The rule of three
Materials for discussion

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19 May, 2011

FROM CHINA TO PARIS:
2000 YEARS TRANSMISSION
OF MATHEMATICAL IDEAS

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Kuppanna Sastry sees the first mention of the Rule of Three in the following verse of the *Vedāṅgajyotiṣa*, "The known result is to be multiplied by the quantity for which the result is wanted, and divided by the quantity for which the known result is given."[4] Sastry goes on to say:

> The instruction is concise and looks like an aphorism. There are four items in a proportion, three known and one unknown, which is obtained from the three known. Hence the rule to get this is called the "Rule of Three." The four items are: (a) If for so much quantity, (b) so much result is got, (c) for this much quantity given now, (d) how much is the result that will be got? The first two are called *jñāta-rāśis* and the next two are called *jñeya-rāśis*. The application of the rule is: Take the known result, i.e. (b), multiply it with the quantity (c) for which the result is to be known, and divide by the quantity (a) for which the result is given; thus the result to be known, i.e. (d), is got.

Kurt Elfering

DIE MATHEMATIK DES ĀRYABHAṬA I

Text, Übersetzung aus dem Sanskrit und Kommentar

Wilhelm Fink Verlag

München 1975

26 Trairāśikaphalarāśiṃ tam athecchārāśinā hataṃ kṛtvā/
labdhaṃ pramāṇabhajitaṃ tasmād icchāphalam idaṃ syāt//

Nachdem man in der Regeldetri die Größe "phala" mit der Größe "icchā" multipliziert hat, wird das Zwischenergebnis durch "pramāṇa" dividiert, von diesem (Ausdruck) sei dieser (Quotient) das gesuchte Ergebnis.

Vor einer Erklärung dieser im Grunde einfachen Regel, müssen die bewußt unübersetzt gelassenen Fachausdrücke erläutert werden: Es sei:

a:	pramāṇa	Maß, Norm
b:	icchā	Wunsch, Fragestellung
c:	phala	Frucht, Effekt, 3. Glied einer Proportion
x:	icchāphala	Ergebnis, gesuchte Größe.

Das erste Wort "trairāśika", wörtlich "die drei Größen" in der Bedeutung "die Regel von den drei Größen", kann man mit Regeldetri bzw. Dreisatz übersetzen.

"rāśi" ist ein Grundbegriff in der indischen Mathematik [2.1.38;1] Die eigentliche Bedeutung ist "Masse, Haufen, Gruppe, Aggregat"; in der Mathematik: "Zahl, Größe, besonders Maßgröße, zusammengesetzte Zahl, Aggregat". In der Astronomie: "Abschnitt auf dem Tierkreis, 1/12 der Ekliptik."

ALGEBRA,

WITH

ARITHMETIC AND MENSURATION,

FROM THE

FROM THE

SANSCRIT

OF

BRAHMEGUPTA AND BHÁSCARA

TRANSLATED BY

HENRY THOMAS COLEBROOKE, Esq.

LONDON:

JOHN MURRAY, ALBEMARLE STREET.

1817.

BRAHMEGUPTA.

10. In the rule of three, argument, fruit and requisition [are names of the terms]: the first and last terms must be similar.[1] Requisition, multiplied by the fruit, and divided by the argument, is the produce.[2]

[1] The middle term is dissimilar. CH.

[2] The rule concerns integers. If there be fractions among the terms, reduce all to the same denominator. CH.

Bhāskara II LI'LA'VATI'.

70. Rule of three terms.[2]

The first and last terms, which are the argument and requisition, must be of like denomination; the fruit, which is of a different species, stands between them: and that, being multiplied by the demand and divided by the first term, gives the fruit of the demand.[3] In the inverse method, the operation is reversed.[4]

[1] The rule of proportion, direct and inverse, simple and compound, including barter, has been similarly treated by BRAHMEGUPTA, Arithm. § 10—13; and by ŚRÍD'HARA (adding, however, as a distinct article, the sale of live animals and slaves, which BHÁSCARA places under the rule of three inverse). *Gaṅ. sár.* § 58—90.

[2] *Trairáśica*, calculation belonging to a set of three terms.—GANG. Rule of three.

The first term is *pramáṇa*, the measure or argument; the second is its fruit, *phala*, or produce of the argument; the third is *ich'há*, the demand, requisition, desire or question. GAṄ.

