The formation of Islamic mathematics
sources and condition

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In memory of GEORGE SARTON

Contents

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To state that the author remains responsible for all errors is not just a matter of academic decorum; given that I come to the subject as an outsider it expresses an obvious matter of fact.

Biographical cues

Based mainly upon ***DSB***; ***GAS***; ***GAL***; [Suter 1900]; [Sarton 1927]; [Sarton 1931]; and [Dodge 1970].

ʿAbbaside dynasty. Dynasty of Caliphs, in effective power in Baghdad from 749–50 to the later 10th / earlier 11th century, formally continued until 1258.

Abraham Bar Ḥiyya (Savasorda). Fl. before 1136. Hispano-Jewish philosopher, mathematician and astronomer.

Abū Bakr. Early 9th century? Otherwise unknown author of the *Liber mensurationum*.

Abū Kāmil. Fl. late 9th and / or early 10th century. Egyptian mathematician.

Abū Maʿšar. 787–886. Eastern Caliphate. Astrologer.

Abū'l-Wafāʾ. 940–997 / 98. Eastern Caliphate. Mathematician and astronomer.

Al-Bīrūnī. *B.* 973, *d.* after 1050. Astronomer, mathematician, historian and geographer from Khwārezm.

Al-Fārābī. C. 870–950. Eastern Caliphate. Philosopher.

Al-Fazārī. Fl. second half of the 8th century. Eastern caliphate. Astronomer.

Al-Ghazzālī. 1058–1111. Eastern Caliphate. Theologian.

Al-Ḥajjāj. Fl. later 8th to early 9th century. Eastern Caliphate. Translator of the *Elements* and of the *Almagest*.

Al-Ḥaṣṣar. Fl. 12th or 13th century, probably in Morocco. Mathematician.

Al-Jāḥiẓ. C. 776–868 / 69. Iraq. Muʿtazilite theologian; zoologist.

Al-Jawharī. Fl. c. 830. Eastern Caliphate. Mathematician and astronomer.

Al-Karajī. Fl. c. 1000. Eastern Caliphate. Mathematician.

Al-Khayyāmī. 1048(?)–1131(?). Iran. Mathematician, astronomer, philosopher.

Al-Khāzin. *D.* 961 / 971. Iran. Mathematician and astronomer.

Al-Khazinī. Fl. c. 1115–1130. Iran. Astronomer, theoretician of mechanics and instruments

Al-Khuwārizmī. Fl. c. 980. Khwārezm. Lexicographer.

Al-Khwārizmī. Late 8th to mid-9th centuries. Eastern Caliphate. Mathematician, astronomer, geographer.

Al-Kindī. C. 801 to c. 866. Eastern Caliphate. Philosopher, mathematician, astronomer, physician, etc.

Al-Māhānī. Fl. c. 860 to c. 880. Eastern Caliphate. Mathematician, astronomer.

Al-Maʾmūn. 786–833. ʿAbbaside Caliph 803–833, ardent *muʿtazilite*, patron of *awāʾil* learning.

Al-Nadīm. C. 935 to 990. Baghdad. Librarian, lexicographer.

Al-Nasawī. Fl. 1029–1044. Khurasan, Eastern caliphate. Mathematician.

Al-Nayrīzī. Fl. early 10th century. Eastern Caliphate. Mathematician, astronomer.

Al-Qalaṣādī. 1412 to 1486. Spain, Tunisia. Mathematician, jurisprudent.

Al-Samaw'al. Fl. mid-12th century, Iraq, Iran. Mathematician, physician.

Al-Umawī. Fl. 14th century. Spain, Damascus. Mathematician.

Al-Uqlīdisī. Fl. 952/953, Damascus. Mathematician.

Al-Yaʿqūbī. Fl. 873–891. Eastern Caliphate. Shiite historian and geographer.

Anania of Širak. Fl. first half of the 7th century. Armenia. Mathematician, astronomer, geographer etc.

Banū Mūsā («Sons of Mūsā»). Three brothers, c. 800 to c. 875, Baghdad. Mathematicians, translators, organizers of translation.

Ḥunayn ibn Isḥāq. 808–873. Eastern Caliphate. Physician, translator.

Ibn Abī Uṣaybiʿa. 1203/4–1270. Syrian lexicographer and physician

Ibn al-Bannā'. 1256–1321. Morocco. Mathematician, astronomer.

Ibn al-Haytham. 965 to c. 1040. Iraq, Egypt. Mathematician, astronomer.

Ibn Khaldūn. 1332–1406. Maghreb, Spain, Egypt. Historian, sociologist.

Ibn Qunfudh. D. 1407/8. Algeria. Jurisprudent, historian. Commentator to ibn al-Bannā'.

Ibn Ṭāhir. D. 1037. Iraq, Iran. Theologian, mathematician, etc.

Ibn Turk. Earlier 9th century. Turkestan. Mathematician.

***Ikhwān al-Ṣafā'* («Epistles of the Brethren of purity»).** 10th century *Ismāʿīlī* encyclopedic exposition of philosophy and sciences.

Kamāl al-Dīn. D. 1320. Iran. Mathematician, mainly interested in optics.

Kūšyār ibn Labbān. Fl. c. 1000. Eastern Caliphate. Astronomer, mathematician.

Māšā'allāh. Fl. 762 to c. 815. Iraq. Astrologer.

Muḥyi'l-Dīn al-Maghribī. Fl. c. 1260 to 1265, Syria and Iran. Mathematician, astronomer, astrologer.

Naṣīr al-Dīn al-Ṭūsī. D. c. 1214. Iran. Astronomer, mathematician.

Qayṣar ibn Abī'l-Qāsim. 1178–1251. Egypt, Syria. Jurisprudent, mathematician, technologist.

Qusṭā ibn Lūqā. Fl. 860 to 900. Eastern Caliphate. Physician, philosopher, translator.

Rabīʿ ibn Zaid. Fl. c. 961. Bishop at Cordoba and Elvira, astrologer.

Severus Sebokht. Fl. mid-7th century. Syrian bishop. Astronomer, commentator on philosophy.

Thābit ibn Qurra. 836–901. Eastern caliphate. Mathematician, astronomer, physician, translator, etc.

ʿUmar ibn al-Farrukhān. 762–812. Eastern Caliphate. Astronomer, astrologer.

ʿUmayyad dynasty. Dynasty of Caliphs, 661–750.

Yūḥannā al-Qass. Fl. first half of 10th century? Mathematician, translator of mathematics.

I. Introducing the problem

When the history of science in prehistoric or Bronze Age societies is described, what one finds is normally a description of technologies and of that sort of inherent practical knowledge which these technologies presuppose. This state of our art reflects perfectly well the state of the arts in these societies: They present us with *no specific, socially organized, and systematic search for and maintenance of cognitively coherent knowledge* concerning the natural or practical world – i.e., with nothing like our own scientific endeavour.

The ancestry of that specific endeavour is customarily traced back to the «Greek Miracle», well described by Aristotle:

> At first he who invented any art whatever that went beyond the common perceptions of man was naturally admired by men, not only because there was something useful in the inventions, but because he was thought wise and superior to the rest. But as more arts were invented, and some were directed to the necessities of life, others to recreation, the inventors of the latter were naturally always regarded as wiser than the inventors of the former, because their branches of knowledge did not aim at utility. Hence when all such inventions were already established, the sciences which do not aim at giving pleasure or at the necessities of life were discovered, [...]
>
> So [...], the theoretical kinds of knowledge [are thought] to be more the nature of Wisdom than the productive.[1]

The passage establishes the fundamental distinction between «theoretical» and «productive» knowledge, between «art» and «science», and thus the break with those earlier traditions where knowledge beyond the useful was carried by those same groups which possessed the highest degree of useful knowledge[2]. Inherent though not fully explicitated is also a fairly absolute social and cognitive separation of the two. Though obvious deviations from this ideal can be found in several Ancient Greek scientific authors (some of which we shall mention below), Aristotle's discussion can be regarded a fair description of the prevailing tendency throughout Greek Antiquity.

On the other hand, it is definitely not adequate as a description of Modern or contemporary attitudes to the relation between science and technology (which we are often disposed to regard as «applied science»[3]). So, we are separated from the Bronze

[1] *Metaphysica* 981^b14–982^a1, [trans. Ross 1928].

[2] I have discussed this relation at some depth for the case of Old Babylonian mathematics in my [1985]. A short but striking illustration for the case of Egypt is supplied by the opening phrase of the Rhind Mathematical Papyrus, the mainly utilitarian contents of which is presented as «accurate reckoning of entering into things, knowledge of existing things all, mysteries [...] secrets all» [trans. Chace et al 1929: Plate 1][2] 12; similarly [Peet 1923: 33].

[3] I shall not venture into a discussion of this conception, which is probably no better founded than its Aristotelian counterpart.

Age organization of knowledge not only by a «Greek Miracle» but also by at least one later break, leading to the acknowledgement of the practical implications of theory. Customarily we locate this break in the Late Renaissance, regarding Francis Bacon as a pivotal figure.

A first aim of the present paper is to show that the break took place earlier, in the Islamic Middle Ages, which first came to regard it as a fundamental epistemological premiss that problems of social and technological practice can (and should) lead to scientific investigation, and that scientific theory can (and should) be applied in practice. Alongside the Greek we shall hence have to reckon an *Islamic Miracle*. A second aim is to trace the circumstances which made Medieval Islam produce this miracle.

I shall not pursue the thesis and the consecutive problem in broad generality, which would certainly be beyond my competence. Instead, I shall concentrate on the case of the mathematical sciences. I shall do so not as a specialist in Islamic mathematics[4] but as a historian of mathematics with a reasonable knowledge of the mathematical cultures connected to that of Medieval Islam, basing myself on a fairly broad reading of Arabic sources in translation. What follows is hence a tentative outline of a synthetic picture as it suggests itself to a neighbour looking into the garden of Islam; it should perhaps best be read as a set of questions to the specialists in the field formulated by the interested outsider.

From this point of view, mathematics of the Islamic culture[5] appears to differ from its precursors by a wider scope and a higher degree of integration. It took up the full range of interests in all of the mathematical traditions and cultures with which it came in contact, «scientific» as well as «sub-scientific» (a concept which I discuss below); furthermore, a significant number of Islamic mathematicians master and work on the whole stretch from elementary to high-level mathematics (for which reason they tend to see the former *vom höheren Standpunkt aus*, to quote Felix Klein). Even if we allow for large distortions in our picture of Greek mathematics due to the schoolmasters of

[4] This role is barred to me already for the reason that my knowledge of Arabic is restricted to some elements of basic grammar and the ability to use a dictionary. Indeed, the only Semitic language which I know is the simple Babylonian of mathematical texts.

[5] I use the term «culture» as it is done in cultural anthropology. Consequently, the Sabian, Jewish and Christian minorities which were integrated into Islamic society were all participants in the «Islamic culture», in *dār al-Islam*.

Similarly, «Islamic mathematics» is to be read as an abbreviation for «mathematics of the Islamic culture», encompassing contributions made by many non-Muslim mathematicians. I have avoided the term «Arabic mathematics» not only because it would exclude Persian and other non-Arabic mathematicians but also (and especially) because Islam and not the Arabic language must be considered the basic unifying force of the «Islamic culture» – cf. below, chapter X.

Late Antiquity[6], similar broad views of the essence of mathematics appear to have been rare in the mature period of Greek mathematics; those who approached it tended to miss either the upper end of the scale (like Hero) or its lower part (like Archimedes and Diophantos, the latter with a reserve for his lost work on fractions, the *Moriastica*).

The «Greek Miracle» would not have been possible, had it not been for the existence of intellectual source traditions. If we restrict ourselves to the domain of the exact sciences, nobody will deny that Egyptian and Babylonian calculators and astronomers supplied much of the material (from Egyptian unit fractions to Babylonian astronomical observations) which was so radically transformed by the Greek mathematicians[7]. Equally certain is, however, that the Egyptian and Babylonian cultures had never been able to perform this transformation, which was only brought about by specific social structures and cultural patterns present in the Greek *polis*[8]. Similarly, if we want to understand the «miracle» of Islamic mathematics, and to trace its unprecedented integration of disciplines and levels, we must also ask both for the sources which supplied the material to be synthesized and for the forces and structures in the culture of Islam which caused and shaped the transformation – the «formative conditions».

II. Scientific source traditions: the Greeks

Above, a dichotomy between «scientific» and «sub-scientific» source-traditions was introduced. Below, I shall return to the latter and discuss why they must be taken more seriously into account than normally done. At first, however, I shall concentrate on the more distinctly visible *scientific* sources, and first of all on the most visible of all, to Medieval Islamic lexicographers as well as to modern historians of science[9]: Greek mathematics.

That this source was always regarded as having paramount importance will be seen, e.g., from the *Fihrist* (*Catalogue*) written by the 10th century Baghdad court librarian al-Nadīm[10]. The section on mathematics and related subjects contains the names and

[6] Cf. [Toomer 1984: 32].

[7] Even the proto-philosophical cosmogonies which precede the rise of Ionian natural philosophy are now known to make use of Near Eastern material – see [Kirk, Raven & Schofield 1983: 7–74, *passim*].

[8] A stimulating discussion of the formative conditions for the rise of philosophy is [Vernant 1982]. An attempt to approach specifically the rise of scientific mathematics is offered in my [1985].

[9] For the same reason, I shall treat this part of the subject with utter briefness, mentioning only what is absolutely necessary for the following. A detailed account of the transmission of single Greek authors will be found in *GAS* V, pp. 70–190.

[10] A recent translation based on all available manuscripts is [Dodge 1970]. Chapter 7, section 2, dealing with mathematics, and the mathematical passages from section 1, dealing with philosophy, were translated from Flügel's critical edition (from 1872, based on a more restricted number of

known works of 35 pre-Islamic scholars. 21 of these are Greek mathematicians (including writers on harmonics, mathematical astronomy and mathematical technology). All the rest deal with astrology (in the narrowest sense, it appears) and Hermetic matters (4 of these belong to the Greco-Roman world, 6 to the Assyro-Babylonian orbit, and 4 are Indians). So, no single work on mathematics written by a non-Greek, pre-Islamic scholar was known to our 10th-century court librarian[11], who would certainly be in good position to know anything there was to know.

Centrally placed in the Greek tradition as it was taken over were the *Elements* and the *Almagest*. Together with these belonged, however, the «Middle Books», the *Mutawassiṭāt*: The «Little Astronomy» of Autolycos, Euclid, Aristarchos, Hypsicles, Menelaos and Theodosios; the Euclidean *Data* and *Optics* and some Archimedean treatises[12]. Even Apollonios and a number of commentators to Euclid, Ptolemy and Archimedes (Pappos, Hero, Simplicios, Theon, Proclos, Eutocios) belong to the same cluster[13].

