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THE FINER STRUCTURE OF THE OLD BABYLONIAN MATHEMATICAL CORPUS

Elements of Classification, with Some Results

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When the existence of "higher" Babylonian mathematics was ascertained almost sixty years ago, the discoverers were *a priori* convinced that Old Babylonian and Seleucid mathematics could not be the same thing, as evident from Neugebauer's words [1929: 80]: "Bei einer solchen Lage der Dinge bereits in altbabylonischer Zeit wird man in Hinkunft auch die spätere Entwicklung mit anderen Augen anzusehen lernen müssen". 1500 years of stagnation was simply unimaginable. As Babylonian mathematics became staple food in general histories of mathematics, however, such sound prejudice was soon discarded; in agreement with the worst stereotypes of the eternal Orient, everything Babylonian (in mathematics) was equated with everything else, and everything was at best understood on the basis of the translations of the *Mathematische Keilschrift-Texte* and the *Mathematical Cuneiform Texts*, too often on the basis of the algebraic formulas by means of which these wonderful volumes explain why the Babylonian procedures are correct (or, when needed, *not* correct).[1]Few were those who between 1950 and 1985 would insist on seeing Babylonian mathematics primarily (or at all) as an constituent of Babylonian culture, and the history of Babylonian mathematics as an aspect of Babylonian history.

This situation has changed; nobody with serious interest in the topic will discuss today texts separated by more than a millennium as if they were contemporary or expressions of a single invariable mode of thought. It is also generally admitted that more than the accidents of excavations distinguishes Old Babylonian mathematics from Ur III computation.

Ein harter Nuß has resisted, however, not due to deficient appetite but because it *is* hard. Old Babylonian mathematics has remained one thing. It is not difficult to point to differences between the texts; some tell that "you see" results, some that a result "is given", some that it "comes up"; some use a relative clause where others have a participle; some are heavily logographic and others predominantly syllabic. It has proved difficult, however, to find any system in the variations. When trying to trace the existence of separate "schools" in Old Babylonian mathematics a few years

[1] I shall not go into details with this historiographical horror story; but see [Høyrup 1996a: 11*ff*].

ago I concluded that a “number of (mostly terminological) indicators *might* point to the existence of separate styles, perhaps so cognitively discrete that one should speak of schools” [Høyrup 1993c: 214]. But except for the series texts I had to admit not to have “been able to associate any of the distinctive characteristics with noteworthy differences in mathematical substance, technique or orientation”.

This may still be true – but only if we understand “substance”, “technique” and “orientation” narrowly. The following pages investigate the distribution of a number of terminological indicators, and shows that text groups defined on the basis of orthography or known geographical provenience often share a whole set of terminological characteristics. They also point to the conclusions that follow regarding the roots of various terminological habits and – more important – regarding the process of synthesis between separate background traditions in which Old Babylonian mathematics emerged. Even though most space is dedicated to terse philological matters it is my hope that these implications for the writing of history will not disappear from view.

My first encounter with Joachim Oelsner (indirect, and not in person at the time) was in December 1980, as I visited Fritz Jürß in Berlin. He told me about the work on *Geschichte des wissenschaftlichen Denkens im Altertum*, published as [Jürß (ed.) 1982]; and as a true historian of thought he told me that the chapter on Babylonian mathematics was written by another true historian – namely Joachim Oelsner. It is with great pleasure that I dedicate this paper to a colleague who knows how to transform philological details into history – not to speak of other qualities of his.

The first groupings

When publishing in 1935–37 his *Mathematische Keilschrifttexte* [MKT], Neugebauer made a main division of the material into table texts and problem texts. As far as the problem texts were concerned, he chose an ordering according to museum number – an obviously choice in as far as no classification according to geographical provenience was possible, and even chronological considerations would have allowed him to detach only a single Kassite and two Seleucid tablets from the bulk of Old Babylonian texts. The only exception to this rule was the group of "series texts",[2] which was not only treated in a separate chapter (still designated "Texte der Yale Babylonian Collection", although two texts were from Berlin) but also ordered within this chapter according to affinity of contents, and a few cases where closely related texts were treated together (VAT 8389 and VAT 8391, VAT 8521 and VAT 8528, Str 368 together with VAT 7532 and VAT 7535 – and evidently BM 85 200 together with VAT 6599, both being parts of the same original tablet).

In volume II (p. 50), a first tentative grouping is found:

> Schon rein äußerlich ist unmittelbar klar, daß die vier Texte BM 85 194 (Kap. III), BM 85 196 (der vorliegende Text), BM 85 200+VAT 6599 (Kap. III) und BM 85 210 (Kap. III) einander sehr nahestehen. Das bestätigt sich auch inhaltlich, bezüglich der beiden ersten und bezüglich des vierten, während BM 85 200+VAT 6599 etwas abseits steht – z.B. schon durch die viel größere innere Zusammenhörigkeit seiner Beispiele im Gegensatz zu der mathematischen Willkürlichkeit in der Aufeinanderfolge der einzelnen Aufgaben bei den drei anderen Texten.
>
> [...]
>
> Es scheint mir, daß sich damit bereits eine gewisse Gruppierungsmöglichkeit innerhalb unseres ganzen Textmaterials ergibt. Eine offenbar eng zusammengehörige Gruppe (zweifellos zu den ältesten erhaltenen eigentlich mathematischen Texten zu rechnen) bilden die Straßburger Texte (Kap. V) und VAT 7532, VAT 7535 (Kap. VI, S. 294 bis 314). Als zweite Gruppe möchte ich jetzt die YBC-Texte [i.e., the series texts / JH] und die Texte BM 85 194, BM 85 196, BM 85 210, BM 85 200+VAT 6599 und VAT 6598 ansehen.

In *Mathematical Cuneiform Texts* [MCT] from 1945, Neugebauer and Sachs grouped the Old Babylonian problem texts (once again the bulk of the material) according to their theme: "Pythagorean numbers" (Plimpton 322,

[2] The texts in question are VAT 7528, VAT 7537, YBC 4668–69, YBC 4673, YBC 4695–98, YBC 4708–15. MCT adds A 24 194 and A 24 195 to the group. With the exception of YBC 4669, all of these texts are catalogues of problem statements (some with, others without answers); the reverse of YBC 4669 contains a mixed collection of problems including one with a prescription. With the exception of YBC 4669, YBC 4696 and A 24 195, all tell in the colophon their number within the series where they belong. All, however, beyond the common style (also found on the obverse of YBC 4669), share the same characteristic ductus and format. There is no reasonable doubt as to their common origin.

no real problem text); “Cube Root”; “Geometrical Problems”; “Excavations”; “Irrigation (Canals, Cistern)”; “Bricks”; and “Equations”, without claiming that this arrangement reflect neither common provenience nor Babylonian disciplinary boundaries. MCT, however, contains a chapter (pp. 146–151) written by Albrecht Goetze, in which variations in dialect and orthography (and, to a slight extent, vocablary) are used to distinguish six different text groups (text with too much logographic writing being either left out or ascribed to one of the groups already established if “connected by external appearance and content with other tablets of the group” – [MCT, 149 n. 356]):[3]

1. “This group is certainly to be localized in the South, in all probability Larsa. It employs PI for both *pi* and *pe*, and shows numerous repeated vowels.”
2. “This is likewise a southern group. It employs PI for both *pi* and *pe*, but exhibits repeated vowels only sparingly.”
3. “This group, likewise southern, is localized in Uruk. It employs BI for *pé*.” It includes the Strasbourg texts, VAT 7532 and VAT 7535, already clustered by Neugebauer.
4. “As far as linguistics is concerned, this 4th group cannot be distinguished from the 3rd. [It is] quite clear, however, that here PI is *pi* and BI *pé*. The provenience may likewise be Uruk.”
5. “The employment of BI for *pi* [*sic*, should be *pí*] and the occurrence of SU make this a northern group”.

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[3] The main division is into a “northern” and a “southern” dialect, the former being that of the Hammurapi code and of texts from Dilbat and Sippar, the latter being described mainly on the basis of Larsa texts. Characteristic northern spellings are the following:

ṭá, ṭe$_4$*, ṭì, ṭú, sa, si, su, ás, is, ús, ba, bi, bu, pa, pí, pu*.

The corresponding southern sequence is

ṭa, –, ṭi, ṭù, sà, sí, sú, as, is, us, ba, bi, bu, pa, pi, pu.

It is mentioned as self-evident (p. 146) that “texts from other places will probably necessitate the positing of additional ‘dialects’”, and pointed out (p. 147) that one group of mathematical tablets “probably at home in Uruk” also uses BI as *pé*. To this comes a number of other characteristics, such as a southern preference for phonetic complements of the CVC-type and a northern preference for the type VC, spelling of /aya/ as *a-i̭a* in the North and *a-a* in the South, etc.

6. “This group combines northern and southern characteristics. It is slightly younger in date than the other groups. Since by now it seems clear that Akkadian mathematics (as the other varieties of Akkadian writings) originated in the South,[4] the situation is satisfactorily explained when it is assumed that the 5th [*sic*; should be 6th] group comprises tablets based on southern originals, but written and modernized in the North. The southern originals were close to the 1st and 2nd group”. The group in question encompasses BM 85 194, BM 85 196, BM 85 10, BM 85 200+VAT 6599 and VAT 6598, clustered by Neugebauer, but not the almost exclusively logographic series texts which according to Neugebauer should be close to them. The series texts, indeed, are not explicitly included in any of Goetze’s groups but mentioned in connection with group 2.

Fortunately, most of the Old Babylonian mathematical problem texts published since 1945 have a well-defined provenience. Today, two further groups can hence be added:

7. The texts from Ešnunna, dated Ibalpiel II and earlier, and thus, according to Thureau-Dangin’s palaeographic sensitivity, the earliest extant Akkadian mathematical texts – [TMB, ix] considers AO 8862 (group 1) and BM 13 901 (by Goetze considered a member of group 2) the earliest texts from the MKT corpus yet hardly earlier than Hammurapi. As we shall see (pp. 35*ff*), AO 8862 may actually be as early as the beginning of the Ešnunna tradition and earlier than the extant Ešnunna tablets. The texts in question have been published by Baqir [1950a; 1950b; 1951; 1962], Goetze [1951] and al-Rawi & Roaf [1984].[5]
8. The Susa texts, dated to the end of the Old Babylonian epoch, and published by Bruins and Rutten as *Textes mathématiques de Suse* [TMS].

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[4] After the discovery of the Ešnunna texts (cf. below) this priority can no longer be taken for granted, and one might be tempted to invert it to the effect that “Akkadian mathematics, like law-codes in Akkadian, originated in Ešnunna”. As we shall see, however, the textual evidence supports simultaneous development in Ešnunna and in the South (Larsa?) – see p. 45.

[5] The index of tablets contains detailed references to publication data.

Evidently, the addition of two well-dated and fairly large text groups allows us to see the problem of categorization in a new perspective. This is what I shall attempt to do in the following, basing myself on terminology in the widest sense (vocabulary in the context of function) rather than on orthography.

Characteristics of the Ešnunna corpus

The Ešnunna texts constitute a convenient starting point, since they are early and of almost the same age,[6] and since they were produced within a relatively small and politically unified area. That they share a number of features is hardly amazing; all the more astonishing is the sharp division of the corpus into two separate subgroups 7A and 7B.

7A is characterized by what we might call a "riddle format". With one exception its problems begin with the formula *šum-ma ki-a-am i-ša-al(-ka) um-ma šu-ú-ma,* "If [somebody] asks (you) thus:" with the marker of direct speech (*umma šū-ma*).[7] This formula is found in IM 53 953, IM 53 957, IM 53 961, IM 53 965, IM 54 010, IM 54 011, IM 54 464, IM 54 478, IM 54 538, IM 54 559; all of these are from Tell Ḥarmal, and belong to the transition years between Daduša and Ibalpiel II. Slightly different is

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[6] With one exception they were found in strata together with dated tablets that differ in age by a few decades. Obviously the mathematical tablets could be slightly older; IM 52 301, moreover, contains typical copying errors and thus descends from an older original – see analysis in [Høyrup 1990a: 338–340]. I use the opportunity to correct a mistake made in that paper for reasons that now escape me: it is true that the tablet is one of the younger Tell Ḥarmal texts (Ibalpiel II); but this evidently does not make it a young member of the Old Babylonian corpus regarded as a whole.

The exception is the "Tell Ḥarmal Compendium" (see below), not found *in situ* but left on the ground after an illicit digging.

[7] Concerning the translations I refer to my earlier discussion of the principle of "conformal translation" [Høyrup 1990a: 60–62]; in some cases, however, second thoughts have persuaded me that the actual translation should be modified. In my translations of numbers written in the sexagesimal place value system I follow Thureau-Dangin's convention, according to which ´, ˝, ‴ (etc.) indicate decreasing and `, ``, ``` (etc.) increasing sexagesimal order of magnitude (and the sign ° is used when necessary to indicate "order zero").

Db_2-146, from Tell Dhiba'i, found together with tablets dated Ibalpiel II years 8 and 9. It is introduced by the formula *šum-ma sí-li-ip-ta-a-am i-ša-lu-ka um-ma šu-ú-ma*, "if, about a [quadrangle with] diagonal, [somebody] asks you thus:". In all texts from the group, the ensuing description of the situation leads to an explicit question marked by one of the phrases *kī masi*, "corresponding to what" and *mīnum*, "what".

All texts in group 7A introduce the prescription with the formula *at-ta i-na e-pé-ši-ka*[8], "you, by your proceeding". No closing formula is present except in Db_2-146, which has *ki-a-am ne-pé-šum*, "thus the procedure". None of the "logical particles" *aššum* ("since"), *inūma* ("as") and *šumma* ("if") are used within the prescription (Db_2-146 uses *šumma* [*viz*, "if" the length is 1 and the width 45´, as has been found] to introduce the proof; after the proof follows a repetition of the formula *at-ta i-na e-pé-ši-ka*. In the many cases where the transition to a new section of the prescription is marked, the phrase used is *na-ás-ḫi-ir*, "turn yourself around" (<*saḫārum*, N-stem). The results of calculations are marked by one of the phrases *ta-mar*, "you see", or *i-li-a-ku-um*, "comes up for you", in both cases often combined with an enclitic *-ma* on the verb for the operation[9].

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[8] We observe the use of BI for *pé*, as in Goetze's groups 3-6. The use of BI for *pí* (*pí-ti-iq-tum* and *e-pí-ri-ka*, IM 54 011 obv. 2, rev. 2; *ša-pí-il-tum*, IM 54 464 rev. 1), however, is only shared by groups 5 and 6.

[9] Only IM 54 559 uses both *ta-mar* and *i-li-a-ku-um*; IM 54 464 makes a "raising"-multiplication (*našûm*) "give you" (*i-na-di-na-ku-um*) the result), but then repeats the calculation making the result "come up" (*i-li*), as if a slip had occurred when the scribe submitted an original to stylistic normalization (or tried to conform to a style which was not fully his, if his composition is original); as we shall see (note 71 and elsewhere), *nadānum*-constructions, used regularly in questions about division by irregular numbers, appear to have had an early connection with sexagesimal multiplication. As we shall see (note 11), another terminological peculiarity of the tablet in question also suggests a normalization gone wrong.

When results "come up", the interrogative phrase of the question tends to be *mīnum*; when they are "seen", it is invariably *kī masi*; a mere coincidence is not very likely, but the statistics is too limited to exclude it: 2 texts against 1 (*viz* Db_2-146) in the first case, 7 texts in the second, excluding the mixed tablet and one (IM 54 464) with a lacuna at the critical point. To a first inspection, the theme of problems does not seem to be a decisive parameter: of two work-force problems, IM 53 961 has *i-li-a-ku-um* and IM 54 011 *ta-mar*; but see below, p. 16.

As regards the operations, group 7A has a clear preference for *ḫarāṣum*, “to cut off”, over *nasāḫum*, “to tear out”;[10] *nasāḫum*, indeed, only occurs in IM 54 464 (rev. 9) in the relative clause *ša ... ta-su-ḫu*, the (damaged) reference of which (obv. 10) seems not to contain this verb at all but only an “excess” (*watrum*).[11] When present, the term for the formation of squares and rectangles is *šutakūlum*, “to make [two line segments] hold each other [as sides of a rectangle]”, (at times with a double, at times with a single object); only IM 54 478 uses *šutamḫurum*, “to make confront itself [as an equal]”, to tell that the base of an excavation is made square.[12] The “equalside” – the square or cubic frame represented by the side as *pars pro toto* – is written íb.si[13] in all cases where a square frame (and hence, numerically, a square root) is intended; in IM 54 478 a cubic “equalside” is designated íb.si$_8$. In the Tell Harmal lot, the term conserves its original verbal character, as revealed by the accusative of the question *mīnam* íb.si / si$_{(8)}$. Db$_2$-146 alternates between ib.sí and ib.si and treats the creature in question as a noun, as something that shall be “taken” (*laqûm*) – perhaps from the table? The *meḫrum* – the “counterpart” or “other

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[10] Both represent the “subtraction by removal”, in which a part is withdrawn from an entity without changing the latter’s “identity”. The other subtractive operation is “by comparison”, the observation that “*A* exceeds *B* by *D*”, at times inverted into “*B* falls *D* short of *A*” (verbs *watārum* / dirig and *maṭûm* / lal, respectively). Further discussion in [Høyrup 1993a].

[11] Even this is thus probably the result of a slip, a deviation from a style deliberately striven for, cf. note 11.

[12] Henceforth, I shall use the term “squaring” when the verb takes a single object and “rectangularization” whenever there are two objects, whether identical or different. Among the terms belonging to the field, *šutamḫurum* (and íb.si$_8$ in the few texts where it occurs in this function – YBC 6504 and the series texts VAT 7537, YBC 4668, 46979, 4709, 4712 and 4713) serve exclusively for squaring; *šutakūlum* and the logograms NIGIN, ì.gu$_7$(.gu$_7$), du$_7$.du$_7$, UR.UR may be used in both ways, but with a preference for the rectangularization formulation even when a square is actually produced.

Both operations denote geometrical constructions, not mere multiplications, in contrast to the “raising (*našûm*) of *A* to *B*” and to *A* a.rá *B*, “*A* steps of *B*”.