[3] *Ich'há-phala*, produce of the requisition, or fruit of the question: it is of the same denomination or species with the second term.

[4] See § 74.

THE

GAṆITA-SĀRA-SANGRAHA
OF
MAHĀVĪRĀCĀRYA

BY

M. RAṄGĀCĀRYA, M.A., Rao Bahadur,

MADRAS:
PRINTED BY THE SUPERINTENDENT, GOVERNMENT PRESS.
1912.

Next we shall expound the fourth subject of treatment, viz., rule-of-three.

The rule of operation in respect thereof is as follows :—

2. Here, in the rule-of-three, *Phala* multiplied by *Icchā* and divided by *Pramāṇa*, becomes the (required) answer, when the *Icchā* and the *Pramāṇa* are similar, (*i.e.*, in direct proportion); and in the case of this (proportion) being inverse, this operation (involving multiplication and division) is reversed, (so as to have division in the place of multiplication and multiplication in the place of division).

Agathe Keller

Expounding the Mathematical Seed

Volume 1: The Translation

A Translation of Bhāskara I on the Mathematical Chapter of the Āryabhaṭīya

Birkhäuser Verlag
Basel · Boston · Berlin
2006

[The Rule of Three]

In order to explain the Rule of Three (*trairāśika*) he states an *āryā* and a half:

> **Ab.2.26. Now, when one has multiplied that fruit quantity in the Rule of Three by the desire quantity|**
> **What has been obtained from that divided by the measure should be this fruit of the desire||**
> **Ab.2.27.ab The denominators are respectively multiplied to the multipliers and the divisor.|**

trairāśikaphalarāśiṃ tam athecchārāśinā hataṃ kṛtvā|
labdhaṃ pramāṇabhajitaṃ tasmād icchāphalam idaṃ syāt||

chedāḥ parasparahatā bhavanti guṇakārabhāgahārāṇām|

Trirāśi is three quantities assembled (a *samāhāva dvigu* compound). *Three quantities assembled are the purpose (prayojana) of this computation (gaṇita), and therefore (iti) it is [called] trairāśika. Trairāśikaphalarāśi* is **the fruit quantity in the Rule of Three** (a locative *tatpuruṣa*). That fruit quantity in the Rule of Three.

The word "**now**" (*atha*) is used when finishing a subject of knowledge and explaining the words that follow. Here such a kind of meaning ⟨is understood⟩.

⟨Question⟩

What is explained in this case?

This is stated: ⟨The word "now" is ⟩ a speech (*paribhāṣa*). And since this ⟨speech⟩ is different, in each situation (*prativiṣaya*), because of the worldly practise (*lokavyavahāra*), it is explained from ⟨its⟩ use in the world. For if not, a different speech ⟨would be required⟩ in each situation and the situations are countless. Therefore this ⟨speech⟩ cannot be specified entirely. Accordingly, with the word "now" he (Āryabhaṭa) sets forth the speech as it has been established in the world.

⟨As for:⟩ "**when one has multiplied by the desire quantity**". That fruit quantity is multiplied by that desire quantity; when one has *hata* that is multiplied (*guṇita*) that by the desire quantity.

"**What has been obtained**" (*labdha*) is what has been gained (*āpta*). How? He says: "**divided by the measure**", that is divided by the measure quantity (an instrumental *tatpuruṣa*).

⟨As for⟩ "**from that**", from such a sort of quantity which has been divided by the measure.

⟨As for⟩ "**the fruit of the desire**"; *icchāphala* is the fruit of the desire (a genetive *tatpuruṣa*); the meaning is: the fruit of the desire quantity.

"**This**" is said when one has made visible what has been obtained.

AL-KHWĀRIZMĪ

LE COMMENCEMENT DE L'ALGÈBRE

R. RASHED

Paris: Blanchard
2007

باب المعاملات

اعلَمْ أن معاملات الناس كلها من البيع والشراء والصرف والأجَر وغير ذلك على وجهين بأربعة / أعداد يلفظ بها السائل وهي: المسعِّر والسعر والثمن والمثمن.

فالعدد الذي هو المسعر مباين / للعدد الذي هو الثمن. والعدد الذي هو السعر مباين للعدد الذي هو المثمن، وهذه الأربعة الأعداد ثلاثة منها أبداً ظاهرةٌ معلومةٌ، وواحد منها مجهول، وهو الذي في قول القائل كم، وعنه يسأل السائل.