Somewhat less central are the Greek arithmetical traditions, be it Diophantos or the Neopythagorean current as presented by Nicomachos (or by the arithmetical books of the *Elements*, for that matter). Still, all the works in question were of course translated; further work on Diophantine ideas by al-Karajī and others is well testified[14], and even though Nicomachean arithmetic was according to Ibn Khaldūn «avoided by later scholars» as «not commonly used [in practice]»[15], it inspired not only Thābit (Nicomachos' translator) but other scholars too[16]. Finally, the treatment of the subject in encyclopaedic works demonstrates familiarity with the concepts of Pythagorean and

manuscripts) by Suter ([1892], supplement [1893]).

[11] Comparison with other chapters in the *Catalogue* demonstrates that the lopsided selection is not due to any personal bias of the author.

[12] A full discussion is given by Steinschneider [1865], a brief summary by Sarton [1931: 1001f].

[13] This can be compared with the list of works which al-Khayyāmī presupposes as basic knowledge in his *Algebra*: The *Elements* and the *Data*, Apollonios' *Conics* I–II, and (implicit in the argument) the established algebraic tradition [trans. Woepcke 1851: 7]. The three Greek works in question constitute an absolute minimum, we are explained.

[14] See Woepcke's introduction to and selections from al-Karajī's *Fakhrī* [1851: 18–22 and *passim*]; [Sesiano 1982: 10–13]; and [Anbouba 1979: 135].

[15] *Muqaddimah* VI,19; quoted from Rosenthal's translation [1958: III, 121].

[16] On Thābit's investigation of «amicable numbers», see [Hogendijk 1985], or Woepcke's translation of the treatise [1852]. Two later treatises on theoretical arithmetic were also translated by Woepcke [1861], one anonymous and one by Abū Ja'far al-Khāzin (a Sabian like Thābit). Among recent publications on the subject, works by Anbouba [1979] and Rashed [1982; 1983] can be mentioned.

Neopythagorean arithmetic[17].

Also somewhat peripheral – *yet definitely less peripheral than with those Byzantine scholars whose selection of works to be laboured upon and hence to survive has created our archetypical picture of Greek mathematics* – are the subjects which we might characterize tentatively as «technological mathematics» (al-Fārābī speaks of *'ilm al-hiyal*, «science of artifices»[18]) and its cognates: Optics and catoptrics, «science of weights», and non-orthodox geometrical constructions (geometry of movement, geometry of fixed compass-opening). They are well represented, e.g., in works by Thābit, the Banū Mūsā, Qustā ibn Lūqā, ibn al-Haytham and Abū'l-Wafā' – detailed discussions would lead too far astray.

III. Scientific source traditions: India

The way al-Nadīm mentions the Indians is a good reflection of the way the Indian inspiration must have looked when seen from the Islamic end of the transmission line, even though he misses (and is bound to miss) essential points. Indian mathematics when it arrived into the Caliphate had, according to all available evidence, become anonymous: The ideas of Indian trigonometry were adopted via Siddhantic astronomical works and *zījes* based fully or in part on Indian sources[19]; Islamic algebra was untouched by Indian influence – which it would hardly have been, had the Islamic mathematicians had direct access to great Indian authors in the style of Āryabhaṭa and Brahmagupta[20].

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[17] So, al-Fārābī's *Ihša' al-'ulūm* (*De scientiis*, trans. [Palencia 1953: 40]); al-Khuwārizmī's *Mafātiḥ al-'ulūm* (translation of the section on arithmetic in [Wiedemann 1970: I,411–428]; theoretical arithmetic is treated amply pp. 411–418); and the encyclopaedic part of the *Muqaddimah* [trans. Rosenthal 1958: III, 118–121].

The treatment in the encyclopaediae is remarkably technical. In itself it seems highly probable that Late Hellenistic Hermeticism, and Sabian, Jabirian and Ismā'īlī numerology would mix up with «speculative» arithmetic. To judge from the encyclopaediae, however, a possible inspiration of interests from that quarter has remained without consequence for the contents of the subject *when understood as mathematics*. Cf. also below, chapter XVI.

[18] In [Palencia 1953: (Arabic) 73].

[19] See, e.g., *GAS* V, 191ff; [Pingree 1973]; and Pingree, "Al-Fazārī", *DSB* IV, 555f. The *Zīj al-sindhind*, the Sanskrit astronomical treatise translated with the assistance of al-Fazārī around CE 773, was mainly built upon the methods of Brahmagupta's *Brāhmasphuṭasiddhānta* – but even influence from the *Āryabhaṭiya* is present. The original authors had become invisible during the process.

[20] The discrepancy between the advanced syncopated algebra of the Indians and the rhetorical algebra of al-Khwārizmī was already noticed by Léon Rodet [1878]. This observation remains valid even if his supplementary claim (*viz.* that al-Khwārizmī's method and procedures are purely Greek and identical with those of Diophantos – p. 95) is unacceptable.

Al-Khwārizmī can be considered a key witness: He is one of the early Islamic workers on astronomy, and mainly oriented towards the *Zīj al-sindhind*, with some connection to the Pahlavi

Below the level of direct scientific import, a certain influence from Indian algebra is plausible. This is indicated by the metaphorical use of *jidhr* («root», «stem», «lower end», «stub» etc.) for the first power of the unknown. Indeed, this same metaphor (which can hardly be considered self-evident, especially not in a rhetorical, non-geometric algebra – cf. below, chapter VI) is found already around 100 BCE in India[21]. In all probability, however, this borrowing was made via practitioners' sub-scientific transmission lines, to which we shall return below; furthermore, the ultimate source for the term need not have been Indian.

Apart from trigonometry, the main influence from Indian mathematics is the use of «Hindu numerals». If the Latin translation of al-Khwārizmī's introduction of the system is to be believed (and it probably is[22]), he only refers it to «the Indians». So does already Severus Sebokht in the mid-seventh century[23]. The earliest extant «algorism» in Arabic, that of al-Uqlīdisī from the mid-tenth century, is no more explicit. Most of its references are to «scribes» or «people of this craft» – evidently, local users of the technique are thought of; as explicit reference to the origin of the craft nothing more precise than «Indian reckoners» occurs[24]. In addition, the dust-board so essential for early «Hindu reckoning» was known under a Persian, not an Indian name (*takht*)[25]. Finally, the methods of indeterminate equations and combinatorial analysis (which both are staple goods in Indian arithmetical textbooks) are not found with the early Islamic expositions of Hindu reckoning (even though examples of indeterminate equations can be found in textbooks based on finger reckoning)[26]. So, the Islamic introduction of Hindu reckoning can hardly have been based on direct knowledge of «scientific» Indian expositions of arithmetic. Like trigonometry, it appears to derive from contact with practitioners using the system.

Zīj al-Šāh and (presumably) to Hellenistic astronomy (cf. Toomer, "Al-Khwārizmī", *DSB* VII, 360f). As we shall see when discussing his treatise on mensuration he would recognize and acknowledge Indian material when using it.

[21] [Datta & Singh 1962: II,169].

[22] The translation conserves the traditional Islamic invocation of God (see [Vogel 1963: 9]), which would in all probability have been cut out before credit-giving references were touched at (as it was cut out in both Gherardo of Cremona's and Robert of Chester's translations of al-Khwārizmī's *Algebra* – see the editions in [Hughes 1986: 233] and [Karpinski 1915: 67], or the quotations in [Høyrup 1985a: 39 n.58].

[23] Fragment published and translated in [Nau 1910: 225f].

[24] Trans. [Saidan 1978]. The various types of references are found, e.g., pp. 45, 104 and 113, respectively.

[25] *Ibid.* p. 351.

[26] Cf. *ibid.* p. 14. On the later (and probably independent) origin of Islamic combinatorial analysis, see [Djebbar 1981: 67ff].

Some inspiration for work on the summation of series may have come from India. Apart from the chessboard problem (to which we shall return below), the evidence is not compelling, and proofs given by al-Karajī and others may be of Greek as well as Indian inspiration.

The two scientific source traditions were mainly tapped directly through translations from Greek and Sanskrit. To some extent, however, the mathematics of Indian astronomy found its way through Pahlavi, while elementary Greek astronomy may have been diffused through both Pahlavi and Syriac[27]. Neither of these secondary transmission channels appears to have been scientifically creative, and they should probably only be counted among the scientific source traditions in so far as we distinguish «scientific» (e.g., astronomical and astrological) practice from «sub-scientific» (e.g., surveyors', master-builders' and calculators') practice.

IV. Sub-scientific source traditions: commercial calculation

This brings us to the problems of sub-scientific sources, which we may initially approach through an example. The last chapter in al-Uqlīdisī's arithmetic has the heading «On Doubling One, Sixty-Four Times»[28]. Evidently, we are confronted with the chessboard problem, to the mathematical solution of which already al-Khwārizmī had dedicated a treatise[29], and whose appurtenant tale is found in various Islamic writers from the 9th century onwards[30]. Al-Uqlīdisī, however, states that «This is a question many people ask. Some ask about doubling one 30 times, and others ask about doubling it 64 times», thereby pointing to a wider network of connections. In the mid-twelfth century, Bhaskara II asks about 30 doublings in the *Lilavati*[31]; so does Problem 13 in the Carolingian collection *Propositiones ad acuendos juvenes* ascribed to Alcuin[32].

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[27] See [Pingree 1963: 241ff] and [Pingree 1973: 34]. Through the same channels, especially through the Sabians, some Late Babylonian astronomical lore may have been transmitted (cf. the Babylonian sages mentioned in the *Fihrist*). Still, the integration of Babylonian results and methods into Greek as well as Indian astronomy makes it impossible to distinguish conceivable more direct Babylonian contributions.

In principle, non-astronomical Greek mathematics *may* also have been conveyed through Syriac learning. Evidence in favour of this hypothesis is, however, totally absent – cf. below, chapter XI.

[28] *Kitāb al-Fuṣūl fī al-Ḥisāb al-Hindi* IV,32, trans. [Saidan 1978: 337].

[29] According to a remark in the third part of Abū Kāmil's *Algebra* (Jan Hogendijk, personal communication).

[30] Relevant passages from al-Ya'qūbī and al-Khazinī are translated in [Wiedemann 1970: I, 442–453].

[31] Trans. [Colebrooke 1817: 55].

[32] Critical edition in [Folkerts 1978].

A newly published cuneiform tablet from the 18th century BCE[33] contains the earliest extant version of the problem, formulated in a dressing which connects it to the chessboard-problem but stated mathematically in analogy with the Carolingian problem.

Evidently, the problem belongs in the category of «recreational problems», defined by Hermelink as «problems and riddles which use the language of everyday but do not much care for the circumstances of reality»[34] – to which we may add the further observation that an important aspect of the «recreational» value of the problems in question is a funny, striking or even absurd deviation from these circumstances. With good reason, Stith Thompson includes the chessboard doublings in his *Motif-Index of Folk Literature*[35]. Seen from a somewhat different perspective, we may look at recreational mathematics as a «pure» outgrowth of the teaching and practice of practitioners' mathematics (which, in the pre-Modern era, spells computation). It does not seek mathematical truth or theory; instead, it serves the display of virtuosity[36].

Other recreational problems share the widespread distribution of the repeated doublings. Shared problem-types (and sometimes shared numbers) and similar or common dressings connect the arithmetical epigrams in Book IX of the *Anthologia graeca*[37], Anania of Širak's arithmetical collection from 7th century Armenia[38], the Carolingian *Propositiones*, part of the Ancient Egyptian Rhind Papyrus, and Ancient and Medieval problem collections from India and China[39]. They turn up without dressing in Diophantos' *Arithmetica*, and they recur in Medieval Islamic, Byzantine and

[33] Published in [Soubeyran 1984: 30]; discussion and comparison with the Carolingian problem and the chessboard problem in [Høyrup 1986: 477f].

[34] [Hermelink 1978: 44].

[35] [Thompson 1975: V, 542 (Z21.1)].

[36] In my [1985], I use the same distinction between theoretical aim and display of virtuosity in a sociological discussion of the different cognitive and discursive styles of Greek and Babylonian mathematics. Even the difference between the arithmetical books VII–IX of the *Elements* and Diophantos' *Arithmetica* is elucidated by the same dichotomy; truly, Diophantos has theoretical insight into the methods he uses, but his presentation is still shaped by an origin of his basic material in recreational mathematical riddles.

We observe that the complex of practical and recreational mathematics can (structurally and functionally) be regarded as a continuation of the Bronze Age organization of knowledge (cf. above, chapter I). The two were, however, separated by a decisive gap in social prestige – comparable to the gap between the Homeric bard and a Medieval peasant telling stories in the tavern.

[37] In [*The Greek Anthology*, vol. V]. The epigrams were edited around CE 500 by Metrodoros.

[38] Trans. [Kokian 1919]. It should be observed that Anania had studied in the Byzantine Empire, and that part of the collection appears to come from the Greek orbit.

[39] A detailed discussion would lead too far. A wealth of references will be found in [Tropfke / Vogel 1980, *passim*].

Western European problem collections. The pattern looks very much like the distribution of folktales (down to the point that Diophantos' adoption of the material can be seen as a parallel to the literate adoption of folk tale material). The geographical distribution is also roughly congruent with that of the Eurasian folktale (*viz*. «from Ireland to India»[40]). This, however, can only be regarded as a parallel, not as an explanation. Firstly, indeed, the recreational problems cover an area stretching into China, beyond the normal range of Eurasian folktales[41]; secondly, mathematics can only be entertaining in an environment which knows something about the subject. The dominating themes and techniques of the problems in question point to the community of traders and merchants interacting along the Silk Road, the combined caravan and sea route reaching from China to Cadiz[42].

«Oral mathematics» is rarely encountered *in vivo* in the sources. Like folktales before the age of folklorists, it has normally been worked up by those who took the care to write it down, adoption entailing adaption[43]. Ordinarily, they would be mathematicians, who at least arranged the material systematically, and who perhaps gave alternative or better methods for solution, or supplied a proof. In a few cases, however, they have also given a description of the situation in which they found the material. So Abū Kāmil in the preface to his full mathematical treatment of the indeterminate problem of «the hundred fowls»[44], which he describes as

> eine besondere art von Rechnungen, die bei Vornehmen und Geringen, bei Gelehrten und Ungelehrten zirkulieren, an denen sie sich ergötzen und die sie neu und schön finden; es frägt einer den andern, dann wird ihm mit einer ungenauen, nur vermutungsweise Antwort geantwortet, sie erkennen darin weder ein Prinzip noch eine Regel.[45]

A similar aggressive description of reckoners who

> strain themselves in memorising [a procedure] and reproduce it without knowledge or

[40] [Thompson 1946: 13ff].

[41] This is illustrated beautifully by the chessboard problem and its appurtenant tale. The *motif* turns up in the Chinese as well as in the «Eurasian» domain; the Chinese *tale*, however, is wholly different, dealing with a peasant and the wages of his servant, determined as the successively doubled harvests from one grain of rice. [Thompson 1975: V, 542, Z 21.1.1].

[42] It is worth noticing that two arithmetical epigrams from the *Anthologia graeca* deal with the Mediterranean extensions of the route: XIV,121 with the land route from Rome to Cadiz, and XIV,129 with the sea route from Crete to Sicily.