[13] Or indeed *íb-se-e* (SI=SE) in most cases, obviously a syllabic writing of a Sumerian loanword in conserved pronunciation.

side” of a square corner – occurs repeatedly. “Breaking” (*ḫepûm*, bisection) only mentions the entity to be broken but not the resulting “natural half”[14] (neither the special term *bāmtum* found in many texts, nor šu.ri.a or $^1/_2$), except in Db$_2$-146, which refers to it as *muttatum*, not known from other mathematical texts. Since no inhomogeneous additions are found in the texts (e.g., sides plus areas) it cannot be seen whether these would have been designated *waṣābum* / daḫ or *kamārum* / gar.gar. In the role where certain texts have the noun *takīltum* (a line which has been made “hold” a square – see, e.g., [Høyrup 1990a: 264]), the present texts employ the equivalent relative clause *ša tuštakīlu*.

Most of the texts from group 7A write the lengths and widths of fields logographically, as uš and sag.ki. The use of logograms for these terms in otherwise mostly syllabic texts is not astonishing but in agreement with the almost invariable pattern of other text groups, irrespective of the amount of syllabic writing. Much more remarkable is the presence of several exceptions: Db$_2$-146 has a syllabic *ši-di* (obv. 3) but uš elsewhere; and IM 53 965 (a “broken reed” problem) has syllabic writings of *šiddum* and *pūtum* throughout.[15]

In view of the early date of the group it is noteworthy that its problems spread over the whole range of Old Babylonian mathematical themes: manpower for the carrying of bricks and for the building of an earth wall; a broken reed problem leading to a mixed second-degree equation; combined commercial rates; a complex problem dealing with a rectangle which is reduced to a standard problem about a different rectangle (*viz*,

[14] A “natural” or “necessary” half is one which could be nothing but the half: the half of the base of a triangle that is multiplied by the height in the area calculation; the average width of a trapezium; the radius as half the diameter of the circle; in “algebraic” texts that half of the excess of rectangular length over rectangular width which is bisected in order to prepare the transformation of the rectangle into a gnomon that can be completed as a square. In Saussurean terms, the normal half is a member of the paradigmatic series $^2/_3$, $^1/_2$, $^1/_3$, $^1/_4$, etc. The natural half is not.

Some texts use a name for the “natural half”, others not. But it is invariably produced by the operation *ḫepûm*, “breaking”.

[15] On the other hand, the *ši-du-um* of IM 54 538 (obv. 2) and IM 54 011 (obv. 2), respectively a carrying distance and the length of a wall to be built, are not exceptional but correspond to a pattern found elsewhere.

given area and excess of length over width); and a cubic excavation. The problem of Db_2-146 (a rectangle with given diagonal and area) is not found elsewhere in the cuneiform record but in later papyri (Greek as well as Demotic) and in medieval practical geometries under circumstances that (except for the Demotic specimens in [Parker 1972: 41–43]) leave little doubt as to the existence of a continuous tradition (see [Høyrup 1996b]).[16]

Even more intriguing is IM 53 957 ([Baqir 1951: 37], corrections and interpretation [von Soden 1952: 52]):

> To $^2/_3$ of my $^2/_3$ I have joined 100 sìla and my $^2/_3$, 1 gur was completed. The *tallum*-vessel of my grain corresponds to what?.

This may be compared to problem 37 of the Rhind Mathematical Papyrus [trans. Chace et al 1929: Plate 59]:

> Go down I [a jug of unknown capacity/JH] times 3 into the *ḥeḳat*-measure, $^1/_3$ of me is added to me, $^1/_3$ of $^1/_3$ of me is added to me, $^1/_9$ of me is added to me; return I, filled am I [actually the *ḥeḳat*-measure, not the jug/JH]. Then what says it?

The coincidences are too numerous to be accidental: firstly there is the shared use of an "ascending continued fraction" – in case even an expression of the type "*p*, and *p* of *p*" (*p* being a simple fraction); such expressions are not only extremely rare in the rich Egyptian record, the RMP example appears to be the only ascending continued fraction occurring at all. To this comes the details of the dress: an unknown measure which is to be found from the process, the reference to a standard unit of capacity, and the notion of filling.

The Egyptian problem is solved in agreement with the normal procedures of Egyptian arithmetic, in a way which depends critically on the fine points of the unit fraction system. The Ešnunna solution, on the other hand, is no solution at all but a sequence of operations which only yield the correct result because the solution has been presupposed – what

[16] The Demotic specimens are suspicious because their method is somewhat different from that of the tradition in general; but since the same papyrus (P. Cairo J.E.81127–30, 89137–43) contains problems (of type "reed against a wall", cf. [Sesiano 1987]) whose relation to the Mesopotamian orbit is beyond doubt, a connection remains likely.

sixteenth-century cossists would call *Schimpfrechnung*, a challenge meant to impress and make fools of the non-initiate. Problems of this type turn up regularly in medieval and Renaissance treatises on applied mathematics that draw directly on oral or semi-oral practitioners' traditions[17] – exactly the traditions where rules for practical computation go together with mathematical riddles that seem to refer to practice but rarely have any practical application. Without pursuing the argument we may deduce that the problem has its origin in a practitioners' environment (merchants?) in touch with both Egypt and Mesopotamia in the early second millennium, and that it was adopted by both Egyptian and Ešnunna scribes[18] – in Ešnunna preserving the eristic form and purpose, in Egypt transformed into "good mathematics".[19]

This inference fits the characteristic introduction of the 7A texts: "If [somebody] asks you thus:", a phrase that recurs in the medieval Arabic practical tradition, as do other characteristic features of the rhetoric of Old Babylonian procedure texts (see, e.g., [Høyrup 1986]). The traces of deliberate stylistic normalization in IM 54 464 (see notes 11 and 11) should warn us, however, against perceiving the texts of the group as nothing but written versions of traditional material. The evidence is certainly delicate, but the scribes (or the scribe) responsible for the production of the ten Tell Ḥarmal tablets belonging to the group seem to have aimed deliberately at stylistic demarcation, imitating an archaic (probably Akkadian) model, borrowing part of its material (*inter alia* the quasi-algebraic problems on rectangular and trapezoidal "fields", but also the

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17 The relation between scholars' "scientific" and apprenticeship-taught practitioners' "subscientific" mathematics is investigated in [Høyrup 1990b] and, more concisely and with some refinements of the conceptual apparatus, in [Høyrup, forthcoming].

18 In [Høyrup 1990c: 315] I weigh the possible diffusion wia trade routes against shared Hamito-Semitic language structures as explanation of the use of continued ascending and other composite fractions in particular contexts in Egypt as well as Babylonia. The Tell Ḥarmal tablet decides in favour of the first possibility.

19 Eventually, the transformation into "good mathematics" also took place in Babylonia, where the problem turns up in somewhat altered shape as YBC 4669 N^os B4–5 ([MKT III, 27], correction [MCT, 103]) – so transformed indeed that the family likeness with the Egyptian problem is no longer obvious.

problem on the *tallum* vessel), and eliminating perhaps some of the references to Ur III computational practice.[20]

Excavation circumstances suggest indeed that the ten Tell Ḥarmal tablets may have been produced either by the same scribe, or – more likely – by scribes in intimate contact with each other: nine were found in the same room, and the tenth in the immediate vicinity.[21] Db_2-146, on the other hand, some ten years younger and found at some 4 kms' distance, is certainly an independent member of the group, and probably – in view of the shift between logographic and syllabic writing of u š / *šiddum* and the varying spellings of í b . s i$_8$ – produced by a stylistically less conscious scribe.[22] It is therefore of some interest to summarize the terminological characteristics of this particular tablet:

– The opening formula contains a reference to the object – "If, concerning a [quadrangle with] diagonal, ...". A drawing of the object – a rectangle with the diagonal drawn and the given numbers written in – is also present.
– The solution is followed by a proof (not found in any of the Tell Ḥarmal texts from the group), introduced by another "If" – in full, "If, [*viz*, as you have found], the length is 1 and the width is 45´, to what

[20] I think in particular of that elimination of *nasāḫum* of which we saw the traces in IM 54 464: z i , the logogram for this verb, is found (as z i . z i) together with g á . g á (in Old Babylonian mathematics replaced by g a r . g a r ~*kamārum*) in Šulgi-Hymn B, l.17 [Nemet-Nejat 1993: 9].

Evidently, this elimination of a reference to the neo-Sumerian tradition (if this is really what is involved, cf. below, p. 37) can only have had symbolic value, since the whole computational technique (with sexagesimals and appurtenant tables of reciprocals, multiples and i g i . g u b factors) was a neo-Sumerian heritage.

One might take the treatment of í b . s i as a verb, and its pronunciation without Akkadian inflection (note 13) as evidence against the deliberate demarcation from the neo-Sumerian tradition – aren't they unmistakeable Ur III borrowings? As we shall see (p. 66), analysis of the evidence seems to suggest that they are not.

[21] Since this tenth tablet (IM 54 559) is precisely the one that mixes *tammar* and *iliakkum* in the announcement of results, we may presume it to have been produced by a different scribe.

[22] Lack of stylistic sophistication is also suggested by the fact that no line break is made between the statement of the solution and the beginning of the proof.

do the area and the diagonal correspond?”
- the habitual formula in the usual spelling (*at-ta i-na e-pé-ši-ka*) introduces both the prescription for the solution and the prescription for the proof.
- the text has a closing formula, *ki-a-am ne-pé-šum*, “thus the procedure”, not present in the Tell Ḥarmal lot.
- the interrogative phrase is *ki ma-a-si*, and results are presented within the phrase *-ma* ... *i-li* – a combination found in none of the Tell Ḥarmal tablets.[23]
- uš, in one place, occurs as *si-di*, elsewhere logographically.
- as in the group in general, subtraction “by removal” is represented by *ḫarāṣum*, “to cut off”, and rectangularization by *šutakūlum*; in obv. 4, where a segment not resulting from a “breaking” (*ḫepûm*) is to be squared, its counterpart (*meḫrum*) is “drawn” (*nadûm*), after which follows the *šutakūlum*-operation.
- in contradistinction to the Tell Ḥarmal texts from the group, the “equalside” (written ib.si and ib.sí) is treated as a noun and “taken” (*laqûm*).
- *wabālum*, “to bring”, is used in a multiplicative sense, corresponding to the use of either *našûm* / íl or *eṣēpum* / tab in other texts.

The remaining published Ešnunna texts are IM 55 357; IM 52 301; Haddad 104; and finally IM 52 916+52 685+52 304, the “Tell Ḥarmal Compendium”. Even though they are much less closely connected than the texts from group 7A I shall refer to them as group 7B. The “Compendium” is a catalogue of problem types (not even problems *stricto sensu*, since the “right-hand side of the equation” is not stated); the rest are procedure texts.

The oldest text is IM 55 357, belonging to Tell Ḥarmal, level III (and thus at least as old as the Tell Ḥarmal tablets from group 7A, found below

[23] Their preference for the pairings *kī maṣi* / *tammar* and *mīnum* / *elûm*, if indeed no accident, is hence more likely to have resulted from some kind of stylistic purism than to correspond to patterns of general validity at the time of writing. Anyhow, since Ešnunna’s independence was already 200 years old when the earliest extant mathematical text (IM 55 357) was produced, original distinct patterns may well have been mixed up in the living discourse of the local scribe school when not resurrected in correct or distorted form by a conscious effort.

the floor of level II). It begins with a diagram and with stating the object (sag.dù). The prescription is preceded by a logographic formula za.e kì.ta.zu.un.dé, "You, by your procedure" (the position of -ta is probably the result of erroneous translation from the Akkadian). Since the prescription is not terminated we cannot know whether a closing formula was intended. Much of the writing is logographic: igi.dù for *tammar*, íl for *našûm*, gaba for *meḫrum*, nam intermittently for *ana*, a.na.àm for *mīnam* (while the nominative *mīnum* is written *mi-nu-um*),[24] ba.zi for *nasāḫum*. *bāmtum*, the "natural half" produced by a "breaking", is abbreviated BA; íb.si$_8$ is spelled in this orthodox way. The underlying grammar, however, is wholly Akkadian, the logograms are allographs and not Sumerian.

In the perspective of group 7A the following features will be noted: As in Db$_2$-104, the object is stated explicitly (name as well as diagram); question and result occur as *mīnum* / *tammar*, the shift to a new section of the prescription is indicated *na-ás-ḫi-ir*; íb.si$_8$ is a verb; the "natural half" is made explicit as in Db$_2$-104 but with the customary term *bāmtum*. Removal occurs as ba.zi and thus presumably as *nasāḫum* (the customary logogram for *ḫarāsum*, the term of group 7A, would be kud).

IM 52 301 dates from the reign of Ibalpiel II; as mentioned in note 6, however, it is a copy of an older tablet. Here, problems are introduced by a *šumma* followed by a description of the situation, in which the object is implicit – "If, to two-third of the accumulation of the upper width and the lower, ...".[25] The prescription starts with the formula za.e TUK.zú.dè –

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[24] Lexical lists give the equivalence a.na.àm~*mīnum* [AHw, 655b], but *mīnam* is evidently required if íb.si$_8$ is a verb – which the word order shows it to be (uš ... *mi-nu-um* in line 5, a.na.àm íb.si$_8$ in lines 8 and 14, in agreement with the verb-final sentence structure of both Sumerian and Akkadian). Analysis of the word (a.na~*mala*, -àm enclitic copula) suggests, however, that the original correspondence is rather with the alternative interrogative phrases *mala masi* [AHw, 621b] and *kī maṣi*. See also [SLa, §120].

[25] As a matter of fact, the implicit definition is not too compelling – *vide* the disagreement between [Gundlach & von Soden 1963: 252f] and [Bruins 1966: 207ff] whether a trapezium or a triangle is dealt with. As argued in [Høyrup 1990a: 338 n.175], however, the absence of a reference to partial areas may support Gundlach

if zú is read as a homophonic variant of zu and TUK as a variant of dug_4 something like "You, in order to know the saying" – and closes *ki-a-am ne-pé-šum*. Question and result are marked *mīnum* / *-ma ... tammar*, shifts of section *tu-ur*, "turn back!". Removal is represented by *ḫarāṣum*. The "equalside" occurs as *ba-se-e*[26] and is made "come up" (*šulûm*) – that is, it is regarded as a noun. As in Db_2-104, a squaring (*šutakūlum*) where the side does not result from a "breaking" operation (*ḫepûm*) presupposes that its *meḫrum* be "drawn" (*nadûm* – rev. 5). No name for the "natural half" occurs. *takīltum* turns up in rev. 9, while obv. 10 has the parallel relative clause *ša tuštakīlu*. When the length of a trapezium is put together in obv. 16–17 from two-third of the sum of the widths and 10 "on my hand", the process is regarded as one of "building" (*banûm*) – a term which in all other occurrences designates the construction of a rectangle.[27]

The last procedure text from Ešnunna is Haddad 104, which contains a total of 10 problems dealing with various topics – all of them of real relevance for professional scribal practice. Its character as a systematic didactical text is revealed by its opening formulas: When a new topic *T* is introduced, the problem starts *ne-pé-eš/eš₁₅ T-im* (N[os] 1 and 4) or just *T* (N[os] 6, 7, 9 and 10); but when a variant on the topic of the preceding problem is introduced, the opening is *šumma*, apparently to be understood "If, [however]" (N[os] 2, 3, 5 and 8). Prescriptions begin with the formula *i-na e-pé-ši-ka*, and N[os] 1, 3, 4, 5, 7 and 10 close *ki-a-am ne-pé-šum* (quite fitting for a text whose introductions tell that its problems teach methods).

šumma also turns up as a genuine "logical particle" within the prescriptions: In I 31, 37, II 16, 39, and III 2, where a preliminary result *R* has been established, it opens a new step in the deduction, "If *R*, what is then ...". Question and result are marked *minum* / *i-lí*, shifts of section *tu-úr*, "turn

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& von Soden (the trapezium). The Babylonian reckoners may have been more sensitive than we in interpreting such cues.

26 *ba-su* when followed in rev. 8 by an enclitic *-šu*.

27 Thus the Tell Ḥarmal Compendium, IM 52 685, 22ff; TMS XVII, 1; YBC 4608, rev. 2; AO 8862, I 2, 31, II 35, III 22; VAT 8390, obv, I 3, II 16. Some series texts (e.g., YBC 4714, rev. II 20) use a so far unexplained a.šà šu.ba.an.tu in the same function.

back!". Removal is represented by *ḫarāṣum*.

As in IM 52 301, the "equalside" appears as a syllabic *basûm* (*ba-sa-šu*, III 7), and as there it is treated as a noun and is "made come up". In the present text, however, "to make come up" (*šulûm*) is used in general in the sense of "calculate" or "find", which fits the use of the phrase *i-lí*, "comes up", for results. This correlation leaves little doubt that the idea of making "equalsides" "come up" belongs originally with the general use of *elûm* for results, and is a borrowing in the "*tammar* text" IM 52 301. Since the problems of Haddad 104 appear to belong to types that will also have been taught in the Ur III school we may even surmise that *elûm* points to the Ur III tradition as it had been received and perhaps transformed in peripheral Ešnunna;[28] *tammar*, on the other hand, may point to the influence from the (lay practitioners') riddle tradition – the "broken reed" problems from group 7A, the filling problem and the complex "algebraic" rectangular problems are indeed all of the *tammar* type.

The last Ešnunna text is the "Tell Ḫarmal Compendium", a catalogue of problem types. It consists, so to speak, of nothing but a list of left-hand sides of equations, for which reason only rather few pertinent observations can be made on the terminology.

The Compendium shares some characteristic features with other Ešnunna texts: uš is sometimes written syllabically though mostly with the logogram; the width of rectangles is designated sag.ki. But a noteworthy difference must also be observed: Removal is *nasāḫum*, both when sides are removed from areas and when lines are removed from lines. On still other accounts no comparison is possible because the remaining

[28] The kind of transformation which we may suspect to have taken place when Sumerian mathematics was implanted or adopted in the Ešnunna region is illustrated by the terms *ib-se-e* and *ba-se-e*: Sumerian in pronunciation, local in their "unorthographic" spelling (the distinction between a "cubic equalside" written íb.si$_8$ and a "plane equalside" *ib-se-e* in the very homogeneous and stylistically conscious Tell Ḫarmal lot from 7A shows that something different from scribal incompetence is involved).

It is far from certain, however, that *íb/ba-se-e* were only imported in the wake of the Šulgi reforms with their place value system, igi-gub coefficients etc. ([Høyrup 1994: 61, 77]; [Robson 1995: 204–209]). They may well point to earlier Akkado-Sumerian interactions – cf below, p. 66.

