Written Mathematical Traditions in Ancient Mesopotamia: Knowledge, ignorance, and reasonable guesses

Høyrup, Jens

Publication date:
2011

Document Version
Tidlig version også kaldet pre-print

Citation for published version (APA):
Høyrup, J. (2011). *Written Mathematical Traditions in Ancient Mesopotamia: Knowledge, ignorance, and reasonable guesses*. Paper præsenteret ved Traditions of Written Knowledge in Ancient Egypt and Mesopotamia, Frankfurt am Main, Tyskland.

Written Mathematical Traditions in Ancient Mesopotamia

Knowledge, ignorance, and reasonable guesses

Jens Høyrup
Roskilde University
Section for Philosophy and Science Studies
http://www.akira.ruc.dk/~jensh

Contribution to the conference
Traditions of Written Knowledge in
Ancient Egypt and Mesopotamia
Frankfurt am Main, 3.–4. December 2011

Preprint
7 December 2011

Peter Damerow in memoriam
1939–2011

Abstract

Writing, as well as various mathematical techniques, were created in proto-literate Uruk in order to serve accounting, and Mesopotamian mathematics as we know it was always expressed in writing. In so far, *mathematics generically regarded* was always part of the *generic written tradition*.

However, once we move away from the generic perspective, things become much less easy. If we look at basic numeracy from Uruk IV until Ur III, it *is* possible to point to continuity and thus to a "tradition", and also if we look at place-value practical computation from Ur III onward – but already the relation of the latter tradition to type of writing after the Old Babylonian period is not well elucidated by the sources.

Much worse, however, is the situation if we consider the sophisticated mathematics created during the Old Babylonian period. Its connection to the school institution and the new literate style of the period is indubitable; but we find no continuation similar to that descending from Old Babylonian beginnings in fields like medicine and extispicy. Still worse, if we look closer at the Old Babylonian material, we seem to be confronted with a small swarm of attempts to *create* traditions, but all rather short-lived. The few mathematical texts from the Late Babylonian (including the Seleucid) period also seem to illustrate attempts to *establish* norms rather than to be witnesses of a survival lasting sufficiently long to allow us to speak of "traditions".

On ignorance and limited knowledge

In Neugebauer's *Vorgriechische Mathematik* from [1934: 204] we find this warning:

> Unser Textmaterial der babylonischen Mathematik ist im ganzen noch viel zu lückenhaft. Es ist gewiß methodisch nicht richtig, die Texte, die wir besitzen, kurzerhand als etwas Einheitliches zu betrachten. Jeder Text (oder jede Textgruppe) hat seine bestimmte Absicht. Wenn der eine sich mit gewissen geometrischen Dingen beschäftigt, so darf man daraus nicht unmittelbar auf die allgemeine Methode schließen, die für gewisse numerische Fragen, etwa Wurzelapproximationen, angewandt worden ist. So kann also die Voraussetzung, die gewissen Textgruppen zugrunde liegt, ganz anders sein als die von anderen Typen.
>
> Man darf bei allen diesen Fragen nicht vergessen, daß wir über die ganze Stellung der babylonischen Mathematik im Rahmen der Gesamtkultur praktisch noch gar nichts wissen.

When this was written, practically nothing was known about anything but the mathematics of the Old Babylonian and the Seleucid periods. Since then we have learned much about the mathematics of the late fourth and the third millennium BCE, and also something about that of pre-Seleucid Late Babylonian times. We have also come to know four geographically localized text groups from the Old Babylonian period, and are now able to distinguish text groups from this time in a way which Neugebauer could only adumbrate.[1]

Sadly (for a discussion about *traditions*), this has only provided us with a larger number of islands in a vast ocean. At times they seem to form a chain, and as in the case of the Aleutian Islands we may assume that they are connected by a submersed mountain ridge; but others stand out in isolation, and even when connections can be suspected, their precise nature (oral/written/...) and geographic location (transmission within Mesopotamia or through peripheral areas) remains hypothetical.

Unless we accept indirect evidence, Neugebauer's second paragraph remains almost as true today as when it was written. I shall therefore *not* restrict myself to written traditions, since we often do not known whether a particular document

[1] What Neugebauer did in [1932: 6*f*] was to propose a division of the Old Babylonian material known by then into two groups, represented respectively by the Strasbourg texts and the CT IX-texts (Louvre). He further suggested the former to be slightly older and the latter slightly younger, and even that the Strasbourg texts are from Uruk, and that AO 8862, though not properly a member of the Strasbourg group, is still likely to be related to it. Everything agrees with the best knowledge of today!

class is really an expression of a generally written practice or only an accidentally written reflection of a non-literate though certainly numerate culture – and in the former case, whether this practice belonged to an environment of scholar-scribes or less educated people. Of course, I shall try when possible to decide in each case what is the situation – but hopefully not be illuded and go much beyond that.

Elementary numeracy and literacy

During Uruk IV, around 3200 BCE[2], writing was created as a means for accounting, and for no other purpose. Accounting needs also gave rise to the development of metrologies with fixed numerical proportions between units and – in the case of length and area metrologies – geared to each other. From the beginning, basic mathematics – numeration, metrologies, and fundamental calculation – was thus not only part but an essential constituent of the Mesopotamian written tradition.[3]

Part of this tradition did not survive the proto-literate period – or at least did not make it into Early Dynastic III (2600–2350 BCE), the next period from which we possess numerate documents. The bisexagesimal system disappeared – it had served for counting bread or grain rations, perhaps also for portions of dairy products, so changes in bureaucratic procedures is a likely explanation; the "grain system" was reshaped, different city states having different factor sequences; and with some exceptions, the markings that indicate the kind of good being measured (barley, malted barley, etc.) vanished. Other systems survived – the area system and its underlying length metrology, and an administrative calendar where each month is counted as 30 days and each 12 months as a year, serving in the distribution of fodder (it was to be used again in Ur III, now also for distribution of rations and calculation of labour obligations, see [Englund 1988]); most important of all the absolute-value sexagesimal counting system persisted – gradually, the curviform shapes were replaced by cuneiform versions, but for long the two were used side by side, and there is no doubt about its continuous existence. Hypothetically, even the proto-literate notion of fractions can be supposed to have been transformed, not replaced – the phrase used from Early Dynastic III onward when fractional notations turn up again, i g i *n* , might mean something like "*n* (dots) placed in eye (i.e., circle)", which would be a

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[2] Here and in the following I use the "middle chronology".

[3] A convenient summary can be found in [Nissen, Damerow & Englund 1993], which also deals with important aspects of the development until Ur III.

description of the proto-literate notation.[4] All in all, basic mathematics survived as part of the same tradition as the lexical lists, with a similar amount of transformation in continuity.