[43] Here, the Carolingian *Propositiones* appear to form an exception. The editor of the collection (Alcuin?) was obviously not more competent as a mathematician than the practitioners who supplied the material.

[44] Its distribution (from Ancient China and India to Aachen) is described in [Tropfke / Vogel 1980: 614–616].

[45] *Kitāb al-ṭarā'if fī'l-ḥisāb* (*Book of Rare Things in Calculation*), trans. [Suter 1910: 100].

scheme [and others who] strain themselves by a scheme in which they hesitate, make mistakes, or fall in doubt

is given by al-Uqlīdisī in connection with the continued doubling asked by «many people»[46].

It is precisely this situation which has distorted the approach to the sub-scientific traditions. The substratum was anonymous and everywhere present, and its procedures would rather deserve the names of recipes than that of methods. Each mathematician when inspired from it would therefore have to employ his own techniques to solve the common problems (or at least have to translate the recipes into his own theoretical idiom) – Diophantos would use rhetorical algebra, the Chinese *9 Chapters on Arithmetic* would manipulate matrices, and the *Liber abaci* would find the answer by means of proportions. We should hence not ask, as commonly done, whether Diophantos (or the Greek arithmetical environment) was the source of the Chinese or *vice versa*. There was *no specific source: The ground was everywhere wet.*

Besides the supply of problems and procedures, the merchants' and book-keepers' community appears to have provided Islamic mathematics with two of its fundamental arithmetical techniques: The peculiar system of fractions, and the «finger reckoning».

The system of fractions is built up by means of the series of «principal fractions» 1/2, 1/3, ... 1/10 (the fractions which possess a name of their own in the Arabic language) and their additive and multiplicative combinations[47]. The system has been ascribed to Egyptian influence and to independent creation within the territory of the Eastern Caliphate[48]. It turns out, however, that already some Old Babylonian texts use similar expressions, e.g., «the third of *X*, and the fourth of the third of *X*» for $^{5}/_{12}X$ [49]. So, in reality we are confronted with an age-old system and at least with a common Semitic usage – but for instance the formulation of problem 37 of the Rhind Papyrus suggests in fact a common *Hamito*-Semitic usage, a usage which had already provided the base on which the Egyptian scribes developed their learned unit fraction system around the turn of the second millennium BCE[50] Since the same «fractions of

[46] Trans. [Saidan 1978: 337].

[47] Described, e.g., in [Saidan 1974: 368]; in [Juschkewitsch 1964: 197ff]; and in Youschkevitch, "Abū'l-Wafā'", *DSB* I, 40.

[48] Saidan [1974: 358] among others proposes Egyptian influence. Youschkevitch quotes M. I. Medevoy for the suggestion of independence.

[49] «*sa-lu-us-ti* 20 *ú ra-ba-at sa-lu-us-ti ú-te₄-tim*» – MLC 1731, rev. 34–35, in [A. Sachs 1946: 205] (the whole article deals with such phenomena). Expressions in the same vein are encountered in the tablet YBC 4652, N^{os} 19–22, ed., trans. [Neugebauer & Sachs 1945: 101].

[50] The problem in question is of typical «riddle» or «recreational» character: «Go down I 3 times into the *ḥekat*-measure, $^{1}/_{3}$ of me is added to me, $^{1}/_{3}$ of $^{1}/_{3}$ is added to me, $^{1}/_{9}$ of me is added to me; return I, filled I am» (the «literal translation», [Chace et al 1929: Plate 59]). It can thus

fractions» are also used occasionally in the Carolingian *Propositiones*[51], they appear to have spread over the whole Near East and the Roman Empire in Late Antiquity, and thus to have been well-rooted in the commercial communities all over the region covered by the Islamic expansion – in agreement with its use in arithmetical textbooks written for practical people in the earlier Islamic period (cf. below, chapter XIV).

The use of «principal fractions» and «fractions of fractions» appears to coincide with that of «finger reckoning», another characteristic method of Islamic elementary mathematics. It was referred to as *ḥisāb al-Rūm wa'l-ʿArab*[52], «calculation of the Byzantines and the Arabs», and a system related to the one used in Medieval Islam had been employed in Ancient Egypt[53]. Various Ancient sources refer to the symbolization of numbers by means of the fingers[54], without describing, it is true, the convention which was applied – but since the very system used in Islam is described around CE 700 in Northumbria by Beda[55], who would rather be familiar with descendants of Ancient methods than with the customs of Islamic traders, we may safely assume that the Ancient system was identical with both.

V. Sub-scientific source traditions: practical geometry

The above dealt with what appears to have been a more or less shared tradition for practitioners involved in book-keeping and commercial arithmetic (*ḥisāb*, as the term was to be in Islam). Another group possessing a shared tradition (*viz*. for practical geometry) was made up by surveyors, architects and «higher artisans»[56].

In the case of this sub-scientific geometry, we are able to follow how the process of mathematical synthesis had begone long before the Islamic era. Indeed, various Ancient civilizations had had their specific practical geometries. The (partly) different

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be seen as a witness of a current more or less popular usage. A close analysis of the (utterly few) instances of rudimentary unit fraction notation from the Old Kingdom (*viz*. from the 24th century BCE) suggests that they stand midway between this original usage and the fully developed system (space and relevance does not permit further unfolding of the argument).

[51] N^{os} 2, 4 and 40 deal with *medietas medietatis* (and N^{os} 2 and 40 further with *medietas [huius] medietatis*, i.e. with ½ of ½ of ½); N^{o} 3 treats of *medietas tertii*. All four problems are of the same type as N^{o} 37 of the Rhind Papyrus, cf. note 50.

[52] [Saidan 1974: 367] (misprint corrected).

[53] See the cubit rod reproduced in [Menninger 1957: II,23].

[54] See *ibid.* pp. 11–15.

[55] *De temporum ratione*, cap. I, ed. [Jones 1943: 179–181].

[56] Once again, the evidence for a shared tradition is found in Islamic sources – e.g., Abū'l-Wafā''s *Book on What is necessary From Geometric Construction for the Artisan* [trans. Krasnova 1966].

characters of Egyptian and Babylonian practical geometry have often been noticed[57]. The melting pots of the Assyrian, Achaemenid, Hellenistic, Roman, Bactrian and Sassanian empires mixed them up completely[58], and through the Heronian corpus some Archimedean and other improvements were infused into the practitioners methods and formula[59]. This mixed and often disparate type of calculatory geometry was

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[57] So, the different ways to find the area of a circle; the Babylonian treatment of irregular quadrangles and of the bisection of trapezia and the absence of both problem types in Egypt.

[58] So, the Demotic Papyrus Cairo JE 89127–30, 137–43 (3d century BCE) has replaced the excellent Egyptian approximation to the circular area (equivalent to $\pi = {}^{256}/_{81} \approx 3.16$) with the much less satisfactory Babylonian and Biblical value $\pi = 3$ (see [Parker 1972: 40f], problems 32–33). The same value is also taken over in pre-Heronian Greco-Egyptian practical geometry, cf. Pap. Gr. Vind. 19996 as published in [Vogel & Gerstinger 1932: 34]. A formula for the area of a circular segment which is neither correct nor near at hand for naive intuition is used in the Demotic papyrus mentioned above (N° 36); in the Chinese *Nine Chapters on Arithmetic*, it is used in N^os I,35–36, and made explicit afterwards [trans. Vogel 1968: 15]. Hero, finally, ascribes it to «the Ancients» (ὀι ἀρχαῖοι – *Metrica* I,xxx, ed. [Schöne 1903: 73] while criticizing it (cf. the discussion in [van der Waerden 1983: 39f, 174]).

The Babylonian calculation of the circular area, which is a deterioration when compared to the Middle Kingdom Egyptian method, was probably an improvement of early Greek and Roman practitioners' methods – Polybios and Quintilian tell us both that the area of a figure was measured by most people erroneously by its periphery (and Thukydides measures so himself – see [Eva Sachs 1917: 174]). Precisely the same method turns up in the Carolingian *Propositiones*, N^os 25 and 29, which find the area of one circle as that of the isoperimetric square, and that of another as that of an isoperimetric non-square rectangle.

To make the mix-up complete, the *Propositiones* find all areas of non-square quadrangles (rectangles and trapezoids alike) by means of the «surveyors' formula» for the irregular quadrangle (semi-sum of lengths times semi-sum of widths). This formula is employed in Old Babylonian tablets. It was used by surveyors in Ptolemaic Egypt (see [Cantor 1875: 34f]), and it turns up in the pseudo-Heronian *Geometrica*, mss S, V [ed. Heiberg 1912: 208]. It was not used by Hero, nor by the Roman agrimensors (nor, it appears, in Seleucid Babylonia); but it turns up again in the Latin 11th century compilation *Boethii geometria altera* II,xxxii [ed. Folkerts 1970: 166]. In the 11th century CE, Abū Manṣūr ibn Ṭāhir al-Baghdādī ascribes the formula to «the Persians» [Anbouba 1978: 74] – but al-Khwārizmī (who does not use it) has probably seen it in the Hebrew *Mišnat ha-Middot* II,1 [ed. Gandz 1932: 23], or possibly in some lost prototype for that work.

[59] So the value $\pi = {}^{22}/_{7}$, represented by Hero as a simple approximation (*Metrika* I, 26 – ed. [Schöne 1903: 66]), is taken over by Roman surveying (Columella and Frontinus, referred in [Cantor 1875: 90, 93f]) and stands as plain truth in Latin descendants of the agrimensor-tradition (e.g., *Boethii geometria altera* II,xxxii – ed. [Folkerts 1970: 166]). The *Mišnat ha-Middot* (II,3, ed. [Gandz 1932: 24]) presents the matter in the same way. So does al-Khwārizmī in the parallel passage of his *Algebra*, but in the introductory remark he represents the factor $3^{1}/_{7}$ as «a convention among people without mathematical proof» (*ibid.* pp 69 and 81f) – telling thereby that he considered at least that section of the Mišnat ha-Middot prototype) a representative of a general sub-scientific environment.

Other Heronian improvements are the formula for the triangular area and his better calculation encountered locally by the mathematicians of Islam, who used it as a basic material while criticizing it – just as they encountered, used and criticized the practice of commercial and recreational arithmetic.

VI. Algebra and its alternative

As pointed out in chapter III, Islamic algebra was in all probability *not* inspired from Indian scientific algebra. A detailed analysis of a number of sources suggests instead a background in the sub-scientific tradition – or, indeed, in two different sub-scientific traditions. I have published the arguments for this elsewhere[60], and I shall therefore only present the results of the investigation briefly.

Al-jabr was performed by a group of practitioners engaged in *ḥisāb* (calculation) and spoken of as *ahl al-jabr* (algebra-people) or *aṣḥab al-jabr* (followers of algebra). The technique was purely rhetorical, and a central subject was the reduction and resolution of quadratic equations – the latter by means of standardized algorithms (analogous to the formula $x = ½b+\sqrt{(½b)^2+c}$ solving the equation $x^2 = bx+c$, etc.) unsupported by arguments, the rhetorical argument being reserved for the reduction. Part of the same practice (but possibly not understood as covered by the term *al-jabr*) was the rhetorical reduction and solution of first-degree problems.

As argued in chapter III, part of the characteristic vocabulary suggests a sub-scientific (but probably indirect) connection to India. An ultimate connection to Babylonian algebra is also inherently plausible, but not suggested by any positive evidence; in any case an imaginable path from Babylonia to the Early Medieval Middle East must have been tortuous, as the methods employed in the two cases are utterly different.

The latter statement is likely to surprise, Babylonian algebra being normally considered to be either built on standardized algorithms or on oral rhetorical techniques. A detailed structural analysis of the terminology and of the distribution of terms and

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of the circular segment, which turn up in various places (see, e.g., [Cantor 1875: 90] reporting Columella, and *Mišnat ha-Middot* V, ed. [Gandz 1932: 47ff]).

[60] [Høyrup 1986]. The essential sources involved in the argument are al-Khwārizmī's *Algebra* [trans. Rosen 1831]; the extant fragment of ibn Turk's *Algebra* [ed., trans. Sayili 1962]; Thābit's Euclidean *Verification of the Problems of Algebra through Geometrical Demonstrations* [ed., trans. Luckey 1941]; Abū Kāmil's *Algebra* [trans. Levey 1966]; the *Liber mensurationum* written by some unidentified Abū Bakr and known in a Latin translation due to Gherardo of Cremona [ed. Busard 1968]; and Abraham bar Ḥiyya's (Savasorda's) *Ḥibbur ha-mešiḥah wetišboret* (*Collection on Mensuration and Partition*; Latin translation *Liber embadorum*, ed. [Curtze 1902]).

On one point, my [1986] should be corrected. P. 472 I quote Abū Kāmil for a distinction between «arithmeticians» (*ba'alei ha-mispar* in the Hebrew translation, [Levey 1966: 95], i.e. «masters of number») and «calculators» (*yinhagu ha-ḥasbanim,* [Levey 1966: 97], «those who pursue calculation). The distinction turns out to be absent from the Arabic facsimile edition of the work [ed. Hogendijk 1986] (Jan Hogendijk, personal communication).

operations inside the texts shows, however, that this interpretation does not hold water; furthermore, it turns out that the only interpretation of the texts which makes coherent sense is geometric – the texts have to be read as (naive, non-demonstrative) constructional prescriptions, dealing really, as they seem to do when read literally, with (geometric) squares, rectangles, lengths and widths (all considered as *measured* entities); they split, splice and aggregate figures so as to obtain a figure with known dimensions in a truly analytic though completely heuristic way. Only certain problems of the first degree (if any) are handled rhetorically, and no problems are solved by automatic standard algorithms[61].

This is most certain for Old Babylonian algebra (c. 17th century BCE) – here, the basic problems are thought of as dealing with rectangles, and they are solved by naive-geometric «analysis». A few tablets dating from the Seleucid period and written in the Uruk environment of astronomer-priests contain second-degree problems too. They offer a more ambiguous picture. Their dressing is geometric, and the method is apparently also geometric (though rather synthetic than analytic); but the geometric procedure is obviously thought of as *an analogy to a set of purely arithmetical relations* between the unknown magnitudes.

Having worked intensively with Babylonian texts for some years I was (of course) utterly amazed when discovering accidentally their peculiar rhetorical organization (characterized by fixed shifts between present and past tense, and between the first, the second and the third person singular) in the Medieval Latin translation of a *Liber mensurationum* written by an unidentified Abū Bakr. The first part of this *misaḥa*-(surveying-)text contains a large number of problems very similar to the ones known from the Old Babylonian tablets: A square plus its side is 110; in a rectangle, the excess of length over width is 2, and the sum of the area and the four sides is 76; etc. The problems, furthermore, are first solved in a way reminding strikingly of the Old Babylonian methods (although the matter is obscured by the absence of a number of figures alluded to in the text); a second solution employs the usual rhetorical reductions and solutions by means of standard algorithms. The second method is spoken of as *aliabra*, evidently a transliteration of *al-jabr*. The first usually goes unlabeled, being evidently the standard method belonging with the tradition; in one place, however, it is spoken of as «augmentation and diminution» – apparently the old splicing and splitting of figures.