Ešnunna corpus encompasses problems on rectangles but none on squares. In square problems, the area of the square is spoken of as a.šà LAGAB (when pluralized, LAGAB becomes a syllabic *mitḫarātum*); square sides, when added to or subtracted from the area, are designated uš / *šiddum*. Noteworthy is that the addition of sides is a concrete joining or "appending" (*wasābum*), which implies that the sides are regarded as "broad lines" or strips[29]; when the areas of two or more squares are added, on the other hand, the operation is "accumulation" (*kamārum*). According to Goetze's reconstructions [1951: 132], *bāmtum* should play the role of an "accidental half", an ordinary member of the series of fractions $^2/_3$, $^1/_2$, $^1/_3$, ..., but he admits that it "is difficult to reconcile this inevitable conclusion with the remnants actually preserved on the tablet". (At the time nobody had observed that this is not the role of *bāmtum*, and that the supposedly "inevitable" conclusion would be highly unusual, YBC 6492 being the only possible parallel.)

Before we leave the Ešnunna texts we may observe that the phrase *rēška likīl*, "may your head retain", occurs in both 7A (IM 53 965) and 7B (Haddad 104, IM 52 301).

The mathematical Susa texts

In total, the Susa corpus consists of 26 texts, of which N^os^ VII–XXVI are procedure texts. Since N° XXVI differs from the others on many accounts, we shall look first at the "typical" group, N^os^ VII–XXV.

In most cases, these texts open by stating the parameters, and only tell in this indirect way which kind of object is dealt with – thus, if only a "length" and a "width" occur and perhaps an area, the object must be a rectangle, the simplest figure (according to Babylonian habits) fully determined by its length and its width. In cases where this is not sufficient, the object is presented explicitly or (N° XVIII) by means of a drawing. No

[29] Unfamiliar as this notion is to us, it is current in many traditional practical geometries – and of course analogous to that notion of "thick areas" on which the shared Babylonian metrology for areas and volumes is based. See [Høyrup 1995]. The Late Babylonian reed metrology for areas has the same basis [Friberg, forthcoming / b].

šumma or other formula occurs.

The prescription, on the other hand, carries an opening formula – a terse z a . e, "you". Closing formulas only occur in IX/*A*, IX/*B* and XVII. IX/*A* and IX/*B* are didactical explanations of a trick to be used in IX/*C* [Høyrup 1990a: 320–328]: *A* shows how to add the length to a rectangular area by extending the width by 1, and closes *ki-a-am ne-pé-šum*, "thus the procedure"; B teaches how to add the length as well as the width by extending both length and width of the rectangle by 1 and augmenting the area by 1×1, a trick presented as "Akkadian"; it closes *ki-a-am ak-ka-du-ú*, "thus the Akkadian [procedure]". N° XVII, too, closes with a reference to a method with a name, *ki-a-am ma-ak-sa-ar-šu*, "thus its bundling".[30] Of these, only XVII can be regarded as a problem *stricto sensu*. Even though precisely the first and last lines of the tablets are often destroyed we may conclude with fair certainty that closing formulas are absent from Susa problems except in very particular cases.

In most cases, the statement includes no explicit question. When it does, the interrogative phrase may be *mīnum* (XII, XVII, XIX), *kī masi* (XIII) or *mīna* g a r (XIV).

Results are regularly followed by *tammar*. Most texts do not apply -*ma* on the preceding operation verb, some do occasionally after a particular operation (VII, XXI and XXIII with *našûm*, XV with z i, IX and XXII with d a ḫ) or at random (XVII, XVIII, XX).

Shift of section in the prescription is marked with high frequency[31], and invariably *tu-úr*. One tablet, moreover, marks a shift within the statement with the corresponding first person singular *a-tu-úr* (N° XII). Five tablets (N^os XI, XIX, XXIII, XXIV, XXV) use the phrase *rēška likīl*, four of which also carry *tu-úr* (and so much of the fifth – N° XXIV – is destroyed that *tu-úr* may well have been present even there). Chances seem relatively

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[30] References by name to this method recur in YBC 6295 (*makṣarum ša* b a . s i) and YBC 8633 (

small that this is due to mere accident, but since many tablets are severely damaged any statistical estimate is treacherous; as complex calculations may easily call for both separation into sections and for the temporary storing of intermediate results, some correlation could be expected.

Some characteristic regularities in the use of logograms can be noted.[32] *kamārum* ("accumulation") invariably occurs as UL.GAR, and *nasāḫum* as a naked zi (not ba.zi), except in N° XVI, 1, where a zi from the statement is quoted as a syllabic infinitive *na-sà-ḫu*. Almost all texts use NIGIN where syllabic texts would have *šutakūlum* or *šutamḫurum*. N° IX, when quoting a NIGIN from the statement, does so with a syllabic infinitive *šu-ta-ku-lu*,[33] whereas N° XVII uses syllabic writings of *šutakūlum* throughout for squaring, and N^{os} XX and XXIII employ *šutamḫurum* systematically in the same function.[34] The "natural half" is written logographically as $^1/_2$, and *meḫrum* as gaba. The "equalside" appears as íb.si, never as íb.si$_8$; it functions as a verb. uš and sag appear as such, never syllabically nor as sag.ki.

The term *takīltum* is used in four texts (XII, XIX, XXI, XXIV), while the alternative relative clause *ša tuštakīlu* is absent. "Inner zeroes" (lacking units or tens within a sexagesimal place) are marked in two texts (XII, XIV) with the separation sign GAM. As is well known, intermediate zeroes occur nowhere else in the Old Babylonian record. Their presence in the Susa texts is one of several indications that these represent a higher level of meta-theoretical awareness than the Old Babylonian corpus in general: even without a separation sign, 30 16 could never be read as 46, nor 30 41 as

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[32] Perhaps it still needs to be repeated that many of the translations of logograms into Akkadian in the edition are linguistically impossible.

[33] The use of the lexicon form in quotations, in agreement with the usage of lexical lists, recurs in X, 12, and in XVII, obv. 10; in these cases the reference is to finite verbal forms in syllabic writing. The references to the logographic zi and nigin of the statements are thus not evidence that they were meant to be read as infinitives, as maintained in [Høyrup 1990a: 304].

To my knowledge, no mathematical text outside the Susa group does anything similar.

[34] It may deserve mentioning that these four texts use the structure *-ma.. tammar* occasionally (IX, XXIII) or regularly (XVII, XX).

71 or 14 3 as 17 – the texts that insert zeroes[35] do so not in order to avoid erroneous readings but for the sake of system.

alākum, "to go", is used in many of the texts in a general sense that may be interpreted "do in repeated steps". Often the use is multiplicative (in agreement with the etymology of the term a.rá and the phrase a.rá *N* ^e^tab familiar from many of the series texts; but in N° VIII the sense is repeated appending (daḫ), and in some cases the context is too damaged to allow interpretation.

The division of a number *A* by an irregular number *B* follows the invariably pattern *mi-na a-na B* gar *šà A i-na-di-na Q* gar, "What to *B* shall I posit that gives me *A*? Posit *Q*".[36]

Among the "logical particles", *aššum* turns up frequently. Its function is to give the reasons for the calculations to come, either with reference to the statement (quoting a complete phrase or a single word), to general knowledge of the characteristics of the object dealt with (N° XIV, obv. 9), or to the situation that has been established so far. *inūma* is found once, *viz* in N° IX, 2, in complete parallel to the use of *aššum* in line 11.[37] *šumma* is absent.

N° XXVI, the last of the Susa procedure texts, deviates from the shared pattern of the others on several accounts. While its problems start as those of the other texts, either by explicit introduction of the object or implicit presentation through specification of the parameters, the prescription carries no introductory formula at all. Nor do results have any formal marker, not even a preceding *-ma*. Shifts of section are indicated in the second person present tense, as *ta-ta-a-ar*, "you turn back", instead of the habitual imperative. Removal is kud, a logogram for *ḫarāsum*, and not zi / *nasāḫum*.

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[35] Not in all texts where they might be used: N° XIX, rev. 9 and 10 has 11 6 and 51 6 without any indication of the zero.

[36] XXIV, 24f; the corresponding but more or less damaged passages in other texts all agree (VIII, 8; IX, 48; X, 20-22; XIII, 13f).

[37] In both cases, the reference is both to the statement and to unstated knowledge about the numerical values of the dimensions of the rectangle dealt with – both passages are located within the didactical exposition of methods that in this tablet precedes the problem *stricto sensu*.

The (plane) "equalside" is a noun and written í b . s i$_8$, not í b . s i ; squaring is written KA+GAR.[38] a . r á , the term for purely numerical multiplication, is found in several places where other texts have *šutakūlum*, *šutamḫurum* or one of the corresponding logograms. The tablet is clearly an outsider in the corpus.

N^{os} V and VI already fall outside the pattern for the trite reason that they are not procedure texts but problem catalogues, and in this respect thus similar to the Tell Ḥarmal Compendium, the series texts, and others. They are interesting in many respects, and I have discussed some of their features elsewhere.[39] In the present context it is noteworthy that the area of a square is designated a . š à LAGAB (N^o V) or a . š à NIGIN (N^o VI; N^o V uses NIGIN as the plural of LAGAB, in a way that reminds of the distinction between a singular LAGAB and a plural *mitḫarātum* in the Tell Harmal Compendium, see p. 17); both speak of the side as u š , as done in the Compendium (and probably in TMS VIII, 10, 18, cf. [Høyrup 1993b: 255f]). While the procedure texts use UL.GAR for "accumulation" (*kamārum*), N^o V uses g a r . g a r ; N^o VI, when adding square sides to the area, "appends" them (d a ḫ) concretely, thus perceiving the side as a "broad line", in the likeness of the Tell Ḥarmal Compendium. Both use z i for removal.

Both N^o XXVI and N^{os} V–VI thus share features that we encountered in group 7 and which distinguish them from N^{os} VII–XXV. But these features are not shared between the two, and while N^{os} V–VI present affinities with the Tell Ḥarmal Compendium, the k u d of N^o XXVI points to the other Ešnunna texts (and to group 1, where even a . r á is used occasionally for *šutakūlum* in one text). Group 8 thus consists of the following subgroups:

8A: N^{os} VII–XXV.

8B: N^{os} V–VI.

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[38] Bruins, inspired by the use of NIGIN for squaring and rectangularization in certain mathematical texts and for the sum-total in the expression š u . n i g i n , believes that KA+GAR stands for both squaring and accumulation [TMS, 125 n. 1], and sees a confirmation in obv. II 8 of the tablet. More likely the second occurrence is a slip for UL.GAR, which would then be a point of agreement with the other Susa texts.

[39] Thus [Høyrup 1990c: 303–305]; [Høyrup 1993a: 48–52].

8C: N° XXVI.

8A, it appears, represents the crystallization of a particular Susa canon.

In term of Goetze's linguistic distinctions, the Susa corpus mixes "northern" and "southern" orthography, with "southern" preponderance.[40] However, since Akkadian was anyhow a scribal language in Susa and not the local tongue [Amiet 1979: 202] this observation is probably not very pertinent. Its relations to the other groups is better decided on the basis of terminology.

Goetze's "northern" groups

Neither *tammar* nor UL.GAR appear in texts from Goetze's "southern" groups apart from marginal use of the former term in YBC 4662 (to which we shall return)[41] but regularly in his group VI. Moreover, as we remember, *tammar* occurs in many of the Ešnunna texts. If we take the

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[40] BI is used for *pé* (*ne-pé-šu*, IX, 9; *ḫe-pé* in IX, 39 and XV, 2, however, are editorial errors for *ḫe-pe*), and for *bi* (*wa-ṣú-bi*, IX, 32; *qa-bi-a-ku-um*, XVII, 5); the value of PI in *ḫe-pe* / *pi* (*passim*) can therefore be assumed to be *pi*, /pe/ being occupied (cf. also the possible *a-ta-ap-pi-šu*, XXIV, 34). These together agree with Goetze's "Uruk" group. Also "southern" are *sà* (*na-sà-ḫu*, XVI, obv, 8; *ta-na-as-sà*[!]*-ḫu*, XI, 5; and *sí* (*na-sí-iḫ*, VII, 35, 40, VIII, 9).

"Northern" are *ṭú* (*pu-ṭú*⟨*-ur*⟩, VII, 10 and *passim*; *ta-aq-ši-ṭú*, XIII, 10); C*a-i̯a* (*ka-i̯a-ma-ni*, XI, 19, XIV, 8, cf. XII, 8, 15); and probably *ṭúr* (*pu-ṭúr*, IX, 45, X, 23, XII, 6).

[41] Both also occur in MLC 1950, which Goetze connects to group 3 for unstated non-orthographic reasons. Probably he has been inspired by the presence of the introductory formula of the prescription, za.e kì.ta[.zu.dè]; as it turns out, however, all "southern" occurrences of this formula (including all other group-3 occurrences) have da instead of ta, which seems to be a characteristic "northern" spelling – an instrumental suffix translating *ina* from the corresponding Akkadian phrase; the southern spelling, on the other hand, could be a phoneteic variation, but might also represent a confusion of cases; cf. [SLa, § 204].

In order not to burden the exposition unduly I shall mostly omit detailed references to the occurrences of terms in texts published in MKT and MCT; they can be tracked without difficulty through the excellent glossaries of these volumes.

Ešnunna texts and group 6 to represent a “northern tradition”,[42] the Susa texts clearly belong in the same context; irrespective of factual geographical latitudes I shall therefore henceforth include them under the “northern” heading, though “peripheral” (with respect to the Ur III core area) would be more precise. I shall, however, preserve the quotes since the usage is not quite appropriate.

Goetze counts the following texts to group 6: BM 85 194; BM 85 196; BM 85 200+VAT 6599; BM 85 210; VAT 6597; VAT 6598; and MLC 1354. Of these, the first four (each of which contains many problems) were already grouped by Neugebauer, who also thought them close to the series texts. They are indeed very close to each other on a number of points which Neugebauer did not mention explicitly but probably used as the basis for his general judgment:

Statements mostly start by telling the object, BM 85 194 and BM 85 196 sometimes also give a diagram, while BM 85 194 and BM 85 210 sometimes (thrice each) start *šumma*. Questions are made explicit, as a rule by means of the interrogative particle e n . n a m , in one problem in BM 85 194 and one in BM 85 196 by *kī maṣi*. Prescriptions start with the formula z a . e and end *nēpešum* – occasionally in BM 85 194 and BM 85 196, once in BM 85 200+VAT 6599 *kīam nēpešum*. Shift of section are indicated n i g í n . n a , and results are marked *tammar* without a preceding enclitic *-ma*. All four texts end by telling the number of sections (*kibsum*, “steps”) contained, BM 85 194 and BM 85 196 referring to this number as “sum-total” (š u . n i g í n).

All four texts also use UL.GAR for “accumulation”, and b a . z i for

[42] Since Akkadian mathematics can no longer be taken to have arisen in the South (cf. note 4), Goetze’s characterization of this group as “based on southern originals, but written and modernized in the North” is no longer credible. The simultaneous use, e.g., of the “northern” *sa* and the “southern” *sà* will therefore have to be explained in a different way. Since supposedly “northern” and “southern” features may coexist within the same line (e.g., BM 85 194, obv. II 39, 43), a mixed origin of the material of these texts (many of which *are* very mixed anthology texts) is probably no adequate explanation; in all likelihood they represent a scribal norm different from that of Dilbat and Sippar. On any account the political history of the southern region makes it implausible that these late Old Babylonian texts should have been produced there.

removal (BM 85 196 has a couple of syllabic forms of *nasāḫum*). *ḫarāṣum* turns up in problem 18 of BM 85 196, but as a physical, not a mathematical operation (cutting off silver from rings). The "natural half" is written $^1/_2$. í b . s i₈ is a verb, in two as well as three dimensions:[43] the question is e n . n a m í b . s i₈, while nouns are asked for with the phrase *X* e n . n a m : the values corresponding to nouns, moreover, are "seen", whereas *tammar* never occurs together with í b . s i₈.[44] When divisions by irregular numbers occur (BM 85 200+VAT 6599; BM 85 210), the format of the question coincides with that of the Susa texts (group 8A), apart from orthography and the use of the logogram s u m in the former text.

The same features recur in VAT 6597 and VAT 6598,[45] confirming that these two tablets belong together with the four tablets grouped by Neugebauer in what we might call group 6A.[46] MLC 1354, on the other

[43] Three dimensions only occur in BM 85 200+VAT 6599, where í b . s i₈ produces the side of a cube; the side *n* of a rectangular prism $n{\cdot}n{\cdot}(n+1)$; and the three sides of a general rectangular prism.

[44] In my analysis of BM 85 200+VAT 6599 [Høyrup 1992], I overlooked these significant details and translated as a noun.

[45] With the variation that VAT 6597 uses the interrogative particle *[k]i-i̭a* at least in problem 2, and perhaps in all problems (the tablet is very damaged) – see [MCT, 50 n. 140]. Since *kii̭ā* seems to go together with a particular problem type (partition between brothers / š e š – thus in VAT 8522 N° 2, VAT 6597, and YBC 4608 (Goetze's group 4, 6 and 3, respectively), too much should not be made of this deviation from the norm.

The only time the term is found in a different thematic context is in MLC 1842 (group 5), a problem on composite rates analogous to TMS XIII.

Division by an irregular number is only present in VAT 6597, which agrees sign for sign with the orthography of BM 85 210, apart from a missing *a-na*, obviously omitted by mistake.

In VAT 6597 as well as VAT 6598, the final segment is destroyed; whether they include a counting of problems is thus unclear.

[46] Within this group, however, the British Museum and the Berlin texts still form subgroups. Both Berlin texts carry the phrase *rēška likīl* (lost in VAT 6597, but a backward reference in rev. 7 shows its presence), while it is found in none of the BM texts. Apart from the occasional use of *šumma* as an opening phrase (including the opening of a proof in BM 85 196, obv. II 12), "logical particles" are rare in the BM texts – *inūma* occurs twice in BM 85 194. On their part, both Berlin texts, much

hand, differs on most of them. Its statement tells the object, but ends with the question *mi-nu*, not e n . n a m . The prescription starts with the full phrase z a . e kì.t a . z u . d è; since the final part of the text is destroyed we do not know whether there was a closing formula; no sections are marked, and results are mostly stated within the structure *-ma ... tammar*. The "natural half" is *bāmtum*, not $^1/_2$. Neugebauer and Sachs reconstruct a lacuna as g a r . g a r -*ma*, but UL.GAR-*ma* seems equally possible. Squaring is *šutamḫurum*, used in only half of the 6A texts.[47]

On these accounts (and in orthography, as far as the brevity of the texts and the use of logograms permits analysis), MLC 1354 is closer to MLC 1950 (whose non-affiliation with group 3 was argued in note 41) than to group 6A. With some caution we may put them together as group 6B.