فالقياس في ذلك: أن تنظر إلى الثلاثة الأعداد الظاهرة، فلا بدّ أن يكون منها اثنان، كل واحد منهما مباين لصاحبه؛ فتضرب العددين الظاهرين المتباينين كل واحد منهما في الآخر، فما بلغ فاقسمه على العدد الآخر الظاهر الذي مباينه مجهول، فما خرج لك فهو العدد المجهول الذي يسأل عنه السائل وهو المباين للعدد الذي قسمت عليه.

ومثال ذلك في وجه / أوّل منه - إذا قيل لك: عشرة بستة كم لك بأربعة؟ فقوله عشرة هو العدد المسعر، / وقوله بستة هو السعر، وقوله كم لك هو العدد المجهول المثمن، وقوله بأربعة هو العدد الذي هو الثمن. فالعدد المسعر الذي هو العشرة مباين للعدد الذي هو الثمن، وهو الأربعة. فاضرب العشرة في الأربعة، وهما المتباينان الظاهران، فتكون أربعين. فاقسمها على العدد الآخر الظاهر، الذي هو السعر وهو ستة، فيكون ستة وثُلثين، وهو العدد المجهول، الذي هو في قول القائل / كم، وهو المثمن، ومباينه الستة التي هي السعر.

Chapitre sur les transactions

Sache que toutes les transactions entre les gens, de vente, d'achat, de change <de monnaies>, de salaire, et toutes les autres, ont lieu selon deux modes, et d'après quatre [B-77r] nombres prononcés par le demandeur, qui sont : quantité d'évaluation, taux, prix, quantité évaluée[76].

Le nombre qui est la quantité d'évaluation n'est pas proportionnel [A-14v] à celui qui est le prix. Le nombre qui est le taux n'est pas proportionnel au nombre de la quantité évaluée, et, parmi ces quatre nombres, trois sont toujours évidents et connus, et l'un d'eux est inconnu, qui, dans les termes de celui qui parle, est « combien », et qui est l'objet du demandeur.

On l'infère ainsi : tu examines les trois nombres évidents ; il est nécessaire que, parmi eux, il y en ait deux, dont chacun n'est pas proportionnel à son associé. Tu multiplies les deux nombres évidents non proportionnels l'un par l'autre ; tu divises le produit par l'autre nombre évident, dont <l'associé> non proportionnel est inconnu ; ce que tu obtiens est le nombre inconnu cherché par le demandeur, et qui n'est pas proportionnel au nombre par lequel tu as divisé.

Exemple pour le premier mode — [I-54] Si on te dit : dix pour six, combien auras-tu pour quatre ? Dix, dans ses termes, est le nombre de la quantité d'évaluation ; [H-25v] six, dans ses termes, est le taux ; et dans ses termes : combien auras-tu ? est le nombre inconnu qui est la quantité évaluée ; et quatre, dans ses termes, est le nombre qui est le prix. Ainsi, le nombre de la quantité d'évaluation, qui est dix, n'est pas proportionnel au nombre qui est le prix, quatre. Multiplie dix par quatre — ce sont les deux non proportionnels évidents ; on a quarante. Divise par l'autre nombre évident, qui est le taux, c'est-à-dire six ; on a six, plus deux tiers, qui est le nombre inconnu, et qui, dans les termes de celui qui parle, [O-17r] est « combien ? », et qui est la quantité évaluée ; et celui qui ne lui est pas proportionnel est le six, qui est le taux[77].

The Algebra of Mohammed ben Musa

Edited and Translated
by Frederic Rosen

LONDON:
PRINTED FOR THE ORIENTAL TRANSLATION FUND:
1831.

ON MERCANTILE TRANSACTIONS.

You know that all mercantile transactions of people, such as buying and selling, exchange and hire, comprehend always two notions and four numbers, which are stated by the enquirer; namely, measure and price, and quantity and sum. The number which expresses the measure is inversely proportionate to the number which expresses the sum, and the number of the price inversely proportionate to that of the quantity. Three of these four numbers are always known, one is unknown, and this is implied when the person inquiring says *how much?* and it is the object of the question. The computation in such instances is this, that you try the three given numbers; two of them must necessarily be inversely proportionate the one to the other. Then you multiply these two proportionate numbers by each other, and you divide the product by the third given number, the proportionate of which is unknown. The quotient of this division is the unknown number, which the inquirer asked for; and it is inversely proportionate to the divisor.*

Examples.—For the first case: If you are told, "ten for six, how much for four?" then *ten* is the measure; *six* is the price; the expression *how much* implies the unknown number of the quantity; and *four* is the number of the sum. The number of the measure, which is *ten*, is inversely proportionate to the number of the sum, namely, *four*. Multiply, therefore, ten by four, that is to say, the two known proportionate numbers by each other; the product is forty. Divide this by the other known number, which is that of the price, namely, six. The quotient is six and two-thirds; it is the unknown number, implied in the words of the question "*how much?*" it is the quantity, and inversely proportionate to the six, which is the price.