A precise reading of the text in question leaves no reasonable doubt that its first part descends directly from the Old Babylonian «algebra» of measured line segments (the second part contains real mensuration in agreement with the Alexandrinian

[61] The arguments for this are, as any structural analysis, complex, and impossible to repeat in the present context. A brief sketch is given in my [1986: 449–456]. A detailed but fairly unreadable presentation is given in my preliminary [1985b]. Another detailed but more accessible exposition will, I hope, be available in near future.

tradition). Once this is accepted at least as a working hypothesis, a number other sources turn out to give meaningful evidence. The geometrical «proofs» of the algebraic standard algorithms given by al-Khwārizmī and ibn Turk will have been taken over from the parallel naive-geometric tradition; Thābits Euclidean proof of the same matter is therefore really *something different*, whence probably his silence about what has seemed to be predecessors; etc.

Especially interesting is Abū'l-Wafā''s report of a discussion between (Euclidean) geometers on one hand and surveyors and artisans on the other[62]. He refers to the «proofs» used by the latter in questions concerned with the addition of figures; these profs turn out to be precisely the splitting and splicing used by Abū Bakr and in the Old Babylonian texts. This confirms a suspicion already suggested by appearance of the «algebra» of measured line segments in a treatise on mensuration: It belonged with the group of practitioners engaged in sub-scientific, practical geometry, and was hence a tradition of surveyors, architects and higher artisans. *Al-jabr*, on the other hand, was carried by a community of calculators, and was considered part of *ḥisāb*, Abū Kāmil seems to tell us[63].

VII. Reception and synthesis

Already in pre-Islamic times, these different source traditions had merged to some extent. The development of a syncretic practical geometry was already discussed above, and the blend of (several sorts of) very archaic surveying formulae with less archaic recreational arithmetic in the *Propositiones ad acuendos juvenes* was also touched at[64]. Still, merging, and especially critical and creative merging, was no dominant feature.

From the ninth century onwards, it came to be the dominant feature of Islamic mathematics. The examples are too numerous to be listed, but a few illustrations may be given.

Among the modest examples, the geometric chapter of al-Khwārizmī's *Algebra* can be pointed at. As shown by Solomon Gandz[65], it is very closely related to the Hebrew *Mišnat ha-Middot*, which is a fair example of pre-Islamic syncretic practical geometry,

[62] *Book on What is Necessary From Geometric Construction for the Artisan* X, xiii [trans. Krasnova 1966: 115].

[63] See note 60.

[64] True enough, the *Propositiones* are not pre-Islamic according to chronology. Still, they show no trace of Islamic influence, and they were collected in an environment where mathematical development was to all evidence extremely slow. We can safely assume that most of the mathematics of the *Propositiones* was already present (if not necessarily collected) in the same region by the sixth century CE

[65] See the discussion and the two texts in [Gandz 1932].

or at least a very faithful continuation of that tradition[66]. Al-Khwārizmī's version of the same material is not very different; but before treating conventionally the circular segments he tells that the ratio $3^{1}/_{7}$ between perimeter and diameter of a circle «is a convention among people without mathematical proof»; he goes on to inform us that the Indians «have two other rules», one equivalent to $\pi = \sqrt{10}$ and the other to $\pi = 3.1416$[67]; finally he gives the exact value of the circular area as the product of semi-perimeter with semi-diameter together with a heuristic proof[68]. So, not only are the different traditions brought together, but we are also offered a sketchy critical evaluation of their merits.

If the whole of al-Khwārizmī's *Algebra* is taken into account, the same features become even more obvious. Just after the initial presentation of the *al-jabr*-algorithms they are justified geometrically, by reference to figures which are inspired from the «augmentation-and-diminution»-tradition but more synthetic in character, and which for the sake of clear presentation make use of Greek-style letter formalism[69]. A little further on, the author attempts on his own to extrapolate the geometrical technique in order to prove the rules of rhetorical reduction.

The result is still somewhat eclectic – mostly so in the geometric chapter. Comparison and critical evaluation amounts to less than real synthesis. But in the work in question, and still more in the total activity of the author, a striving toward more than random collection and comparison of traditions is clearly visible. Soon after al-Khwārizmī, furthermore, other authors wrote more genuinely synthetic works. One example, in the same field as al-Khwārizmī's naive-geometric proofs of the *al-jabr*-algorithms, is Thābit's treatise on the «verification of the rules of *al-jabr*» by means of *Elements* II.5–6[70]; another, in the field of practical geometry, is Abū'l-Wafā''s *Book on What is Necessary From Geometric Construction for the Artisan*, where methods and

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[66] In his English summary, Sarfatti [1968: ix] claims that Arabic linguistic influence «although not evident prima facie, underlies [the] mathematical terminology» of the *Mišnat ha-Middot*. If this is true, the work must be dated in the early Islamic period. The main argument of the book is in Hebrew, and I am thus unable to evaluate its force – but since Arabic and Syriac (and other Aramaic) technical terminologies are formed in analogous ways, and since no specific traces of Arabic terms are claimed to be present, it does not seem to stand on firm ground.

[67] The former is an Ancient Jaina value, the second is given by Āryabhaṭa – se [Sarasvati Amma 1979: 154].

[68] Arabic text and translation in [Gandz 1932: 69f]; *ahl al-handasa* («surveyors» and later «geometers», translated «mathematicians» by Gandz) corrected to *ahl al-hind* («people of India») in agreement with [Anbouba 1978: 67].

[69] Trans. [Rosen 1831: 6–20]. The whole technique of the proofs has normally been taken to be of purely Greek inspiration, partly because of the letter-formalism, partly because neither the Old Babylonian naive-geometric technique nor its early Medieval descendant was known.

[70] Ed., trans. [Luckey 1941].

problems of Greek Geometry (including, it now appears, Pappos' passage on constructions with restricted and constant compass opening[71]) and Abū'l-Wafā''s own mathematical ingenuity are used to criticize and improve upon practitioners' methods, but where the practitioners' perspective is also kept in mind as a corrective to otherworldly theorizing[72]. The examples could be multiplied *ad libitum*. Those already given, however, will suffice to show that the Islamic synthesis was more than the bringing-together of methods and results from the different source traditions; it included also an explicit awareness of the difference between theoreticians' and practitioners' *perspectives* and of the legitimacy of both, as well as acknowledgement of the possible relevance and critical potentiality *of each* when applied to results, problems or methods belonging to the other. While the former aspect of the synthesizing process was only much further developed than in other Ancient or Medieval civilizations, but not totally unprecedented, the latter *was* exceptional (cf. also below, chapter XIII).

What, apart from a violent cultural break and an ensuing cultural burgeoning (which of course explains much, but does so unspecifically), accounts for the creative assimilation, reformulation, and (relative) unification of disparate legacies as «mathematics of the Islamic world»? And what accounts for the *specific* character of Islamic mathematics as compared, e.g., with Greek or Medieval Latin mathematics?

VIII. «Melting pot» and tolerance

I shall not pretend to give anything approaching an exhaustive explanation. Instead, I shall point to some factors which appear to be important, and possibly fundamental.

On a general level, the «melting-pot-effect» was at least an important precondition for what came to happen. Within a century from the *Hijra*, the whole core area of Medieval Islam had been conquered, and in another century or so the most significant strata of the Middle Eastern population were integrated into the emerging Islamic culture[73]; this – and also the travels of single scholars as well as the movement of larger

[71] See [Jackson 1980].

[72] Trans. [Krasnova 1966]. Interesting passages are, e.g., chapter I, on the instruments of construction; and X.i and X.xiii, which discuss the failures of the artisans as well as the shortcomings of the (too theoretical) geometers. Consideration of practitioners' needs and requirement is also reflected in the omission of all proofs.

Though more integrative than al-Khwārizmī's *Algebra*, Abū'l-Wafā''s work is not completely free from traces of eclecticism. This is most obvious in the choice of grammatical forms, which switches unsystematically between a Greek «we» and the practitioners' «If somebody asks you ..., then you [do so and so]».

[73] According to Bulliet's counting of names [1979], the majority of the Iranian population was converted around 200 A.H. (816 CE), while the same point was reached in Iraq, Syrian and Egypt some 50 years later. The socially (and scientifically!) important urban strata (artisans, merchants, religious and state functionaries) were predominantly Muslim some 80 years earlier (cf. also [Waltz

population groups, especially toward the Islamic center in Baghdad – broke down earlier barriers between cultures and isolated traditions, and offered the opportunity for cultural learning. Here, the religious *and cultural* tolerance of Islam was also important. Of course, Muslims were aware of the break in history created by the rise of Islam, and in the field of learning a distinction was upheld between *awāʼil*, «pristine» (i.e. pre-Islamic) and Muslim / Arabic «science» (i.e. *ʽilm*, a term grossly congruent with Latin *scientia* and better translated perhaps as «field of knowledge»). Since, however, the latter realm encompassed only religious (including legal), literary and linguistic studies[74], the complex societal setting of learning in the mature Islamic culture prevented attitudes like the Greek over-all contempt for «barbarians». Furthermore, the rise of people with roots in *different* older elites to elite positions in the Caliphate may have precluded that sort of cultural exclusiveness which came to characterize Latin Christianity during *its* phase of cultural learning (which, even when most open to Islamic and Hebrew learning, took over only translations and practically no scholars, and which showed little interest in translating works with no relation to the «culturally legitimate» Greco-Roman legacy[75]).

Whatever the explanations, Islam kept better free of ethnocentrism as well as culturocentrism than many other civilizations[76]. Due to this tolerance, the intellectual and cognitive barriers which were molten down in the melting pot were not replaced in the same breath by new barriers, which would have blocked up the positive effects of cultural recasting. It also permitted Islamic learning to draw (both in its initial phase and later) on the service of Christians, Jews and Sabians and on Muslims rooted in different older cultures – let me just mention Māšāʼallāh the Jew, Thābit the heterodox Sabian, Ḥunayn ibn Isḥāq the Nestorian, Qusṭā ibn Lūqā the Syrian Christian of Greek descent, Abū Maʽšar the heterodox, pro-Pahlavi Muslim from the Hellenist-Indian-Chinese-Nestorian-Zoroastrian contact-point at Balkh, and ʽUmar ibn al-Farrukhān al-Tabari the Muslim from Iran. In later times, the late conversion of the Jew al-Samawʼal appears to have been independent of his scientific career[77]. Still later, the

1981]). A correlation of Bulliet's geographical distinctions with the emergence of local Islamic scholarly life would probably be rewarding.

[74] See ibn Khaldūn, *Muqaddimah* VI, *passim*, esp. VI. 9 (trans. [Rosenthal 1958: II–III], esp. II, 436–439); and [Nasr 1968: 63f].

[75] I have discussed this particular culturocentrism in my [1985a: 19–25 and *passim*]. One of the rare fields where it makes itself little felt appears to be mathematics, where the requirements of mathematical astronomy, «Hindu reckoning» and commercial arithmetic and algebra in general may have opened a breach of relative tolerance.

[76] This is of course not to say that it kept totally free. The originally conquering Arabs, e.g., felt ethnically superior to others, as conquerors have always done. The lack of ethnocentrism is only relative.

[77] Cf. the biographies in *DSB*, as listed in the appendix to the bibliography.

presence of Chinese astronomers collaborating with Muslims at Hulagu's observatory at Maragha underscores the point[78].

Still, «melting-pot-effect» and tolerance were only preconditions – «material causes», in an approximate Aristotelian sense. This leaves open the other aspect of the question: Which «effective» and «formal» causes made Medieval Islam scientifically and mathematically creative?

IX. Competition?

It has often been claimed that the early ninth century awakening of interest in pre-Islamic (*awā'il*) knowledge[79] «must be sought in the new challenge which Islamic society faced» through the «theologians and philosophers of the religious minorities within the Islamic world, especially the Christians and Jews» in «debates carried on in cities like Damascus and Baghdad between Christians, Jews and Muslims», the latter being «unable to defend the principles of faith through logical arguments, as could the other groups, nor could they appeal to logical proofs to demonstrate the truth of the tenets of Islam»[80]. One problematic feature of the thesis is that «we have very little evidence of philosophical or theological speculation in Syria [including Damascus] under the 'Umayyad dynasty», as observed by De Lacy O'Leary[81], who is otherwise close to the idea of stimulation through intellectual competition. Another serious challenge is that Islamic learning advanced well beyond the level of current Syriac learning in a single leap (not least in mathematics). Here, the cases of the translators Ḥunayn ibn Isḥāq and Thābit should be remembered. Both began their translating activity (and Thābit his whole scientific career) in the wake of the 'Abbaside initiative. Most of their translations and other writings were in Arabic; what they made in Syriac was clearly a secondary spin-off, and included none of their mathematical writings or translations (be it then Thābit's «verschiedene Schriften über die astronomischen Beobachtungen, arabisch und syrisch»[82]. Especially Ḥunayn's translations show us that the Syriac environment was almost as much in need of broad Platonic and Aristotelian learning as the Muslims by the early ninth century CE[83] (cf. also below, chapter XI). Admittedly,

[78] See [Sayili 1960: 205–207].

[79] The beginnings of astronomical interests in the later 8th century is different, bound up as it is with the practical interests in astrology. The same applies to the very early interest in medicine – cf. *GAS* III, 5.

[80] The formulations are those of Seyyed Hossein Nasr [1968: 70].

[81] [O'Leary 1949: 142].

[82] [Suter 1900: 36].

[83] Cf. Rosenfeld & Grigorian, "Thābit ibn Qurra", *DSB* XIII, 288–295; and Anawati et al, "Hunayn ibn Isḥāq", *DSB* XV, 230–249.

if we return from philosophy to mathematics, a Provençal-Hebrew translation from CE 1317 of an earlier Mozarabic treatise claims that an Arabic translation of Nicomachos' *Introduction to Arithmetic* was made from the Syriac before CE 822[84]. However, even if we rely on this testimony it is of little consequence: Essential understanding of scientific mathematics is certainly neither a necessary condition for interest in Nicomachos nor a consequence of even profound familiarity with his *Introduction*. Equally little can be concluded from the existence of a (second-rate) Syriac translation of Archimedes' *On the Sphere and Cylinder*, since it may well have been prepared as late as the early ninth century and thus have been a spin-off from the ʿAbbaside translation wave[85].