For use in the following we may take note of a few further terminological peculiarities of group 6A.

Firstly, the use of t a b . Mostly it is used as a logogram for *eṣēpum*, "to double" or "to repeat", in phrases of the type *X ana* 2 t a b . b a . But a couple of times BM 85 194 combines it with a . r á , *X* a . r á *Y* t a b . b a (obv. II 44, 50).[48]

Secondly, a particular way to speak about squares in BM 85 194, BM 85 196, BM 85 200+VAT 6599 and BM 85 210. All employ the Gt-form *imtaḫḫar*, as *X imtaḫḫar*, "*X* stands against itself", *X* t a . à m *imtaḫḫar*, "*X*, each, stands against itself]", or *X* í b . s i$_8$ *imtaḫḫar*, "*X* as equalside stands against itself". The emphasis on the notion of "each" side recurs in the

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shorter though they are, use *aššum*.

47 BM 85 194 and BM 85 196 use NIGIN for squaring; BM 85 200+VAT 6599, BM 85 210 and VAT 6598 use *šutamḫurum* for squaring but ì . g u$_7$(. g u$_7$) when referring to it in relative clauses. Outside group 6A we have encountered *šutamḫurum* in one text from group 7A (IM 54 478) and in several from group 8A. All these are "northern"; the only occurrence in a "southern" text is YBC 6504 (Goetze, group 1). In general, *šutamḫurum* is thus a "northern" but not specifically a group 6 term.

48 In both cases *Y* is 3, and *X* a circle diameter, and the result thus the corresponding perimeter. The meaning is thus likely to be the usual concrete repetition; that it is "gone" in three steps (a . r á) points to the use of *alākum* in the Susa corpus. The phrase itself, as we shall see, points to the series texts.

Mediaeval Arabic continuation of the lay tradition – see [Høyrup 1996b].

Thirdly, the form g a r . r a , used here as well as widely in groups 3 and 4. In the present group it serves not only as an imperative ("posit!") but also (at least in BM 85 200+VAT 6599 and BM 85 210) as a precative.

Fourthly, BM 85 196 and VAT 6598 speak about "leaving" (í b . t a g_4) the remainder after a removal (cf. below, note 54).

As regards the mathematical substance of the texts it may be observed that volumes (including brick calculations) are in focus; only VAT 6597 is an exception to this rule, dealing with the division of silver between brothers.

Goetze's other "northern" group (5) contains only three texts: YBC 6967; MLC 1842 (heavily damaged); and YBC 10 522 (a fragment excerpted from the middle of the prescription of a longer procedure text). Their common origin is confirmed by terminological parameters: all use *elûm* for results (mostly "comes up for you"); all use *šutākulum* (YBC 6967 for rectangularization, YBC 10 522 for squaring, MLC 1842 undecidable). Both YBC 6967 and MLC 1842 introduce the statement implicitly, by giving the parameters; YBC 6967 opens the prescription with *at-ta*, while MLC 1842 has *at-ta i-na e-pé-š*[*i-ka*]. YBC 10 522 marks shift of section *tu-úr-ma*, as does probably MLC 1842.

The group is thus clearly related in style to Haddad 104, and in general to that part of the Ešnunna corpus which points to the Ur III tradition as it had been received and transformed in the periphery (see p. 16). A further point of contact with Haddad 104 is the explicit statement in MLC 1842 that a number is to be "inscribed" (*lapātum*) on a calculation tablet,[49] while YBC 10 522 demonstrates affinity with Db_2-146 through its use of *wabālum*, "to bring", in a multiplicative sense, and YBC 6967 does so by "taking" (*laqûm*) the equalside (a lacuna prevents us from knowing whether b a . si_8 or í b . si_8, but in any case a noun) and by "drawing" (*nadûm*) the "counterpart" (*meḫrum*).

But there are differences too, at least in YBC 6967: *ḫarāṣum* has been replaced by *nasāḫum*, *takīltum* replaces the relative clause *ša tuštakīlu*. The

49 See [Robson 1995: 36 n.95], correcting [Høyrup 1990a: 58 n.83].

"natural half" is *bāmtum*, found in only one Ešnunna text (as b a, in IM 55 357; Db_2-146 uses the unique *muttatum*). The former two features correspond to the Susa group 8A, the reference to the *bāmtum* does not.

The "Uruk" groups

Since Goetze says explicitly that his groups 3 and 4 cannot be distinguished as far as linguistics is concerned, we may suspect terminology (or rather, as we shall see, vocabulary) to have been one of his keys (in some cases we need not suspect because it is told). The two groups do indeed differ strongly on this account.

Group 4 as constructed by Goetze contains the following tablets: VAT 8389, VAT 8390, VAT 8391, VAT 8512, VAT 8520, VAT 8522, VAT 8523, VAT 8528, YBC 4186, YBC 6295, YBC 8588, YBC 8600, YBC 8633; VAT 8521 is affiliated to the group for non-linguistic reasons.

As Goetze points out, the phrases *inaddinam*, "it gives me", and *ittaddikkum*, "it gives to you",[50] are used in many of these texts. He does not discuss the contexts in which the terms occur, which might have made him discover that the use in VAT 8521 is extra-mathematical and thus no strong argument for the suggested affiliation.[51]

Even in YBC 6295, the use of *nadānum* is not mathematical (moreover, the grammatical forms that turn up are different from those occurring when

50 Reading *it-ta-di-kum* as a Gt-form (with suffixed *-kum*), and expressing the inherent directionality of that form. If the correct reading is as a perfect *ittadikkum*, the reasons for its use would probably be the relative consecutiveness of "positing" followed by "giving", which would suggest the translation "will it give you" (unless some interference with a Sumerian formula should be in play, which I doubt).

51 In VAT 8521, indeed, *nadānum* is not used as a mathematical term but about the interest "given" by a capital; the grammatical forms that occur, moreover, all happen to be different from those that turn up in texts from the group when the verb functions as a mathematical term. Since *nadānum* is the standard term in connection with interest, the connection is as tenuous as could be. To make things even worse, the only text certainly belonging to the group and dealing with interest problems – *viz* VAT 8528 – does so from the perspective of the creditor and uses *nadānum* and the logogram s u m about the *borrowing* of the capital, while interest is "taken" (*laqûm*).

the word functions as a mathematical term): “Since [the table][52] did not give you the (cubic) equalside of 3°22´30˝”, it is found by the *maksarum* (“bundling”) method from “7´30˝ whose equalside [the table] gives you”.

As in groups 6 and 8, the main use of *nadānum* in group 4 is in connection with divisions by an irregular divisor. In VAT 8389, VAT 8391, VAT 8512 and VAT 8520 we find a more complex structure than we have seen so far: *mi-nam a-na A lu-uš-ku-un ša B i-na-di-nam / Q* g a r . r a *Q a-na A* í l *B it-ta-di-kum*, “What shall I posit to *A* which gives me *B*? Posit *Q*, *Q* to *A* raise, *B* it gives to you”. VAT 8522, rather a collection of problem recapitulations than a genuine problem text, has the laconic ellipsis *A mi-nam ša B* / *Q* g a r . r a , “*A* what which *B*? *Q* posit”.

In YBC 8588 and YBC 8633, however, we find a different use of the term (*it-ta-di-kum* and *i-na-di-ku*, “it gives to you” and “it gives you”, respectively, the latter twice), namely for the outcome of a calculation. In both cases, as already in IM 54 464 (see note 11), the calculation in question is a “raising” (í l , *našûm*).

Except in these three passages, results are only (if at all) marked as such by a preceding enclitic *-ma* (which of course also serves in the general meaning “and then”) – in strong contrast to all the groups so far examined.

As in group 6A, g a r . r a is used logographically for the imperative of *šakānum* (*šukun*). The precative *luškun*, on the other hand, is written syllabically (as we have just seen), and so is the subjunctive *taškunu*. This corresponds to a general tendency of this group: the imperative of *kamārum*, “to accumulate”, is g a r . g a r , while the corresponding sum is a syllabic *kumurrum*; most texts also write the nominative *mīnum* logographically, as e n . n a m , while giving the accusative *mīnam* syllabically. The exceptions to the latter rule are VAT 8512, YBC 8600 and YBC 8633, which write *mīnum* syllabically and which are also the only ones to start the prescription

[52] Neugebauer and Sachs write “they” for the subject; however, since both occurrences of the verb (*id-di-nu-kum, i-na-di-nu-kum*) are in the subjunctive, a third person singular is equally possible and seems to me more plausible than a sudden appearance of Basil Bernstein’s “restricted code”. When the calculator tries to “take” an “equalside” from the table it may obviously happen that it does not “give” it to him.

with an introductory formula (*at-ta*). VAT 8521, on the other hand, agrees to the full; we may probably conclude that Goetze was rigth in including it although his only explicit argument for doing so was vulnerable.

Statements mostly introduce the object implicitly, through specification of the parameters; in some cases, the object is stated explicitly (in VAT 8523 in the first problem, after which the others open *šumma*, in agreement with a recurrent pattern; even VAT 8391 uses *šumma* in this sense of "If, [however]"). The interrogative particle is mostly e n . n a m , but a few cases of *kī masi* and one of *kiịā* (VAT 8522 N° 2) occur. Except in three cases, as mentioned, no formula introduces the prescription; closing formulas are totally absent. Shifts of section are not marked by any formula. The phrase *rēška likīl* occurs in exactly half of the 14 tablets.

The most common logical particle is *šumma*; apart from the use in the beginning of statements considered as variants, its main function (VAT 8389, VAT 8390, VAT 8391, VAT 8520, VAT 8521) is to open the proof; in VAT 8512, VAT 8522, and YBC 8588 it serves within the prescription to open a new piece of reasoning after the establishment of a preliminary result. *aššum* serves as a reference to the statement (VAT 8390), or to the given situation (VAT 8523, YBC 6295).

Removal is *nasāḫum*;[53] in VAT 8389, VAT 8390, VAT 8391, VAT 8512, VAT 8520, VAT 8523 and YBC 8633, the process is said to "leave" (*ezēbum*) the remainder – a word which is used nowhere else in this function.[54] *ḫepûm*, "breaking", is always "into two", and the "natural half" is never mentioned. *šutakūlum* is used for rectangularization, and no other term for squaring or rectangularization occurs; YBC 4186, however, uses the phrase 10 n i n d a n *imtaḫḫar* to tell that the base of a prismatic cistern is a square with side 10 n i n d a n, and must have contained a term for squaring in a lacuna in line 10. In the only places where the procedure allows its occurrence (VAT 8512, VAT 8520), *takīltum* is used instead of

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[53] The b a . z i possibly occurring in VAT 8522, obv. II 1c, is certainly no subtraction.

[54] BM 85 196 and VAT 6598, both from group 6, use the Sumerogram í b . t a g$_4$. So does the Ešnunna text IM 55 357 (group 7B) and a number of series texts (the latter often as a noun, designating the remainder).

the alternative possibility *ša tuštakīlu*.[55]

í b . s i$_8$ is treated as a noun. So is b a . s i when it is used:[56] about the cubic "equalside" in VAT 8521 and YBC 6295; and in the expression b a . s i 1-l a l, "equalside, 1 diminished", about the side n of a prism $n \times n \times (n-1)$ in VAT 8521. VAT 8528 uses í b . s i$_8$ alternating with b a . s i in an even more generalized sense (the number of periods in a sequence of successive doublings, mathematically speaking the "dyadic logarithm").

Summing up we may conclude that the group is coherent, and that only the agreement between the presence of *atta* and the syllabic writing of *mīnum* allows us to single out a possible subgroup 4B (VAT 8512, YBC 8600 and YBC 8633) from the main group 4A, consisting of VAT 8389, VAT 8390, VAT 8391, VAT 8520, VAT 8521, VAT 8522, VAT 8523, VAT 8528, YBC 4186, YBC 6295, YBC 8588.

In Goetze's group 3, the following texts are included: Str 362, Str 366, Str 368, VAT 7530, VAT 7531, VAT 7535, VAT 7620, YBC 4608; the following tablets are affiliated to the group for non-linguistic reasons: Str 363, Str 364, Str 367, VAT 7532, VAT 7621 – and finally MLC 1950, which however differs from the shared norms of the group on all significant accounts (some of which were mentioned in and around note 41), and which I propose be discarded and moved to group 6B (see p. 25). The Strasbourg texts, VAT 7532 and VAT 7535 were already grouped by Neugebauer, we remember.

55 Neugebauer also restores two damaged passages in VAT 8389, obv. II 4 as *ta-ki-il-tam*. Since all other occurrences of the term refer back to *šutakūlum*, which cannot be the case here, I have strong doubts about this reconstruction – cf. [Høyrup 1990a: 293].

56 The spelling with SI should be observed. The form í b . s i is only used in Ešnunna and Susa (the "outer periphery"), whereas b a . s i is standard in the group 4, and probably in the Old Babylonian South in general: it also occurs in AO 8865, a table from Larsa (Samsuiluna year 1). In this table, and in YBC 6295 as well as VAT 8521, it designates the spatial equalside (cubic and otherwise), while í b . s i$_8$ alone is used in all "southern" texts when a quadratic equalside is intended.

In group 6A, as we remember (see note 43), í b . s i$_8$ was used even in three dimensions.

Some of the texts contain only questions (VAT 7530, VAT 7531, VAT 7621) or only questions and answers (Str 364). Leaving these aside for a moment, all procedure texts belonging to the group turn out to share several characteristic features.

Firstly, the prescription always opens za.e kì.da.zu.dè; no shifts of section are marked within the prescriptions,[57] and no closing formulas are present.

Secondly, results are followed in all texts by the phrase IN.sum, "it gives"; twice in Str 363 and twice in YBC 4608 we find instead the syllabic *i-na-di-nam*, which suggests the reading ^{in}sum for the logogram; three of the syllabic cases concern divisions by irregular numbers.

Questions are always made explicit, mostly with the particle en.nam, sometimes in Str 362 with *kī maṣi*, once in YBC 4608 *kii̯ā*. In Str 368 and VAT 7535, it is also told that nu.zu, "I do not know" the entity asked for, while YBC 4608 has the corresponding syllabic *ú-ul i-de*.

Other features are shared by all texts whose themes allow them to turn up:

"Breaking" is never "into two" but a process which brings forth a single natural half – mostly written $^1/_2$ but *bāmtum* in YBC 4608. Rectangularization is absent, and squaring is always du$_7$.du$_7$ (read ZUR.ZUR in MKT), referred to in Str 363 with a subjunctive *ša tuštakīlu*[58] and in Str 368 as *ša* du$_7$.du$_7$. The *takīltum*, the possible alternative to these relative clauses, does not occur; but VAT 7532 and VAT 7535 use a different relative clause, *ša tēzib*, "which you have left", referring thus not to the half of the broken entity which was moved around in order to form the other side of a square but to the one that was left in place when the other half was broken

[57] VAT 7532 and VAT 7535 (complex "broken-reed" problems) seem to indicate shifts of section in the statement by the word *a-tu-úr*, as TMS XII (see p. 18). But since they separate the measurement of the length and of the width they may also be interpreted concretely, as "I turned [90°]". Cf. also below, n. 62.

[58] The origin of the term, however, is rather as a logogram for *itkupum*, "to butt" or "push" each other – semantically closer to *šutamḫurum*, "to confront each other", with which it alternates in YBC 6504.

off.[59] The same verb, we remember, was used in group 4 in connection with the *nasāḫum*-subtraction.

íb.si$_8$ is a verb: in Str 363, Str 366, VAT 7532 and VAT 7535 this is revealed by the full clause *A*.e *B* íb.si$_8$ (in VAT 7535, *B* is en.nam), in Str 368 by the word order of the question en.nam íb.si$_8$.

Division by irregular numbers is treated with some variation. A representative formulation, however, is *mi-nam a-na A* ḫe.gar *ša B* ^{in}sum *Q* ḫe.gar, "What may I posit to *A* which gives me *B*? You may posit *Q*" (VAT 7532, rev. 4–5); in Str 367, *mi-nam* is replaced by en.nam; in Str 363 and YBC 4608, the first ḫe.gar may become a syllabic precative *lu-uš-ku-un*, the second a syllabic imperative *šu-ku-un*, and / or ^{in}sum may become *i-na-di-nam*. Logographic equivalences apart, the formula coincides with the first part of the formula from group 4, while the second part of the latter – *Q a-na B* íl *A it-ta-di-kum*, "*Q* to *B* raise, *A* it gives to you" – is always omitted in group 3. Two texts (VAT 8390, YBC 4608) employ the rare locution a.šà *abni*, "a [rectangular] surface I have built" (cf. note 27).

Most texts from the group make strong or very strong use of logographic writing, and often provide the verbal roots with a Sumerian grammatical affix; prepositions, however, always remain syllabic and are never replaced by the Sumerian case suffixes, which allows us to conclude that the language is indubitably Akkadian and neither Sumerian nor some attempted ideographic supralinguistic symbolism; this also follows from the occasional syllabic writing of oblique forms of terms which are elsewhere represented by logograms, and by the phonetic complements on ^{in}sum (*inaddinam*) and ^{e}tab (*ēsip, esip* – the latter form is always represented by tab.ba in group 4).

The only logical particle to be found is *aššum*, which introduces an argument by "single false position" in VAT 7532 and VAT 7535 (also the only texts which possibly marked a shift of section, *viz* in the statements, see however note 57), "since $^1/_6$ of the original reed was broken off, inscribe

[59] The same idea is found in the pseudo-Heronic *Geometrica*, ms S, **24**,3 (square area plus perimeter equal to 896), a problem with evident roots in the surveying tradition [ed. Heiberg 1912: 418]. As revealed by the footnotes, Heiberg does not understand.

[*lapātum*, i.e., write down] 6, let 1 go away, ...".[60] The phrase *rēška likīl* is found only twice, in Str 362 and YBC 4608.

