, and the great engineer.

The untamed style of the *Practica* corresponds well to the approach of the *De subtilitate*, and can hence be said to be a result of Cardano's philosophy. It was, if we regard the detailed contents, idiosyncratically Cardano's own. The general tendency of both works is, however, close to what can be found in Michael Stifel

[132] I used the original edition (1550).

[133] Also to be found as a theoretical astronomer in *Practica generalis*, chapter XLVI, which shows him to be identical with the »Gebar hispanus« of Regiomontanus' Padua lecture, and hence with Jābir ibn Aflaḥ, from whom Regiomontanus had borrowed freely for his *De triangulis* (cf. Lorch, Jābir ibn Aflah, *DSB* VII, 39).

(ca. 1487-1567), both his principal work, the *Arithmetica integra* from 1544[134] and the *Deutsche Arithmetica* from 1545[135]. The former, presenting »all that was then known about arithmetic and algebra, supplemented by important original contributions«[136], included also magic squares and extensions of the theory of irrationals, while the latter, popular book tried to propagate more high-level methods in the field of the *practica*, arguing that everything which could be done by false position could be done more easily by algebra (»coß«).

Stifel did not share Cardano's specific philosophy; while the latter was an astrologically minded, heterodox Catholic physician, Stifel was a Cabalist and a Lutheran Pastor (whose Biblical numerology was only spared the heterodox epithet because of Luther's personal intervention). In both, then, a general occultist orientation, a deep interest in the secret forces of nature or number, can be found, and in both it was coupled to their unification of *all levels of* and *all approaches to* mathematics. Because both were extraordinarily gifted mathematicians, this unification (and hence ultimately this common orientation) brought the inherent tendency of early Renaissance mathematics to a culmination.

It is well-known that many contemporary figures shared the occultist orientation of our two eminent mathematicians, mostly without sharing their natural giftedness. *They* did not understand the meaning of those traditional disciplinary delimitations which they disrespected, which then led only to confusion.

Up to this point, the early Renaissance was dealt with from the perspective of active mathematicians (including non-professionals like Alberti). Other groups too, however, took part in the shaping of the new mathematics, moved by philosophical or quasi-philosophical ideas.

Above, a number of *mathematicians with Humanistic affinities* were discussed. Moving from them toward the centre of the Humanistic movement we meet a number of *Humanists with mathematical affinities*[137]. We meet them in increasing

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[134] Rather detailed description in Cantor 1900: 431-443. It was published by Ioh. Petreius in Nürnberg, who also published Cardano's *Ars magna* and *De subtilitate* and Copernicus' *De revolutionibus*.

[135] I used the original edition (Stifel 1545), equally published by Petreius.

[136] Vogel, "Stifel", *DSB* XIII, 59.

[137] Cf. Rose 1975: 26-75 (the chapter "Patrons, Collectors and Translators [...]") and passim.

numbers toward the second half of the period and further on in the mature scientific Renaissance, as translators and diffusors of Greek mathematics and as patrons of translation and diffusion. Early examples are Cardinal Bessarion (1403-1472), protector of Peurbach and Regiomontanus, whose desire to spread the full Gospel of Greek learning was the motive force behind their common abridgement of the *Almagest*[138]; and Pope Nicholas V (1377-1455), who, along with Homer, Herodotos and Greek Fathers, had Archimedes translated once again from the Greek[139]. Later names are Giorgio Valla (c. 1447-1500), physician, translator of Aristotle, and author of a bulky encyclopedic work[140] fusing mathematics (the regent discipline), medicine, and natural philosophy with the *studia humanitatis* and encompassing much Euclidean and Archimedean material; and his student Bartolomeo Zamberti (b. 1473, d. after 1539), violently anti-Medieval translator of the *Elements* from the Greek. Continuators of the tradition into the next period were Maurolico (1494-1575), commentator on Archimedes and Apollonios, translator of Autolycos, Theodosios, and Menelaos; Commandino (1509-1575), translator of Ptolemy, Archimedes, Apollonios, Euclid, Aristarchos, Pappos and Hero; and Baldi (1553-1615), friend of Commandino and author of more than 200 *Lives* of mathematicians[141].

Detailed accounts of the activities of these Humanists would lead much too far. I shall therefore just note that they confirm the impression gained from the Humanist mathematicians discussed above: Mathematics is *in itself* a part of and a path to Ancient splendour; interest in mathematics is not in itself coupled to Platonic (or Aristotelian) predilections, but rather to that mood which saw both philosophers as great men who could easily go in company (a conception which in itself is of course a break with scholastic Aristotelianism, and which is hence often considered as »Platonism«[142]). Their common evaluation of the relative

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[138] See Rosen, "Regiomontanus", DSB XI, 348.

[139] See Clagett 1978: 297f and 321ff.

[140] *De expetendis et fugiendis rebus*, see Rose 1976: 300f. Extensive information on Valla, including the catalogue of his library, is given by Heiberg (1896).

[141] 50 of these were published in various volumes of Boncompagni's *Bulletino*--most by Steinschneider (1872) and Narducci (1886); a complete list is given in vol. **20** (1887), p. 731.

[142] This is, in fact, the supposed »Platonism« dominating the source quotations in Crombie 1977. See e.g. the quotation from Clavius pp. 66f, and that from Possevino on pp. 70-72.

merit of Ancient mathematicians is expressed by Baldi in the last paragraph of his extensive biography of Archimedes: »Archimedes was the Prince of mathematicians; whence Commandino said quite rightly that *he* can hardly call himself a mathematician who hasn't studied Archimedes works with diligence«[143].