A possible quest for intellectual competitiveness will thus hardly do, and definitely not as *sole* explanation of the scientific and philosophical zeal of the early ninth century ʿAbbaside court and its environment. Similarly, the ʿAbbaside adoption of many structures from the Sassanian state and court and the concomitant peaceful reconquest of power by the old social elites[86] may explain the use of astrologers in the service of the court, e.g., at the famous foundation of Baghdad. But then it does not explain why Islamic astrologers were not satisfied with the *Zīj al-Šāh*, connected precisely to the past of the reborn Sassanian elite. Truly, the acquisition of Siddhantic and Ptolemaic astronomy directly from the sources is only a mild surprise, being only a quantitative improvement of what was already known indirectly through Pahlavi astronomy[87]. Continuity or revival of elites and general cultural patterns are, however, completely

[84] The Mozarabic work is a paraphrase of Nicomachos written by the mid-tenth century Corduan bishop Rabīʿ ibn Zaid. In his preface, Rabīʿ refers to commentaries which al-Kindī should have made to a translation from Syriac. The evidence can hardly be considered compelling; on the other hand, *some* Arabic translation antedating Thābit's appears to have existed. See [Steinschneider 1896: 352] and *GAS* V, 164f.

[85] Cf. *GAS* V, 129. Positive evidence that Syriac learning was close to mathematical illiteracy is found in a letter written by Severus Sebokht around 662 [ed., trans. Nau 1910: 210–214]. In this letter, the Syrian astronomer *par excellence* of the day quotes the third-century astrologer Bardesanes extensively and is full of contempt for those who do not understand the clever argument – which is in fact nothing but a mathematical blunder, as enormous as it is elementary.

[86] «The people will become subject to the people of the East and the government will be in their hands», as it was expressed by the contemporary Māšāʾallāh in his *Astrological History* [trans. Kennedy & Pingree 1971: 55]. Or, in Peter Browns modern expressive prose [1971: 201]: «Khusro I had taught the *dekkans*, the courtier-gentlemen of Persia, to look to a strong ruler in Mesopotamia. Under the Arabs, the *dekkans* promptly made themselves indispensable. They set about quietly storming the governing class of the Arabic empire. By the middle of the eighth century they had emerged as the backbone of the new Islamic state. It was their empire again: And, now in perfect Arabic, they poured scorn on the refractory bedouin who had dared to elevate the ways of the desert over the ordered majesty of the throne of the Khusros».

[87] See [Pingree 1973: 35].

incapable of explaining a sudden *new* vigour of scientific culture in the Irano-Iraqian area – a truly qualitative jump. In particular, they are unable to tell why a traditional interest in the astronomico-astrological applications of mathematics should suddenly lead to interest in mathematics *per se* – not to speak of the effort toward synthesis between separate traditions.

X. Institutions or sociocultural conditions?

Sound sociological habit suggests one to look for explanations at the institutional level. Yet, a serious problem presents itself to this otherwise reasonable «middle range» approach to the problem (to use Robert K. Merton's expression[88]): The institutions of Islamic learning were only in the state of making by the early ninth century CE, and hardly that. In this age of fluidity and fundamental renewal, Islamic learning formed its institutions quite as much as the institutions formed the learning[89]. In order to get out of this circle of closed pseudo-causality we will hence have to ask why institutions became shaped the way they did. The explanations should hold for the whole core area of Medieval Islam and should at the same time be specific for this area.

Two possibilities appear to be at hand: Islam itself, which was shared as a cultural context even by non-Muslim minorities and scholars; and the Arabic language. Language can be ruled out safely; truly, the general flexibility of Semitic languages makes them well suited both to render foreign specific ways of thought precisely in translation and as a basis for the development of autochthonous philosophical and scientific thought – but Syriac and other Aramaic dialects are no less Semitic than the Arabic; they had been shaped and grinded for philosophical use for centuries, and much of the Arabic terminology was in fact modelled upon the Syriac[90]. The Arabic tongue was an

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[88] See his kindly polemical defence of a «middle range theory» whose abstractions are «close enough to observed data to be incorporated in propositions that allow empirical testing» against such precocious total systems whose profundity of aims entails triviality in the handling of all empirical details ([Merton 1968: 39–72], especially the formulations pp. 39 and 49).

[89] As «institutions» of learning in the widest sociological sense (i.e. socially fixed patterns of rules, expectations and habits) one can mention the short-lived «House of Wisdom» at al-Ma'mūn's court, together with kindred libraries; the institutions of courtly astronomy and astrology and, more generally, the ways astronomy and astrology were habitually carried out; that traditional medical training which made medicine almost a monopoly of certain families (cf. Anawati and Iskandar reporting ibn Abī Usaybiʿa in *DSB* XV, 230); the fixed habits and traditions of other more or less learned practical professions; the gatherings of scholars; the fixed form in which science could already be found in Byzantium; the Mosque as a teaching institution (the madrasah was only developed much later); and practical and theoretical management of Islamic jurisprudence, including the handing-down of *ḥadīth*. Only one of these institutions, *viz*. Byzantine science, can be claimed to have been really fixed by the early ninth century. Cf. [Nasr 1968: 64–88]; [Makdisi 1971]; and [Watt & Welch 1980: 235–250].

[90] See [Pines 1970: 782].

adequate medium for what was going to happen, but it replaced another medium which was just as adequate. Language cannot be the explanation. Islam remains.

Of course, the explanation need not derive from Islam regarded as a system of religious teachings – what matters is Islam understood as a specific, integrated social, cultural, and intellectual complex. Some factors of possible importance can be singled out from this complex.

XI. Practical fundamentalism

One factor is the very character of the complex as an integrated structure – i.e., those implicit fundamentalist claims of Islam which have most often been discussed with relation to Islamic law as

> the totality of God's commands that regulate the life of every Muslim in all its aspects; it comprises on an equal footing ordinances regarding worship and ritual, as well as political and (in the narrow sense) legal rules, details of toilet, formulas of greeting, table-manners, and sick-room conversation.[91]

True enough, religious fundamentalism in itself has normally had no positive effects on scientific and philosophical activity, and it has rarely been an urge toward intellectual revolution. In ninth century Islam, however, fundamentalism was confronted with a complex society in transformation; religious authority was not segregated socially as a «Church» – *hadīth* (jurisprudentially informative traditions on the doings and sayings of the Prophet) and Islamic jurisprudence in general were mainly taken care of by persons engaged in practical life, be it handicraft, trade, secular teaching, or government administration[92], and a jealous secular power would do its best to restrain the inherent tendencies of even this stratum to get the upper hand[93].

Fundamentalism combined with the practical engagement of the carriers of religious authority may have expressed itself in the recurrent tendency in Islamic thought to regard «secular» knowledge (*scientia humana*, in the Medieval Latin sense) not as an

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[91] [Schacht 1974a: 392].

[92] See [Cohen 1970].

[93] There were at least two (tightly coupled) good reasons for this jealousy. Firstly, traditionists and jurisprudents might easily develop into a secondary centre of power; secondly, they might inspire, participate in or strengthen popular risings, which were already a heavy problem for the ʿAbbaside Caliphs.

The destruction of the Baghdad «House of Learning» (*Dār al-ʿilm* – «Residence of Knowledge» would perhaps be a more precise translation) in a Sunnite riot in CE 1059 shows what could be the fate even of scholarly institutions when religious fervour and social anger combined. See [Makdisi 1961: 7f].

alternative but as a *way* to Holy Knowledge (*kalām al-dīn*, «the discourse of Faith[94], *scientia divina*) and even to contemplative truth, *ḥāl* (*sapientia*, in yet another Latin approximate parallel). Psychologically, it would be next to impossible to regard a significant part of the activity of «religious personnel» as irrelevant to its main task[95], or as directly unholy (as illustrated by the instability generated by scandalous Popes etc. and leading to the Reformation revolts; in Islam, the ways of the Umayyad Caliphs provided as effective a weapon for *Khārijite* radicals and for the 'Abbaside take-over as that given to Anabaptists and to Lutheran Princes by the Renaissance Popes). That integration of science and religious attitude was not just a *Mutakallimūn*'s notion but was shared to a certain extent by active mathematicians is apparent from the ever-recurring invocation of God in the beginnings and endings of their works (and the references to Divine assistance interspersed inside the text of some works[96]).

The legitimization of scientific interests through the connection to a religion which was fundamentalist in its theory and bound up with social life in its practice may also by analogy have impeded the segregation of pure science from the needs of daily life without preventing it from rising above these needs, thus provoking not only the phenomenon that even the best scientist would occasionally be concerned with the most practical and everyday applications of their science[97], but also the general appreciation of theory and practice as belonging naturally together – cf. above, chapter VII, and especially below, chapter XIII.

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[94] The expression is quoted from the ninth century *Mutakallim* al-Jahiz via Anton Heinen [1978: 64], who sums up (p. 57) his point of view in the formula «*Knowledge (ʿilm) = kalām al-dīn + kalām al-falsafah*», the latter term meaning «the discourse of philosophy», i.e. secular theoretical knowledge.

[95] In a similar way, practical charity, the management of ritual and sacraments, and religious teaching are understood as belonging *by necessity* together in Christian environments where the Church (and perhaps the same priest) takes care of them all.

[96] One work containing such copious references to God is Abū Bakr's *Liber mensurationum*, which was discussed above. Normally, the invocations were abridged or left out in the Latin translations (not least in Gherardo's translations); in this case, however, they have survived because of their position inside the text (while the compulsory initial invocation is deleted).

Of course, routine invocation is no indicator of deep religious feeling. What imports is that the invocation *could* develop into a routine, and that it was thus considered *a matter of course* or *decorum* even in mathematical texts. You may perhaps persevere in an activity which you fear is unpleasant to God – but then you rarely invite him explicitly (routinely or otherwise) to attend your sins.

[97] Among the numerous examples I shall just mention Abū'l-Wafā''s *Book on What is Necessary from Geometric Construction for the Artisan*, which was discussed above; al-Uqlīdisī's *Arithmetic*, the mathematical level of which suggests that the author must have been beyond the rank-and-file; and ibn al-Haytham's works on the determination of the *qiblah* and on commercial arithmetic (N^os^ 7 and 10 in ibn Abī Usaybiʿa's list, trans. [Nebbia 1967: 187f], cf. [Rozenfeld 1976: 75]).

The plausibility of this explanation can be tested against some parallel cases. One of these is that of Syriac learning, which belonged in a religious context with similar fundamentalist tendencies. Syrian Christianity, however, was carried by a Church, i.e. by persons who were socially segregated from social practice in general, and Syriac learning was carried by these very persons. The custodian and non-creative character of Syriac learning looks like a sociologically trite consequence of this situation – as Schlomo Pines explains,

> pre-Islamic monastic Syriac translations appear to have been undertaken mainly to integrate for apologetic purposes certain parts of philosophy, and perhaps also of the sciences, into a syllabus dominated by theology. In fact great prudence was exercised in this integration; for instance, certain portions of Aristotle were judged dangerous to faith, and banned.[98]

Other interesting cases are found in 12th–13th century Latin Christianity. Particularly close to certain ninth-century Islamic attitudes is Hugh of Saint-Victor, the teacher and rationalistic mystic from the Paris school of Saint Victor[99].He was active during the first explosive phase of the new Latin learning (like the Islamic late eighth and early ninth century CE the phase where the *Elements* were translated), in a school which was profoundly religious and at the same time bound up with the life of its city. The sociological parallels with ninth century Baghdad are striking, even though no Caliph was present in Paris. Striking are also the parallel attitudes toward learning. In the propaedeutic *Didascalicon*[100], Hugh pleads for the integration of the theoretical «liberal arts» and the practical «mechanical arts»; his appeal «learn everything, and afterwards you shall see that nothing is superfluous»[101] permits the same wide interpretation as the Prophet's saying «seek knowledge from the cradle to the grave»[102]; and he

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[98] [Pines 1970: 783].

[99] Cf. [Chenu 1974].

[100] I use the edition in *PL* 176, col. 739–952. A recent English translation is [Taylor 1961] (it should be observed that the chapters are counted somewhat differently in the two versions).

[101] *Omnia disce, videbis postea nihil esse superfluum* (*Didascalicon* VI, iii). Strictly speaking, «everything» is «everything in sacred history», since this is the subject of the chapter; but in the argument Hugh's own play as a schoolboy with arithmetic and geometry, his acoustical experiments and his all-devouring curiosity are used as parallel illustrative examples.

[102] Quoted from [Nasr 1968: 65]. In this case, as in that of Hugh, the intended meaning of «knowledge» is probably not quite as wide as a modernizing reading might assume. This, however, is less important than the open formulation and the optimism about the religious value of knowledge which was read into it, and against which the opponents of *awā'il* knowledge had to fight (cf. [Goldziher 1915: 6]) – for centuries with only limited success.

A major vehicle for the high evaluation of knowledge in Medieval Islam (be it knowledge in the narrow sense, *viz*. knowledge the God's «Uncreated Word», i.e. of the Koran, and of the Arabic language), and at the same time a virtual medium for the spread of a high evaluation of knowledge in a more general sense was the establishment of education of a large scale in Koran considers Wisdom (the study of which is seen in I.iii as «friendship with Divinity») a combination of moral and theoretical truth and practitioners' knowledge[103] – one can hardly come closer to al-Jāḥiẓ' «formula» as quoted in note 94.

Hugh was, however, an exception already in his own century. The established Church, as represented by the eminently established Bernard of Clairvaux, fought back, and even the later Victorines demonstrate through their teaching that a socially segregated ecclesiastical body is not compatible with synthesis between religious mysticism, rationalism, and an open search for all-encompassing knowledge. As a consequence, the story of 12th and 13th century Latin learning is mainly a tale of philosophy as potentially subversive knowledge, of ecclesiastical reaction, and finally of a subsequent synthesis where the «repressive tolerance» of Dominican learning blocked up the future development of learning[104]. It is also a tale of segregation between theoretical science and practitioners' knowledge: Lay theoretical knowledge gained a subordinated autonomy, but only by being cut off from the global world-view[105] and concomitantly also from common social practice. The cost was hence loss of that unsubordinated, mutually fecundating integration with practical concerns which was a matter of course in Islam[106].

XII. Variations of the Islamic pattern

If we look upon formulated theological schools or currents, the coupling between the «discourse of philosophy» and the «discourse of Faith» was strong among the *Mu'tazila*. Truly, according to earlier interpretations the *Mu'tazilite* attitude to

schools etc. (I am grateful to Jan Hogendijk for reminding me of this point, which I had not mentioned in the «Ghent» version of the paper).

A wealth of anecdotes illustrating an almost proverbial appreciation of knowledge (from the level elementary education to that of genuine scholarship) will be found in [Tritton 1957: 27f and *passim*].

[103] I.ix. «Practitioners' knowledge» translates *scientia mechanica*, while «moral truth» renders *intelligentia practica / activa*.

[104] This picture is of course unduly distorted. The subject is dealt with in somewhat greater detail in my [1985: 32–38].

[105] The situation is expressed pointedly by Boethius de Dacia in his beautiful *De eternitate mundi* (ed. [Sajo 1964: 46 and *passim*]), when he distinguishes the truth of natural philosophy (*veritas naturalis*) from «Christian, that is genuine, truth» (*veritas christianae fidei et etiam veritas simpliciter*).

