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A new analytical framework for the understanding of Diophantus's *Arithmetica* I–III. (English)

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The aim of the paper under review is to lay the "foundation of a new interpretation of the introduction and the three first books of Diophantus's Arithmetica, one that opens the way to a historically correct contextualization of the work". One may doubt the notion of absolute correctness of historical interpretations, and also observe that the paper deals with Diophantus's text alone and contains no hint of contextualization apart from a peripheral reference to contemporary rhetoric. These general objections notwithstanding, the paper offers promising analytical tools for analysis of the Arithmetica; the concluding pages, it should also be said, are far less absolutist in their claims than the opening.

The analysis is based on a new division of the text. Instead of the enumerated "problems" of the textual tradition also used in Tannery's critical edition, each containing a statement and one or more solving procedures, the "problems" of the present division consist of single procedures with appurtenant statement (which may in consequence belong to several problems). This division makes it is possible to discuss whether Diophantus really progresses from the simple to the difficult, as he promises in the introduction. It also entails that a solution that results in a dead end and calls for solution of a subordinate problem and finally a successful variation of the original approach stands clearly out as three problems, and is not seen as a single one solved by a roundabout procedure.

A number of terms are delineated more precisely than in traditional discussions of problem- based mathematics or as used by Diophantus himself; some are actually new. The essential term is position (hypostasis) and its cognate words ("to posit", etc.). It designates the identification of the magnitudes that are asked for in the statement (or of composites of these) with one of the traditional "elements of arithmetical theory" (as Diophantus calls them in the introduction) arithmós, dýnamis, etc., or composite expressions in these. The analysis is concentrated on a classification of the ways this position is reached – the core of Diophantus's "method of invention", as the authors argue.

One or (in the case of indeterminate problems) several positions may be independent; thus, in I.22, a first number is taken conveniently to be 3x (x standing for the arithmós of the "arithmetical theory", abbreviated ζ), the second to be 4. Other positions (not always spoken of as such by Diophantus) are derived from the independent positions(s), through considerations that may be rather direct; or which may depend on manipulations which since the Middle Ages would have been considered "algebraic", and which the authors designate "simulations" (namely of what could be done in the "arithmetical

theory"), or on the explicit or implicit use of arithmetical (or "algebraic") identities (which the authors for some reason speak of as "algorithms"); finally, they may result from the "formation" of a whole "algebraic" expression that is identified with some quantity from the statement, in a way which allows elimination from a resulting equation. The grid offered by the authors is actually much more detailed but not fit to be repeated here.
Applying the outcome to a complete conspectus of the problems of books I–III, the authors are able to conclude that Diophantus does indeed progress from the simple to the more difficult, as he promises, and that he does teach a kind of method (albeit not a mechanical one). Occasional instances which do not agree with this principle can be argued to represent interpolations or displaced problems; partial agreement with Tannery's identification of interpolated/displaced problems (derived from different considerations) suggest that the method is sound.
Greek terms are transcribed but not always translated; readers who are not familiar with basic Diophantine Greek may need a dictionary but nothing more. Readers should also keep the text of the Arithmetica itself at hand (Tannery's edition with Latin translation can be found at `http://gallica.bnf.fr/`).

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