If we go to Northern Humanists with mathematical affinities but not themselves mathematicians, *their* level would (at most) permit them to see *Euclid* as the »Prince of mathematicians«. An illustrative example is Melanchton, who generously borrowed his name to many a mathematical book. His preface to the Basel Euclid (*Euclidis Megarensis ... Elementorum geometricorum libri XV*) shows him to be much better versed in Aristotle and Plato and their discussions of arithmetically versus geometrically proportionate justice than in the subject-matter of the particular object of his praise; in another book to which he granted his favours, Vögelin's *Elementale geometricum ex euclidis geometria*, the author himself claims that his excerpts from *Elements* I-IV (the very books once commented upon by Albert) were »almost sufficient to lead to the summit of learning«[144]. »Armseliger Gipfelpunkt, aber noch armseligere genügsamkeit der Zeit, welche Vögelin's kleinen Auszug in wiederholten Nachdrucken förderte und an den verschiedensten Anstalten mit Vorliebe benutzen liess! Geometrie, das sehen wir auch aus dieser Thatsache wieder, war nicht die starke Seite der deutschen

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The status of mathematics as (practically) *first* philosophy is curiously seen in Bessarion's defence of Plato. While the High Middle Ages would argue for the value of mathematics from the words of the philosophers, Bessarion uses mathematicians as authorities and mathematics as an argument for his favourite philosopher (see Rose 1975: 44f). Others, when arguing from philosophical writings in favour of mathematics, would do so not so much from the philosophers' authoritative words as from a claim that mathematics was the real foundation on which philosophers had built (cf. Possevino as quoted in Crombie 1977: 70).

[143] *E stato Archimede il principe de'matematici; onde con molta ragione diceua il Commandino, a pena potersi chiamare matematico chi con diligenza non haueua studiato l'opere d'Archimede.* Ed. Narducci 1886: 453, my translation.

[144] *Satis prope [...] ad disciplinarum culmen perducere*--my translation from fol. 21^r of the volume containing first Bradwardine's *Geometria speculativa* and next the second edition of the *Elementale geometricum* (1530). According to Cantor (1900: 409), the 1546 edition prefaced by Melanchton was virtually unchanged.

Mathematiker im Allgemeinen«, as Cantor[145] observes with affectionate irony.

As soon as we disregard the few creative mathematicians, we need not restrict ourselves to geometry nor to Germany alone. In France of the earliest sixteenth century, a small century of Renaissance innovations in mathematics had passed with as few traces as in Germany even among those who were mathematically interested without being mathematicians themselves. As witnesses we can take Jacques Lefèvre d'Étaples (c. 1455-1536), the circle surrounding him and the books published in this environment.

Lefèvre d'Étaples himself was responsible for a number of mathematical editions, including Sacrobosco's *De sphaera*, a common edition of Campanus' and Zamberti's *Elements*, and two editions of Jordanus' *Arithmetica*. He also wrote a number of mathematical manuals and compendia himself. All was done in an attempt to raise the mathematical level of the Parisian university environment, and from a broadly Neo-Platonic perspective[146]--and all was very traditional though when evaluated on that background of good quality. In spite of Italian acquaintances[147] and inspiration the Neo-Platonic paragon of French mathematical publishing of 1500 could do nothing better than resurrect the best quality of thirteenth century mathematics[148].

The low pretensions of Vögelin and the more brilliant but not much more innovative accomplishments of Lefèvre d'Étaples are not astonishing when seen in the light of the *real* success of the early sixteenth century scholarly book-market: The *Margarita philosophica* written in 1496 by Gregor Reisch (Cartusian

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[145] 1900: 394f.

[146] Lefèvre's preface to the Jordanus-edition (1514, but probably unaltered since the first edition from 1496, which I have not seen) refers to the public utility of mathematics, to the *Prisca theologia*, to Pythagoras (quoted for the opinion that nothing can be known without numbers), to Plato's inscription over the entrance to the Academy and to Book VIII of his *Republic*, and to Theon of Smyrna. For once Aristotle and Boethius are not referred to, and the Isidorean *tolle numerus* only turns up indirectly and in a way which shows that the traditional formulation is consciously cast in new form.

[147] Prominent among whom are, it is true, philosophers like Ficino, Pico, and Ermolao Barbaro rather than mathematicians (cf. Randall 1962: 92).

[148] Though, as Cantor (1900: 364) observes, Jordanus' *Arithmetica* was printed and not the much more »modern« *De numeris datis*, Jordanus' theoretical algebra, which did not fit into a simple quadrivial scheme.

from Freiburg, and future collaborator in Lefèvre d'Étaples' edition of Cusanus' writings; c. 1470-1525), printed first in 1504 and reprinted in France, Switzerland and Germany numerous times during the following decennia[149]. If anything is described by the concept of *Gewohnheitsaristotelismus*, this work is. For this reason; because its popularity demonstrated it to be a good exponent of the moods of its times; and because it was especially read as a mathematical textbook[150], I shall describe it in some detail.

First of all comes a table of the division of philosophy. The subject falls in two parts, *theorica/speculativa* and *practica*. The former is divided into *realis* and *rationalis*, and the former of these into *metaphysicam* (including both theology and normal philosophical metaphysics), *mathematicam* (identical with the quadrivium), and *phisicam* (containing title for title the traditional curriculum of Aristotelian natural philosophy). *Practical philosophy* is either *activa* (with subdivisions *ethica, politica, oeconomica*, and *monastica*), or *factiva*, the subdivisions of which are nothing but those »mechanical arts« once presented by Hugue de Saint Victor in *Didascalicon* III, i (same sequence, same terms). Only the inclusion of theology as the main part of metaphysics and the specification of a moral science of monastic life is something new, the rest is a mix-up of twelfth- and thirteenth-century lore, Humanistic only in so far as the strict scholastic conceptual organization of the whole has been dissolved into general benevolence toward everything revered and old. The inclusion of mechanical arts as »productive philosophy« is perhaps an expression of the Renaissance upgrading of applied knowledge; if so, the mere repetition of an almost 400 years older list (which furthermore had already been bookish at birth) shows the upgrading to be totally empty.

A pictorial representation of the castle of philosophy shows the uppermost floor to be occupied by Petrus Lombardus, representing *Theologia seu Metaphysica;* the next by *Philosophus* (alias Aristotle, one must presume) representing *Physica*, Euclid (representing geometry) and Ptolemy (astronomy); third come Aristotle,

[149] Some information on its publishing history is given by Scriba (1985: 38 and *passim*). More is available in Geldsetzer 1967: vii-ix, which only got into my hands when the bulk of the work was finished. Reisch's participation in the Cusanus-venture is mentioned in Wilpert 1967: viii.