BARNABAS B. HUGHES

ROBERT OF CHESTER'S LATIN TRANSLATION OF AL-KHWĀRIZMĪ'S *AL-JABR*

BOETHIUS

TEXTE UND ABHANDLUNGEN ZUR GESCHICHTE DER EXAKTEN WISSENSCHAFTEN

FRANZ STEINER VERLAG WIESBADEN GMBH
STUTTGART 1989

<PARS QUARTA:> REGULA DE TRIBUS

Res autem venales et omnia que ipsis attinent, duobus modis et 4 numeris disponuntur. Horum vero numerorum primus iuxta Arabes, Almuzarar[1] id est primus propositus nominatur. Alter vero Alszarar[2] id est secundus per primum dinotus appellatur. Tertius Almuthemen id est ignotus. Quartus Althemen id est per primum et secundum dinotus. Sed et hii 4 numeri sic disponuntur, vt eorum primus qui est Almuzarar, vltimo qui est Althemen, opponatur. Horum eciam 4 numerorum 3 semper noti atque certi ponuntur. Quartus vero ignotus ponitur et incertus, et ipse est ille cum quo quantum inquiritur. Talis quoque ad hanc artem regula datur, vt in omni huius inquisicione tres numeri qui noti ac certi sunt positi considerentur, quoniam eorum duo semper adinuicem oppositi inuenientur. Horum ergo duorum vnus in altero multiplicandus est, et eorum multiplicacionis summa per tercium notum atque certum positum, qui ignoto opponitur, erit diuidenda. Nam quod ex diuisione exierit, erit numerus de quo dubitabatur. Et ipse ei numero opponitur per quem facta est diuisio. Sed ne per hanc artem aliquis errorem manere arbitretur, tale damus exemplum.

<1> Secundum ergo primum modum sic dicas, "10 pro 6, quot pro 4?"[65]

Vide nunc, quomodo secundum quod diximus, prefati numeri disponuntur. Nam quoniam 10 dixisti, numerum Almuzarar pronunciasti. Et quando pro 6 dixisti, numerum Alszarar protulisti. Et quando quot dixisti, numerum Almuthemen seu Magul id est ignotum nunciasti. Et quando pro 4 dixisti, numerum Althemen edidisti. Vide ergo qualiter eorum tres id est 10 et 6 et 4 noti ac certi ponuntur. Et quomodo de quarto adhuc incognito dubitatur. Si ergo ad regulam prius datam respexeris, primum in vltimo, id est 10 in 4, multiplicabis. Sunt etenim oppositi noti quoque ac certi. Et quod ex multiplicacione excreuerit, id est 40, per alterum numerum notum ac certum qui est Alszarar, id est per 6, oportet diuidere. Et erunt 6 et 2/3 vnius numeri Almagul id est incognitum designantes. Et huius numerus numero senario est oppositus qui Arabice Alszarar nominatur.

<2> Secundus modus huius artis est vt dicas, "10 pro 8, quot pro 4?"[66]

Barnabas Hughes, O.F.M. *Mediaeval Studies* **48**, 211–263

GERARD OF CREMONA'S TRANSLATION OF AL-KHWĀRIZMĪ'S *AL-JABR*: A CRITICAL EDITION

<VIII.> Capitulum conventionum negociatorum

Scias quod conventiones negociationis hominum omnes, que sunt de emptione et venditione et cambitione et conductione et ceteris rebus, sunt secundum duos modos, cum quattuor numeris quibus interrogator loquitur. Qui sunt pretium et appretiatum secundum positionem, et pretium et appretiatum secundum querentem. Numerus vero qui est appretiatum secundum positionem opponitur numero qui est pretium secundum querentem. Et numerus qui est pretium secundum positionem opponitur numero qui est appretiatum secundum querentem. Ho(P 115rb)rum vero quattuor numerorum tres semper manifesti et noti, et unus *est* ignotus. Qui est ille qui verbo loquentis notatur per quartum, et de quo interrogator querit. Regula ergo in hoc est ut consideres tres numeros manifestos. Impossibile est enim quin duo eorum sint quorum unusquisque suo compari est oppositus. Multiplica igitur unumquemque duorum numerorum apparentium oppositorum in alterum. Et quod proveniet, divide per alterum numerum cui numerus ignotus opponitur. Quod ergo proveniet, est numerus ignotus pro quo querens interrogat. Qui etiam est oppositus numero per quem dividitur.