Where Sumerian was not or no longer the administrative language – for instance, in Old Babylonian and later Babylonia and Assyria – we still find the system, but now coupled with Akkadian number words for one hundred (*mē*) and one thousand (*līmum*). The mathematical tradition – as could be expected – could not be totally stable when the habits of the environment where it served were different or changing.[5]

The place-value system and complex

During Ur III, probably in the wake of Shulgi's administrative reform (2075 BCE),[6] the place-value notation for intermediate calculation was introduced together with the whole spectrum of tools without which it would be useless: tables for metrological conversion (and the "metrological lists", didactical preliminaries to the metrological tables); tables of technical constants; tables of reciprocals; and multiplication tables.

Our evidence that the whole complex goes back to Ur III is indirect but compelling: as Eleanor Robson [1999: 182] has shown, some of the technical constants taught in the Old Babylonian school had gone out of use after Ur III.

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[4] This connection – the only plausible one ever advanced – was first proposed as a possibility by Jöran Friberg [1978: 45], though with an erroneous (and less adequate) interpretation of g̃ á l as "to open".

The occasional Old Babylonian interpretation (i g i ~*pani*, "in front of", namely in the table of reciprocals), found for instance in Haddad 104 [al-Rawi & Roaf 1984: 22] (also proposed by E. M. Bruins [1971: 240]) is certainly a mistaken folk etymology. The phrase was used in Lagash around 2400 BCE [Bauer 1967: 508–511; Lambert 1953: 60, 105*f*, 108, 110; Allotte de la Fuÿe 1915: 132], preceding the creation of tables of reciprocals by more than 300 years.

[5] The numerate culture of Assyria being already in the Old Assyrian period on the whole rather different from what we know from central and southern Mesopotamia, I shall only refer in the following to Assyrian material on a single occasion.

[6] Since the system was used for intermediate calculation, never surviving, and since texts containing only numbers are difficult to date palaeographically, the Ur III date was only indirectly attested until recently; some years ago, however, Eleanor Robson [personal communication] discovered tables of reciprocals found in dated contexts, which definitively settles the matters.

Long before that, small stylistic differences allowed to distinguish older (presumably Ur III) from normal Old Babylonian specimens; see [Oelsner 2001] and [Steinkeller 1979].

But from Old Babylonian Nippur we have direct indications of how the complex was taught as a coherent curriculum [Robson 2002a; Proust 2008].

We also have evidence – though not in detail – that the complex spread (at least in part) as a constituent of the scribal curriculum to regions that had only been submitted for a shorter period (Eshnunna) or not at all (Mari) to Ur III,[7]. Even after the Old Babylonian period, we find traces outside the Babylonian area. Of particular interest is Assurbanipal's assertion [Ungnad 1917: 41*f*, revised interpretation] that he is able to "find reciprocals and make difficult multiplications", which shows that the scholar-scribes of his times (these, indeed, must be the ones who had inspired his literate pretensions) kept the tradition alive – whether in genuine continuity or as part of the same antiquarian interest which sometimes made them emulate the script of the mid-third millennium, which the king claims to understand in the same text [Fincke 2003: 111].

Late Babylonian (fifth-century as well as Seleucid) mathematical texts produced within the environment of scholar-scribes, though insufficient in number to let us know much about traditions at a higher mathematical level (see below), also show that the place-value system and the use of reciprocals were still alive there (as does mathematical astronomy).

What Marvin Powell [1990: 458] calls the "standard (scientific) system" of metrology was largely present already in Shuruppak, to some extent already in proto-literate Uruk; during the Sargonic epoch it underwent some regularization, to be ultimately stabilized by becoming part of the place-value complex during Ur III. The very purpose of that complex was indeed to harmonize the metrological system with the principle of sexagesimal place-value computation; integration of a change in the factor structure of a metrology would only be possible if new metrological lists and tables were created. During the Kassite

[7] Eshnunna broke loose in 2025; interestingly, texts from Eshnunna (to be dated c. 1775 BCE) often use deviant ("unorthographic", that is, phonetic) spellings of íb.si$_8$. They also often use ba.si referring to a square; this (though written ba.si$_8$) is also found in texts from 19th-century Ur, while other Old Babylonian texts only use it about a cube – see [Høyrup 2002a: 253].

Mari had never been directly subjugated, though certainly for a while under Ur III influence; this, however, is a type of political bond which would not automatically entail adoption of administrative or scribal techniques; that tables of reciprocals belonging to the earlier decades of the 18th century are none the less found in the palace archives from Mari is thus evidence of a deliberate adoption of the system – parallel to, but not necessarily concomitant with the deliberate adoption of Eshnunna orthography and syntax [Michel 2008: 255].

period new measures arose, but these were never integrated systematically – Jöran Friberg's survey [1993] of known texts from the later period somehow related to metrological tables show this. Some of them refer to the traditional system, which was thus still, to an extent that cannot be precisely determined, part of the tradition carrying sexagesimal computation; others include some of the new sequences, in a format which reflects the idea of a metrological table but is hardly thought of as an aid to intermediate calculation (at least not place-value computation) – one [Friberg 1993: 391], probably of Late Babylonian date, for instance, expresses "the successive units of length [...] as multiples of one or two of the nearest smaller unit". As Marvin Powell [1990: 469] argues from scattered occurrences in non-mathematical contexts, the new units were probably "more widely used that our sparse evidence suggests".[8] The Late Babylonian scholar-scribes, when taking up interest in basic (and sometimes less basic) mathematics, probably combined whatever was still handed down from the "scientific" system with what was actually used in the world around them, producing something which was neither really the tradition nor a faithful representation of what was done by those who measured and counted professionally.

Area computation

As mentioned, the proto-literate area metrology was geared to the length system, and rectangular areas were determined correspondingly, as product of length and width (where we have no indication of the conceptualization of "multiplication" as an arithmetical operation before Ur III). One model document (that is, a teaching text shaped as a real administrative document) shows that approximately rectangular areas were determined by the "surveyors' formula", as average length times average width [Damerow & Englund 1987: 155 n. 73].

This latter tablet must have served teaching, and we can thus safely presume that the very restricted circle of manager-priest were caring for such matters, which in consequence were part of the incipient written tradition (whether there was any specialization we cannot know); already in Shuruppak, however, surveying and scribal management were no longer fully coincident, and specialization within the scribal profession appears to have taken place – one

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[8] Some fifth-century BCE "sophisticated" texts to which we shall return, which combine "scientific" length metrology with seed measure also show that no genuine integration has been achieved – instead of using the methods connected to the place-value complex they translate by means of a technical constant [Friberg 1997: 260].

contract about the sale of a house [Visicato & Westenholz 2002: 2], for instance, refers to the um.mi.a lú.é.eš.gar, "the schoolmaster who measured the house"; this professional figure is also present in many other contracts, as is the dub.sar.aša$_5$, "surveyor-scribe" [Visicato 2000: 22–25 and *passim*].