The following account of the work is based upon the (unpaginated) Straßburg edition from 1512; the woodcuts described are also found in the first edition.

[150] A separate edition of the book on geometry appeared as late as 1549 in Paris (Reisch 1549).

this time under his own name, representing *logica;* Tullius (Cicero) representing rhetorics and poetry; and Boethius with arithmetic. No more innovative, and no less Medievally-eclectic, than the preceding verbal description.

Book IV on arithmetic is also opened with a woodcut, representing Boethius and Pythagoras, respectively, as a young modish, apparently Italian merchant calculating with Arabic numerals, and as an elderly colleague in old-fashioned northern dress performing his computations on the ruled abacus-board (traditional lore on the invention of the two notations). The description itself starts by praising the quadrivium, with unspecific references to Boethius and to Cusanus' *De docta ignorantia*, for being the key to many arcane places in the Scripture.

The discussion of speculative arithmetic is in the main taken over (directly or indirectly) from Isidore's *Etymologiae* III, ii-vii, but compares sometimes badly with this source[151]. Some formulations come from Boethius, but even the modicum of theoretical reflection offered in the latter's *Arithmetica* is absent from the *Margarita*.

Practical arithmetic is mainly an algorism, dealing with both integers and fractions, »vulgar« as well as »physical« (sexagesimal), covering the subject up to the extraction of a cube root, and including calculation on the ruled abacus and a presentation of the rule of three with select examples. On this subject Reisch is hence as modern as the anonymous Vienna algorism mentioned above in note 101.

The chapter on speculative geometry is liable to provoke as much bewilderment as the depreciated Isidore on arithmetic. Not that it is taken over from elsewhere. At least I have found no probable source for its peculiarities; I also doubt that any source can be found to maintain that no circle can be drawn through *two* or three points on a straight line, or to regard *diameter, axis, chorda, costa, latus, basis, cathetus, corauscus* (the leg of an isosceles triangle), *hypotenusa, diagonalis, perpendicularis, orthogonalis (et alia plura)* as different sorts of lines; the author can

[151] So, when Isidore tells that »Pythagoras is supposed to be first among the Greeks to have written about the discipline of numbers, but then it was ordered more broadly by Nicomachos and was transferred to the Latins first by Apuleius, and then by Boethius« (*Etymologiae* III, ii; PL 82, 155; my translation), Reisch simply claims that Apuleius and Boethius are told to have translated Pythagoras. When Isidore derives the term *arithmetica* from ├*a?riymo'v*┤, Reisch gives a double derivation from ├*a?riymo'v*┤ and ├*a?reth'*┤, an explanation also given in Hugue's *Didascalicon* II, viii (PL 176, 755).

have possessed little understanding of the terms he uses, and conveyed no more.

There is no reason to say more on the contents of this *Pearl of Philosophy*. It will probably be clear from the above that the work mixes up the traditional Medieval authorities (some recognized, others unrecognized) in a totally uncritical way, misunderstanding furthermore much of what they have to say. Its immense success demonstrates that the mathematical *avantgarde* dealt with above had left the main corps of the universitarian army of northern Europe far behind. Readers satisfied with a work like this cannot have taken its references to philosophical authorities as anything more than tokens of submission to revered old traditions--a queer sort of *poor scholar's Humanism*. Any finer philosophical points--including all such points which would make it meaningful to ask about philosophical influence on the discourse of mathematics--will have been beyond their horizon; so will, however, everything in mathematics which made it a fruitful field.

Hence, neither Northern Humanism nor its reflection in writings like the *Margarita* are engaged directly in the development of new kinds of mathematics, as sometimes seen in Italy. I shall, however, point briefly to more indirect influences contributing to later developments in the area: First the idea that an educational system without mathematics was incomplete, which procured a chair at the Collège Royal for the »encyclopedic, elementary, and unoriginal« astronomer and mathematician Oronce Fine[152], but even then contributed to broaden the basis for local mathematical activity and hence to make the general reception of new thinking on the subject possible. Second the Humanist interest in cosmography, which was coupled to map making and mathematical geography and hence to mathematics in general; this had effects of a similar sort. Third, finally, the activity of certain printers, who worked systematically in favour of the new currents: Petreius has already been mentioned, who (in partial collaboration with Osiander) managed to print Copernicus' *De revolutionibus* in 1543, Stifel's *Arithmetica integra* in 1544, and Cardano's *Ars magna* in 1545 (to be followed by the *De subtilitate* in 1550); another name which returns time and again is Ioh. Schöner (1477-1547) from Nürnberg, himself a teacher and writer on mathematics and astronomy[153], who *inter alia* printed most of Regiomontanus' works (which an early and sudden death had prevented the printer-astronomer himself from publishing). Petreius' personal preface to *De subtilitate* shows him

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[152] Characterization quoted from Poulle, "Fine", *DSB* XV, 156; the biography as a whole gives ample support for the harsh words.

[153] A brief biography is Rosen, "Schöner", *DSB* XII, 199-200.

to share the philosophical attitudes of Cardano, while Schöner's presentation of his edition of the *Algoritmus demonstratus*[154] demonstrates him to be a true follower of Regiomontanus. Discussion of the scholar-printers will hence add no new dimensions to the above. The only quasi-philosophical attitude to derive from the uncommitted goodwill toward mathematics exemplified by the Collège Royal and cosmography is also a repetition: The tendency to consider mathematical practice a legitimate entrance into theory, and theory the best tool for practice. I shall therefore discuss none of these subjects any further.

In an over-all discussion of this period, the first question will naturally be for the place of the prominent philosophers of the time. Where is Marsilio Ficino? Where is Pico della Mirandola? Nicoletto Vernia? Pomponazzi?