In spite of certain differences between these procedure texts there is no doubt that they belong together not only for linguistic reasons but also because of their mathematical style, nor that the texts which Goetze included for non-linguistic reasons should be included in the group (apart from MLC 1950). Since Goetze's linguistic criteria turn out to be so certain when membership of the group can be cross-checked with the terminology, we may safely accept even VAT 7530 and VAT 7531. Str 364 and VAT 7621, admitted by Goetze for non-linguistic reasons, are so close in terminology (including the use of Sumerograms which Goetze did not consider) to Str 367 and VAT 7531, respectively, that there is no reason to doubt their legitimate affiliation; Str 364 also shares n u . z u with Str 368 and VAT 7535 (Str 362 has a corresponding syllabic *ú-ul i-de*),[61] while its use of *ezēbum*, "to leave" (rev. 9) reminds of the function of this term in VAT 7532 and VAT 7535.

The indubitable discrepancies between the texts do not allow us to isolate one or more subgroups; texts which should belong together according to one criterion are always separated by others.

The first "Larsa" group

Goetze's group 1, "certainly to be localized in the South, in all probability Larsa", comprises the tablets AO 6770, AO 8862, YBC 4675, YBC 5022, YBC 6504, YBC 7243, YBC 7997, YBC 9852, YBC 9874, and, conjecturally, YBC 9856 and Plimpton 322. Of these, YBC 5022 and YBC 7243 are i g i . g u b tables, and even Plimpton 322 is a table text; in

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[60] *aššum* also occurs twice in YBC 4608, but in the construction *aš-šu X a-ma-ri-i-ka*, "in order to see [i.e., find] *X*", and not as a logical particle.

[61] Though not decisive, this is a fairly strong argument. Outside group 3, n u . z u and its syllabic equivalents are only found in this function in a few series texts; in IM 53 965 (group 7A), a *kī maṣi*-text dealing with a broken reed and very close to Str 368; and in AO 6770 (group 1). In VAT 8391, rev. I 27 *ú-ul i-de* also appears, but telling that "I do not know" the i g i of 35´; in TMS XX, 5 and TMS XXV, 4, the relative clause *šà la ti-du-ú* is used within the prescription to characterize a quantity referred to in the argument as unknown.

connection with the present investigation the only observation to be made on them is that íb.si$_8$ occurs in the latter two in orthodox spelling, and that *nasāḫum* turns up in one of the headings of Plimpton 322. YBC 9852 is a copy of some lines from YBC 4675, deviating from this model only in a few spellings and by two omissions. In the following, we shall therefore concentrate on the texts AO 6770, AO 8862, YBC 4675, YBC 6504, YBC 7997, YBC 9856, and YBC 9874.

Beyond the shared orthographic characteristics noticed by Goetze, there is little that keeps the texts in question together as a coherent group. We may notice the absence of logical particles within the prescriptions, and that these never close with a formula; *tammar* is never used with results.

Apart from this, we shall have to present the particularities of the texts one for one – beginning with the three texts whose prescriptions carry introductory formulae.

AO 8862 has always been regarded as one of the earliest Old Babylonian mathematical texts (see p. 5). First come four "algebraic" problems on rectangular fields (whose area we may designate a, while u is uš / the length, and s sag / the width). In N^{os} 1–3, $u+s$ is given together with a combination $a+\alpha u+\beta s$; in N^{o} 4, $u+s = a$, $u+s+a = 9$, different but similar). 3 problems on brick carrying follow, of which the last is of the second degree and very close to a rectangular problem from the Tell Harmal Compendium (given $a+u+s$ and s:u; see [Thureau-Dangin 1937: 90f] and [Goetze 1951: 148, D l. 18]). Only N^{os} 1–3 include prescriptions.

N^{os} 1–4 start by stating the object: uš sag, "length width", i.e., "rectangle" or "rectangular field" (cf. p. 17); the statement is subdivided into sections by means of the words *a-sà-ḫi-ir* and *a-tu-úr* (in N^{os} 1–3 both); since the laying out of fields is dealt with, the meaning may well be concrete, to "walk around" the field and to "turn back" to the starting point – *a-sà-ḫi-ir* follows upon the phrase uš *ù* sag *uš-ta-ki-il*$_5$*-ma* a.šà^{lam} *ab-ni-i*, "length and width I have made hold each other [as sides of a rectangle], a surface I have built".[62] The question is *mīnum*. The brick-carrying

[62] The use of a completely different phrase – *lā watar*, approximately "no further!" – to demarcate sections in the prescriptions (II 13, III 13) supports the assumption that the author of the present text meant *asaḫḫir* and *atūr* differently, i.e., concretely;

problems instead start by stating the work norm and corresponding wage for one man; then follows the word *inanna*, "now", introducing the description of the actual organization of the work[63]; the question is asked with the phrase *kī maṣi*.

The prescriptions of N^os^ 1–3 open with the phrase *at-ta i-na e-pe-ši-i-ka*. Normally, the statement is held in the first person, singular, preterite tense, and the prescription in the second person singular, present tense or imperative. This structure is so all-pervasive that I have not discussed it up to now – it is indeed shared by all groups[64]. On this account, N^os^ 1 and 3 are quite regular, while interesting aberrations are found in N° 2; $a+{}^{1}/_{2}u+{}^{1}/_{3}s$ is referred to (II 10) as "my sum" (actually a plural, see below), which might of course mean that the person who instructs is thought of as identical with the one who stated the problem – if only $u+s$, equally referring to the statement, had not been referred to as "your sum" (II 16). Similarly, "you" add 25´ to 3°25´ (II 27) in order to find the length of a modified rectangle, while "I" subtract (II 30) in order to find its width. No system can be found in the anomalies; they appear to have resulted

cf. also note 57 on the use of *atūr* in the broken-reed problems VAT 7532 and VAT 7535.

63 Separation of general information from description of the actual situation is the general function of the term. This general information may be an i g i . g u b factor (AO 8862 N^os^ 5–7, YBC 4673 N^os^ 2–3, YBC 10 772 – all dealing with brick carrying and having the same factor); it may be the rent to be paid per bùr for different fields (VAT 8389 and VAT 8391, *passim*); or the dimensions of an old dike, where the specific information concerns the reparation to be performed (YBC 4673 N^os^ 14–15).

The only apparent exception is YBC 4669 N° B6, where the word has been moved so as to be coupled with the question.

64 In many texts, it is true, the statement seems neutral, in particular because logograms conceal grammatical person and tense; but not rarely, an occasional possessive suffix -*i̭a* or a reference in the prescription to a "he" who has stated the problem reveals that the seeming neutrality was not intentional. I have found it too risky to conclude from the absence of such accidental clues from other statements that these should really be read as neutral. In the prescriptions, the only deviation from the "you" in the texts dealt with up to now are those connected with division problems – on which presently.

from the author's failure to adapt completely to a style.

The way rectangularizations are spoken of tells more about the nature of this adaptation. In I 24 we find a quite regular 15 u š 12 s a g *uš-ta-ki-il$_5$-ma*, followed, however, by a most unusual numerical computation 15 a . r á 12 3 a . š à , "15 times 12, 3` is the surface" (similarly II 13–14); in I 12, on the other hand, the *bāmtum* is broken from 29, after which follows immediately the numerical calculation 14.30 a . r á 14.30 3.30.15 (similarly, III 13–15); in II 19–23, after a breaking the *bāmtum* is to be "inscribed twice" (*a-di ši-ni-šu ta-la-pā-at-ma*); only then follows the numerical computation. This multiple variation does not fit the editing of an existing written text; it suggests, instead, a scribe who brings into writing the methods of the non-scholastic practitioners, in part using their terminology (*šutakūlum*) but then combining this with the terminology for sexagesimal computation, in part describing their procedures in his own words.

This scenario becomes somewhat more than a just-so story if we look at other peculiarities of the text. Firstly, there is the very structure of the basic (first) problem, on which N^{os} 2–4 are variations – in symbolic translation, a+(u–s) = 3`3, $u$+$s$ = 27. Addition transforms this into a problem $A$ = 3`3+27 = 3`30, u+S = 27+2 = 29, where A is the area of a rectangle with sides u and S = s+2. No other Babylonian text contains a problem of this structure, nor *a fortiori* the characteristic way to solve it; both problem type and trick, on the contrary, are well represented in Abū Bakr's *Liber mensurationum*, an Arabic descendant from that lay surveyors' tradition which also provided the Old Babylonian scribe school with inspiration for its "algebra"[65]. The problem, we may conclude, was certainly present in the lay source tradition, but appears not to have been adopted with much success into the scribe school – probably because the trick, elegant though it be, did not lend itself directly to generalization[66] – or, in a

[65] This is not the place to repeat the arguments for this interpretation of the historical process set forth in detail in [Høyrup 1996b]. Cf. also above, p. 11.

[66] Precisely the same holds for BM 13 901 N° 23 (below, p. 49), given sum of the four sides and the area of a square field. Even this problem is only found once in the scribe school texts, and in a deliberately archaizing formulation; but it turns up time and again in treatises derived from the lay practitioners' tradition – cf. [Høyrup 1996b].

different formulation, the generalized form of the problem $a+\alpha u+\beta s = P$, $\gamma u+\delta s = Q$ asked for different methods.[67] Elegant tricks fit a riddle, but easily turn out to be dead ends in a systematic enquiry.

Secondly, the excess of *u* over *s* is "appended" (*waṣābum*) to the area, as done in the Tell Ḥarmal Compendium and in TMS VI (cf. pp. 17 and 21) and nowhere else in the Old Babylonian corpus. The notion of broad lines implied by this was obviously abandoned when the school tradition matured: inhomogeneous additions (lines and areas, areas and volumes) were treated as "accumulations" of measuring *numbers*; in order to make possible the geometrical joining required for the solution, lines were provided in one text (BM 13 901) with an explicit *waṣītum*, a breadth of 1 "sticking out" from the line; the Susa text TMS IX does not use the word but explains the technique (see above, p. 18).

Thirdly, removal is represented by both *nasāḫum*, "to tear out", and *ḫarāṣum*, "to cut off", but with a tendency to distinguish according everyday connotations, using the former when a part of an area is removed and the latter when lines are involved – cf. [Høyrup 1990a: 319]. As we have seen, *ḫarāṣum* was important in the Ešnunna corpus; the texts of group 7A even appear to have eliminated *nasāḫum* intentionally (cf. n. 11)). This purism may rather be a school phenomenon than a characteristic of the lay practitioners' parlance – an oral culture is more likely to possess a picturesque language rich in concrete connotations than to be purist.[68] In any case, *ḫarāṣum* will surely have been part of the lay parlance, whereas it was soon eliminated from the school tradition (even from those "northern" texts which on other accounts are clearly related to the early Ešnunna group); even on this account the first part of AO 8862 is thus close to the original inspiration – probably much closer than the texts from group 7A.

Striking are finally some singularities which suggest that the author

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[67] Methods demonstrated, e.g., in TMS IX and VAT 8520, and consisting in the transformation into a rectangular problem in $U = \gamma\cdot(u+\beta)$ and $S = \delta\cdot(s+\alpha)$

[68] On formal occasions, however, the discourse of an oral culture tends to be formulaic – a feature which, when emulated by *literati* (romanticist and others), is easily transformed into purism.

of the text was not working within an already established tradition but rather at the intersection between a tradition to be exploited and a tradition *in statu nascendi* – that "interface between the oral and the written" which Jack Goody [1987] took as a book title: Sums by accumulation appear as a plural *kimrātum*, "the things accumulated", appearing in no other mathematical text[69]. Moreover, the numerical values – which in the perspective of modern mathematics and mathematics teaching are arbitrary and not worth caring about, but which in pre-Modern and particular Babylonian mathematics were highly standardized – are unusual and even unstable: in N° 1, the sides are 15 and 12, in N° 2 they are 4 and 3, in N° 3 they are 1` and 40.

The way results are indicated has turned out to be an important parameter in the preceding pages. In the present text, as in group 4 (A as well as B), a preceding enclitic *-ma* is normally all there is. Twice, however, a result is followed by *inaddikkum*, "it gives you": In II 15, i g i 6 g á l "gives you" 10´; and in II 20, "breaking" the *bāmtum* from 6°50´ "gives you" 3°25´. Normally, it will be remembered, *nadānum* when found in texts that for the rest have a different or no marking of results,[70] was connected with divisions by irregular numbers and thus somehow with the "raising" multiplication;[71] on rare occasions (IM 54 559, cf. note 11; and YBC 8588,

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[69] The tentative reconstruction of VAT 8512, obv. 6 [MKT I, 341] should indeed be [*ta-w*]*i-ra-tum*, cf. [von Soden 1939: 148]. As we shall see (below, pp. 40 and 47), however, a similar idea is expressed in a different way in YBC 4675 and the twin text YBC 4662, YBC 4663.

[70] I.e., outside group 3, where s u m is used regularly about results. Even in group 3, however, 3 of the 4 syllabic occurrences were within division questions – cf. p. 31.

[71] This connection is obvious in the full division question of group 4 (see p. 28). Since "raising is not involved explicitly in the division question in any other text it appears, however, that the form given to the question in group 4 is a secondary development, and that "positing to" was originally rather an alternative to the "raising" terminology. "Positing" will presumably have referred to the inscription of the two factors on a round clay tablet [Robson 1995: 281–284], and thus to the computational procedure; "raising", a generalization of the way the "standard thickness" 1 k ù š of the base of a prismatic volume is "raised" to the real height, instead refers to the concrete meaning of the operation, to the finding of something

YBC 8633, cf. p. 28) it directly marked the outcome of raisings. At a pinch, to find the igi may be seen as belonging to the same domain as “raising”, namely sexagesimal calculation – though with the difference that the multiplications make use of tables but rarely find the result given directly, while igi 6 is certainly in the table. But “breaking”, as distinct from multiplication with 30´, certainly does not belong to the field. Even on this account the present text thus follows a pattern of its own – which would fit a transition between traditions. In view of the scarce attention which the author has dedicated to terminological uniformity, the striking difference between the styles of problems 1–4 and 5–7 is finally evidence, not only that the two problem groups were ultimately derived from different sources but also that he was drawing on these sources in still independent form.

For comparison with other texts from the group, four further observations on the terminology of AO 8862 may be made:

- The remainder after a removal (*ḫarāṣum* as well as *nasāḫum*) is spoken of as *šapiltum*.
- The “natural half” is *bāmtum*.
- *wabālum*, “to bring”, is used, not as a multiplication (as in Db_2-146, see p. 13) but to tell a step in the cut-and-paste procedure.
- íb.si$_8$ is a verb, for instance in the phrase 15.e 30 íb.si$_8$.

AO 6770 consists of 5 problems of mixed contents. The prescriptions are brief and in part held in general terms, which makes the interpretation dubious. For the present purpose, however, this presents no serious problem. As the first group of AO 8862, the problems start by presenting the object. N[os] 2 and 4 (interest on a loan, and determination of an amount of bitumen by means of an igi.gub coefficient) ask the question *kī maṣi*, while N[os] 3 and 5 (a riddle on the weight of a stone and a reed broken in arithmetical series) ask *mīnum*. N[os] 3 and 4 also notify that “I do not know” (respectively *ul īde* and nu.zu) the weight of the stone and the amount of bitumen.[72] In N[os] 1 and 2, the prescription opens with the formula

via an argument of proportionality – see [Høyrup 1992: 351*f*].

[72] This usage, we remember, is also found in Str 364 (group 3) and in the broken-reed problems Str 368 and VAT 7535 (group 3) and IM 53 965 (group 7A). Cf. note

at-ta i-na e-pe-ši-i-ka. No closing formula is present.

In N[os] 1–3, the final result is given “to you” (*i-na-ad-di-ik-kum*, N[o] 1) or “to me” (*i-na-ad-di-nam*, N[os] 2–3). In N[o] 2 (obv. 15), even an intermediate result (after ba.zi) is given “to me”; as in AO 8862 N[o] 2, the deviation from the normal “I-you” pattern is noteworthy. Other results are at most marked by a preceding *-ma*.

Removal is represented both by *nasāḫum* (ba.zi in the stone riddle) and by *ḫarāṣum*; the remainder from the latter process is spoken of as *šapiltum*. Twice (obv. 7, rev. 7), *šutakūlum* denotes multiplications that cannot be rectangularizations – the first time regularly with the preposition *itti*, the second time, as if a total mix-up with *našûm* has taken place, with *ana*.

YBC 4675 contains a single problem on a (supposedly) bisected trapezoid. The statement opens *šumma* and asks the question *kī maṣi*. The prescription carries neither opening nor closing formula. A shift of section is demarcated *ta-as-sà-ḫa-ar*. Results “come up for you” *i-*(*il-*)*li-a-*(*ak-*)*kum*, except after two raising multiplications, where they are “given you”, in the nasalized spelling *i-na-an-di-kum*. Removal is *ḫarāṣum*; íb.si$_8$ is a noun and “taken”. Thrice, when the *bāmtum* is broken from an accumulation with two components, it is spoken of as *ba-a-ši-na*, “their” natural half – which implies that the accumulation is understood as the plurality of components and not as *one* entity, as also in AO 8862. *šutakūlum* is used (obv. 12) in standard fashion (for squaring), but also in the purely constructive sense of making the sides of a trapezoid “hold each other”.[73]

61.

[73] Rev. 15, in parallel with rev. 6, where the verb *epēšum*, “to make”, is used instead; similarly, in obv. 1 (the statement), uš uš ^{i}gu$_7$ is used in the sense that a length and a (different) length “hold” the trapezoid. Presumably the reading of ^{i}gu$_7$ is *īkullū*: as I have argued elsewhere (e.g., [Høyrup 1990a: 49]), the mathematical term *šutakūlum* cannot derive from *akālum* but must come from *kullum* – *takīltum* and *ša tuštakīlu* are equivalent, but *takīltum* cannot come from *akālum*; the logographic use of ì.gu$_7$.gu$_7$ must be explained as a kind of pun, due to the almost-coincident St-forms of the two verbs. In the present use, this logographic value will have been transferred “backwards” to the D-stem.

YBC 7997 is a brief text calculating the brick capacity of a cylindrical oven. It starts by stating this object, but has neither explicit question nor formulas of any kind. In several ways it is particularly close to the preceding text: Results "come up" (*i-li-a-am*, "for me", not "you", though other operations are in the conventional second person singular); the final result, coming from a raising, is "given", without specification of the receiver but with the same nasalized spelling *i-na-an-di-in* as in YBC 4675. But there are also similarities with other texts from the group: As in AO 6770, the prescription attempts to be general and not a mere paradigmatic example (rev. 6, *ma-la i-li-a-am*, "as much as comes up for me" instead of the actual result of the computation).