At least in Old Babylonian times, surveying appears to have been to some extent incumbent on a "lay", that is, non-scribal profession. However, it also remained part of the scribal curriculum for long. Firstly, the Sargonic school texts that have been identified all deal with (mostly rectangular) areas and their sides [Foster & Robson 2004]; like a grain distribution problem from Shuruppak,[9] their question is invariably marked by the possessive suffix .bi – "its area", etc. Secondly, the same mark (completed however now with the pseudo-Sumerogram en.nam) is sometimes found in Old Babylonian school tablets from Nippur in which square areas are determined – examples in [Proust 2008: 181, 183].[10] It seems reasonable to assume continuity within the scribal educational tradition.

But the school tradition cannot have been the only carrier of agrimensorial calculation between the Sargonic and the Old Babylonian period. This follows *inter alia* from other aspects of the way to ask or answer the question. In many of the Sargonic texts, results are either *seen* or *to be seen* (using pàd or the unorthographic pad) [Foster & Robson 2004: 6]. The same term is used in many of the texts from 19th-century Ur [Friberg 2000], cf. below, but never afterwards in any Old Babylonian text we know about. Instead, the texts from early 18th-century Eshnunna (and later texts from the periphery, not least Sippar and Susa) use Akkadian *tammar*, "you see", when announcing results. A few Old Babylonian texts from the periphery use a new (and not very adequate) Sumerographic writing;[11] texts from the southern former Sumerian core avoid the expression consistently, but a slip in the text YBC 4608, probably from Uruk, shows it to have been known.[12] This suggests (and other evidence corroborates the suspicion) that a lay environment of Akkadian-speaking surveyors was also engaged in area computation; that it used the idiom of "seeing" results; and that this was adopted by the school tradition in the periphery while being known but mostly

[9] A granary of 40·60 gur.maḫ, each of 8·60 sila, of which "each man" receives 7 sila. The question is formulated "Its men". See [Høyrup 1982].

[10] On p. 194, the same phrase is used in a problem about the weight of a brick.

[11] IM 55357, the earliest text from Eshnunna, uses igi.dù, an unorthographic writing of igi.du$_8$. The latter spelling is used in the probably late Old Babylonian "series texts" YBC 4669 and YBC 4673.

[12] It asks what to do *aš-šu* X *a-ma-ri-i-ka*, "in order to see [i.e., find] *X*".

The preceding discussion draws on [Høyrup 2002a: 319–361, *passim*].

avoided in the south. Since the texts from 19th-century Ur never use .bi to indicate questions we may presume that its use of pàd was also no intra-school heritage from the Sargonic period but a translation from Akkadian (after all, it is the regular Sumerian translation, better indeed than igi.du$_8$ – not to speak of igi.dù).

As already hinted at in the discussion of metrologies, the end of the Old Babylonian period probably deepened the split between the scholar-scribes taught in scribal families and those who "measured and counted professionally"; those of the latter who measured land were probably responsible for the area metrologies created in Neobabylonian times [Powell 1990: 482–483]: the "reed measure" based on "broad lines" and thus allowing the measurement of areas in length units[13], and the two slightly different "seed measures", measuring land in terms of the amount of seed needed to plant it and to feed the plough oxen; the modes of thought inherent in broad lines as well as seed measures are those of people engaged in real surveying and agricultural management, not of scholars producing the counterpart of the "rational mechanics" of more recent times.[14]

The sophisticated level: "Babylonian mathematics"

What is spoken of in general histories of mathematics as "Babylonian mathematics", and what together with the arithmetical tables belonging with the place value system occupies almost all space in the famous source editions on which general histories are ultimately based – MKT, TMB, MCT, TMS – is the sophisticated mathematics of the Old Babylonian period, together with a few texts of a similar kind from the Seleucid era.

In the general histories, all of this is treated as one homogeneous body; the text editions, on their part, seem to suggest that at least the Old Babylonian material is homogeneous (apart from Evert Bruins' unfounded claim that the Susa texts distinguish between the Susian and the Akkadian methods – see [Høyrup 2002a: 98 n. 128]).

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[13] On the notion of "broad lines" and its widespread occurrence in pre-modern practical metrologies, see [Høyrup 1995].

[14] The "scientific system" measures volumes in terms of "thick surfaces" provided with a default height of 1 cubit, but "broad lines" are only visible in certain substructures of Old Babylonian mathematics – in particular the use of *našûm* for the multiplication involved in area calculation, suggesting an operation of proportionality.

Actually, MKT and MCT are more perceptive. As pointed out in note 1, Neugebauer had already suggested a separation of the material into two groups in 1932 (corresponding to my preceding distinction between texts from the periphery and from the core), and this was carried over to MKT. MCT contains a whole chapter written by Albrecht Goetze [1945], in which he divides the Old Babylonian corpus as known by then into six groups, purportedly on the basis of Akkadian orthography but in fact also from considerations of vocabulary.

More systematic investigation of the terminology and phraseology has confirmed Goetze's classification, moving only a few dubious texts from one group to another one (and dividing a group which even Goetze had difficulty in seeing as being really a group). Beyond that, several groups of texts found *in situ* (though sometimes badly excavated) and not on the antiquity or black market have been added. The situation as it looks now is described in [Høyrup 2002a: 319–361], on which I shall draw heavily in the following.

In an introduction to a discussion of the shaping of extispicy as a literary form Seth Richardson [2010: 225] writes as follows:

> The Old Babylonian period [...] was a time in which many third millennium cultural forms were being transformed by programmatic revision and political appropriation in the contest to restore geopolitical equilibrium,

This appears to be also relevant for mathematics. The small lot of mathematical texts from (probably) 19th-century Ur mentioned above looks as evidence for the beginnings of the process. Most of the texts are elementary number exercises – four of them, as Friberg [2000: 147*f*] observes, seemingly coming from a small private school teaching only part of the classical curriculum. But there are a few genuine problems. None of them correspond to the favourite types from the mature Old Babylonian period,[15] but they are interesting because they are in a rudimentary problem *format*, which appears to have been absent from

[15] Particularly striking is a problem about the bisection of a trapezium by a parallel transversal (UET V, 858, [Friberg 2000: 142]), a problem whose correct solution goes back to Sargonic times [Friberg 1990: 541], and which has a certain family connection with the "algebra" of the following centuries. In the present case, the ratio in which the sides has to be divided is taken to be given, for which reason the solution becomes trivial.