The fact seems to be that if we approach the question of relations philosophy-mathematics from the mathematical side, all these major figures are absent. Among the characters from the period counted as philosophers e.g. in Randall's *Career of Philosophy from the Middle Ages to the Enlightenment*[155] only Cusanus, Bessarion, Cardano and Lefèvre d'Étaples are of any importance for mathematics--and among these only Cardano is of real importance, Cusanus being either regarded as a mathematical fool (by Regiomontanus) or used together with Isidore or Augustine as an advocate for the general importance of mathematics. Disregarding Cardano, who as a philosopher wholly of his own must count as a special case, philosophers of importance were simply not oriented in contemporary mathematics. An illustrative example is Pomponazzi, who *did* take up a mathematical discussion in 1514--but a discussion with Swineshead,

[154] Schöner 1534. The algorism itself is a mathematically stringent presentation of the subject written in the mid-thirteenth century by an anonymous follower of Jordanus. In the preface Schöner quotes Greek authors and Virgil, and on the title page he declares that the book »will put the mathematical demonstrations of that calculating art which is popularly called algorism, and thereby its source and origin as well as its reason and certitude, clearly (as it is usual in all branches of mathematics) to the eyes« of the reader (my translation).

An appendix "De proportionibus" demonstrates the tendency of Humanist mathematics to merge different levels and approaches, presenting the theory of arithmetical, geometrical and harmonic means, and the 18 Ptolemean »rules of six«.

[155] Randall 1962.

belonging in the context of the early fourteenth century[156].

Conversely, mathematicians were (Cardano being again by necessity an exception) not very well oriented in the actual philosophy of the day--either because occupational specialization separated the two, or because the precise discussions of contemporary philosophy were uninteresting to them (probably both, in varying balance and interplay).

Only as a very general background were the philosophical developments of the time of importance for mathematics. First, of course, most or all currents participated somehow in the Humanist movement, which *was* of importance--this is a question to which we shall return in a moment. Second, the eclectic appreciation of everything in Ancient philosophy and especially of Greek texts (at times regarded as »Platonism«) would further work on all available Ancient mathematical texts. Third, the break-down of old philosophical fences between different ontological levels and categories--just by being *new* philosophy--contributed to break down old barriers. Fourth, finally, *some* currents at least would not only break down barriers but also create connections. So, Ficino declared about the soul, that

> *... she ascends to higher things and descends to lower. And when she ascends, she does not forsake the lower, and when she descends she does not leave the higher. For if she forsook either, she would fall into the other extreme, nor would she be the true link between both worlds.*[157]

Similarly, in his *Oration on the Dignity of Man* Pico declared *Man* the mean binding heaven and earth, the higher and the lower, together[158]. Both represent a variant of Neo-Platonism, where the *Great Chain of Being* was no longer a unidirectional system channeling Divine emanations, and where *Man* (or the human soul) as an active being had the task to mediate the daily and the supreme levels of reality. If a mathematician would perform this task inside his profession he could do no better than regard practical applications as a way to theory and theory as the best tool for practice[159]. On the other hand: Should a Neo-Platonic

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[156] See Wilson 1953.

[157] Theologia platonica III, ii, quoted from Randall 1962: 60.

[158] *Ibid.* p. 62.

[159] In my 1987 I discuss a similar relation between early Islamic non-institutionalized religious fundamentalism and the character of early Islamic science (especially mathematics).

philosopher have translated an unspecific belief in the unity of all cosmic levels into the terms of *his* discipline, his way would be the one chosen by Ficino and Pico.

Nowhere have I, furthermore, seen a mathematician argue directly for the unified treatment of theoretical and applied mathematics in philosophical terms. So, rather than really speaking of »philosophical influence« we have to do with an all-pervading and unspecific, quasi-philosophical conviction that *reality is One*[160], which philosophers and mathematicians made specific, each part in its own terms. Ficino's philosophy will be no *cause* but rather a parallel highlighting a »mathematicians' quasi-philosophical attitude«.

Such attitudes can be discussed under two broad headings: *Humanism* and *naturalism*. Dependent on the climate of Humanism are a number of interrelated issues reflected in many mathematical works, included many of the works discussed above.

A recurrent theme is the creed that mathematics is in practice the real *first philosophy*. This is definitely not Aristotelianism; nor can it, however, be regarded as serious Platonism--after all, Plato had considered mathematics only a *prolegomenon*, a training and mental preparation by analogy to the real insight gained through dialectics. But the writings of both philosophers (and their commentators) abound in references to mathematics, and indirectly their writings would then carry the message that mathematics was fundamental.

Closely related to the appraisal of mathematics as practical first philosophy was hence the estimation of the field as itself an expression of and a path to Ancient splendour. In this connection it should not be forgotten that the acquisition of the mathematical heritage from Antiquity had been an essential aspect of the High Medieval reorganization of learning (and, as we have seen, that mathematics had rather been an *alternative to* than real *part of* strict scholasticism). As Bessarion, Gherardo di Cremona and the anonymous translator from the Greek had seen the *Almagest* as an indispensable work.

Closely related is also the character of Humanism as a citizens movement, a cultural current carried by people actively engaged in the higher and highest levels of civic life and emphasizing the public utility of their cultural skills. In many ways, mathematics had proved itself publicly useful since the late Middle Ages: As commercial arithmetic; in the administration of the city states; in

[160] The expression used by Garin (1975: 96) to characterize Ficino's Platonic theology.

architecture; in »machines« used in military and architectural engineering as well as the clockwork »machines« referred to be Cusanus; in painting; in surveying and cartography; in other sorts of calculation combined with measurement (gunnery, *Visierkunst*, etc); and--not to forget--as medical and courtly astrology (all of it fields which in some way or other turn up in the works discussed above, and most of which are discussed in Regiomontanus' Padua lecture[161]). If utility in the cultural context of the Renaissance city state was the yardstick, no wonder that mathematics came to be regarded, with rhetorics, *belles lettres* and *beaux-arts*, as a major constituent of the Ancient heritage;- and no wonder that the Ancient philosophers' references to mathematics led not only Luca Pacioli but even Bessarion to treat mathematics as something more fundamental than philosophy.

However, if utility was the gauge, the traditional disrespect for applied mathematics as being of secondary rank would be an unbearable paradox; similarly, the quadrivial scheme was, in spite of its Ancient legitimacy, an unacceptable straightjacket precisely for many of the new applications of mathematics. We remember Luca Pacioli's protest that the mathematics of painting was no less important than the mathematics of acoustical harmony. Even in the *Margarita philosophica* Reisch shows the quadrivium to be outdated by investing his real interest in algorism and practical geometry while filling the mandatory »theoretical« chapters with traditional lore and nonsense.