šutakūlum appears twice regularly as squaring, after which the resulting areas are "brought" (*wabālum*), in one case to the height, in the other to the factor 5´ (= $^1/_{4\pi}$). The resulting two numbers – of which the former is a quasi-volume, *viz* a ratio between areas times a height, and the second a circle area – are then taken as the objects of another *šutakūlum*, which is grammatically regular as a rectangularization but is evidently non-standard in its use (*našûm* would be the standard choice, since an operation of proportionality is involved).

YBC 9874 is a short text on the maintenance of a canal. It starts by stating the object, and has neither explicit question nor formulas of any kind. The only observation to be made in the present connection is that the results of the three raising operations are "given to you" (*i-na-*(*ad-*)*di-ik-ku*). YBC 9856 (included conjecturally by Goetze in the group) contains two problems with answers but no prescriptions. Of interest for the present analysis is only that both problems start by stating the object, and the first asks the question *kī maṣi*.

The last text from the group is YBC 6504. While all the others (or at least AO 8862, AO 6770, YBC 4675 and YBC 7997,) are mutually connected, this one differs from all of them on almost all accounts.

The tablet contains four quasi-algebraic problems dealing with a particular mutilated rectangle. The object is introduced implicitly, through specification of the parameters; N^os^ 1–2 leave even the question implicit, whereas N^os^ 3–4 have a question e n . n a m . N^os^ 1–2, on their part, start the prescription with the formula *i-na e-pe-ši-i-ka*, while N^{os} 3–4 have no opening formula. Closing formulas are altogether absent. In contrast to the other tablets from group 1, the present text is predominantly logographic – often in singular ways: results (intermediary as well as final) are followed by the phrase in.gar, used in no other mathematical text[74] (but with the same prefix as *in*sum, used in most of group 3, although it cannot serve on *šakānum*~gar as a phonetic complement corresponding to an adequate grammatical form[75]); "appending" is bi.daḫ (elsewhere occurring only in Str 363, group 3), removal ba.zi (used in groups 4 and 6, in series texts, in IM 55 357 (group 7A), and in one instance in AO 6770 from the present group); for *watārum* serves the truncated logogram SI instead of the normal dirig (=SI.A);[76] the natural half is šu.ri.a, elsewhere the Sumerogram for *mišlum*, the ordinary half; *patārum*, *našûm* and *ḫepûm*, however, are syllabic. íb.si$_8$ is a verb when (in arithmetical interpretation) it tells a square root (10.25-e 25 íb.si$_8$). Squarings are *šutamḫurum* in the prescriptions of N^{os} 1–2 (as in various texts from groups 6A, 7A and 8A, see note 47), while N^{os} 3–4 have du$_7$.du$_7$ (as most of group 3).

In statements, N^{os} 1–2 have *X* íb.si$_8$ where N^{os} 3–4 have *X* du$_7$.du$_7$ in N^{os} 3–4, probably as logograms for *mitḫartum* and in the sense of "the square constructed on the side *X*" (as in YBC 4709 and other series texts).

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[74] The closest we get is the "positing" of given numbers in the beginning of many procedures in groups 3, 4, 6 and 8, and the *mīna* gar of TMS XIV (see p. 18). The usage of the present text appears to be a generalization of this idea (perhaps borrowing only the word but changing the meaning, perhaps telling that intermediate as well as final results should be recorded on a piece of clay).

[75] A hint that in.gar is simply a mistake? That *in*sum was already in use, and in.gar a calque? Or that IN.sum should not be read *in*sum but in.sum, and thus reflect the general tendency to disregard the animate/inanimate distinction in Old Babylonian Sumerian [SLa § 292d]? The fairly consistent use of íb in connection with other verbs in the mathematical texts (si$_8$, tag$_4$) speaks against the latter assumption.

[76] Even this is unique, in spite of Bruins's claim [TMS, 54] that zi (=SÍ) in TMS VII, 23, must be a writing error induced by dictation for SI meaning dirig.zi, "to be torn out", is quite regular: the number 5´ characterized by this epithet (actually, what it becomes when the equation is multiplied by 4) *is* torn out in line 26.

X is the expression *mala* u š u . g ù s a g SI, "so much as (that by which) the length exceeds the width", in symbols (*u–s*). With the logogram a . n a, this expression is common in the series texts. Elsewhere this kind of "parenthesis function" of *mala* is rare; it is found in AO 8862 N[os] 1 and 3; in VAT 8390 N[os] 1 and 2 (group 3); and in BM 13 901 N[o] 19 (exceptionally, enclosing the excess of one square side over another; group 2, see below). This, however, is probably not a mere terminological peculiarity; it rather expresses the fact that problems involving "nested" operations are common in the series texts but rare elsewhere. If we add that there are obvious mathematical affinities between YBC 6504 N[o] 2, AO 8862 N[o] 3, and BM 13 901 N[o] 19 (all make use of the same standard diagram, the square on the sum of the entities whose difference is spoken of), we may surmise that even YBC 6504 belongs with the rest of group 1 for more than reasons of orthography – anyhow somewhat tenuous in this case because of the preponderance of logograms. If so, however, its position is surely somewhat apart.

Looking back at group 1 as a whole (whether YBC 6504 be included or not) we reach the conclusion that it repeats the characteristics of AO 8862 in larger scale. Few "positive" features connect the texts belonging to the group, but all the more the "negative" characteristic that most deviations from normal usage are concentrated here: that the outcome of an accumulation is regarded as a plural; aberrant uses of *šutakūlum*, and conversely the use of a . r á for rectangularization (with or without preceding "twofold inscription" (*lapātum*) of the side); appearance of *wabālum* as an all-purpose term; shifts from the second to the first person singular within prescriptions in connection with results "coming up" or being "given". Here we also find attempts to present procedure prescriptions not through paradigmatic examples alone but in general terms.[77]

Group 1 is not the exclusive repository of such oddities. *wabālum* was also used as a multiplication in Db_2-146 (group 7a) and in the fragment

[77] In strong contrast to what the ideals of Greek and present-day mathematics would make us expect, general formulations were thus *not the end point* of the development but *a possibility that was deliberately discarded* in mature Old Babylonian scribe school mathematics, at least from its written expression.

YBC 10 522 (group 5); a description in general terms (*viz*, of the "surveyors' formula" for the area of a trapezoid) is found in IM 52 301 (group 7B); as we shall see (p. 47), YBC 4662–63 (group 2) apprehends an accumulation as a composite entity; CBM 12 648, an early Old Babylonian tablet from Nippur in unusually grammatical Sumerian (see below, p. 55), uses gu_7 not about rectangularization but with triple object, finding a rectangular prismatic volume from length, width and height.[78]. Intrusions of the first person singular in prescriptions are widely spread in connection with division questions though only there.

Precisely this scattered appearance of the peculiar features found densely in group 1 shows that the authors of its texts were not mere bunglers; but they wrote in a situation where it was not clear that the construction of a rectangle should not be spoken of just as "inscription" of its sides, etc. The group as a whole is witness of a period of assimilation of traditions, a phase of creativity which would ultimately give rise to a new tradition governed by rather strict canons – not the same canon everywhere, as illustrated by the difference between the geographically close groups 3 and 4, but still variations on the same pattern and with high local uniformity, as illustrated by the internal homogeneity of each of these groups. In as far as one of the traditions to be integrated was that of the lay, semi-oral practitioners, the group as a whole and not only AO 8862 is situated, if not at the "interface between the oral and the written", at least at the interface between semi-oral and literate culture. YBC 6504 may express the attempt to establish new standards – some of them to reappear in group 3, the closest kin (in particular the replacement of the logographic pun ì.gu_7.gu_7 by the semantically more appropriate du_7.du_7, if this is the explanation of the choice; cf. note 73), others unsuccessful and replaced in group 3 by other standards (^{in}sum instead of in.gar, the simple $^1/_2$ instead of the complex šu.ri.a as the logogram for *bāmtum*).

To some extent, the variations within the group coincide with those of the Ešnunna texts – *nasāḫum*/*ḫarāṣum*, íb.si_8 as a verb or a noun

[78] Other texts invariably find the base by squaring or rectangularization and then "raise" it to the height or the depth [Høyrup 1992: 351f]; the only other exception is YBC 7997, which "brings" the base to the height.

designating an entity to be "taken". As will be remembered (note ? and preceding text), the use of *banûm* for constructions (in group 1 only represented by repeated occurrences of the regular a.šà[lam] *abni* in AO 8862) was also vacillating in the Ešnunna corpus. Even the heterogeneity of the Ešnunna corpus is thus to be explained from an analogous situation, only with the difference that the ten Tell Harmal texts from group 7A expressed the same attempt to define and stick to a standard as we have found in YBC 6504 and probably in no other group 1 text. The conspicuous differences – not least the total absence of *tammar* from group 1 (in all other respects so eclectic) and the different ways to deal with the outcome of a "breaking" – exclude that one of the two text groups could be a mere adaptation of texts from the other group (even though some inspiration cannot be excluded); we are forced to conclude that Goetze was wrong when maintaining that "Akkadian mathematics [...] originated in the South", but that it would be equally wrong to maintain that "Akkadian mathematics, like law-codes in Akkadian, originated in Ešnunna" (cf. note 4); Akkadian school mathematics arose from synthesis of the surviving Ur III traditions and the lay, probably Akkadian-spoken practitioners' tradition in parallel processes in the North and the South (and to all we know about the crystallization of the Old Babylonian culture, Ešnunna and Larsa are likely foci). Evidently. some mutual inspiration cannot be excluded

Group 2 – a non-group?

As the core of this group, Goetze points to YBC 4662 and YBC 4663, two theme texts on "excavations" (ki.lá) with almost purely logographic statements and fairly syllabic prescriptions, whose "close connection [...] needs hardly any comment" [MCT, 148 n. 354]. To this core he joins YBC 7164, a catalogue of heavily logographic statements and answers on the maintenance of small canals (pa$_5$.sig), omitting – tacitly and for unexplained reasons – not only YBC 4666, of which this latter tablet is a direct continuation, but also YBC 4657, a similar collection of statements on excavations – actually the statements for YBC 4663, YBC 4662 (in this order) and a missing tablet in between.[79] Finally, since the status of YBC 7164 as a continuation of another tablet reminds of the series texts, and because the predominantly syllabic procedure text BM 13 901 was pointed out by Neugebauer [MKT III, 10] to possess already the systematic order and progression found in the later series texts, even this text is included, though no linguistic clues connect it to the other syllabic texts from the group rather than to groups 3 or 4; as admitted in note 354, "the argument presented may be regarded as circular".

The close connection between YBC 4662, YBC 4663 and, we may add with Neugebauer and Sachs, the statement catalogue YBC 4657, is indeed beyond doubt. The affiliation of other statement catalogues to the same family – apart from YBC 4666 and YBC 7164 also YBC 4607, YBC 4652 and YBC 5037 – is inherently plausible, not only because of the common style of the statements (an argument which, like coinciding logographic terminology, tends to become circular) but also because of a shared format and similar ductus.[80] Whether the supposed group is really one therefore hinges on the comparability of BM 13 901 with the procedure texts YBC 4662–63. We shall start by describing these latter texts.

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79 The three procedure tablets correspond to Problems N^{os} 1–8, 9–18, and 19–28 of YBC 4657. Each sequence constitutes a coherent series of variations on a common basis, while there is a break between N^{os} 8 and 9 and between N^{os} 18 and 19 (in the latter case also a seeming stylistic break); since there is space enough left in the end of YBC 4663 for another problem we may infer that the procedure texts were made first and the statements of the catalogue copied from them; this agrees with the sudden appearance of *tammar* towards the end of YBC 4662 (see presently), a shift that (given the absence of *tammar* from the vocabulary of all other texts of plausible southern origin) could hardly have occurred if the scribe had filled out a collection of statements with procedures that were totally of his own making.

80 All the tablets in question except YBC 7164 indicate the number of statements contained in larger writing, either at the end or on the edge. YBC 7164, which is also the only member of the group not to insert a line between the single statements, is a companion piece to YBC 4666, and therefore has to go with the rest.

YBC 4612, a catalogue of statements for simple rectangle problems and similar in style, is in a much coarser hand than the others and does not count its problems. Its affiliation with the group cannot be excluded, nor can it however be regarded as reasonably established.

Statements start by presenting the object, a ki.lá,[81] and ask the question en.nam, with one appearance of *kī maṣi* in YBC 4662, rev. 2. Prescriptions open with the formula za.e kid$_9$/ kì.da.zu.dè (kid$_9$ in YBC 4663, kì in YBC 4662); in YBC 4663 they close *ki-a-am né-pe-šu*, in YBC 4662 without any formula. As a rule, results are "given" (*inaddikkum*, varying spellings in both tablets), but in YBC 4662 a first *tammar* turns up in rev. 22; in the last problem (rev. 31–36), *tammar* is used systematically for intermediate results, and *inaddikkum* is reserved for the solution.

As far as mathematical operations are concerned, both tablets employ *ḫarāsum* for removal from lines; YBC 4663 employs *nasāḫum* when areas are involved, while YBC 4662 uses *tabālum*. Rectangularization (and a single squaring in YBC 4663, rev. 8) is UR.UR, found nowhere else in the mathematical texts[82], in YBC 4663 alternating with *šutakūlum*. íb.si$_8$ is a noun and "taken", and then when appropriate "inscribed to 2" (*a-na* 2 *lu-pu-ut-ma*). For the "detachment" of an igi, YBC 4663 alternates between a syllabic *paṭārum* and the logogram du$_8$, while YBC 4662 sticks to du$_8$ alone. Both tablets tend to connect *ḫarāṣum* with *ana* instead of the regular *ina* (cutting off "to" instead of "from") and to combine daḫ with a.rá instead of *ana* (appending "times" instead of "to"). The natural half is written logographically as $^1/_2$ – in cases where the entity to be broken is an accumulation the whole phrase runs $^1/_2$ uš *ù* sag *ša* gar.gar*ru* ḫe pe-ma, "$^1/_2$ of the length and the width which you have accumulated break" (YBC 4663, rev. 7; YBC 4662 obv. 7). Since other occurrences of *ḫepûm* do not always involve a similar description or identification of the entity to be broken (and never any similarly complex description), we are led to a double conclusion: Firstly, the scribes of the two tablets (evidently different scribes, given the systematic divergences on several points) must have worked on the basis of a written source; secondly, this written original

81 At times, this word comes first, at times after the statement of the wage to be paid. But it is always there.

82 The closest relative is UR.KA, used for squaring in the Kassite text AO 17 264. Since UR is somehow semantically close to si$_8$ and *maḫārum* – in the fragment Ist.S. 428 it is used about the equalside of 2 02 02 02 05 05 04, while íb.si$_8$ designates the equalside of a factor which is split out, cf. [Friberg 1990: §5.3] – the use for rectangularization, at times with unequal sides, is somewhat awkward.

must have apprehended the accumulation not as a single entity but somehow as “things accumulated”, in the likeness of AO 8862. The rare occurrences of *šutakūlum*, *kī maṣi* and *tammar* suggest that the actual texts result from an attempt to rewrite the contents of the original according to a new standard, in which *šutakūlum* was replaced by u r . u r (perhaps an *ad hoc* construction),[83] *kī masi* by e n . n a m , and *tammar* by *nadānum* in generalized use. The original is also likely to have had the closing formula of YBC 4663, and to have used *ḫarāṣum* – perhaps also for surfaces, as the revised texts diverge on this point. Since *tammar* seems to have been absent from the southern environment – as already pointed out it does not occur in any of the group 1 texts, which otherwise seem to have collected all variant terminologies at hand – we may assume that the original belonged to the “northern” type.[84]

Even BM 13 901 is a theme text, containing “algebraic” problems of the second degree about one or more squares; on almost all accounts, however, it differs from the two Yale texts. Leaving aside for a moment problem N° 23, statements specify the object only implicitly, and close without any explicit question. Introductory and closing formulas are equally absent from the prescriptions. Results are at most preceded by an enclitic *-ma* on the preceding operation verb, and as a rule the answer is already involved in the following operation.[85] Statements as well as prescriptions are overwhelmingly syllabic.

As regards the terminology, removal is exclusively *nasāḫum*. The outcome of a “breaking” is a syllabic *bāmtum*; when the accumulation of

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83 Such a replacement of the normal verb by an ideogram constructed *ad hoc* and with no precise logographic counterpart may perhaps explain that neither the *takīltum* nor the corresponding relative clause nor any other equivalent appears in places where they would be appropriate.

84 Corroborative evidence pointing in the same direction (but certainly no proof) is constituted by the mathematical substance of the texts. The only other text accumulating the base and the volume of a prismatic excavation is BM 85 200 +VAT 6599, a theme text belonging to group 6A.

85 The division *question* runs *mīnam ana X luškun ša Y inaddinam*, but the answer is a naked number.

two distinct magnitudes is “broken” (e.g., in N° 9), there is no trace in the formulation that its composite nature should be thought of. Rectangularization is *šutakūlum*, and the object of the process is later referred to with the relative clause *ša tuštakīlu*. íb.si$_8$ is a verb (15-e 30 íb.si$_8$); when the outcome of the process of “making equalsided” is “inscribed”, this is done *a-di ši-ni-šu* (a phrase also used in AO 8862), not *a-na* 2 as in YBC 4662–63.

The text progresses rather systematically from simple to complex problems, and follows a very homogeneous stylistic canon, quite different however from those pursued in YBC 4662–63 and YBC 6504, and no less from those achieved in groups 3 and 4 and the “northern” groups.[86] It is certainly already well integrated in a school tradition, well at a distance from that transition between traditions where AO 8862 was located. Unless the two texts should come from distinct localities and not both from Larsa it is hence not likely that they should be equally old (notwithstanding the shared opinion of Neugebauer and Thureau-Dangin). Culturally, at least, BM 13 901 is certainly younger, if not with full necessity in terms of chronology.