[106] And so, when integration was needed by a group of practitioners, as was the case in 13th and 14th century astronomy, the need was satisfied by means of simplifying compendia, in striking contrast to the development in Islam – cf. below, chapter XVI. Latin science, when applied, was subordinated, and hence not fecundated by the interaction with the questions and perspectives of practice – for which reason, on the other hand, applications were bound to remain on the level of common sense. Cf. [Beaujouan 1957], in particular the conclusion.

philosophy should have been as lopsided as that of Syriac monasticism[107]; but from Heinen's recent analysis it appears that the *Mu'tazila* in general did not derive the Syro-Christian sort of intellectual censorship on philosophy from the theological aims of *kalām* (cf. above, note 94). Under al-Ma'mūn, who used *Mu'tazilism* in his political strategy, the attitudes of this theological current were strengthened by the ruler's interest in clipping the wings of those traditionalists whose fundamentalism would lead them to claim possession of supreme authority, first concerning knowledge but implicitly also in the moral and political domain (cf. above, note 93 and appurtenant text). Among the *Ismā'īlī* there was an equally strong (or stronger) accept of the relevance of *awā'il* knowledge for the acquisition of Wisdom, even though the choice of disciplines was different from that of the *Mutakallimūn* (cf. the polemic quoted in note 107): To judge from the *Ikhwān al-safa)*, Neoplatonic philosophy, Harranian astrology and the Hermeticism of Late Antiquity were central subjects[108]; but the curriculum of the *Ismā'īlī* al-Azhar madrasah in Cairo included philosophy, logic, astronomy and mathematics[109], i.e. central subjects of Ancient science. The same broad spectrum of religiously accepted interests (to which comes also Jabirian alchemy) can be ascribed to the Shiite current in general. In the *Aš'arite* reaction to *Mu'tazilism*, and in later Sunna, the tendency was to emphasize fundamentalism and to reject non-Islamic philosophy, or at least to deny its relevance for Faith. Accordingly, the curriculum of the Sunnite Nizamiyah madrasah in Baghdad included, besides the traditionalist disciplines (religious studies, Arabic linguistics and literature) only «arithmetic and the science of distributing bequests»[110] (the latter being in fact a «subdivision of arithmetic», as ibn Khaldūn explains[111]).

In the long run, *Mu'tazilism* lost to Sunnism, and in the very long run the dominance of traditionalist Sunnism (and of an equally traditionalist Shiism) was probably one of the immediate causes that Islamic science lost its vigour (this is not

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[107] So, according to Albert Nader, «les mu'tazila touchent à la sphère physique avec des mains conduites par des regards dirigés vers une sphère métaphysique et morale: la raison cherchant à concilier les deux sphères» ([1956: 218], quoted from [Heinen 1978: 59]). As pronounced enemies of the *Mu'tazila*, the 10th century Ismā'īlī *Ikhwān al-safa)* are still more emphatical, claiming that the *Mu'tazila* «die medizinische Wissenschaft für Unnütz, die Geometrie als zur Erkenntnis des wahren Wesens der Dīnge unzuständig halten, die Logik und die Naturwissenschaften für Unglauben und Ketzerei und ihre Vertreter für irreligiöse Leute erklären» (IV,95, quoted from [Goldziher 1915: 25]).

[108] See Marquet, "Ikhwān al-safa)", *DSB* XV, 249–251.

[109] See [Fakhry 1969: 93].

[110] [Fakhry 1969: 93].

[111] *Muqaddimah* VI,19, trans. [Rosenthal 1958: III, 127]. It will also be remembered that «inheritance calculation» occupies just over one half of al-Khwārizmī's *Algebra* (pp. 86–174 in Rosen's translation).

the place to investigate the ultimate causes). During the Golden Age, however, when institutionalization was still weaker than a vigorous and multidimensional social life, the attitudes of the formulated theological currents were not determinants but rather reflections of ubiquitous dispositions (cf. also above, note 102). So, *Muʿtazilism* was only the most clear-cut manifestation of more general tendencies, and the reversal of the *Muʿtazilite* policy in 849 did not mean the end to secular intellectual life in the Caliphate nor to the routinely expression of religious feelings expressed in the opening and closing sentences of scholarly works. Furthermore, through *Sufi* learning, and in the person of al-Ghazzālī, secular knowledge gained a paradoxical new foot-hold – more open, it is true, to quasi-Pythagorean numerology than to cumulative and high-level mathematics[112], but still a factor of encouragement even for more serious scientific and mathematical study. It appears that the *Sufi* mathematician ibn al-Bannāʾ did in fact combine mathematical and esoteric interests[113]; and even though al-Khayyāmī's *Sufi* confession can be suspected not to reflect his inner opinions as much as his need for security[114], his claim that mathematics can serve as part of «wisdom»[115] must either have been an honest conviction or (if it was meant to serve his security) have had a plausible ring in contemporary ears. At the same time, the two examples show that the rôle of Gnostic sympathies was only one of *external* inspiration: Ibn al-Bannāʾ's works are direct (and rather derivative) continuations of the earlier mathematical and astronomical tradition[116], and al-Khayyāmī's treatise is written as part of a running tradition for metamathematical commentaries to the *Elements*, and in direct response to ibn al-Haytham. Gnostic sympathies might lead scholars to approach and go into the mathematical traditions, but it did not transform the traditions, nor did it influence the way work was done inside traditions.

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[112] The similarity with the *Ismāʿīlī* orientation is clear; according to ibn Khaldūn, good reasons for such similarity exist through the close relations between the early *Sufis* and «Neo-Ismâʿîlîyah Shîʿah extremists» (*Muqaddimah* VI,16, trans. [Rosenthal 1958: III, 92]).

The paradoxical (or at least quite vacillating) attitude of the mature al-Ghazzālī toward mathematics is illustrated through a number of quotations in [Goldziher 1915, *passim*].

[113] See [Renaud 1938].

[114] See Youschkevitch & Rosenfeld, "Al-Khayyāmī", *DSB* VII, 330; and [Kasir 1931: 3f].

[115] The claim is even given emphasis by a somewhat clumsy repetition in the introduction to his «Discussion of Difficulties of Euclid» [trans. Amir-Móez 1959: 276].

[116] See Vernet, "Ibn al-Bannāʾ", *DSB* I, 437f.

XIII. The importance of general attitudes: the mutual relevance of theory and practice

So, the analysis according to specific religious currents is not to be taken too much at the letter when the conditioning of mathematics is concerned. As long as religious authority was not *both* socially concentrated and segregated *and* in possession of scholarly competence (as it tended to be in 13th century Latin Christianity), the attitudes of even dominating religious currents and groups could only influence the internal development and character of learning indirectly, by way of influencing overall scholarly dispositions and motivations (what it could do directly without fulfilling the two conditions was to strangle rational scholarship altogether – such things happened, but they affected the pace and ultimately the creativity of Islamic science, and these are different questions). Provided scholars could find a place in an institution under princely protection (of type «library with academy», i.e. *Dār al-ʿilm* and the like, or an observatory) or covered by a religious endowment (of type madrasah, hospital etc.), the absence of a centralized and scholarly competent Church permitted them to work in relative intellectual autonomy if only they kept inside the limits defined by institutional goals[117]: princes, at least, were rarely competent to interfere with learning by more subtle and precise means than imprisonment or execution[118]. The general attitude – that mathematics qua knowledge was religiously legitimate and perhaps even a way to Holy Knowledge, and that conversely the Holy was present in the daily practice of this world – could mold the disposition of mathematicians to the goals of their discipline; but even a semi-Gnostic conception of rational knowledge as a step toward Wisdom appears not to have manifested itself as a direct claim on the subjects or methods of actual scientific work – especially not as a claim to leave traditional subjects or methods.

Accordingly, explicit religious reference in Islamic mathematical works is normally restricted to the introductory dedication to God, the corresponding clause at the end of the work, and perhaps passing remarks mentioning his assistance for understanding

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[117] In a case like the Sunnite Nizamiyah madrasah in Baghdad, the institutional goals were of course already quite restricted. They would permit you to teach *al-jabr* but not Apollonios.

Formally, the situation was thus not very different from that of Syriac monastic learning. The Syriac learned monk, too, should merely stick to the institutional goal of Church and monastery. Materially, however, the difference was all-important, because the institutional goal of the Church included the defence of an already established theological opinion – we may think of the difference between the obligation to *teach biology instead of sociology* and the *prohibition to teach anything but creationist biology.*

[118] Cf. the anecdotes on Hulagu and Nasīr al-Dīn al-Tūsī reported by Sayili [1960: 207] and the story of the closing of the Istanbul observatory when its astrological predictions had proved catastrophically wrong (*ibid*, pp. 291–293). At most, the ruler was able to make cuts in a program which was too ambitious to his taste, or to close an institution altogether.

the matter or his monopoly on supreme knowledge. Apart from that, the texts are as secular as Greek or Medieval Latin mathematics. It is impossible to see whether the Divine dedications in Qusṭā ibn Lūqā's translation of Diophantos[119] are interpolations made by a Muslim copyist, or they have been written by the Christian translator with reference to a different God – they are completely external to the rest of the text. The ultimate goals of the activity were formulated differently from what we find in Greek texts (*when* we find a formulation – but cf. the initial quotation from Aristotle). The mathematician would not be satisfied by staying at the level of immediate practical necessity – he would go beyond these and produce something higher, *viz.* principles, proofs, and theory; nor would he, however, feel that any theory however abstract was *in principle* above application[120], or that the pureness of genuine mathematics would be polluted by possible contact with more daily needs. Several exemplifications were discussed above, in chapter VII (al-Khwārizmī, Abū'l-Wafā') and chapter XI, note 97 (al-Uqlīdisī, ibn al-Haytham). An example involving a non-mathematician (or rather a philosopher-not-primarily-mathematician) is al-Fārābī's chapter on ʿ*ilm al-ḥiyal*, the «science of artifices» or «of high-level application» (see above, text to note 18). We should of course not be surprised to find the science of *al-jabr wa'l-muqābalah* included under this heading – algebra *was* already a high-level subject when al-Fārābī wrote. But even if we build our understanding of the subject on Abū Kāmil's treatise, we might well feel entitled to wonder when seeing it intimately connected to the complete Ancient theory of surd ratios, including both that which Euclid gives in *Elements* X and «that which is not given there»[121]. More expressive than all this is, however, the preface to al-Bīrūnī's trigonometrical treatise *Kitāb istikhrāj al-awtār fī'l-dā'irah bi-khawāṣṣ al-khaṭṭ al-munḥanī al-wāqiʿ fīhā* (*Book on finding the Chords in the Circle* ...), which I quote in extensive excerpts from Suter's German translation[122] (emphasis added):

> Du weißt wohl, Gott Stärke dich, was für eine Ursache mich bewog, nach einer Anzahl von Beweisen zu forschen zur Bewahrheitung einer Behauptung der alten Griechen betreffend die Teilung der gebrochenen Linie in jedem [beliebigen] Kreisbogen mittels der auf sie von seiner Mitte aus gezogenen Senkrechten, und was ich für eine Leidenschaft für die Sache empfunden habe [...], so daß du mir wegen der Beschäftigung mit diesen Kapiteln der

[119] Openings of Book IV, V, VI, and VII, and the closing formula of the work (trans. [Sesiano 1982: 87, 126, 139, 156, 171], or [Rashed 1984: III, 1; IV, 1, 35, 81, 120]. Only in the final place, the praise which ends the work is followed by the date of copying, which again is followed by another praise of God and a blessing of the Prophet, in a way which (through comparison with other treatises with Muslim author and Muslim copyist suggests (but does not prove) that the first praise go back to Qusṭā himself.

[120] «In principle» – for of course much theory went unapplied in practice, and theory was developed regardless of possible application.

[121] My translation from the Spanish of [Palencia 1953: 52].

[122] [Suter 1910a].

> Geometrie Vorwürfe gemacht hast[?], ohne Kenntnis zu haben von *dem wahren Wesen dieser Gebiete, das eben in jeder Sache in dem hinausgehen über das Genügende besteht*. Wenn du doch, Gott stärke dich, genau achten wolltest auf die Ziele der Geometrie, die in der Bestimmung der gegenseitigen Verhältnisse ihrer Größen mit Rücksicht auf die Quantität bestehen, und das sie es ist, durch die man zur Kenntnis der Größe *aller Meßbaren und wägbaren Dīnge gelangt, die zwischen dem Mittelpunkt der Welt und der äußersten Grenze der sinnlichen Wahrnehmung liegen*. Und wenn du doch wüßtest, das mit ihnen [den geometrischen Größen] gemeint sind die [bloßen] Formen, losgelöst von der Materie [...] Was immer für einen Weg er [der Geometer] beschreiten möge, so wird er durch das mittel der Übung *von den physischen Lehren emporgeführt zu den göttlichen*, die schwer zugänglich sind wegen der schweren Verständlichkeit ihres Sinnes [ihrer Bedeutung] und der Subtilität ihrer Wege [Methoden] und der Majestät ihrer Sache, und wegen des Umstandes, daß es nicht jedem möglich ist, sich eine Vorstellung von ihnen zu machen, und *besonders dem nicht, der von der Kunst der Beweisführung sich abwendet*. Du hättest recht, Gott stärke dich, mich zu tadeln, für das, was ich erwähnt habe, wenn ich *die Aufsuchung dieser Wege [Methoden] unterlassen hätte, und die Zeit für etwas verschwendet hätte, von dem das leichtere schon genügen würde*; oder wenn die Arbeit *nicht bis zu dem Punkte gelangt wäre, der die Grundlage für die Astronomie bildet*, nämlich bis zur Berechnung der Sehnen im Kreise und der Verhältnisse ihrer Größen zu der angenommenen des Durchmessers [...]
> Nur bei Gott, dem Allmächtigen und Allweisen ist die Hilfe!

So, by going beyond the limits of immediate necessity and by cultivating the abstract and demonstrative methods of his subject, the geometer worships God – but in full only on the condition that (like God, we may add) he cares for the needs of astronomical everyday.

Al-Bīrūnī's formulation is unusually explicit, which perhaps reflects an unusually explicit awareness of current attitudes and their implications. In most other texts, these stand out most obviously if we compare with texts of similar purpose or *genre* from neighbouring cultures. Particularly gratifying in this respect is the field of technical literature. As mentioned in note 106 in the case of astronomy, the prevailing tendency in Latin learning was to escape the easy way by means of simplifying, non-demonstrative compendia. Most illustrative are also the various Anglo-Norman treatises on estate management. One such treatise was compilated on the initiative of (or even by) the learned Robert Grosseteste[123]; yet it contains nothing more than common sense and thumb rules. Not only was the semi-autonomous playing-ground granted to philosophical rational discussion in the 13th-century compromise not to encroach on *sacred* land; nor should it divert the attention of *practical* people and waste their time. In contrast, a handbook on «commercial science» written by one Šaykh Abū'l-Faḍl Ja'far ibn 'Alī al-Dimisqī somewhere between CE 250 and CE 1174 combines general economic theory (on the distinction between monetary, movable and fixed property) and Greek political theory with systematic description of various types of goods and with good

[123] Several of the treatises were edited by Oschinsky [1971]. Grosseteste's involvement is discussed pp. 192ff, cf. the texts pp. 388–409.

advice on prudent trade[124]. Knowledge of the delicacies of trade, like mathematics and any coherently organized, systematic knowledge, was considered natural part of an integrated world-view covered by *al-Islam* – and, in agreement with the basic (fundamentalist but still non-institutionalized) pattern of this world-view, the theoretical implications of applied knowledge were no more forgotten than the possible practical implications of theory.