Traditionally, *music* had been regarded as the discipline of proportions and balance, and hence *of good life*[162]. Luca (and others with him, not least of course his friend and collaborator Leonardo da Vinci) had come to regard the theory of proportions as a theoretical discipline to which music, visual harmony etc. were subordinated. Melanchton (only to mention one example), in his Euclidean preface, shows that the old connection to moral and political balance was also present to Renaissance minds. Through the concept and theory of proportions mathematics was hence not only a theoretical and utilitarian but also a *moral science*, a *scientia activa*. Comparison of fourteenth and fifteenth century Italian painting gives an immediate impression that the political shift from communal to princely government in the Italian city states was reflected (and not only accompanied by) a shift in interest in art from naturalistic truth and detail to balance, stability and harmony (»realism« in the philosophical sense, as opposed

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[161] Regiomontanus 1537: 3^r ff.

[162] Explicitly stated e.g. in Cassiodorus, Institutiones II.v.2 (transl. L. W. Jones 1969: 190).

to the »nominalist« acceptance of the phenomenal world). The arguments of mathematicians and philomaths on the importance of proportions, connecting itself to both aspects of the development, demonstrates that it is in fact not coincidental, and hence also that mathematical interest in precisely that field was founded (at least in part--Boethian tradition and internal developments presumably played their rôles too) on ideas deeply rooted in the political attitudes and dispositions of the aristocratic and princely environment. (Yet another point where the cultural conditions and attitudes influencing both philosophy and mathematics are easily mistaken for philosophical inspiration of mathematics).

Archimedes was known to have served his city and his king, biographical facts often referred to (e.g. by Luca Pacioli). He was known as a fabulous military engineer; and his mathematical works would speak for themselves to anybody able to grasp their sophistication, making him the supreme representative of Ancient mathematical splendour. From every aspect of Humanist mathematics he was thus the paramount figure to be called upon as a witness. When personified, Humanist mathematics therefore appears in the guise of *Archimedism*. On the other hand the Archimedes invoked was a Protean figure, able to legitimate almost every kind of mathematical activity. Archimedism was hence no strict program or philosophy; it *was* apparently influential all the same, and no pure epiphenomenon, but only so because invocation of one aspect would evoke the others too: If architecture was supreme because represented by Archimedes, a really good architect had to be an eminent geometer too. Archimedism was hence the conceptual institutionalization of the unity of courtly or civic science, utilitarianism and theory at the highest level.

While *Humanist* attitudes were a specific Renaissance influence in mathematics, *naturalism* is an old acquaintance, which will require fewer words, though it took on new forms in the Renaissance period.

In as far as astrology is concerned these new forms were primarily social, *viz*. the institutionalization of court astrology. Thereby astronomy was transformed into a lucrative career permitting specialization, permitting but also requiring a much higher level of sophistication, technicalization and reflection on the subject. (Socially, the contact to the courtly environment was also a contact to Humanist currents, which makes distinction between influences difficult.)

By being less established than astrology, the broader naturalist and occult[163]

[163] My conceptualization of the field is vague with purpose. Occultism is most often concerned with *the hidden forces of Nature*, and hence to be characterized

currents are more interesting. They are often Neo-Platonic or at least tainted by Neo-Platonism, and as such they are of course related to the Neo-Platonic currents of precedent periods. As Ficino (a connecting figure, both a most important philosopher and a representative of the occult current) gave *Man* an active rôle as the mediator between the various levels of the Great Chain of Being, so occult Renaissance naturalism in general gave to him a central rôle as the executant of *natural magic*.

Renaissance occultism has been amply discussed since the early 1960es, and its influence in mathematics can easily be overrated. Direct reflection in the new trends of mathematical thought are no more visible than were direct reflections of philosophical stances. Once again, of course, Cardano is (with Stifel) sort of exception, in so far as his undisciplined *Practica generalis* mixes up things which are unconnected from almost any viewpoint except his own hylozoic naturalism. But already in his *De numerorum proprietatibus* had he sorted things out; the *Ars magna* is totally free of them, and so is the reception of his work by other mathematicians. The effect of Cardano's distinctive naturalism was, in the end, only that of *epistemological optimism*--an observation which (with emphasis on the utility of knowledge) seems to hold throughout for the core of occult philosophy (to which Cardano is not to be counted) and its effects on mathematics. Archimedes, so important a figure in renascent mathematics, was more or less a non-person to the occultists[164].

Furthermore, »epistemological optimism with emphasis on the utility of knowledge« was not specifically characteristic of the occult currents. I have already discussed it above, as a Humanist mathematicians' attitude. Naturalism, in as far as influencing the development of mathematics, dissolves in broader

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as naturalism. During the sixteenth century it came to support itself increasingly on the *Corpus hermeticum*, which has made many speak indiscriminately of Hermeticism, and will still permit us to speak about naturalism albeit turned specifically toward Neo-Platonism. When we come to *Cabala*, numerology and the magical use of geometric figures, however, a relevant concept of nature is not easily found, though they of course often mixed up with naturalist occultism (*vide* e.g. Pico and Agrippa von Nettesheim). A useful survey is Shumaker 1972; sound caution on »Hermeticism« is recommended by Westman & McGuire (1977; cf. Schmitt 1978).

164 It is characteristic that neither Yates 1979 nor Shumaker 1972 mention Archimedes in their index. The two references in the index of Yates 1964 are immaterial, and the single reference in Yates 1972 almost so.

currents; and occult naturalism, in its most specific form, was rarely shared by important mathematicians and had no influence on mathematics. While the specialization made possible by the emergence of courtly astrology had given astrological naturalism the possibility to go into fruitful dialogue with the advances of mathematics and with the best of Ancient astronomy, general Neo-Platonic naturalism, bending increasingly toward occultism, lost the contact which had been fruitful in preceding centuries.