Problem N° 23, by being a deliberate archaism, highlights the distance between the text in general and the lay tradition.[87] I have discussed this in depth elsewhere [Høyrup 1996b]; here I shall only mention that N° 23 starts b stating the object, identified explicitly as an accusative (a.šà^{lam}), and hence probably to be understood as an ellipsis for [*šumma*] *eqlam* [*išâluka umma šū-ma*], “[if somebody asks you thus about] a field”, in the likeness of Db$_2$-146 (see p. 7);[88] that the side of the square is designated

[86] In this connection, the use of the term *wasītum* for the width *1* with which sides are provided in order that they may be appended concretely to a surface is also characteristic, as a way to make rigorous the traditional use of “broad lines”. This precise usage is unique, even though the phrase 1 *wa-ṣi-am* found in VAT 8391 (rev. I 12, 18) and VAT 8528 (obv. 20; both group 4) will probably have the same meaning in a different context.

[87] It is not excluded that N° 22 share this character; but too little remains to allow any certain conclusion.

[88] Mostly, when texts starts by stating the object, the term is a logogram deprived of phonetic complements. When syllabic writing occurs (e.g., in some of the problems of BM 85 196), the form is always the lexical form, i.e., a nominative.

pūtum in syllabic writing (*pa-a-at*, plural) – found nowhere else in this function except in IM 53 965, the Ešnunna broken-reed problem (even there thus referring to a supposedly "real" field and no "algebraic variable"); that the solution tells that 10 n i n d a n *im-ta-ḫa-ar*, "10´, n i n d a n , stands against itself", in a phrase which else belongs with group 6A (and with the group-4 cistern problem YBC 4186, see p. 29) and is totally absent from the rest of the tablet.

Since even spellings do not correspond too well, we may conclude that YBC 4662–63 and BM 13 901 do not belong within a common group. The twin text reflects an attempt to organize the material at hand (in the actual case apparently a "northern" written text) in agreement with a local canon; BM 13 901 belongs within an already well developed tradition obeying a wholly different canon – closer to group 4 than to anything else, but different on many accounts. We may keep the label "group 2" for YBC 4662–63 and associated logographic catalogue texts (certainly YBC 4657, most probably also YBC 4607, YBC 4652, YBC 4666, YBC 5037 and YBC 7164, perhaps YBC 4612). BM 13 901 must be considered the only known representative of what might be termed "group ii".

The series texts

Neugebauer connected the series texts[89] to the texts of group 6A because of the thematic similarity between the problems of BM 85 196 and some of the series texts. The similarity between the excavation texts YBC 4662–63 from group 2 (with the appurtenant catalogue YBC 4657) and the excavation text BM 85200+VAT 6599 from group 6 undermines the argument. As we have just seen, YBC 4662–63 offer evidence that problems did not only travel between schools and regions but were systematically borrowed and adapted to the local canon.

Without including the series texts explicitly in any of his groups, Goetze used them to link BM 13 901 with the rest of his group 2, thus presupposing that even they were tied to that group. The argument was risky already

[89] That is, VAT 7528, VAT 7537, YBC 4668–69, YBC 4673, YBC 4695–98, YBC 4708–15, and A 24 194–195 – cf. note 2.

in 1945 (“circular”, as he admits) and is easily disposed of in the light of texts published since then. It assumes that the systematic order of texts like BM 13 901 and YBC 4662–63 and that of problem catalogues – with or without a series number – was a local specialty. The existence of the Tell Ḥarmal Compendium and of TMS V–VI shows that this was not the case.

In [MKT I, 387f], Neugebauer had argued that the series texts would probably come from Kiš because of a terminological peculiarity – the supposed use of sig$_4$ in the sense of “volume” – shared with the catalogue text AO 10 822, excavated by Genouillac in 1911 in this place. As pointed out by Neugebauer and Sachs, however, their decipherment of the brick measuring system dissolved the peculiarity into nothing [MCT, 95].

The association with Kiš was supported by other arguments – writing and format of the tablet, general terminological agreement of AO 10 822 with the series texts VAT 7528, YBC 4669 and YBC 4673, and appearance on the antiques market in the years following upon Genouillac’s excavation. With hindsight, even the format and the terminology turn out to be weak arguments. The lines of the series texts are densely spaced and short, containing some 5 signs; those of AO 10 822 are twice to thrice as long and widely spaced. Terminological similarities do not go beyond what can now be seen to follow automatically from the subject-matter. Taken alone, the acquisition year has little demonstrative force.

Terminology thus seems the only possible cue to the affiliation of the series texts, even though the absence of prescriptions eliminates many of the interesting parameters. Happily, the reverse of YBC 4669 contains one brief problem provided with a prescription, in which results are marked igi.du$_8$, the logogram for *tammar*. This already points to the “northern” orbit, as represented by groups 6, 7 and 8.

A single term in a single problem proves nothing – as we have seen, even the last problem of YBC 4662 uses *tammar*. But other details, partly mentioned already, point in the same direction.

Firstly, the use of the term íb.tag$_4$ for the remainder after a removal in YBC 4668, YBC 4669, YBC 4697, YBC 4710, and YBC 4713. Outside the group of series texts this was found in BM 85 196 and VAT 6598 (both group 6), and in IM 55 357 (group 7B).

The phrase *X* a.rá *Y* ^e^tab is used in VAT 7537, YBC 4668, YBC 4695–97, YBC 4708–4713, YBC 4715, and A 24 194–195, and elsewhere only (in the slightly different shape *X* a.rá *Y* tab.ba) in BM 85 194, obv. II 44, 50.

YBC 4708 asks repeatedly EN ta.àm íb.si$_8$, "What is each equalside?". Similar references to "each" of the sides of a square are found in BM 85 194 and BM 85 196 (both group 6); NBC 7934 (not linked to any group); YBC 4607 and YBC 5037 (group 2 catalogues, and thus possibly recasts of "northern" material in the likeness of the group 2 excavation texts); and BM 80 209 (on which below, p. 54). As mentioned above (p. 25), the phraseology seems to belong with the lay surveyors' tradition, which carries it into the Middle Ages.

But the series texts certainly do not belong to any of the "northern" text groups established so far. They never employ UL.GAR for *kamārum* or NIGIN for *šutakūlum* / *šutamḫurum*, which sets them apart from groups 6 and 8A; the sophistication of many of the series texts – regarding mathematical substance as well as the pluridimensional variation of statements – shows them to belong to a more mature phase of scholastization than the Ešnunna corpus; we may also observe the absence of kud (~*ḫarāṣum* – common in groups 7A and 8C) from the series texts.

The distribution of a few other terms and expressions may serve to complete the picture:

ba.zi, the normal writing of the removal in the series texts, is used in group 6A; in IM 55 357 from group 7A; and in AO 6770 (only the stone problem) and YBC 6504 from group 1.

nu.zu, "I do not know", is used in connection with the question in YBC 4668, YBC 4673, YBC 4698, YBC 4710 and YBC 4713. Outside the series text group the same phrase or its syllabic equivalent is used in Str 362, Str 364, Str 368, VAT 7535, and YBC 4608 (all from group 3); further in IM 53 965 (group 7A), a *kī maṣi*-text dealing with a broken reed and very close to Str 368; and in AO 6770 (group 1), the stone problem and in one other place. In TMS XX, 5 and TMS XXV, 4, the relative clause *šà la ti-du-ú* is used as an identifier within the prescription.)

The phrase a.na uš u.gù sag dirig, "as much as that by which the length exceeds the width", is used in YBC 4668, YBC 4697, YBC 4709, YBC 4711 and YBC 4713. As mentioned above (p. 42), the same phrase in more or less syllabic writing turns up in groups 1 and 3, with a single similar expression in BM 13 901 (=group ii). The Susa texts (group 8A) reduce it to d i r i g .

We may conclude that the series texts are less closely related to group 6A than believed by Neugebauer; that they will have been produced somewhere in the “northern” orbit – that is, outside the ancient Ur III core area. If we look at the problem types where n u . z u and a . n a u š u . g ù s a g d i r i g and their syllabic equivalents turn up in groups 1 and 3 (broken-reed and stone riddles, etc.) we may also infer that the series texts, in spite of their sophistication and highly technical language, were produced in a place where the riddle tradition was closer to the surface than in the school where (e.g.) group 6A was produced and used.[90]

Varia

On the whole, the inclusion of new parameters has confirmed Goetze’s original categorization, moving a few tablets from one group to another or away from the established groups;[91] it has also allowed us to distinguish a couple of subgroups. As demonstrated by the series texts, it does *not* allow us to link all tablets to the existing groups. One reason

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[90] Close connections, if not specifically to the riddle tradition then at least to the practical background of Old Babylonian mathematics is also suggested by the fact that even those series texts that contain “algebraic” problems on rectangles indicate the units explicitly, and that the normal dimensions of the rectangles are 30 n i n d a n ×20 n i n d a n , not 30´×20´ as preferred elsewhere (thus in YBC 6504, group 1; BM 13 901, group ii; and various Susa texts). This is no absolute rule, it is true – YBC 4668 N[os] A35–47, YBC 4695 and A 24 195 are of the 30´×20´ type, unless we assume that these, like N[os] B7–10 of YBC 4668, simply take advantage of the floating-point character of the numerals and make formal additions without regard for the proper order of magnitude.

Many of the series texts, finally, deal with genuinely practical problems and not with artificial “algebraic” matters – perhaps to a larger extent than other texts groups apart from 2, 6 and 7.

[91] Conversely, the agreement with a categorization based on orthographic criteria confirms the correlation between our terminological parameters and the origin of tablets.

for this is probably that the major groups come from very few very specific locations – tablets that are as alike as the sequence VAT 8389–8391 and which were bought in the same lot (as is evident from the museum numbers) are likely to belong as closely together as the group 7A texts – and comparison of the 7A Tell Ḥarmal texts with the 7B texts from the same city shows that even the same locality may have produced texts of widely diverging character during the same decades. Many of the remaining texts may simply have come from other locations. But another reason is that none of the terminological parameters determines anything when taken alone, and that texts containing one or two problems are likely to make use of only a few of the interesting operations.

None the less, a few texts should be discussed in the light of the preceding analysis – at first the catalogue text BM 80 209 (problems concerned with squares and circles, mostly second-degree "equations") discussed by Friberg [1981]. Terminology leaves little doubt that the text belongs to group 6A – unexpected from its contents, because the 6A texts concentrate on volume determinations, and give no systematic treatment of quadratic equations on their own (even though they certainly enter into many problems, in particular but not exclusively in BM 85 200+VAT 6599 – see, e.g., the analysis in [Høyrup 1992]).

Accumulation, indeed, is UL.GAR, as found in groups 6 and 8A and nowhere else. That a configuration is square is told by the phrase *X* TA *im-ta-ḫ̮ar*, in agreement with the general style of group 6A (see p. 25) and obviously an abbreviation of the precise phrase used in BM 85 196 (and with a small variation in BM 85 194), *X* t a . à m *im-ta-ḫ̮ar*, "*X*, each, confront each other". The reference to the notion of "each" side of a square is shared with the series texts and probably in general with texts in close contact with the lay tradition, but no text except BM 85 194, BM 85 196 and BM 80 209 has the precise combination of *imtaḫ̮ḫ̮ar* with t a . à m . Removal is written b a . z i as generally in group 6A; in the series texts (with which the present catalogue has little in common beyond being a catalogue); and in only three other tablets (see p. 52). As in BM 85 194 and BM 85 196, some of the statements start with *šumma*.

BM 15 285 is less easily categorized. The text is a catalogue of problems about the inscription of geometrical figures in a square. Problems start 1

UŠ *mi-it-ḫa-ar-tum* (at times written 1 UŠ íb.si$_8$). Normally UŠ is read uš, "length", and translated "1 (ist) die Länge. Ein Quadrat" [MKT I, 138], "Un carré: le flanc est 1" [TMB, 53], or "The length of the square is 1" [Robson 1995: 248]. The reference to the "length" of a square is used in the Tell Harmal Compendium (group 7B) and TMS V+VI (group 8B), perhaps also in TMS VIII (group 8A); if the interpretation is correct the expression hence suggests the "northern" orbit. But it hardly is. Robson's translation does not fit the nominative of *mitḫartum*;[92] Neugebauer and Thureau-Dangin instead read *mitḫartum* / íb.si$_8$ as an indication of the object, which is then in a most unusual second position (which Thureau-Dangin corrects in his translation).[93] A reading of UŠ as the unit 60 nindan, and the whole phrase as "1 UŠ [= 1` nindan] is the equalside" seems a more likely alternative. This reading, however, has no implications as to the origin of the tablet.

Most of the terminology is either too narrowly connected with the themes dealt with or (like *libba* / *ina libbi*) too widely used to be informative. Only two observations can be made. Firstly, that the spelling íb.si$_8$ is not that of the "outer periphery", i.e., Ešnunna and Susa.[94] Secondly, the logographic use of íb.si$_8$ for *mitḫartum* is a rare phenomenon outside the series texts (plausibly from the "inner periphery", as we have seen); but it does occur in YBC 6504 (group 1) and Str 363 (group 3), both of them from the South.

CBM 12 648 was referred to above (p. 43) for its aberrant use of gu$_7$

[92] If we try instead (no obvious choice!) to read *mitḫartum* as a Gt-verbal adjective (and the whose expression thus as a stative variant of the *imtaḫḫar*-construction), the singular feminine form of *mitḫartum* does not fit an interpretation of uš neither as a singular (whence masculine), nor as a feminine (whence plural).

[93] In YBC 4662–63, it is true, the object (the ki.lá) was sometimes told after the wage to be paid, cf. note 81. But the dimensions of the object still follow its presentation.

[94] íb.si$_8$, we remember, was only used in one text from group 7A (IM 54 478), where it produced the cubic "equalside". Another tablet from Tell Haddad to be published by Friberg and al-Rawi has BA.SI$_8$.E *šu-li-ma* [Friberg, forthcoming / a, section 9f], which, however, is shown by the final E (that can be no agentive suffix) to be a syllabic writing *ba-si$_8$-e* (or *ba-sá-e*, cf. p. 16).

in the production of a rectangular prismatic volume. The text is from Nippur; the writing makes Neugebauer [MKT I, 234] suggest "early first Babylonian dynasty" as the date. Both the certified origin and the early date recommend that we look at the terminology.

The writing is almost purely logographic, and uses enough grammatical complements to be regarded as (Old Babylonian) Sumerian. In the function where, e.g., YBC 6492 writes simply $^2/_3$ uš sag, the present text has $^2/_3$.bi uš.a.kam sag, "$^2/_3$ of (the) length, it is (the) width"; in particular the use of a composite complement like -a.kam (<ak.àm) is most unusual in a mathematical text. None the less, the appearance of the Akkadian *u* and *-ma* suggest that the Sumerian phrases translate an underlying Akkadian structure. The text seems not to represent directly an original Sumerian mathematical terminology; it rather shows how its author thought Sumerian mathematical terminology should look or had looked; like BM 13 901 N° 23 and the Tell Ḥarmal texts from group 7A (though certainly in search for a different ideal), it is an archaization rather than archaic.

The text thus does not tell us that íl is a traditional Sumerian term for "raising" multiplication; but it does tell that it was thought to be the adequate Sumerian term in early Old Babylonian Nippur; and it shows the same to hold for the designation of the cubic "equalside" as íb.si$_8$, treated as a noun and "coming up" (e$_{11}$.dè).

gu$_7$, as stated, is used for *šutakūlum* in the unusual "prismatic" function. In this case we may be sure not only that the expression translates Akkadian thought but also that the Sumerian is of the do-it-yourself kind. The full form of the verb is UB.TE.gu$_7$, which is not easily analyzable, but which at least is a composite verb, whose first element probably stands for ub, "corner".[95] The object (uš sag *ù* bùr.bi, "length, width, and its depth") should therefore stand in a dimensional case, which (according to the general style of the text) should be indicated by the pertinent suffix. Instead, the construction follows the Akkadian model word for word. It does not show that after all *šutakūlum* should be *šutākulum* and derive from

[95] TE could be te, "to approach closely", or temen, "fundament". Neither seems to fit grammatically, unless ub.TE itself constitute a composite word.

akālum, nor that the metaphor of letting a length and a width "eat" each other is a Sumerian loan translation. It has probably been constructed backwards from an already established use of ì.gu$_7$.gu$_7$ for *šutakūlum* (cf. note 73).

In contrast, the fully grammatical phrase *X*.še ba.e.íl, "you raise to *X*", and the parallel with the use of *šulûm* in IM 52 301 and (particularly) Haddad 104 (see p. 16) might imply that "raising" and "coming up" belong, if not to some age-old Sumerian mathematical terminology (if they did, there would probably be *some* traces in Ur III documents) then at least to the teaching of practical arithmetic in the wake of the Ur III tradition – not only in Ešnunna, as already argued above, but even in the core area (cf. also note 71).

A small group of four text (VAT 672, VAT 6505, and the palaeographically similar fragments VAT 6469 and VAT 6546) belong together as a group. All exhibit "northern" features. VAT 672 uses igi for results, while VAT 6505 and VAT 6546 have *tammar* (no results remain in VAT 6469). With the possible exception of VAT 6546, they all open the prescription with the formula za.e kì.ta.zu.dè; VAT 6469 and VAT 6505 close it *ki-a-am ne-pé-šum* (in the others, the passages where it could occur are destroyed). So far, everything (apart from the igi of VAT 672) looks like group 6A, with which other features (mostly present on only one of the tablets, and therefore inconclusive[96]). Other characteristics, however, set them apart: VAT 6546 marks the division into sections by *tu-ur*, not nigín.na; and VAT 672 as well as VAT 6505 use rá (~*alākum*, "to go"[97]) when multiplying by a reciprocal – a function where other texts would use *našûm*/íl. The usage reminds somewhat of the use of *alākum*

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[96] We may mention the use of *ḫarāṣum* in VAT 6546 not as a normal subtraction but referring to the curtailment of a profit, analogous to the appearance of the term in BM 85 196 N° 18 (see p. 23); and the final counting of the šu.nigín, "sum-total", of *kibsu*, "steps", at least in VAT 6505 (in the three other texts, the end is missing), in the manner of BM 85 194 and BM 85 196 (see p. ?).

[97] Lexical lists give readings *gi-in*, *du-u*, *ri-i*, and *ra* for DU$_8$. In order to keep in mind that connection to a.rá which no Babylonian calculator could miss, and whose pronunciation follows from Akkadian loanwords, I have chosen the interpretation *rá*.

in certain Susa texts (see p. 20) and the combination of a.rá with tab in BM 85 194 and a number of series texts; but no other Old Babylonian text replaces "raising" by "going" and the interesting connection is to the Seleucid texts. It seems sensible to label the group 6C, while keeping in mind that VAT 672 and VAT 6505 form one cluster, VAT 6469 and VAT 6546 another.