Cuius exemplum secundum primum modum eorum est ut querens interroget et dicat: 'Decem caffícii sunt pro sex dragmis; quot ergo provenient tibi pro quattuor dragmis?' Sermo itaque eius, qui est decem caffícii, est numerus appretiati secundum positionem. Et eius sermo, qui est sex dragme, est numerus eius quod est pretium secundum positionem. Et ipsius sermo, quo dicitur quantum te contingit, est numerus ignotus appretiati secundum querentem. Et ipsius sermo, qui est per quattuor dragmas, est numerus qui est pretium secundum querentem. Numerus ergo appretiati qui est decem caffícii opponitur numero qui est pretium secundum querentem, quod est quattuor dragme. Multiplica ergo decem in quattuor, qui sunt oppositi et manifesti, et erunt quadraginta. Ipsum itaque per alium numerum manifestum divide, qui est pretium secundum positionem, quod est sex dragme. Erit ergo sex et due tertie qui est numerus ignotus. Qui est sermo dicentis quantum. Ipse namque est appretiatum secundum querentem, et opponitur sex qui est pretium secundum positionem.

Other Arabic authors

Hochheim, Adolf (ed., trans.), 1878. *Kâfî fîl Hisâb (Genügendes über Arithmetik)* des Abu Bekr Muhammed ben Alhusein Alkarkhi. I-III. Halle: Louis Nebert.

Cap. XLII. Die Proportionen[5]). Von den vier Grössen der Proportion verhält sich die erste[6]) zur zweiten, wie die dritte[7]) zur vierten[8]). Hast Du diese Beziehung gefunden, so erhältst Du durch Vertauschung der Glieder: Das erste verhält sich zum dritten, wie das zweite zum vierten[9]). Ferner erhältst Du, wenn Du vereinigst, ebenfalls eine Proportion: Es verhält sich die Summe des ersten und zweiten Gliedes zum zweiten Gliede, wie die Summe des dritten und vierten Gliedes zum vierten[1]). Ausserdem kannst Du Differenzen bilden, dann verhält sich die Differenz des ersten und zweiten Gliedes zum zweiten, wie die Differenz des dritten und vierten Gliedes zum vierten[2]). Weiterhin erhältst Du durch Umstellung: Das zweite Glied verhält sich zum ersten, wie das vierte zum dritten[3]). Nach dieser Umstellung verhält sich das erste Glied znr Differenz des ersten und zweiten Gliedes, wie das dritte zur Differenz des dritten und vierten[4]). Stehen nun vier Glieder in solcher Beziehung zu einander, so ist das Product des ersten und vierten Gliedes gleich dem Producte des zweiten und dritten[5]).

Ist das erste Glied unbekannt, so multiplicirst Du das zweite ins dritte Glied und dividirst das Product durchs vierte. Ebenso dividirst Du, wenn das vierte Glied unbekannt ist, jenes Product durch das erste Glied. Ist das zweite oder dritte unbekannt, so multiplicirst Du das erste ins vierte und dividirst das Product durch das bekannte der beiden andern Glieder[6]).

Wenn drei Zahlen eine Proportion bilden, so verhält sich die erste zur zweiten, wie die zweite zur dritten. Das Product des mittlern Gliedes in sich selbst ist gleich dem Producte der beiden äussern Glieder[7]).

Cap. XLIII. Die Geschäftsrechnung. Wisse, dass Du bei den Aufgaben der Geschäftsrechnung vier Grössen, von denen je zwei homogen sind, haben musst, nämlich den Preis, das Mass[8]), die Kaufsumme und die Quantität[9]).