Equally devoid of specific influence was the northern universitarian tradition of »Aristotelianism by habit and convention«. Of course, the continued teaching of fundamental mathematical skills and knowledge (*Elements* I-IV, algorism, *De sphaera*) supplied a certain basis of recruitment for professional astronomers and mathematicians and a living for a number of mathematics professors; but e.g. the incipient integration of the rule of three into algorisms (which was of course in harmony with the general unifying tendency in Renaissance mathematics) came to the universitarian environment from the outside, and was rather an influence of renascent mathematics on university teaching than the other way round. As illustrated by the *Margarita Philosophica*, the whole environment was in general too philosophically flabby to be able to exert any influence on the increasingly vigorous field of mathematics.

EPILOGUE: MATHEMATICS AS PART OF THE FOUNDATION OF NEW PHILOSOPHIES

We are approaching the end of the study, which I shall round off with some sketchy observations on the transformations of the structures discussed up to now during and immediately after the mature scientific Renaissance.

One astonishing development is a temporary union between Hermetic-occult-naturalistic interests and mathematics. It was just argued that no such marriage took place before the mid-sixteenth century--but then it did. Three rather prominent figures can be mentioned as its representatives[165]: Bishop Foix de Candale (c. 1502-1594); John Dee (1527-1608), and Faulhaber (1580-1635). Candale, who made a new Greek edition of the *Corpus hermeticum* and was convinced of Hermes' sovereign wisdom (but did not accept Hermetic magic)[166], also made a large commentary on the *Elements*[167]. Dee, no less a universal genius than Cardano, was a Cabalist, an astrologer and a magician; he translated an Arabic version of Euclid's work on the partition of figures; he cooperated with Commandino, studied with Gemma Frisius and was friend of Pierre de la Ramée and Mercator; he took care of the first English translation of Euclid (which involved the writing of an extended and influential *Mathematicall Praeface*[168], of introductions to the single books, and of many commentaries and additional theorems), and was the proponent of

[165] A number of secondary figures are mentioned in Feingold 1984. Stifel, whose private Cabalism appeared as an exception when discussed in the context of Early Renaissance mathematics, could be seen as only premature.

[166] See Yates 1964: 173, and Westman 1977a: 42ff.

[167] Description in Cantor 1900: 554.

[168] Separate facsimile edition in Debus 1975.

educational reform, of English imperialism, and of the application of science[169]. Faulhaber, an extraordinary self-taught mathematician, engineer and teacher of engineering mathematics, developed his own version of Cabala in which he was a firm believer[170].

In the preceding chapter *Archimedism* was used as a key-word embodying the new developments in Renaissance mathematics. It is characteristic that all three figures just mentioned pass the Archimedist test literally. Foix de Candale was regarded by contemporaries as »le grand Archimède de nostre age«[171]; Dee refers to Archimedes time and again in the *Mathematicall Praeface*[172]; and among those authors whom Faulhaber translated from Latin into German for his own use was, besides Euclid, Apollonios, Regiomontanus and Cardano, Archimedes.

In Foix de Candale the only connection between his Hermeticism and his Euclidean commentary appears to be a common search for pristine truth behind later errors and distortions[173]. For both Dee and Faulhaber, however, magic and applied mathematics were parts of a continuum: For Dee, as quoted, the squaring of the circle was arcane knowledge; Faulhaber too used similar words to speak of new geometrical instruments and of his new wonderful Cabala[174]. For both, the very great emphasis placed on applied disciplines, as well as the explosion of the number of such disciplines, was directly dependent on their general utilitarian occultism (and *vice versa*).

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[169] See French 1972, *passim*. Some extra information on the English *Elements* are drawn from Easton, “Dee”, *DSB* IV, 5. The Arabic translation and the collaboration with Commandino is discussed in Rose 1972.

[170] See Kirschvogel, “Faulhaber”, *DSB* IV, 549-553. The width of his engineering mathematics as depicted in copperplate in the two editions of his *Ingenieurs-Schul* is reproduced in Scriba 1985: 50f. In the second edition from 1537, the four quadrivial disciplines have been increased to no less than 18.

[171] Jean Bodin, quoted from Westman 1977a: 42.

[172] So inter alia fol. b.iiiirev-c.iobv, quoting in full six theorems of hydrostatics; fol. c.irev, telling that Archimedes sought and found the squaring of the circle, that »great Secret: of him, by great trauaile of minde«; fol. c.iiiirev and d.iobv, presenting him as the engineer sans pareil.

[173] This, at least, is my impression gained from the secondary literature; I have seen none of his works in original.

[174] Cf. the list of his titles in Kirschvogel, “Faulhaber”, *DSB* IV, 552.

The sudden occurrence of such major mathematicians involved in Hermeticism or occultism puts the absence of mathematically influential occultists in the previous period in a new light. It proves that early Renaissance mathematics was not at a technical or epistemological level making magical approaches impossible or necessarily unfruitful; the impotency must be found on the side of early Renaissance occultism, whose cultivators, far below the level of a Dee, were not even competent to grasp that of living fifteenth century mathematics. It seems that the continuity of Neo-Platonic and related traditions (from Joachim of Fiore, Arnaldo di Villanova and Meister Eckhart onwards) was only philosophical and mystical. Mathematical results gained by one generation (as thirteenth century optics) might be accepted as mathematics and then handed down through the continuity of the mathematical tradition (uneven as this continuity was); they were, however, not carried on by the philosophically and religiously heterodox current itself, and fifteenth century *magi* had to start afresh with elementary number symbolism etc.

Above, Dee and Faulhaber were grouped together as equally technologically minded. Socially, however, they differ. While Dee was an outstanding figure in a broad Hermeticist and occultist movement, Faulhaber was an eccentric in his occultism. In his generation, occultism was a *possible but strictly private* inspiration for a mathematician and no longer something to be regarded as influence of an intellectual current on mathematics regarded broadly. The Kepler-Fludd[175] debate supplies another illustration of this: Fludd (1574-1637) the Rosicrucian had lost contact with serious mathematics; only Kepler's »Pythagorean« bent was still compatible with broad orientation and influence in the field[176],-- but even that was on the wane.

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[175] See Westman 1984.