This is not the place to take up the discussion whether Old Babylonian "algebra" was "an algebra" or not, the answer to which will anyhow depend on definitions; see, for instance, [Høyrup 2002a: 278–282] or [Høyrup 2010: 103–106]. For the sake of simplicity, I shall refer in the remainder of the present article to the technique dealing with square and rectangular areas and their sides (as well as its extensions) as "algebra", retaining the quotes.

the mathematical curriculum of the Ur III period.[16] The question may be made explicit (depending on grammatical case by the regular Sumerian a.na.àm or by the pseudo-Sumerogram en.nam); a few times results are "seen" (pàd or pad).[17]

In a general sense, these texts seem to inaugurate a "tradition" of mathematical problems. However, everything specific is so different from what turns up elsewhere in the Old Babylonian record that it is preferable to see them as an early expression of a "mood" or "culture" characterizing Old Babylonian school mathematics; it appears that the 19th-century Ur expression of this mood left no traces in the later record, and thus did not give rise to (or participate in) a genuine tradition. Its interest lays in its way to show how the general mood could express itself in a reshaping of Ur III mathematics.

Another lot of mathematical texts, published by Denis Soubeyran [1984], is from the palace archive of early 18th-century Mari. It mainly consists of arithmetical tables, but one text (pp. 30–35; seen as a an exponential table by Soubeyran) is of a different kind:[18] an early version of the "chess-board problem" about continued doublings of a grain of barley. There is no hint of a problem format, only the mere calculation; but there is no doubt that the text deals with the well-known and widely circulating problem; it has 30 steps, as was the standard until the spread of familiarity with the chess-board game (after which 30 and 64 were competing).[19] The appearance of this problem is thus another expression of a new mood in the school, and an example of how this

[16] The evidence for this is complex, coming mostly from the presence / absence of Sumerographic writings for terms for operations and terms structuring the format in Old Babylonian mathematical texts. See [Høyrup 2002c].

[17] Except for the appearance of a few Akkadian loan words, the texts are written in grammatical Sumerian – but so grammatical that they seem to be written "grammar book in hand": grammatical elements are not always contracted as they would be in regular Sumerian writing (thus ù.ub. instead of ub., cf. [Thomsen 1984: 208]).

[18] Some of Soubeyran's texts seem not to be mathematical at all. One, for instance (pp. 41–45), deals with the loss of weight of various amounts of precious metal during refinement, not according to expectation but apparently in material processes – the amounts do not form an ordered list, and the relative loss changes from case to case.

[19] A papyrus from Roman Egypt [Boyaval 1971] thus has 30 steps; the *Propositiones ad acuendos iuvenes*, a Carolingian problem collection, also has 30 [Folkerts 1978: 51*f*]; al-Uqlīdisī, Damascus, CE 952 / 53 [Saidan 1978: 337], states that "many people ask [...] about doubling one 30 times, and others ask about doubling it 64 times".

mood led to the adoption of circulating mathematical riddles and "recreational" problems.

The mathematical texts from Eshnunna (Tell Harmal, Tell Dhibaʾi, Tell Haddad) are much more informative. With the possible but unlikely exception of the undated "Tell Harmal compendium" [Goetze 1951], the earliest mathematical text from the region is IM 55357 [Baqir 1950a] from c. 1790 BCE. It deals with the subdivision of a triangle with sides 45´, 1 and 1°15´ into triangles that are similar to it. The choice of parameters is strong evidence that the author was aware of the "Pythagorean rule", at least for these proportions, but the rule is not made use of in the solution. For our purpose, it is perhaps more interesting that we have a rudimentary indication of format: after the presentation of the data follows an explicit question, and the prescription is introduced by the phrase za.e ak.ta.zu.un.dè, "You, to know the proceeding". The writing makes heavy use of logograms, and shares one peculiarity with one of the texts from Ur – namely the use of a.na.àm for the accusative of the question "what" (the nominative is a syllabic *minûm*, whereas the texts from Ur have en.nam). The outcome of calculations are "seen", but the term employed is igi.dù, not pàd. The use of a.na.àm, though not present in other mathematical texts I remember, is therefore hardly evidence of any specific link to the Ur group. Because of the predominance of logograms, we cannot ascertain to which extent the later change of grammatical person[20] was intended.

The remaining Eshnunna texts [Baqir 1950b; Baqir 1951; Baqir 1962; al-Rawi & Roaf 1984] date from c. 1775 BCE. They span much of the thematic spectrum known from later Old Babylonian mathematics, and are characterized by more elaborate problem formats that the text just discussed.

With minor variations depending on exact context, the majority (the ten texts published in [Baqir 1951] and the one in [Baqir 1962]) start "If somebody asks you thus", after which follows the statement in the first person singular, "I have done so and so". This is not the format known from later texts (see note 20) but that of a riddle. This connection to non-school riddle traditions is confirmed by one of the problems, namely IM 53957 ([Baqir 1951: 37], corrections and interpretation [von Soden 1952: 52]):

[20] That is: Statement in the first person singular, past tense; prescription in the imperative or the second person singular, present tense, occasionally with references to the statement as what "he" has said. The implied voices are thus those of the teacher and the instructor – the šeš.gal, "big brother", of edubba texts [Kramer 1949: 209 n.187 and *passim*].

> If [somebody] asks (you) thus: To $^2/_3$ of my $^2/_3$ I have appended 100 silà and my $^2/_3$, 1 gur was completed. The *tallum*-vessel of my grain corresponding to what?

Problem 37 of the Rhind Mathematical Papyrus [trans. Chace et al. 1929: Plate 59] instead runs as follows:

> Go down I [a jug of unknown capacity – JH] times 3 into the *hekat*-measure, $^1/_3$ of me is added to me, $^1/_3$ of $^1/_3$ of me is added to me, $^1/_9$ of me is added to me; return I, filled am I [actually the *hekat*-measure, not the jug – JH]. Then what says it?

The affinities are too numerous to be accidental. Firstly, we notice the shared use of an ascending continued fraction; in the rich Egyptian record of texts using fractions, RMP #37 appears to contain the only ascending continued fraction ($^1/_3$ and $^1/_3$ of $^1/_3$) occurring at all.[21] Secondly, there are the details of the topic: an unknown measure which is to be found from the process, the reference to a standard unit of capacity, and the notion of filling.