The use of theory to improve on practice looks like a fulfillment of the Ancient Heronian and Alexandrinian project[125]. It had not been totally inconsequential – the acceptance of $\pi = {}^{22}/_{7}$ was discussed above; «Archimedes' screw» and Alexandrinian military and related techniques were also reasonably effective[126]. On the whole, however, the project had proved beyond the forces of Ancient science and of Ancient society – for good reasons, we may assume, since fruitful application of theory presupposes a greater openness to practitioners' specific problems and perspective than current in Greek science[127]. And so, as it was claimed in the introductory chapter, the *systematic* theoretical elaboration of applied knowledge was a specific creation of the Islamic world. It was already seen in the early phase of Islamic mathematics, when the traditions of «scientific» and «sub-scientific» mathematics were integrated. The great synthetic works in the vein of al-Khwārizmī's *Algebra* or Abū'l-Wafā''s *Book on What is Necessary From Geometric Construction for the Artisan* were already discussed above (chapter VII), as was their occasionally eclectic character. One step further was taken in such cases where a problem taken from the sub-scientific domain was submitted to theoretical investigation on its own terms (i.e. not only used as inspiration for an otherwise independent investigation, as when Diophantos takes over various recreational problems and undresses them in order to obtain pure number-theoretical problems). Thābit's Euclidean «verification of the rules of *al-jabr*» was mentioned above (same chapter), and Abū Kāmil's preface to his investigation of the recreational problem of

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[124] A discussion and an incomplete translation of the treatise is given by Ritter [1916]. The treatise is not only a contrast to 13th century European handbooks on prudent management but also to Greek common-sense deliberations like Hesiod's *Works and Days* or Xenophon's *Oeconomica*.

A similar contrast is obvious if we compare Ovid's *Ars amoris* or the pseudo-scholarly treatises to which it gave rise in the Latin Middle Ages with the development of regular sexology in Islam.

Closer to mathematics we may compare Villard de Honnecourt's very unscholarly reference to *figures de lart de iometrie* (*Sketchbook*, [ed. Hahnloser 1935: Taf. 38]) to the serious study of Euclidean geometry by Islamic architects (see [Wiedemann 1970: I,114]).

[125] Cf. Hero's introductions to the *Metrika* and *Dioptra* [ed. Schöne 1903: 2ff, 188ff].

[126] See [Gille 1980].

[127] We may remember Benjamin Farrington's observation, that «it was not [...] only with Ptolemy and Galen that the ancients stood on the threshold of the modern world. By that late date they had already been loitering on the threshold for four hundred years. They had indeed demonstrated conclusively their inability to cross it» [1969: 302].

«hundred fowls» was quoted in chapter IV[128]. A final and decisive step occurred when the results of theoretical investigation were adopted in and transmitted through books written for practitioners. A seeming first adoption of Thābit's *Verification* is found in Abū Kāmil's *Algebra*. At closer inspection, however, the reference to Euclid is ornamental – the argument itself is purely naive-geometric. But in Abraham bar Hiyya's (Savasorda's) *Collection on Mensuration and Partition*, where Euclid is not even mentioned by name, the actual argument can only be understood by somebody knowing his Euclid by heart[129].

XIV. The institutionalized cases (i): madrasah and arithmetical textbook

The two most interesting cases of infusion of theory into an inherently practical mathematical tradition appear to be coupled not only to the general dispositions of Islamic culture but also to important institutions which developed in the course of time. The first institution which I shall discuss is the writing of large-scale, reasoned arithmetical textbooks. Al-Uqlīdisī's work was already discussed above (chapter IV and note 97), and in his introductory commentary to the translation Saidan describes a number of other works which have come down to us[130]. In the early period, two largely independent types can be described: The «finger-reckoning type» and the «Hindi type», using, respectively, verbal and Hindu numerals. The two most important finger-reckoning books are those of Abū'l-Wafā' and of al-Karajī[131]. The two earliest extant Hindi books are those of al-Khwārizmī (extant only in Latin translation) and al-Uqlīdisī. Among lost works on the subject from the period between the two, al-Kindī's treatise in four sections can be mentioned[132]. Later well-known examples are Kūšyār ibn Labbān's explanation of the system for astronomers[133] and al-Nasawī's for accounting

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[128] A parallel case is ibn al-Haytham's investigation of the «purchase of a horse» (partially translated in [Wiedemann 1970: II, 617–619]). This treatise too opens with a polemic against practitioners who do not justify their procedures.

[129] Latin (and German) translation in [Curtze 1902: 38,40]. It should be observed that Abraham's text is meant most practically. It is in the same tradition as Abū Bakr's *Liber mensurationum*, cf. above, chapter VI.

[130] [Saidan 1978: 19–31]. In the following, I follow Saidan's typology.

[131] The first is described in [Saidan 1974], the second translated in [Hochheim 1878].

[132] Listed in al-Nadīm's *Fihrist* [trans. Dodge 1970: 617]. It is not clear whether the "Introduction to Arithmetic, five sections" also mentioned there is a finger-reckoning treatise, a commentary on Nicomachos, or the two combined.

[133] Trans. [Levey & Petruck 1965].

officials[134].

After the mid-11th century, it becomes difficult to distinguish two separate traditions. While al-Nasawī, when examining around CE 1030 earlier treatises on his subject, would still (according to his preface) restrict the investigation to Hindi books, his contemporary ibn Tāhir inaugurated an era where the traditions were combined, writing himself a work presenting «the elements of hand arithmetic and the chapters of *takht* [dust board, i.e. Hindi – JH] arithmetic» together with «the methods of the people of arithmetic» (apparently his section 6 on Greek theoretical arithmetic) and the «arithmetic of the zīj» (sexagesimal fractions)[135].

The same combination is found again in the Maghrebi arithmetical tradition as we know it from works of al-Ḥaṣṣar[136], ibn al-Bannā'[137], al-Umawī[138] and al-Qalasādī[139], and through ibn Khaldūn's report[140]. It is interesting in several respects, not least for its systematic development of arithmetical and algebraic symbolism[141], the former of which was also taken over by Leonardo Fibonacci in the *Liber abaci*[142].

The early writers of large arithmetical textbooks appear to have been relatively independent of each other. During the initial phase of synthesis, they collected, systematized and reflected upon current methods and problems, and eventuaqlly they

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134 To be precise, the Arabic treatise which has come down to us is written for the Buyid vizier Saraf al-Muluk; but we must assume that the author keeps close to the earlier treatise written in Persian of which he speaks himself in the preface. See the translation of the preface in [Woepcke 1863: 492–495], and Saidan, "Al-Nasawī", *DSB* IX, 614. The report of the treatise in [Suter 1906] covers only the brief section dealing with the extraction of roots.

135 Quoted from [Saidan 1978: 24] (pp. 24–29 give an extensive abstract of the whole work).

136 Reported extensively in [Suter 1901].

137 Ed., trans. [Souissi 1969].

138 See [Saidan 1978a] and Saidan, "Al-Umawī", *DSB* XIII, 539f. Al-Umawī taught in Damascus, but he came from the West, where he had been taught, and whose methods he brought to the East.

139 His «rather extended and rich summary» of arithmetic (as he describes it himself in the introduction) was translated by Woepcke [1859].

140 *Muqaddimah* VI, 19, trans. [Rosenthal 1958: III, 122f]. Ibn Khaldūn had been taught himself by a disciple of ibn al-Bannā' (see Vernet, "Ibn al-Bannā'", *DSB* I, 437).

A systematic investigation of certain sides of Maghrebi mathematics has been undertaken by Djebbar [1981].

141 A general account is given in [Djebbar 1981: 41–54]. The explanation given by ibn al-Bannā''s commentator ibn Qunfudh is translated in [Renaud 1944: 44–46]. [Woepcke 1854] deals mainly with al-Qalaṣādī's symbolism.

142 Compare Leonardo's various complicated fractions [ed. Boncompagni 1857: 24] with the similar forms in [Djebbar 1981: 46f].

used earlier treatments which were accessible as books. The first possibility appears to have been realized in al-Uqlīdisī's work, while al-Nasawī quotes the written works he has consulted. The integration of finger- and Hindi reckoning, on the other hand, appears to depend upon the more continuous teaching tradition of the madrasah. Already ibn Tāhir is reported to have taught at the mosque[143], and as mentioned above (chapter XII) the only non-«traditional» subject permitted at the Baghdad Nizamiyah madrasah was arithmetic. In the Maghreb tradition, ibn al-Bannā' was taught and himself became a teacher of mathematics and astronomy at the madrasah in Fez[144]. This makes it inherently plausible that even al-Hassar, upon whose works he commented, had relations to the madrasah (at the very least his works must have been used there). Ibn al-Bannā''s network of disciples also appears to cohere through the social network of madrasah learning. Al-Qalasādī must (as a writer of commentaries to al-Bannā') be presumed to belong to the same context, and in fact he tells himself that his arithmetic is written as a manual for the most bright among his students[145]. Finally, even al-Umawī was active as a teacher in Damascus. The theoretical elevation of the subject of arithmetic was hence not only a product of the general dispositions of Islamic culture; according to all evidence it was also mediated by the madrasah, which in this respect came to function as an institutional fixation and materialization of these same attitudes.

That theoretical elevation of this practical subject requires a specific explanation becomes evident if we compare the Islamic tradition with the fate of its «Christian» offspring: The *Liber abaci*, which carried the elevation of practical arithmetic to a summit. This is not to say that Leonardo's book was a cry in the desert. Its algebra influenced scholarly mathematics in the fourteenth century, so Jean de Murs[146]; besides, it is plausible that it inspired Jordanus de Nemore[147]. Part of the material was also taken over by the Italian «abacus schools» for merchant youth. The scholars, however, took over only specific problems and ideas, and the abacus teachers only the more elementary, practically oriented facets of the work. Western Europe of the early 13th century was in possession of no institution which could appreciate, digest and continue Leonardo's work. Only in the fifteenth century do similar orientations turn up once

[143] Saidan, "Al-Baghdadi", *DSB* XV, 9.

[144] Vernet, "Ibn al-Bannā'", *DSB* I, 437.

[145] Trans. [Woepcke 1859: 231]. Cf. also Saidan, "Al-Qalaṣādī", *DSB* XI, 229f.

[146] Se [G. l'Huillier 1980: *passim*].

[147] An alternative possibility is that Leonardo was drawing on Jordanus for the revised edition written to Michael Scot – see my [1985a: 7f].

again – apparently not without renewed relations to the Islamic world[148].

XV. The institutionalized cases (ii): astronomy and pure geometry

The other case of a whole tradition integrating theoretical reflection and investigation into a branch of practical mathematics is offered by astronomy. Because of its ultimate connection to astrology, astronomy was itself a practical discipline[149]; the *mathematics* of astronomy was, of course, practical even when astronomy itself happened to be theoretical.

In the Latin 13th through 15th centuries, this practical aim of the mathematics of astronomy had led to reliance on compendia, as it was observed above (note 106). In Islam, however, astrology was the occasion for the continuing creation of new *zījes* and for the stubborn investigation of new planetary models. Islam was not satisfied with using good old established models like the *Zīj al-Šāh* and the *Zīj al-Sindhind* in the way the Latin Late Middle Ages went on using the *Theorica planetarum* and the *Toledan Tables* for centuries.

Astronomy can even be seen to have been the main basis for mathematical activity in Medieval Islam. This appears from even the most superficial prosopographic study. The immense majority of Islamic mathematicians are known to have been active in astronomy[150]. Since astronomy was (together with teaching at levels which did not exceed that of the madrasah and hence hardly that of the large arithmetic and

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[148] An early example is a provençal arithmetic written c. 1430. A certain affinity to Islamic traditions is suggested by an initial invocation of God, of Mary his Mother, and of the Patron Saint of the city (see [Sesiano 1984] – the invocation is on pp. 29–31).

Later examples are Chuquet's *Triparty* and Luca Pacioli's *Summa de arithmetica*. The *Triparty* is told (in its first line) to be divided into three parts «a lonneur de la glorieuse et sacree trinite» [ed. Marre 1880: 593] – perhaps a jocular reference to familiar invocations in related treatises? In any case, the same author's *Pratique de geometrie* [ed. H. l'Huillier 1979] has no religious introduction.

[149] Two other practical aims for astronomy can also be mentioned: Finding the *qiblah* (the direction toward Mecca), and fixing the prayer times. None of them called for astronomy of such sophistication as developed around the princely observatories.

[150] Among those referred to above, only Abū Kāmil, al-Karajī, al-Samaw'al and al-Qalaṣādī stand out as exceptions. Al-Samaw'al, however, is at least known to have written a refutation of astrology, involving both mathematical arguments and knowledge of observations (Anbouba, "Al-Samaw'al", *DSB* XI, 94).

Strictly speaking, even ibn Turk, al-Uqlīdisī, al-Hassar and al-Umawī might also be counted as exceptions, since no works from their hand are known. However, our knowledge of these scholars is so restricted that they fall outside all attempts at statistical analysis.

A last important mathematician who appears definitely to have been a non-astronomer is Kamāl al-Dīn, whose important work concentrates on optics (cf. [Suter 1900: 159, N° 389], and Rashed, "Kamāl al-Dīn", *DSB* VII, 212–219).

mensuration textbooks) the most obvious way a mathematician could earn a living, one is forced to conclude that the astronomer's career involved quite serious work on mathematics, and at times serious work *in* mathematics.

This is also clear from al-Nayrīzī's introductory explanation to his redaction of the al-Ḥajjāj-version of the *Elements*: here it is stated that «the discipline of this book is an introduction to the discipline of Ptolemy's *Almagest*»[151]. Later, the same connection was so conspicuous that the Anglo-Normal writer John of Salisbury could observe in 1159, that «demonstration», i.e. the use of the principles expounded in Aristotle's *Posterior Analytics*, had

> practically fallen into disuse. At present demonstration is employed by practically no one except mathematicians, and even among the latter has come to be almost exclusively reserved to geometricians. The study of geometry is, however, not well-known among us, although this science is perhaps in greater use in the region of Iberia and the confines of Africa. For the peoples of Iberia and Africa employ geometry more than do any others; they use it as a tool in astronomy. The like is true of the Egyptians, as well as some of the peoples of Arabia.[152]

So, it was not only the factual matter of the *Elements* which was reckoned part of the astronomical curriculum. According to the rumours which (via the translators?) had reached John of Salisbury, the geometry of astronomy was concerned even with the metamathematical aspects and problems of the *Elements*.