[176] The difference in attitude is illustrated by Kepler's criticism of customary astrology. He dismissed zodiacal astrology, the division of the zodiac being purely human and conventional, devoid of physical reality, and hence unable to influence anything. Planetary *aspects*, on the other hand, depending on the harmonic proportions between the angle between planets and the full circle, had physical reality; the possibility of aspectual astrology was therefore not only a theoretical possibility but almost necessary in a universe governed by harmony and proportion (»Since God the Creator derived the structure of the corporeal world from the form of body [...] it is reasonable to suppose that the positions, the spacing and the bulk of bodies should bear to one another

Archimedism, the specific complex of admiration for Ancient mathematics symbolized by supreme reverence for Archimedes the engineer and geometer was another inherited structure. Much was already said about it; here I shall just remind that the change of emphasis in the current interpretation of Archimedes (from the engineer to the subtle theoretician) was only a result of the mature Renaissance, to which both Baldi and Commandino belong (cf. above, text to note 143). As we have just seen, the involvement of occultism with mathematics at the professional level implicated its involvement with Archimedism, a fact which in itself shows that this attitude was still of fundamental (if not necessarily very specific) importance in mathematics.

A third structure of importance was the utilitarian orientation and persuasion of mathematicians. We met it in intensified form in Dee and Faulhaber, but it is a much more general characteristic. It found new support in the development of new or improved mathematical techniques, from *prosthaphairesis* (the use of trigonometric formulae and tables to transform multiplications into additive operations), logarithms and the invention of new instruments to military architecture, cartography and navigation[177].

The conviction that genuine mathematics was, or was modelled upon, Ancient mathematics, suggested the tools for two attempts to reform algebra. Both were French, and chronologically they were separated by only by three decades. The difference between their choice of Ancient ideas and their corresponding fate is illustrative of the ways in which inspiration from quasi-philosophical attitudes can direct, and of the extent to which it cannot determine, mathematical development.

One work is Pierre de la Ramée's (1515-1572) *Algebra*[178]. In its set-up it is

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the proportions that arise from the regular solid figures«--*De fundamentis astrologiae certioribus* XXXVII, transl. Field 1984: 250). Cf. also Simon 1979: 36-48.

[177] Cf. the survey in Keller 1972. The construction of automata (also discussed extensively by Keller), especially of planetary clocks, as a new branch of courtly science bringing higher artisanate and mathematics into contact is the subject of Moran 1977.

[178] I used the original Paris edition from 1560, the existence of which appears to be virtually unknown. Neither Cantor (1900: 612, 641) nor Mahoney ("Ramus", DSB XI, 289) knows anything but an edition by Lazarus Schoner from 1591, which they regard as dubious.

basically different from cossist or Italian algebra. Algebra is defined as »a part of arithmetic, which from imagined continued proportions establishes a certain form of counting of its own«[179]. The imagined continued proportion is of course the sequence of algebraic powers *unitas* (x^0), *latus* (x^1), *quadratus* (x^2), etc. (An example using the powers of *2* goes to *215*). For all these, as for addition and subtraction, symbols are introduced. Part 1 is then a set of examples illustrating the use of schemes for the calculation with algebraic expressions--e.g. the scheme

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 8q -- 9
       8q
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64bq--72q
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where *q* means *quadratus* and *bq biquadratus* (x^4), and the whole scheme therefore $(8x^2-9)\cdot 8x^2=64x^4-8x^2$. The rules are given merely *as rules*, proofs are absent. Part 2 *in aequatione* falls into two subsections, the first of which deals with problems of the first degree and the second with second-degree equations. Here at least reasons are given for the standard algorithms used for the solution, viz. geometrical representations and references to *Elements* II, 4-6. If we compare the work with al-Khwârizmî's classical treatise, the use of abbreviated names and of schemes for algebraic reductions are new; so are the Euclidean references and the use of the Ancient concept of magnitudes in continued proportion. As far as mathematical substance is concerned, however, Ramus does not surpass the first Islamic treatise, nor its twelfth century Latin translations (in spite of the possibilities offered by abbreviations and schemes). The idea to use the framework of Ancient mathematics in order to replace the old algebraic *experimenta* with something more indubitable was good; but his choice was uncritical, depending on familiarity (what could be more familiar from Ancient mathematics than the elementary concepts of proportion and the early books of the *Elements?*) and not on mathematical adequacy. Jordanus, when modelling *his* theoretical algebra on the analytic idea of the *Data* and on the mathematics of *Elements* VII-IX had chosen much better more than three centuries before.

Decisively better was also done by Viète (1540-1603) in his *In artem*

[179] »Algebra est pars Arithmeticae, quae é figuratis continué proportionalibus numerationem quandam propriam instituit«--my translation.

analyticam isagoge[180], published three decennia later, and in a number of other works. Viète considered existing algebra »so defiled and polluted by barbarians« that he found it necessary »to bring it into a completely new form«. The enterprise was necessary, because, as all mathematicians knew, »under their *Algebrâ* or *Almucabalâ*, which they extol and call the great art, incomparable gold is concealed, which however they cannot find at all«[181]. The way was, once again, a return to the cleanliness of Ancient mathematics--but for thorough reform, not just to dress up single concepts and procedures of the existing discipline in Ancient garb as done by Ramus. Everything had to be recast, and the mould was provided not by the works nearest at hand[182] but by Pappos' discussion of the concept of *analysis* (making possible the formulation of the theory and of the metatheoretical status of algebra), by the stringent differentiation between quantities of different kind (contributing the principle of homogeneity), and even by a twisted borrowing from Aristotelian ontology, permitting Viète to conceive of the unknown not as something »imagined« but as pure »form«.

Both representatives of mature French Humanism thus show us the legacy of Archimedism: The way to correction of errors and to progress in mathematics was the return *ad fontes*[183]; fruitful result, however, would of

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[180] Ed. Hofmann 1970. Translation (together with selections from other algebraic writings) in Reich & Gericke 1973.

[181] *Ecce ars quam profero nova est, aut demùm ita vetusta, et à barbaris defoedata et conspurcata, ut novam omninò formam ei inducere [...] excogitare necesse habuerim. [...] At sub suâ, quam praedicabant,et magnam artem vocabant, Algebrâ vel Almucabulâ, incomparabile latere aurum omnes agnoscebant Mathematici, inveniebant verò minimè*. My translation from the dedicatory letter, ed. Hofmann 1970: XI.