The Rhind papyrus solution proceeds in agreement with the normal ways of Egyptian arithmetic, making elaborate use of the system of aliquot parts and the appurtenant "red auxiliary numbers". The Eshnunna solution, on the contrary, is a mock solution, a sequence of operations which only yield the correct result because the solution has been presupposed. It is nothing but a challenge meant to impress and make fools of the non-initiate and teaches no useful mathematical procedure. In other words, it is a genuine riddle posing as a mathematical riddle – a type which also turns up in other sources drawing on oral or semi-oral practitioners' traditions.[22]

We thus have good evidence that the creators of the Old Babylonian school tradition did in mathematics as the diviners had done in their field (according to Seth Richardson): borrowing from oral practices, and putting into order. The similarity with divination is also (though only superficially) reflected in the language. Taha Baqir explains [1951: 29] that

> In a preliminary classification, these tablets and some others which will be dealt with in coming issues of "Sumer", were wrongly labelled as, "probably religious or omen texts", probably because they start with the phrase,"shumma ishalka" etc.[23]

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[21] In Semitic languages, Akkadian as well as Arabic, it is instead a standard way to express difficult fractions, see [Høyrup 1990].

[22] One example, contained in the Carolingian *Propositiones ad acuendos iuvenes* [ed. Folkerts 1978: 47*f*], explains how two merchants selling swines at the same price as they bought them for none the less make a profit.

[23] It should be observed, however, that the opening *šumma*, standard in legal, divinatory and medical texts, only characterizes a subset of the mathematical problems. The similarity

The filling problem and the continued doublings from Mari may have been adopted from a merchants' environment – the presence of the same problem structure in Eshnunna and Pharaonic Egypt suggest travelling merchants. Much more important than these, however, are problems that refer to surveyors' practice: problems dealing with rectangles, trapezia, and measuring reeds that break.

Not all the Eshnunna texts are derived from riddles or formulated as riddles (not the same thing, formats may be borrowed). The long text Haddad 104 published in [al-Rawi & Roaf 1984] mostly contains rules and problems falling within the range of Ur III scribal calculation (capacity of containers, quantity of labour needed for a specified piece of work, etc.). The format here is similar to that of the early triangle division IM 55357, but in syllabic Akkadian and more elaborate: grammatically neutral explanation of the situation (though at times preceded by *nēpeš*, "procedure of", or, if a variant is concerned, by *šumma*, meaning "if [instead]"); and prescription preceded by *atta ina epēšika*, "you, by your making". Mostly, the prescription closes by *kīam nēpešum*, "thus the procedure".

The effort to develop the problem format can also be seen in the texts published in [Baqir 1951]. Nine of these ten texts were found in the same room in a private house,[24] and the tenth in the immediate vicinity; one of them is the mock filling calculation mentioned above, and all are in riddle format "if somebody ...". Prescriptions open with the phrase *atta ina epēšika*; closing phrases are absent.

The ten texts have other characteristics in common, several of which are not even shared with other texts from Eshnunna. Results may either be "seen" or "come up" (*elûm*); in the latter case, they are invariably asked for by the word *minûm*, in the former always with the phrase *kī maṣi*, "corresponding to what"; only the tablet not found in the same room as the other nine uses both. Length and width of rectangles occurring in "algebraic" problems are invariably written with the logograms u š and s a g, never with grammatical or phonetic comple-

concerns a style which in mathematics was neither compulsory nor connected by necessity to substance; it is not evidence of properly parallel intellectual endeavours.

[24] A complete list of the texts found in this room (and of whatever else may have been found) could be our first hint of the social setting of Old Babylonian sophisticated mathematics – which, though written in "school format", appears not to have been part of the normal scribal curriculum as we know it from Nippur and as analyzed by Christine Proust [2008]

ments;[25] if real distances are meant (including the dimensions of a field measured by a reed which breaks), the writing is phonetic, as *šiddum* and *pūtum*. The "logical particles" *aššum* ("since"), *inūma* ("as") and *šumma* ("if") are absent (except for the appearance of the latter word in "if somebody"). The plane "equalside" (the square parametrized by its side) is always treated as a verb ("what is equal"), and always appears in unorthographic (or rather, phonetic) writing as ib.si or ib.se.e. The cubic equalside, on the other hand, is ib.si$_8$ (still a verb) the only time it appears. Subtraction by removal is usually *ḫarāṣum*, "to cut off", a term apparently without Sumerographic equivalent in the mathematical texts.[26]

The text Db_2-146 [Baqir 1962] has much in common with these ten texts, not least the riddle introduction.

Looking at the whole Eshnunna corpus we find, firstly, outspoken efforts to create terminological and structural uniformity; secondly, that authors even a few kilometres and at most a decade apart did not agree on *how* this uniformity should look.

Eshnunna was conquered by Hammurapi in 1761 BCE, after which we know about no more mathematical texts from the area. The beginning of sophisticated mathematics in the south may perhaps be dated just *after* this event. In any case, the prism AO 8862, according to internal criteria probably an early exponent of this development[27] is almost certainly from the same place and approximately the same time as a prism carrying tables of squares, inverse squares and inverse cubes which was written in Larsa in 1749 BCE [Robson 2002b].[28]

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[25] This was to be the general norm. However, precisely in the early Eshnunna texts we see that it was a *choice*. The "Tell Harmal compendium" ([Goetze 1951] – a catalogue of problem types, undated because it was found on the ground (left behind after an illegal digging) but probably contemporary with the other texts – sometimes writes uš with a phonetic complement (the possessive suffix *-ia*), and sometimes uses phonetic writing. This alternation corresponds to the pattern we find with terms where no strict technicalization is attempted; its virtual absence from the "algebraic" texts is thus evidence of precise awareness of the particular technical role of uš and sag as "algebraic variables".

[26] Lexical lists give kud, which to my knowledge appears only with this possible meaning in the atypical mathematical Susa text TMS XXVI [TMS, 124*f*] – but the intention there might just as well be *nasāḫum*, as supposed by Bruins in his transcription and commentary, even though this would also by singular. Normally, kud when used in Old Babylonian mathematical texts stands for *nakāsum* or *ḫasābum*.

[27] See the analysis in [Høyrup 2002a: 162–174].

[28] As Robson points out, we have a mathematical text from Larsa from the late 19th century

If this dating (of the prism, and of the beginning of sophisticated mathematics in the south) is correct, it leaves a short time span only for its development. From around 1720 BCE, that is, from the successful secession of the Sealand, we have very few dated documents, and the main cities appear to have been depopulated; there is no reason to assume that this is not evidence of a general decline of high literate culture in the area. Already twenty years before that, after an earlier rebellion, the emigration of scholar-priests toward the north seems to have begun.

The sophisticated mathematical texts produced in the south thus represent something like snapshots of local "styles" or "schools".[29] Most of them belong to four more or less well-defined text groups, two of which are likely to come from Uruk and one from Larsa. They are described in [Høyrup 2002a: 333–349]. Beyond certain orthographic characteristics, they all have in common the avoidance of the idea of "seeing" the outcome of calculations. Other conspicuous features, however, allow us to differentiate.