In the initial eager and all-devouring phase of Islamic science (say until al-Nayrīzī's time, i.e. the early 10th century CE), the general positive appreciation of theoretical knowledge may well have laid the foundation both for the extension of astrology into the realm of high-level theoretical astronomy and for the extension of astronomy into that of theoretical mathematics. Down-to-earth sociology of the astronomers' profession may be a supplementary explanation of the continuation of *the first* tradition: The importance of the court astronomer (and, in case it existed, of the court observatory) could only increase if astronomy was a difficult and inaccessible subject. But even if this common-sense sociology is correct, it is not clear why intricacy should be obtained via the integration of metamathematics, the difficulties of which would only be known to the astronomer himself, and which would therefore hardly impress his princely employer. Why then should the integration survive for so long?

It appears, once more, that the original positive appreciation of (mathematical) theoretical knowledge was materialized institutionally, in a relatively fixed curriculum for the learning of astronomy. This curriculum started (as stated by al-Nayrīzī) with the *Elements*, and it ended with the *Almagest*. In between came the *mutawassiṭāt*, the «Middle Books» (cf. above, chapter II).

It is not clear to which degree this fixation was developed at different times. A

[151] My translation from the Latin of Besthorn & Heiberg [1897: I, 7].

[152] *Metalogicon* IV, vi; quoted from McGarry's translation [1971: 212].

full codification of the corpus of Middle Books is only known from the Naṣīrean canon[153], and the precise delimitation of the concept may have varied with time and place. Most remarkable are perhaps the indications that books I–II of the *Conics* may also have been considered normal companions of the *Elements* in the times of ibn al-Haytham and al-Khayyāmī[154]. It appears, however, that Ḥunayn ibn Isḥāq made a translation of the «Little Astronomy» which already served the purpose, and that Thābit had a similar concept[155]. Al-Nayrīzī too, we remember, appeared to have a fixed curriculum in mind.

So, from the ninth century CE onwards, it appears that astronomical practice and interest kept the focus upon pure and metatheoretical geometry not only because of a vague and general appreciation of the importance of theoretical knowledge[156] but also because of the institutional fixation of this appreciation. Evidently, this does not imply that, e.g., the long series of investigations of the foundational problems of the *Elements* were all made directly (or just presented) as astronomical *prolegomena* – the opposite is evident both in the case of al-Khayyāmī's *Discussion of Difficulties in Euclid* (cf. note 115) and in the case of Thābit's two proofs of the parallel postulate[157]. Other metatheoretical investigations, however, were expressly written for recensions (*taḥrīr*) of the *Elements* for the introductory curriculum of astronomy – this is the case of Muḥyi'l-Dīn al-Maghribī's and Naṣīr al-Dīn al-Ṭūsī's proofs of the same postulate[158]. With utterly few exceptions, the authors of such metamathematical commentaries appear to have been competent in mathematical astronomy[159].

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153 See [Steinschneider 1865: 467 and *passim*]; and Nasr, "Al-Ṭūsī", *DSB* XIII, 509.

154 For ibn al-Haytham, see [Sesiano 1976: 189]. For al-Khayyāmī, cf. above, note 13.

155 See [Steinschneider 1865: 464, 457], respectively. The Greek «Little Astronomy» was to form the backbone of the *mutawassiṭāt* even in the Naṣirean canon where, however, Euclid's and Thābit's *Data* and Archimedes' *Measurement of the Circle*, *On the Sphere and Cylinder* and *Lemmata* are included together with some other works.

156 Such general attitudes, too, remained effective – they are expressed in the praise of Archimedes' *Lemmata* formulated by al-Nasawī, who speaks of the «beautiful figures, few in number, great in *utility*, on the fundaments of geometry, in the highest degree of *excellency* and *subtility*» (quoted from [Steinschneider 1865: 480]; emphasis added). Cf.also al-Bīrūnī as quoted in chapter XIII.

157 Both translated in [Sabra 1968].

158 See [Sabra 1969: 14f, 10 n.59], respectively.

159 So, all those mentioned above, as well as others mentioned in [Sabra 1969] (Qayṣar ibn Abī'l-Qāsim, Yūḥannā al-Qass, al-Jawharī) and [Folkerts 1980] (which, besides some of the same, mentions al-Māhānī) – with the ill-documented al-Qass as a possible exception [Suter 1900: N° 131].

XVI. A warning

The above might look like a claim that the global character and all developmental trends of Islamic mathematics be explainable in terms of one or two simple formulae. Of course this is not true. Without going into details I shall point to one development of a character puzzlingly different from those discussed above: that of magic squares[160]. Their first occurrences in Islam are in the Jabirian corpus, in the *Ikhwān al-Safā'*, and (according to ibn Abī Usaybiʿa) in a lost treatise from Thābit's hand. Various Islamic authors ascribe the squares to the semi-legendary Apollonios of Tyana[161], or even to Plato or Archimedes. An origin in Classical Antiquity is, however, highly improbable: A passage in Theon of Smyrna's *On the Mathematical Knowledge which is Needed to Read Plato* is so close to the idea that he would certainly have mentioned it had he heard about it[162]; but neither he nor any other Ancient author gives the slightest hint in that direction. On the other hand, an origin in Late Hellenistic or Sabian Hermeticism is possible, though still less probable than diffusion along the trade routes from China, where magic squares had been known and used since long. This doubt notwithstanding, it is obvious that the subject was soon correlated with Hermeticism and *Ismāʿīlī* and related ideas. Truly, at least one mathematician of renown took up the subject – *viz.* ibn al-Haytham, with whose omnivorous habits we have already met on several occasions. Truly, too, some progress took place, from smaller toward larger squares and toward systematic rules for the creation of new magic squares. On the whole, however, the subject remained isolated from general mathematical investigations and writings. The exceptional character of ibn al-Haytham's work is revealed by an observation by a later writer on the squares, that «I have seen numerous treatises on this subject by crowds of people. But I have seen none which speaks more completely about it than Abū ʿAlī ibn al-Haytham»[163]. The treatise just quoted combines the subject with arithmetical progressions; but integration into larger arithmetical textbooks or treatises seems not to have occurred – and so, Islamic mathematics did not integrate *every* subject into its synthesis. Instead, magic squares appear to have conserved an intimate connection to popular superstition and illicit sorcery[164].

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[160] A good and fairly recent overview of magic squares in Islam is [Cammann 1969]; but see also [Ahrens 1916]; [Bergsträsser 1923]; [Hermelink 1958]; [Sesiano 1980, 1981]; and [Sarton 1927, 1931], index references to «magic squares».

[161] The most widespread assumption to judge from the *Fihrist* [trans. Dodge 1970: 733].

[162] Ed. [Dupuis 1892: 1966]. The passage shows the square $\begin{matrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{matrix}$.

[163] The anonymous author is quoted from [Sesiano 1980: 188].

[164] This is clearly the point of view of ibn Khaldūn in the *Muqaddimah*, every time he approaches the subjects of talismans, letter magic and magic squares (which mostly go together). In one place

It is not plausible that the exclusion of magic squares from the mathematical mainstream shall be explained by any inaccessibility to theoretical investigation – other subjects went into the arithmetical textbook tradition even though they were only known empirically and not by demonstration[165]. So, the exclusion of magic squares from honest mathematical company must rather be explained by cultural factors – be it that the subject did not belong inside the bundle of recognized subdisciplines which had been constituted during the phase of synthesis; that its involvement with magic and sorcery made it a non-mathematical discipline[166]; or that the involvement of mainstream mathematicians with practically oriented social strata made them keep away from a subject (be it mathematics or not) involved with *sufi* and other esoteric (or even outspokenly heretical) currents. I shall not venture into any definite evaluation of these or other hypotheses (even though ibn Khaldūn makes me prefer N° 2), but only conclude that the place of magic squares in the culture of Medieval Islam is *not* explainable in the same terms as the synthesis, the integration of practical mathematics and theoretical investigation, the development of the arithmetical textbook tradition, or the interest in the foundations of geometry. No culture is simple.

XVII. The moral of the story

The above is hence no complete delineation of Medieval Islamic mathematics; nor was it meant to be. The purpose was to demonstrate that Islamic mathematics possessed certain features not present in any earlier culture (but shared with Early Modern science) and to trace their causes. I hope that I have succeeded in demonstrating the existence of these features, and hence of an «Islamic Miracle» just as necessary for the rise of our modern scientific endeavour as its Greek namesake, and to have offered at least a partial explanation of what happened.

This leaves us with a question of a different order: Was the integration of theory and advanced practice in Renaissance and Early Modern Europe a set-off from Islam, or was it an independent but parallel development?

Answering this question involves us with the recurrent difficulty of diffusionist explanations. «Miracles» and other cultural patterns *cannot* be borrowed simply: they

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he also claims that a work based on such things «most likely [...] is incorrect, because it has no scientific basis, astrological and otherwise» (III, 52, trans. [Rosenthal 1958: II, 224]).

165 So al-Karajī's summation of square numbers in the *Fakhrī* (see the paraphrase in [Woepcke 1853: 60]).

166 Ibn Khaldūn does not mention the subject at all during his discussion of arithmetic (*Muqaddimah* VI,19, trans. [Rosenthal 1958: III, 118–129]). Like amicable numbers (once investigated mathematically by Thābit but now only mentioned as a talisman producing love) it is relegated to the chapters on magic and sorcery (VI,27–28, trans. [Rosenthal 1958: III, 156–227]). The silence on amicable numbers is all the more striking as the circle of Maghrebi mathematicians was in fact interested in that subject – cf. [Rashed 1983: 116f].

can only inspire developments inside the receiving culture. Even a piece of technology can only be borrowed if the receiver possesses a certain preparedness. The experience of cargo cults shows to which degree the receiver determines the outcome of even a seemingly technological inspiration, and investigations of any process of cultural learning will show us radical reinterpretations of the original message (and we may ask whether Charlemagne's identification of the Palace school of Aachen with the resurrection of Athenian philosophy was less paradoxical than the cargo cults).

We know the eagerness with which the European Renaissance tried to learn to the letter from Ancient Rome and Greece – and we know to which enormous extent the social and cultural conditions of Europe made it misunderstand the message. In contrast, no serious effort was made to understand the cultural messages of the Islamic world; on the contrary, great efforts were invested to prove that *such messages were morally wrong*. We can therefore be confident that no general cultural patterns or attitudes (be it the attitudes toward rational knowledge and technology) were borrowed wholesale by Christian Europe. Nor was there any significant borrowing of institutions[167], including those institutions which materialized the attitudes to knowledge. The only way Renaissance and Early Modern Europe could learn from the «Islamic miracle» was through acquaintance with its *products*, i.e. through scholarly works and technologies which it had produced or stamped. Because they were received in a society which was already intellectually and technologically mature to make an analogous leap, part of the «Islamic message» could be apprehended even through this channel. Primarily, however, Renaissance Europe developed its new integrative attitudes to rational and technological knowledge autochtonously; transfers were only of secondary importance.

This conclusion does not make the Islamic miracle irrelevant to the understanding of modern science. *Firstly* two relatively independent developments of analogous but otherwise historically unprecedented cultural patterns should make us ask whether similar effects were not called forth by similar causes. Here, the sources of Islamic and Renaissance mathematics were of course largely identical (not least because Christian Europe supplemented the utterly meager direct Greco-Roman legacy with translations from such Arabic works which were accessible in Spain, i.e. mainly works dating from the 9th century). These sources had, however, not been able to produce the miracle by themselves before the rise of Islam. Were there then any shared «formative conditions» which helps us explain the analogous transformation of the source material?

Probably the answer is «Yes». Truly, Western High Medieval Christianity had been dominated by a powerful ecclesiastical institution; moreover, after the 12th century it could hardly be claimed to be fundamentalist. Yet precisely during the critical period

[167] A few exceptions, e.g., in commercial law can be found – but the difference between the Maghrebi arithmetical textbook tradition and the Italian abacus school shows that even the institutions of *commercial teaching* could not be transferred.

(say, the period of Alberti, Ficino, Bruno and Kepler) the fences of the Thomistic synthesis broke down, and rational knowledge came to be thought of both as a way to ultimate truths concerning God's designs and to radical improvements of practice. At the same time the ecclesiastical institution lost much of its force, both politically and in relation to the conscience of the individual; religious feelings were, however, rather stronger than weaker than in the 13th century. It would therefore not be astonishing if patterns like the non-institutionalized practical fundamentalism of 9th century Islam could be found among Renaissance scholars and higher artisans. It would also be worth-while to reflect once more in this light upon the «Merton thesis» on the connections between Puritanism, social structure and science[168].

Secondly the whole investigation should make us aware that there are no privileged heirs to the cultural «miracles» of the past. It is absurd to claim that «science, as we know it and as we understand it, is a specific creation of the Greco-Occidental world»[169]. Firstly, Greek «science» was radically different from «science as we know it and as we understand it». Secondly, with relation to science (and in many other respects, too), it is no better (and no worse) to speak of a «Greco-Occidental» than of a «Greco-Islamic» world, and not much better to claim a «Greco-Occidental» than an «Islamo-Occidental» line of descent.

In times more serene than ours, these points might appear immaterial. If Europe wants to descend from Ancient Greece and to be her heir *par excellence*, then why not let her believe it? Our times are, however, not serene. The «Greco-Occidental» particularity always served (and serves once again in many quarters) as a moral justification of the actual behaviour of the «Occident» toward the rest of the world, going together with anti-Semitism, imperialism and gunboat diplomacy. In theory it might be different, and the occidentalist philosopher just quoted finds it «unnecessary to specify that no "practical" or "political" conclusions should be drawn» from «our» privileged place in world history[170]. It is, alas, *not* unnecessary to remind of Sartre's observation that the «intellectual terrorist practice» of liquidation «in the theory» may all too easily end up expressing itself in *physical liquidation of those who do not fit the theory*[171].

As Hardy once told, «a science is said to be useful if its development tends to accentuate the existent inequalities in the distribution of wealth, or more directly promotes the destruction of human life». The ultimate drive of the present study has been to undermine a «useful» myth on science and its specifically «Greco-Occidental»

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[168] A supplementary approach might compare the institutions of «courtly science» and the patterns of princely protection in the two settings.

[169] [Castoriadis 1986: 264] (my translation).

[170] *Ibid*: 263 n.3.

[171] [Sartre 1960: 28].

origin – whence the dedication to a great humanist.

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Supplement

The following articles from* DSB *are referred to in the text:

David Pingree, “Abū Maʿšar”. I, 32–39.
A. P. Youschkevitch, “Abū’l-Wafāʾ”. I, 39–43.
A. S. Saidan, “Al-Baghdadi, Abū Mansur ... ibn Ṭāhir ...”. XV, 9–10.
J. Vernet, “Ibn al-Bannāʾ”. I, 437–438.
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G. C. Anawati & A. Z. Iskandar, “Ḥunayn ibn Isḥāq”. XV, 230–249.
Yves Marquet, “Ikhwān al-Ṣafāʾ”. XV, 249–251.
Roshdi Rashed, “Kamāl al-Dīn”. VII, 212–219
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G. J. Toomer, “Al-Khwārizmī”. VII, 358–365.
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