[182] Not even by Diophantos, the prominent »algebraist« of Greek Antiquity, although he supplies a number of problems for Viète's *Zeteticorum libri quinque*--see the tabulation in Reich & Gericke 1973: 93-96. Diophantos is characterized as *subtilissime* in the »zetetic art« (the derivation of the equation expressing a problem); but even though his *working* is algebraic his *presentation* is numerical, by which his subtlety and skill is made even more admirable, but which also loads the field with unnecessary abstruseness (*Isagoge* V,14, ed. Hofmann 1970:10).

[183] Both are also Archimedist in the most literal fashion. In book I of the Scholae mathematicae (1569), where Ramus sets forth a general view of mathematics, Archimedes is discussed over 8 consecutive pages; no other

course only follow when the Ancient source was used in critical integration with the urgent problems of mathematics as a living discipline. Archimedist ideology alone wouldn't do.

The break-through in algebra created by Viète and later by Descartes contributed to make the Early Modern age conscious of its scientific advantage over Antiquity, and to make it consider mathematics a major point of this advantage. Thus understood, even Viète's *algebrâ novâ* contributed to the foundation of the new philosophy. The contribution was, however, both modest, indirect and peripheral, and insufficient to motivate the heading of the present chapter, and Viète is unlikely to be mentioned in even a broad-minded history of philosophy; on the other hand, Gilbert, Galileo, Kepler, Hooke, Boyle and Newton can be expected to turn up, some of them as principal actors[184].

I shall not venture into an investigation of these founders of Modern science. It would double the size of the present paper, which is already bulky. Instead I shall recall the unifying concept used in the seventeenth century as a common description of this somewhat uneven bunch (and used as a slogan by some of them): *Experimental philosophy*. Experimental philosophy was related to the *experimenta* of unorthodox Medieval currents, and to the naturalism of the sixteenth century (including such figures as Cardano, Telesio and della Porta). On the whole, however, sixteenth century non-astronomical naturalism had not been mathematicized[185]; its use of mathematics had been symbolic and figurative, not descriptive and calculating. This was precisely what came to distinguish *experimental philosophy* from the precursor[186]. The mathematics of experimental philosophy was not

author is mentioned on more than two pages. Viète is more parsimonious in his references to authorities; but while Euclid and Apollonios are mentioned 7 times each in the index to the collected mathematical works (and Plato 4 times and Aristotle twice), Archimedes turns up 13 times (Hofmann 1970: XXXII*-XXXIX*).

[184] I checked my prophecy in Randall 1962 and Copleston 1963. It proved correct, apart from the absence of Hooke from Copleston.

[185] Exceptions can of course be found, as Della Porta's optics.

[186] Cf. also Vickers 1984a on the »rejection of occult symbolism, 1580-1680« and on the differentiation of the experimental from the late occult tradition on

necessarily of great sophistication[187]; but it was connected to actual and potential measurement, and was thus a modelling of sensible reality (as the mathematics of astronomy had been since Antiquity). *Often*, furthermore, its modelling function came to require comprehensive mathematical deduction, i.e. directly involvement of description with the production of mathematical theory. This whole rôle of mathematics was something new; it was fundamental for the new philosophy; and it was understood as such. As *Experimental philosophy* was itself an essential constituent of Early Modern philosophy, mathematics came to be constituent part of its foundations.

It can easily be argued, texts in hand, that the new rôle for mathematics was understood by these actors along Archimedist lines; Archimedism, once a mathematicians' private quasi-philosophy, was dissolved inseparably into Early Modern philosophy. But it was also adopted more directly and independently, as a general, paradigmatic »geometrical method«[188] . It is perhaps no wonder that Galileo's *Discorsi* are organized in part in *theorems*, *problems*, *lemmata*, *corollaries* and *scholia*, nor that Newton's *Principia* follow the same pattern with still greater consequence. But in Descartes' replies to the critics in the *Meditationes* (1641) we find that he was urged to propound his »arguments in a geometrical fashion, in order that the reader may perceive them as it were with a single glance«--which he then did perfectly, with definitions, axioms and propositions in his *Arguments Demonstrating the Existence of God [...] Drawn Up in Geometrical fashion*[189]. Later in the century, both Spinoza's exposition of Descartes philosophy (1663) and his *Ethica* (1675)[190] were *more geometrico demonstratae* as perfectly as anything, and in

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account of their distinction or non-distinction between *word* and *thing*, between *signifier* and *signified.*

[187] See e.g. Gilbert's *De magnete* from 1600 (ed., transl. Thompson 1900).

[188] The »geometric method« could of course be identified with Euclidean as well as with Archimedean method; I prefer a continued reference to Archimedes, because *he* continued to be regarded (with Apollonios) as the supreme geometer, the *Elements* being primarily *prolegomena.*

[189] "Objections and Replies", transl. Haldane & Ross 1931: II, 48 and 52-59. Remarkably, Descartes organized his Geometrie (facsimile ed. & transl. Smith & Latham 1954) discursively and not more geometrico.

[190] Ed., transl. Caillois, Francès & Misrahi 1954.

1658 Pascal wrote an essay “De l’esprit géométrique et de l’art de persuader” setting forth the reasons why the geometrical method possessed this paradigmatic status,--namely that geometry complies with a method »consisting principally in two things, one of them to prove every proposition in particular, the other to dispose all propositions in the best order«[191]. Evidently a perfect methodology, and according to many seventeenth-century thinkers a goal within reasonable reach. After 500 years and many disruptions of continuous development, the phantasmagorically optimistic belief in Euclid, Ptolemy and Nature which had once sent scholars to the outposts of Christian Europe and beyond had proved convincingly--if only temporarily--true.

VALE

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[191] Ed. Chevalier 1954: 575-604, quotation from p. 576. Precisely the same two qualities were emphasized by Descartes when *he* characterized the method (*Objections and Replies*, transl. Haldane & Ross 1931: II, 48).

Once more, the *weight, number, and measure* of *Wisd.* XI, 21 turn up in Pascal’s argument (p. 583)--but in a way which shows him to be closer to d’Alembert and Kant than to Augustine (not to speak of Isidore).

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