We may look first at the two Uruk groups – labelled "group 3" and "group 4" by Goetze [1945]. The mathematical language is characterized by multiple possibilities to express the same operation or process – we have already encountered some of them. The "equalside" (the side of a square area or cubic volume) may be treated as a verb or as a noun; in group 3, it is consistently a verb, in group 4 a noun. Bisection (*ḫēpum* / g a z , literally "breaking") may be explained to be "into two"; so it is consistently in group 4, but never in group 3. The prescription may open with an elaborate formula "you, by your making", and always does so in group 3; or this may be reduced to a mere "you" or be totally absent, which are the two possibilities used in group 4. Similarly concerning a number of other features of the terminology as well as of the way to structure problems by means of logical operators.

Each group is so internally consistent that its texts are likely to come from the same school (and perhaps school room), and thus also to have been produced within a rather short time span. On the other hand, the formats of the two groups differ so clearly from each other that none of them can have descended from the other, neither by reduction nor by elaboration. They represent two different

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BCE – but a multiplication table (YBC 11924, [MCT, 23]). Edubba texts on similar prisms from Larsa reflecting the ideology of the school are dated 1739 BCE.

29 This discussion concerns *formats*, which best characterize particular written traditions. Similar *problems*, *problem types*, and *methods*, on the other hand, are found in all groups, in the south as well as the northern periphery; they can thus be seen to have travelled, and to have provided that shared cultural framework of which we speak as "Old Babylonian mathematics".

ideas of how a mathematical problem should look, and two different attempts at norm-setting. Maybe they even express mutual deliberate rejection (Uruk being a large city this is not certain).

Group 1 is probably from Larsa. AO 8862, just discussed, belongs to this group. The group is less uniform that groups 3 and 4, and even on the same tablet different problems may use different formats – see [Høyrup 2002a: 337–345]. Sometimes these differences may point to differences in inspiration – in AO 8862, "algebraic" problems about rectangles and their sides differ from those that deal with bricks, a traditional scribal concern; but even the "algebraic" problems do not fully agree on the choice of terminology. Similarly, four "algebraic" problems about the same geometric configuration in YBC 6504 do not agree with each other in this respect. The texts from the group are likely to have been produced within a single environment, but perhaps over some time; in general the group appears to offer evidence of experimentation rather than codification.

Group 2 [Høyrup 2002a: 345–349], not even hypothetically located better than "in the south", presents us with a new phenomenon – extensive "theme texts". Since it is a theme text (containing 24 "algebraic" problems about one or more squares), Goetze also included the text BM 13901 in the group, but the inclusion can now be seen to be ill-founded. The theme texts that remain, and which are almost certainly made within the same environment though not by the same author, deal with excavations (ki.lá) and small canals (pa$_5$.sig). They are characterized by combining geometric or "algebraic" calculation with determination of the labour costs of producing the objects. Beyond the theme texts, the group encompasses a number of statement catalogues, in part corresponding to known theme texts – an extraordinary luck, and a strong indication that the texts really come from the same find spot.[30] Some slips indicate that the texts

[30] The catalogue YBC 4612 [MCT, 103*f*], dealing with simple rectangle problems, is written in a courser ductus than the catalogues that with certainty belong to the group; otherwise it is similar to them, but not sufficiently similar to eliminate all doubts concerning its appurtenance.

In any case, this simple text may be of singular interest, as it seems to provide the missing link between the area computations that represent the culmination of the normal mathematical syllabus and the sophisticated "algebra" problems. It contains 15 problem statements about rectangles, with answers. As in the catalogues certainly belonging to group 2, the format is rudimentary: a grammatically neutral and almost purely logographic presentation of the situation, the question marked by .bi en.nam, and answer – precisely as in the Nippur area problems solved by students that were discussed above, before note 10. Everything is stated in specified length and area metrology, whereas the were inspired by material of northern origin, and that the authors attempted to reformulate this material in a way prescribed by their own norm.

The creation of theme texts and of corresponding catalogues is evidently a parallel to what happened in other domains of Old Babylonian scholarship like extispicy, astrological divination and medicine [Maul 2005: 71; Rochberg 2004: 63; Rochberg 2006: 347; Glassner 2009: 3; Geller 2010: 42]. The affinity is enhanced by the fact that the catalogue texts indicate the number of sections they contain (normally on the edge, which would allow this number to be read when the tablet was on the shelf).

Among the northern texts, Goetze's group 5 [Høyrup 2002a: 332] is too small to say much – it consists of one complete and fairly well-preserved text, a fragment and a heavily damaged text. It exhibits some similarities to Haddad 104 [al-Rawi & Roaf 1984], referred to above as containing "rules and problems falling within the range of Ur III scribal calculation", that is, of the Ur III tradition as digested in Eshnunna. It cannot be decided whether the similarities between the three texts that constitute the group reflect a deliberate attempt to adjust to or develop a norm or merely reflect loose local habits.

Goetze's group 6 (as augmented with texts belonging to the same family and published or identified as such in the meantime) is much more extensive [Høyrup 2002a: 329–332]. One of its members mentions a name in the colophon which is likely to be from Sippar [Robson 1999: 240 n. 26], which agrees well with the shared orthographic habits of the group.

A subgroup (labelled 6A in [Høyrup 2002a]) is so uniform that it certainly comes from a single school with a particular norm. A few more texts differ from this subgroup on several accounts but are still sufficiently close to allow us to distinguish a local style.

"algebra" texts usually leave units implicit (or, said in another way, remain within the domain of place value calculation, where all units have been transformed into tacitly assumed basic units). The problems fall into three groups, the last of which varies the two sides and ask for the area; they correspond precisely to the student exercises. The former two groups both: (1) start by stating the sides, asking for the area, and then go on with four problems where the area is given together with (2) the length, (3) the width, (4) the sum of length and width, or (5) the difference between them. (1), (2) and (3) are already present in the Sargonic school texts. (4) and (5) are not, but they are the basic "algebra" problems.

In other Old Babylonian text – e.g., YBC 6504 – we see that the types (2)–(5) were regarded as a closed group, but too elementary to presented directly; therefore they had to be embedded in more complicated situations, or submitted to variation.

6A encompasses both theme texts (including BM 85200 + VAT 6599, famous for treating irreducible cubic problems about "excavations" but indeed also problems of the first and second degree about this configuration) and a catalogue (BM 80209, see [Friberg 1981]); the theme texts indicate the number of sections, as did the catalogues from group 2, but they are much less orderly than these (bordering upon the class of "anthology texts") and in so far less related to the omen and medical series emerging at the time.

Certain features of the texts show a still living contact with the lay surveyors' environment. Some of these features (and a number of others) also indicate affinity with the texts from Eshnunna – not least the use of *tammar*, "you see", for the results of calculations. Goetze's claim [MCT, 151], advanced before the Eshnunna texts were known, that the "6th group comprises northern modernizations of southern (Larsa) originals" can be put safely to rest.