Later texts

The terminology of later mathematical texts differs from that of any Old Babylonian text group. Turning upside down a famous phrase, however, we may observe that some text groups are more different than others. This allows us to say something (though not much) about the historical process.

AO 17 264 is a procedure text on the partition of a field (supposedly a trapezium) in six strips (latest analysis in [Brack-Bernsen & Schmidt 1990]). According to the dealer it is from Uruk, and the writing suggests a Kassite date.

The statement starts by telling the object, and asks an explicit question en.na. The prescription starts with the formula za.e kì.da.zu.dè (i.e., in the characteristic "southern" spelling used in groups 2 and 3 – cf. note 41), and ends *kīam nēpešum* – a formula known only from Old Babylonian texts belonging to groups 6A, 6C, 7 and 8A. The plane "equalside" is a noun and is made "come up" – *ba-se-e-šu šu-li-ma*, the phrase as well as the spelling pointing to group 7B. Results are followed by I.DÙ, which Neugebauer and Thureau-Dangin understand as ^{i}dù~ibanni, "it produces";the complement *i* leaves little doubt that this corresponds to the scribe's own understanding. But the spelling igi.dù instead of igi.du$_8$ in IM 55 357 suggests that this terminological innovation stems from a reinterpretation of the unorthographic Ešnunna spelling – a "scholars' folk etymology".

Unorthographic spelling also seems to explain íb.TUG, used twice after a removal: As proposed by Thureau-Dangin, the word is likely to stand for *šapiltum*, regularly written with the logogram íb.tag$_4$ (thus in group 6A, in IM 55 357, and in various series texts, see note 54). In syllabic spelling, it is used in AO 8862 and AO 6770 (both group 1) and IM 54 464 (group 7A).

Accumulation is UL.GAR, as in groups 6A and 8A, while squaring is UR.KA – a missing link between the UR.UR of YBC 4662–63 (group 2) and the KA+GAR of TMS XXVI (group 8C)? LAGAB is used instead to tell the equality of shares. “Breaking” is treated as in the Tell Ḥarmal texts from group 7A, mentioning neither that it is “into two” (as in group 4) nor the resulting natural half (as habitual in the rest of the Old Babylonian corpus).

Apart from the spelling of the introductory formula, the features are thus definitely “northern”, but vacillating between groups 6A, 7A+B and 8A+C, with preponderance for the links to group 7. If the tablet is really from Uruk (as are the Old Babylonian groups 3 and 4, if we are to believe Goetze) this leads to the interesting result that the southern tradition was so effectively interrupted by the disorders of the late Old Babylonian period that mathematics was imported anew into the former Sumerian heartland from the periphery in the Kassite period. If we choose not to trust the dealer we can go no further than the observation that the only problem text from the Kassite period is related to the mathematics of the Old Babylonian “North”, in particular to its most “primitive”, least sophisticated branch.

The Late Babylonian (but probably pre-Seleucid, cf. below, p. 60) Uruk text W 23 291-x shows similar affiliations but no significant traces of descent from texts akin to the Kassite text just discussed. It is what Friberg, Hunger & al-Rawi [1990] call a “metro-mathematical topic text” on the mensuration of surfaces. A few observations will make the point.

Statements either start by telling the object directly or (more often) implicitly through the parameters. Questions are asked explicitly by e n. From II 31 onwards prescriptions start m u n u . z u ú (~*aššum lā tidû,* “since you do not know”). This phrase itself is not known from Old Babylonian texts, but evidently connected to the appearance of n u . z u and *lā edûm* in the statements of various series texts and of texts from groups 1, 3 and 7A, and to the clause *ša lā tidû* in TMS XX and TMS XXV (see p. 52). No closing formula is given. From the point where the phrase m u n u . z u ú is introduced, the prescription starts by a computation rule formulated in more or less general terms, after which follows the particular computation –

most often in two variants, one where the unit is n i n d a n (*šum-ma* 5 *am-mat-ka*, “if your cubit is 5´”) and one where it is k ù š (*šum-ma* 1 *am-mat-ka*, “if your cubit is 1”). The general rules are more transparent than the examples from groups 1 and 7B, and the coupling of a general rule with an example is certainly a didactical improvement; but the very idea of making general formulations points back to the intersection between lay and scribe school mathematics (cf. note 77 on the elimination of general formulations from mature Old Babylonian mathematical texts). Results are mostly marked by nothing but a preceding *-ma*, but often the last step of the general rule is followed by i g i *mar* (*tammar*, “you see [the quantity asked for]” – e.g., II 33[98]); within the general rule, intermediate results are referred to as *šá a-na* i g i *-ka* e_{11}*.a*, “what comes up for your eye” (e.g., III 25)[99] – a noteworthy combination of the two apparently original styles, *tammar* and *elûm* / e_{11}.

Accumulation is g a r . g a r , not UL.GAR as in the Kassite text and in groups 6 and 8a. Subtraction has become a counting of the *a–b* steps from *b* to *a*, and multiplications (including rectangularization) are expressed as *X* a . r á *Y* r á , “*X* steps of *Y* go” (both *X* and *Y* being nominatives!). The “equalside” is UR. a (seemingly meant as *mitḫartum*, but in any case related to the Kassite UR.KA and to UR.UR as used in group 2); finding the “equalside” of *A* is expressed à m *A* t i *qé*, “each [equalside] of *A* take” – reminding of the í b . s i$_8$ t a . à m used in series texts and texts from group 6A and of a parlance that is to turn up again in Mediaeval Arabic mensuration – see above, p. 25.

“Appending” is still *waṣābu* (IV 16, 19), but the form that appears (*teṣṣib*, 2nd person singular, present tense in the inflection of the age) is such that it can no longer be distinguished in writing from the corresponding form of *eṣēpu* (a circumstance to be mentioned because of possible

98 In one place (IV 35) appearing in syllabic writing as *tam-mar*. In Old Babylonian texts, the invariable spelling is *ta-mar*.

In general, the many phonetic complements used in the text leave no doubt that the logograms were to be read in Akkadian.

99 To be compared particularly with *mala illiam*, “as much as comes up for me”, used in the same function in YBC 7997 (group 1).

relevance for the Seleucid texts).

Three Seleudic procedure texts are known: AO 6484, BM 34 568 and VAT 7848 (much of which is destroyed). Like W 23 291-x, AO 6484 is known to be from Uruk. Since various features of the terminology of these texts look like further developments from W 23 291-x, a pre-Seleucid date for that text is corroborated.

AO 6484 is of mixed contents, the others (except one problem in BM 34 568) deal with geometry. Statements either start by telling the object or (more often) suggest it through the parameters. AO 6484 has some questions *kīma maṣi*, but mostly its questions are implicit, as are those of VAT 7848. BM 34 568 asks with the same abbreviated en as W 23 291-x. BM 34 568 starts the prescription with the formula mu nu.zu^{ú}, "since you do not know"; none of the texts employ any closing formula. Results are, if marked at all, preceded by an enclitic *-ma*.

Accumulation is gar.gar, and subtraction a counting-off as in W 23 291-x ("going up"/nim in BM 34 568 and "going down"/lal in AO 6484, while VAT 7848 uses the neutral e_{11}); interestingly, however, the result of the operation is said to "remain" (*riāḫu*, thus a reference to the concrete meaning of the operation, not to the calculational technique). As in W 23 291-x, multiplication is sometimes expressed *X* a.rá *Y* rá, but more often the abbreviation *X* GAM *Y* rá is employed. No specific operation for rectangularization or squaring is distinguished, nor any term for the "equalside" – finding the square-root of *A* is a purely arithmetical operation, "what steps of what should be gone in order to get *A*?". The square figure occurs in AO 6484 as *tamḫirtum*, with the corresponding logogram UR.a.

The reappearance of the interrogative phrase *kī maṣi* after its absence from the record since the end of the Old Babylonian period should not cause amazement: we only have two texts from the intervening centuries, which anyhow are of a kind (area problems) where this question was never the rule. More remarkable is the reappearance in BM 34 568 and AO 6484 of the spelling sag.ki for the width, which we have not encountered in any mathematical text since group 7. The tradition to which these texts refer is evidently not exclusively the one that is represented by the mathematical school texts.

The same conclusion is suggested by another terminological oddity. *waṣābu*, "to append", has been replaced by *ṭepû*, used in the same function and within the same grammatical construction. When written logographically, however, the term becomes t a b . In the Old Babylonian texts, this Sumerogram was used for *eṣēpum*, "to double" / "to repeat". Both doubling / repetition and addition fit the original Sumerian meaning of the word: "to be / make double" / "to clutch / clasp to" [SLa, 318]. Since there is no tradition for the equivalence *tepû*~t a b before or outside the Late Babylonian (perhaps only Seleucid?) texts, the most likely explanation of the usage is a re-Sumerianization of the Akkadian terminology (or Aramaic, which has *tpā*?), made from general knowledge of Sumerian and in ignorance of earlier mathematical customs. Another possibility that cannot be fully excluded is an interference with the coincident spelling (and probably pronunciation) of forms of *waṣābu* and *eṣēpu* which we have encountered in W 23 291-x. However, since this latter text did not seem to know about the possibility of using t a b , I tend to regard this explanation as implausible.

Similarly innovative is the indication of "inner zeroes" in AO 6484 – probably induced by their use in contemporary table texts and mathematical astronomy. One might believe the opposite, given that even some Susa texts have inner zeroes (cf. p. 19). However, the idea to use one of the standard separation signs in this function is too close at hand to exclude independent invention. The Susa material, moreover, uses the sign GAM, which our Seleucid texts employ as an abbreviation for a . r á , using instead ABZ#378 to represent the zero. Most significant of all: what we understand as "inner zeroes" is not the same thing in the two cases and does not serve the same purpose: The Susa texts use it to indicate missing ones or missing tens, in agreement with a decimal-seximal understanding of the numeral system, and for order's sake; such zeroes are not marked in the Seleucid texts.[100] Seleucid zeroes indicate missing orders of magnitude in a truly sexagesimal system, and they serve to avoid possible misunderstandings.

[100] E.g., AO 6484 has 2. | .15 in the sense of 2°15″ but no "zero" in 2.3 (rev. 24 and rev. 15, respectively).

The outcome

The outcome of the preceding investigation is first of all a confirmation of Goetze's classification, to some extent a refinement; and, most important, a demonstration that the categories, originally based on criteria that have nothing to do with the contents of the texts,[101] reflect specific professional environments, endeavours, and even "schools" with particular canons. Once this has been established, the classification may function as a grid for the ordering of further observations:[102] a *sine qua non* if we want to understand the development of Babylonian mathematics and not just to see the mathematical texts as manifestations of a general average.

I shall not venture into a general exploration of these possibilities – the many details have already made the present paper exorbitantly long. However, certain conclusions emerge directly from the argument as presented, and I shall sketch them briefly.

A first conclusion has to do with the character of our knowledge of Babylonian – in particular Old Babylonian – mathematics. As far as the substance – themes, techniques, mode of thought – is concerned, the extant corpus seems to offer a rather coherent picture. With some noteworthy exceptions (mixed cubic problems present only in one group 6 text, didactical explanations present only in certain Susa texts, etc.), problem

101 As argued well by Goetze, the orthographic habits that were his main argument reflect different pronunciations, which are certainly external. But even his isolated observations on the vocabulary were so, in particular because they did not approach the vocabulary in its function of a technical terminology – treating, for instance, at the same level a capital *giving* interest and a calculation *giving* a result (see note 51).

102 A grid, it should be kept in mind, that does not abrogate the need for scrupulous reflections and second thoughts: As we have seen, particular terms have a tendency to travel widely between groups together with particular problems types – thus *ki̯ā̄* together with the partition between brothers (see note 45), *inanna* with brick carrying (note 63), and n u . z u with the broken reed (note 72). The grid of local professional environments is thus tied irreparably in with a different grid, that of problem types. Any safe conclusion based on shared or discordant terminology must take both into account.

types and techniques are more or less the same in all or at least several groups, and nothing indicates that the authors of the different text groups thought differently about the matters they dealt with or had different purposes in mind. Since this question has not been argued explicitly above, I shall not pursue the subject, just remind that YBC 4662–63 seems to be the outcome of a deliberate adaptation of material from a "northern" location to a southern canon.

As regards stylistic canons, on the other hand, a very different picture emerges. Most of the Old Babylonian material falls within a few internally coherent groups – but the later texts descends from none of these but from stylistic types of which we only know that they must be mutually different, "northern", and often closer to the early (or even non-scholastic) types than to the sophisticated varieties. Old Babylonian mathematics as we know it seems a handful of semi-researched islands in an otherwise unknown archipelago. For that reason, even our trust in the substantial representativity of the extant sources should not be too great – if one island (group 6) shelters griffins (third-degree equations) and the others not, how can we be sure that unicorns were nowhere present?

All the way through the analysis, observations on the possible traces of pre-Old-Babylonian traditions have been made. I shall remind of a few essentials and outline a few perspectives:

Old Babylonian mathematics is a synthesis of several earlier traditions: the teaching of the mathematics of the Šulgi reform (the sexagesimal place value system and the igi.gub tables); a lay, semi-oral tradition carried by practical geometers and reckoners (the "riddle tradition")[103]; and, as we shall see presently, probably a pre-Šulgi school tradition at least in

[103] Speaking of one lay tradition may be a simplification; it is not evident that the surveyors' riddles on areas and sides giving rise to the development of second-degree "algebra" were carried by the community that transported the filling riddle between Egypt and Mesopotamia, or the 30 successive doublings from Old Babylonian Mari [Soubeyran 1984: 30] into the Middle Ages. For simplicity; because the sources do not allow us to distinguish; and because we only know this lay mathematics as seen and rendered by scholars who (then as now!) are not likely to have distinguished neatly between kinds of non-scholars: for these reasons I shall none the less refer to the lay tradition in the singular.

the periphery. Group 1 shows us the very enterprise of integration; group 7A (or at least its Tell Ḥarmal section) as well as BM 13 901 N° 23 seem to reflect an archaizing awareness of the characteristics of background traditions;[104] other groups present us with the outcome of the synthesis in the form of already established canons.

In the periphery (the "North"), the use of *tammar* for results seems to point originally to the lay tradition; the use of *elûm* in the same function appears to be connected everywhere to the post-Šulgi scholastic tradition. Later in the Old Babylonian period, *tammar* becomes the canonic expression in most of the "North" (group 5 being a remarkable exception, sticking to *elûm*), which suggests a continuous interaction with the non-school environment; in the South, where *tammar* is absent from the beginning, two different canons develop: preceding enclitic *-ma* or nothing in groups ii and 4, syllabic forms of *nadānum* in group 2, *in*s u m (occasionally syllabic forms) in group 3.

The origin of *nadānum* as a marker of results is with multiplications in the sexagesimal system (and thus with the post-Šulgi tradition); the main use of the term in texts where it does not serve for results in general is in the division question; second comes the indication of the result of "raising" multiplications; thirdly that of final results. It is probably significant that even group 3 tends to use the syllabic forms in this original domain.

It is also significant that the use of *nadānum* goes together with the only systematic breaking of the normal switch between grammatical persons. This switch between the statement formulated by an "I" and a prescription of what "you" should do (at times involving quotes from what "he" has said) becomes a general characteristic of mature Old Babylonian mathematics, the only texts that have some difficulty with the principle being indeed those of group 1. But if it is systematically broken in connection with a term pointing towards the post-Šulgi school tradition, it must come from elsewhere – and the format of the 7A texts shows how it fits the riddle tradition: The "he" of the prescription is indeed the "someone" who has

104 Since the semi-oral traditions survives into the Middle Ages, BM 13 901 N° 23 may also imitate the language of an active practitioners' environment.

posed the problem and the "I of the statement", whence different from the speaking person of the prescription who explains the procedure.[105]

So far, the integration of a lay tradition and a school tradition derived from the Ur III teaching institution seemed an adequate framework for the discussion. The reason that the school tradition may need to be split into two (and thus the reason for which I have spoken of a "post-Šulgi" and not of a general school tradition) is the thorny problem íb.SI/SI$_8$. Early texts that refer closely to post-Šulgi techniques (thus Haddad 104, CBM 12 648) treat it as a noun, having it "come up". Other texts that have results "come up" in the manner of Haddad 104 (and presumably in that of the post-Šulgi tradition) "take" it – thus YBC 6967 (group 5) and YBC 4675 (group 1). In texts closer to the riddle tradition – *tammar* texts, the Tell Harmal texts from 7A with their deliberate riddle imitation, AO 8862 – the term is a verb. This verb, however, functions within a definitely Sumerian phrase structure, 15´.e 30´ íb.si$_8$, and even texts that are almost purely syllabic always write it logographically or with a syllabic spelling of the Sumerian pronunciation. Somehow, the riddle tradition must have taken it from the school, and – since the Post-Šulgi "equalside" is a noun – from a pre-Šulgi school. The (at least to some extent deliberate) unorthographic spelling in some of the Ešnunna texts and the syllabic writing in others also shows that the term must have had a local life of its own and cannot have been a fresh borrowing from the standard formulation of the square root table (I know of no published tables from Ešnunna, but the Mari tables published by Soubeyran [1984] have íb.si$_8$).

The question is intertwined with the problem íb.si$_8$/ba.si$_8$. Haddad 104 and IM 52 301, both of which treat the "equalside" as a noun and have it "come up", have a syllabic *ba-se-e* for the quadratic "equalside". In southern texts, the ba.si (in this spelling, whereas they always have íb.si$_8$) is reserved for the role of a spatial "equalside" (see note 56); in group 6 as well as group 7A, in contrast, the spatial "equalside" is íb.si$_8$,

105 This origin does not exclude that the three persons got a different interpretation in the school. The reason that the structure was generally adopted may well have been its correspondence with the well-known organization of the class room, with master, "big brother" or instructor, and student – see, e.g., [Lucas 1979: 312f].

orthographic even in Ešnunna.

No obvious scenario emerges from this intricate situation; all we can say for the moment is that at least in the Ešnunna focus a synthesis between a lay and a single school tradition does not to describe the historical process satisfactorily. Possibly the publication of further texts will improve the situation.

Index of terms

[NB: In the present electronic version of the preprint, page breaks are up to 5 lines off their original position; some index references are thus not fully precise.]

Index of texts

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