Indexical Hybrid Tense Logic

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Modality and Modalities

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Part I

The Next 25 Minutes

- Introducing the problem and the project
- Minimal hybrid tense logic
- Contextual semantics
- Basic facts about contextual models
- Results for *now*

Afterwards Patrick will talk about *yesterday*, *today* and *tomorrow*.

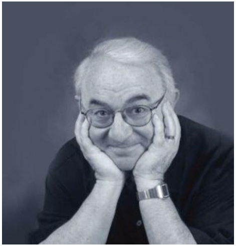
The Logic of Indexicals: Background

Hans Kamp:
"Formal Properties of 'Now' " (1971)

"I learned last week that there would now be an earthquake"

David Kaplan:
"On the Logic of Demonstratives" (1978)

"I am here now"



Kaplan-Kamp Semantics Meets Hybrid Logic

Hybrid Logic started in the context of tense logic (of course). We want to be able to refer to points in time

"It is five o' clock 10 May 2007"

and to define, say, irreflexivity.

As we shall see, Hybrid Logic is a natural context for analysing indexicals like *now, yesterday, today* and *tomorrow*. The project started here:

Patrick Blackburn: "Tense, Temporal Reference and Tense Logic", *Journal of Semantics:* **11** (1994).

Basic Hybrid Tense Logic (1/2)

Syntax. We work with a set $\{p, q, r, \ldots\}$ of propositional symbols and a set $\{i, j, k, \ldots\}$ and diamonds P and F.

$$\phi ::= i \mid p \mid \neg\phi \mid \phi \wedge \psi \mid P\phi \mid F\phi \mid @_i\phi$$

We call this language $\mathcal{L}$.

Basic Hybrid Tense Logic (2/2)

Semantics. Models are given by a frame (T, R) together with a valuation function V. Nominals name points in time. Therefore:

- $V(p)$ is a subset of T, for propositional symbols.
- $V(i)$ is a singleton subset of T, for nominals.

$$
\begin{array}{lll}
\mathfrak{M}, t \models a & \text{iff}, & a \text{ is atomic and } t \in V(a) \\
\mathfrak{M}, t \models \neg\phi & \text{iff}, & \mathfrak{M}, t \not\models \phi \\
\mathfrak{M}, t \models \phi \wedge \psi & \text{iff}, & \mathfrak{M}, t \models \phi \text{ and } \mathfrak{M}, t \models \psi \\
\mathfrak{M}, t \models P\phi & \text{iff}, & \text{for some } t',\ t'Rt \text{ and } \mathfrak{M}, t' \models \phi \\
\mathfrak{M}, t \models F\phi & \text{iff}, & \text{for some } t',\ tRt' \text{ and } \mathfrak{M}, t' \models \phi \\
\mathfrak{M}, t \models @_i\phi & \text{iff}, & \mathfrak{M}, t' \models \phi \text{ and } t' \in V(i)
\end{array}
$$

Adding *Now*

We add the special atomic formula *now* and obtain the language $\mathcal{L}(now)$. Syntactically *now* behaves like a nominal; the following are formulas in the extended language.

$$\phi ::= now \mid i \mid p \mid \neg\phi \mid \phi \wedge \psi \mid P\phi \mid F\phi \mid @_{now}\phi \mid @_i\phi$$

Kamp's Idea: A Second Index

We want $\phi \leftrightarrow @_{now}\phi$ to come out as a *kind of* a validity. If we only work with one index, then

$$\mathfrak{M}, t \models @_{now}\phi \quad \text{iff,} \quad \text{something}$$

This won't work, as we also want

$$\phi \rightarrow G@_{now}\phi$$

to be a validity of this kind.

Semantics for *now* (1/2)

A contextual model $\mathfrak{M}$ is a 5-tuple

$$\mathfrak{M} = (T, R, V, C, \eta)$$

η is a mapping from contexts to points in T. The function η is crucial: it specifies, for any context $c \in C$, what the utterance time (or temporal location) of that context is.

Semantics for *now* (2/2)

The valuation function:

- $V(c, p)$ is a subset of T, (propositional symbols)
- $V(c, i)$ is a singleton subset of T, (nominals)
- $V(c, now) = \{\eta(c)\}$.

Some of the clauses for *satisfiability* in a contextual model $\mathfrak{M} = (T, R, V, C, \eta)$ are:

$$
\begin{array}{lll}
\mathfrak{M}, c, t \models now & \text{iff,} & t = \eta(c) \\
\mathfrak{M}, c, t \models \phi \wedge \psi & \text{iff,} & \mathfrak{M}, c, t \models \phi \text{ and } \mathfrak{M}, c, t \models \psi \\
\mathfrak{M}, c, t \models P\phi & \text{iff,} & \text{for some } t',\ t'Rt \text{ and } \mathfrak{M}, c, t' \models \phi \\
\mathfrak{M}, c, t \models @_{now}\phi & \text{iff,} & \mathfrak{M}, c, \eta(c) \models \phi
\end{array}
$$

Remember, η specifies for any context the moment of that context. Thus, η is what Kaplan calls the *character* of *now*.

Two Kinds of Validities

Logical validity. ϕ is *logically valid*, if ϕ is true in any model $\mathfrak{M}$ at any pair c, t, i.e., if for any $\mathfrak{M}, c, t$

$$\mathfrak{M}, c, t \models \phi.$$

Contextual validity. ϕ is *contextually valid*, if ϕ is true in any model $\mathfrak{M}$ in any context c (evaluated at the moment of utterance), i.e, if for any $\mathfrak{M}, c$

$$\mathfrak{M}, c, \eta(c) \models \phi.$$

Semantical Facts (1/2)

Let $\mathcal{L}$ be the ordinary language without *now*, and let $\mathcal{L}(now)$ be the extended language with *now*.

1. Any ordinary model can be extended to a contextual model which with respect to $\mathcal{L}$ is semantically equivalent to the ordinary model.
2. Formulas in $\mathcal{L}$ are semantically insensitive to how the contexts are hooked up with the frame.

Semantical Facts (2/2)

We see that *now* really just is a special nominal (*play with substitutions*):

3. Any nominal can be swapped with *now*. More precisely, any nominal i in any formula ϕ where ϕ is satisfiable in some $\mathfrak{M}$ can be substituted with *now*, and $\phi[i \leftarrow \mathit{now}]$ is satisfiable in a corresponding model $\mathfrak{M}'$.

4. *now*-consistency reduces to a ordinary consistency. More precisely, consistency of $\phi \wedge \mathit{now}$ reduces to consistency of $\phi[\mathit{now} \leftarrow j] \wedge j$, where j is a fresh nominal.

Results for *now* (1/2)

Let K_h^t be some (complete) standard tableau proof system for minimal hybrid tense logic.

	$\mathcal{L}$	$\mathcal{L}(now)$
Logical validity	K_h^t	$K_h^t(now)$
Contextual validity	K_h^t	$K_h^t(now) + now$

Results for *now* (2/2)

Theorem. Let Λ be K_h^t extended with pure axioms, and $\Lambda + \textit{now}$ be its contextualized counterpart. Then:

1. $\Lambda + \textit{now}$ is contextually complete with respect to the same classes of models as that Λ is logically complete for.
2. Λ-satisfiability has the same complexity as $\Lambda + \textit{now}$ satisfiability.
3. There is a terminating tableau system for $\Lambda + \textit{now}$ iff there is a terminating tableau system for Λ.

Part II