The mathematical texts from Susa,[31] presumably from the outgoing Old Babylonian period, are also in "northern" style and with a single exception coherent enough to be regarded as expressions of a particular normative ideal. Their being found together already shows them to have belonged to the same archive; the explicit didactic character of several of the texts (explaining concepts, not solving problems, see [Høyrup 2002a: 85–95]) confirms that this must have been some kind of school archive. It contains some of the most intricate problems ever dealt with in Old Babylonian mathematics – not least TMS XIX, which solves a bi-biquadratic problem – and also presents us with the first known experiments with intermediate zeroes (in text XII).[32] The sign is sufficiently close to what is used in Seleucid texts to make us suspect a link; but since it is nothing but the separation sign, reinvention is not to be excluded.

We shall close the discussion of Old Babylonian mathematical text groups by the "series texts", which certainly constitute the closest parallel to the scholarly series produced in domains like divination and medicine. The texts were given

[31] Published and (often badly) translated and commented upon in TMS – and also badly excavated by an expedition that was not interested in mud-brick structures or the provenience of tablets, see [Robson 1999: 19] and [MCT, 6 n. 28].

[32] In order to understand that these are intermediate zeroes one should realize that the place value system was not really sexagesimal but seximal-decimal, as the Roman number system is dual-quintal. So, it stands (three times) where a 1-place is empty between two 10-places: 1.30▴▴16.40, 5.7.30▴▴41.40, 1.30▴▴16.40. The "zeroes" are there not in order to eliminate (non-existent) ambiguity but as a matter of principle.

the name by Neugebauer [MKT I, 383*f*] because the tablets are indeed numbered as members of series.[33]

The texts are written in an utterly compact logographic style, and often the single statement can only be understood in the context of those that precede it, since it often just indicates the variation with respect to what comes before and not the complete set of data. The variation is highly systematic, organizing the variation of up to four parameters in Cartesian product.[34] Similar aims can be found in other fields where series were produced – but their subject-matter did not permit a similar unfolding of the principle, as illustrated by this excerpt from the "Diagnostic Handbook" going back to c. 1700 BCE [Geller 2010: 90, cf. p. 42]:

> [If] his urine is like ass urine, that man suffers from "discharge".
> [If] his urine is like beer dregs, that man [...]
> [If] his urine is like wine dregs, [...]
> [If] his urine is like clear paint, [...]
> If his urine is like *kašu*-juice, [...]
> If his urine is yellow-green, [...]
> If his urine is white and thick, [...]
> If his urine is like *dušû*-stone, [...]
> If his urine is as normal, but his groin and epigastrium cause [him] pain, [...].

It is difficult to determine with precision the geographical origin of the series texts. Neugebauer in MKT suggested Kish, with arguments that he himself and Sachs eliminated in [MCT, 95], together with the whole category [MCT, 37].[35]

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[33] Since such serialization was a widespread phenomenon in late Old Babylonian scribal culture, it is not to be excluded that serialization of mathematical texts was initiated in several places. Friberg [2000: 164] suggests to move VAT 7528, YBC 4669, YBC 4698 and YBC 4673 (all classified as series texts in MKT) to a "group 2b", related to the expurgated group 2 (which he calls "2a", following [Høyrup 2000], and which already Neugebauer [MKT I, 506] has regarded as a separate "Gruppe C"). He could be right – apart from the absence of serial numbering from the group-2a catalogues there are outspoken similarities.

[34] Christine Proust, who is undertaking a new profound study of the text group, speaks of "tree-structured lists" [2010] or "schéma arborescent à 4 niveaux" [Proust 2009], which is adequate if (and only if) we think of all branches at the same level splitting up in the same way.

[35] Neugebauer and Sachs argue that the same number might be given to different texts (which however only shows that no single canonical series similar to *Enūma Anu Enlil* existed in mathematics), and that therefore

> the numbering of these texts implies nothing more than an arrangement of tablets of various groups by a scribe to keep them in order.

However, as it has turned out, even the mature *Enūma Anu Enlil* exists in several variants, In [Høyrup 2002a: 351*f*] I conclude from a sequence of arguments of which none are fully coercive when taken in isolation

> that the series texts are less closely related to group 6A than believed by Neugebauer; that they will have been produced somewhere in the peripheral orbit – that is, outside the ancient Ur III core area. If we look at the problem types where n u . z u and a . n a u š u g u s a ĝ d i r i g and their syllabic equivalents turn up in groups 1 and 3 (broken-reed and stone riddles, etc.) we may also infer that the series texts, in spite of their sophistication and highly technical language, were produced in a place where the riddle tradition was closer to the surface than in the school where (e.g.) group 6A was produced and used.

Friberg [2000: 172] concludes from analysis of the use of Sumerograms that

> the general impression one gets is that the Sumerian terminology of the mentioned [main] group of series texts, Group Sa, is closest to that of Group 3, the one assumed to be from Uruk (in spite of what Hoyrup claims, op. cit.)

while Proust [2010: 3] suggests

> that the structure of the colophons might speak in favor of a connection between the mathematical series texts and a tradition which developed in Sippar at the end of the dynasty of Hammurabi

more precisely, during Ammiṣaduqa's reign – cf. also [Proust 2009: 195].

As to the time when the mathematical series texts were produced, we also have to content ourselves with indirect argument. Proust's observation of the similarity with dated colophon's from the late 17th century is supported by the observation that the utterly intricate elaboration of the texts show them to be the end product of a long development. This, on the other hand, can be combined with our general knowledge of history: mathematical texts written at that moment can hardly have been made in the Sealand, and thus not in the former Sumerian core (Ur, Larsa, Uruk); they may, on the other hand, have been produced by scholarly emigrants from the south, which would explain the features shared with texts from groups 1 and 3.

In the end, there turned out to be a fundamental difference between the genre of mathematical series texts and other incipient serializations like k i + *n*. The latter were adopted by the scholar-scribes of the Kassite and later times, giving rise to the large series we know from the Assyrian libraries. The former, like

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and in general the attempt to create standardized ("canonical") series seems to belong to the Kassite period [Rochberg-Halton 1984: 127*f*]. The extispicy texts k i + *n* [Glassner 2009: 24–29] would fit Neugebauer's and Sachs's discussion no less well than the mathematical texts.

the whole fabulous enterprise of Old Babylonian sophisticated mathematics, did not survive the breakdown of the Old Babylonian cultural complex. Mathematics may serve for warfare and already did so in the Bronze Age,[36] but it appears to be better served itself by peace. At the conquest of Eshnunnna, it could follow the victors to the south and flourish in the *pax babyloniaca* (relative as it was), even though the choices of format indicate that it was a general idea and not a precise written tradition nor a well-defined professional carrier group that made the transfer. When the southern cities fell to the Sealand, some carriers of the tradition might still go north – but at the Kassite take-over, there was nowhere left to go. Divination and magic could survive in "inner emigration" within the scribal families and eventually re-emerge; mathematics, if admitted, withered away.

Late Babylonian sophistication

One seemingly sophisticated – but actually pseudo-sophisticated – text does seem to come from the Kassite period: AO 17264. It deals with a topic dear to Old Babylonian calculators: a trapezoidal field divided by parallel transversals into strips – here six strips that are pairwise equal in area. As Lis Brack-Bernsen and Olaf Schmidt conclude after analyzing the text and the mathematics of the problem, it

> is beyond the capability of Babylonian mathematicians, and it looks as if they have given up in despair in their attempt at solving this problem and just given some meaningless computations that lead to a correct result.

The solution is indeed another mock solution, not mathematics but just mystifying calculations. The Kassite date, originally suggested by Thureau-Dangin [1934: 61] for palaeographic reasons, is supported by the terminology and format [Høyrup 2002a: 387*f*]. It is of vaguely northern type, but not similar in details to anything known to be Old Babylonian. It suggests (nothing more!) conservation within a scribal family of some memory of the high level of Old Babylonian mathematics and a rather vane ambition to show that the author was still at that level. In any case, we have to wait until the fifth century BCE before we find a few texts which are somehow akin to Old Babylonian "algebra".

The texts in question have been published in [Friberg, Hunger & al-Rawi 1990] and [Friberg 1997]. According to [Friberg 2000: 175*f*]

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[36] The "siege calculations" of mathematical texts are certainly artificial, but they are none the less witness of a kind of practice where volume calculation (etc.) was applied.

> these texts contain what must be Late Babylonian reformulations of Old Babylonian mathematical problems, with the ninda as the basic unit of length and the square ninda as the basic unit of area, as well as obviously Late Babylonian mathematical problems, with the cubit as the basic unit of length and surface extent measured in terms of either seed measure or reed measure.

However, the texts that combine the "standard" length metrology (still present in lists, we remember) with the new area metrology show in other respects *not* to be mere reformulations of Old Babylonian texts except in a very vague sense. They deal with rectangles for which the area is known together with one of the sides; the sum of the sides; or the difference between them. As we remember from note 30, these are the simple problems that so to speak hide below Old Babylonian algebra but were too simple to be presented directly; half of them, we also remember, were already taught in the Sargonic school. More decisively, they belong to that small set of surveyors' riddles that was borrowed by the Old Babylonian mathematics teachers and developed by them into the "algebraic" discipline.[37]

As can be read in a colophon, the texts in question belonged to a scholar-scribe from the fifth century BCE.[38] It is therefore informative that one of the Sumerian terms occurring in the texts (n i m , "lift up") is used differently than in the Old Babylonian period. In Old Babylonian mathematics it had been one of the logograms that could designate the "multiplication by proportionality" (Akkadian *našûm*), by now it meant "subtract" (namely, by lifting up from the counting board[39]). In corroboration of what was said above, it appears that the Late Babylonian scholar-scribes, when taking up interest in mathematics, probably combined whatever was still handed down from the "scientific" system with what was actually used by "those who measured and counted professionally" and with substance borrowed from these. What these people did was probably already carried out in Aramaic, and written not on clay but on wax tablets or on papyrus; the colophon just mentioned states indeed that the text is copied from a wax tablet [Friberg, Hunger & al-Rawi 1990: 545]. It is therefore not

[37] This set of riddles, together with its widespread influence and duration until the Sanskrit, Islamic and even Latin / Italian Middle Ages, is discussed in [Høyrup 2001].

[38] See ([Friberg, Hunger & al-Rawi 1990: 545]; dating from [Robson 2008: 227–237]).

[39] This was also the original (Ur III) sense of z i, in Old Babylonian times used as a logogram for *nasāḫum*, "to tear out", the concrete, "identity-conserving" subtraction. Irrespective of language change and interruption of textual traditions, material calculational practice had remained the same. Not all traditions in Mesopotamian mathematics were written.

possible to claim that these texts are really part of a written tradition belonging to the scholar-scribes, they may as well represent an attempt to *re-establish* a tradition which was known to have been lost – in the way twelfth-century (CE) Latin scholars struggled to reconquer a Greek scientific and philosophical heritage whose existence they only knew about from late ancient Latin encyclopediae. Perhaps they represent a temporarily successful attempt, whose continuation we only have not been fortunate enough to find, perhaps they are nothing but the remains left over after a failure.

In any case, the next small group of sophisticated texts we know about, written some 200 years later, is again quite different in character. Apart from a particular kind of second-degree "algebra" asking for the value of a pair of reciprocal numbers (*igûm* and *igibûm*) whose sum or difference is given (an application of the simple rectangle problem structure), already popular in the Old Babylonian period and probably handed down together with the place value system, what we find in the Seleucid texts are again geometrical riddles – now in pure numbers, as in the Old Babylonian period, but involving for instance the sum of the sides and the diagonal of a rectangle and using new (but still geometric) techniques. Since the same problems turn up at approximately the same time in sources from Demotic Egypt [Høyrup 2002b], they cannot have been developed and kept within a closed environment of scholar-scribes, as supposed by Eleanor Robson [2008: 261*f*]. One problem in a text which otherwise contains "algebraic" rectangle diagonal problems (BM 34568 #16, in [MKT III, 16]), moreover, deals with a cup consisting of an alloy of gold and copper – a type which was to become very popular in medieval merchants' arithmetic. Once again, what we see is a reflection of the impact of external traditions (literate, semi-literate or oral, we do not know) on the cuneiform-scholarly environment, in an interesting replay of the influence of similar traditions on Old Babylonian (and, to a much more restricted extent, Pharaonic) mathematics – but with the difference that this time no lasting tradition or mathematical culture resulted within the cuneiform literate world (which by then was reduced to a tiny though stubborn *arrière-garde*); Marx's adage about history being played twice, first as tragedy and then as farce, comes to mind.

Summing up

As we have seen, basic mathematical techniques were handed down within the cuneiform literate tradition over very long periods, some of them (part of the metrology) over a time span longer than the one which separates us from Homer. We may suspect but often cannot specify interactions between this literate and other less literate ("lay") traditions.

When looking instead at what is mostly thought of as "Babylonian mathematics", namely the sophisticated level, it is much more difficult to distinguish true traditions. The Old Babylonian period presents us with a mathematical culture of high level, and when we look at details we find attempts to *establish* standards and traditions – but all of them apparently short-lived, for internal or external reasons. Parallels to the omen or grammar traditions beginning in Old Babylonian times and still alive in the later first millennium cannot be found.

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