Der Preis ist der Werth einer Masseinheit, welche im Handel gilt, z. B. des Dînâr, des Kurr, der Manâ. des Dscharîb[10]), des Kaffîz, der Elle[11]) oder des Verdienstes innerhalb eines Monats oder eines Jahres oder des Umfangs, den ein Dscharîb Land haben muss, oder dessen, was bei der Steuereinschätzung der Sultan auf ein Dscharîb zu beanspruchen hat. Was die Zehnten[12]), die Monatsabgaben und die Tribute anlangt, so richtet sich dies ganz nach der Behandlung des Uebrigen, was hier angeführt worden ist. Der Preis ist also der Geldbetrag, welcher jeder dieser Masseinheiten entspricht; er ist im Verkehr der Menschen feststehend und bekannt.

Die Quantität ist das, was der Handeltreibende seinen Kunden überlässt. Die Summe endlich ist der Werth der Quantität. Von diesen vier Grössen sind immer drei bekannt, eine unbekannt. Die Unbekannte findest Du, indem Du eine der Bekannten, z. B. die Summe oder die Quantität in die ihr unhomogene, nämlich Mass oder Preis, multiplicirst und das Product durch die ihr homogene Grösse dividirst. Was dabei herauskommt, ist das Resultat[1]).

Oder wenn Du willst, setzest Du eine der bekannten Grössen, z. B. Quantität oder Summe, zu der ihr homogenen ins Verhältniss und suchst damit das Verhältniss der unhomogenen Grösse[2]).

Souissi, Mohamed (ed., trans.), 1969. Ibn al-Bannā', *Talkhīṣ a'māl al-ḥisāb*. Texte établi, annoté et traduit. Tunis: L'Université de Tunis.

OPÉRATIONS PAR RAPPORTS

Ces opérations sont de deux sortes : par les quatre nombres proportionnels, et par les plateaux (de la balance).

Les quatre nombres proportionnels sont tels que le premier est au deuxième comme le troisième est au quatrième (1).

Le produit du premier par le quatrième est égal au produit du second par le troisième (2).

Lorsqu'on multiplie le premier par le quatrième et qu'on divise le produit par le deuxième, on obtient le troisième ; en divisant par le troisième, on obtient le deuxième (3).

Lorqu'on multiplie le second par le troisième, et qu'on divise le produit obtenu par le premier, on obtient le quatrième ; en divisant par le quatrième on obtient le premier.

N'importe quelle inconnue, parmi ces nombres, s'obtient, par ce procédé, à partir des trois autres nombres connus. La méthode consiste à multiplier le nombre donné isolé, hétérogène des deux autres, par celui dont on ignore le correspondant, et à diviser par le troisième nombre connu ; il en résulte l'inconnue (1).

Rebstock, Ulrich, 1993. *Die Reichtümer der Rechner (Ġunyat al-Ḥussāb) von Aḥmad b. Ṯabāt (gest. 631/1234). Die Araber – Vorläufer der Rechenkunst.* (Beiträge zur Sprach- und Kulturgeschichte des Orients, 32). Walldorf-Hessen: Verlag für Orientkunde Dr. H. Vorndran.

Das Buch über die ‚Geschäftsrechnungen' (*al-mu'āmalāt*)

Trotz ihrer Verschiedenheiten enthalten sämtliche Geschäftsrechnungen (*al-mu'āmalat*) vier [Verhältnis]größen (*maqādīr*). Jeweils zwei von ihnen sind gleichartig. Drei sind immer bekannt, eine unbekannt.
Das Verhältnis der ersten zur zweiten ist gleich dem Verhältnis der dritten zur vierten; umgekehrt ist das Verhältnis der zweiten zur ersten gleich dem Verhältnis der vierten zur dritten. Zwischen der zweiten und der dritten gibt es kein Verhältnis.
Das Produkt der beiden Außenglieder (*ṭarf*) ist gleich dem Produkt der beiden Innenglieder (*wasṭ*).
Z.B. ergeben die Zahlen 2, 8, 5 und 20

Grundlage aller ‚Geschäftsrechnungen' ist, daß du eine gegebene Größe mit einer ungleichartigen multiplizierst und das Ergebnis durch die gleichartige dividierst; oder du setzt eine gegebene Größe ins Verhältnis zu der ihr gleichartigen und nimmst dann dieses Verhältnis von der ungleichartigen Größe. Das Ergebnis von Division und Proportion ist die Antwort.

Rebstock, Ulrich (ed., trans.), 2001. 'Alī ibn al-Ḫiḍr al-Qurašī (st. 459/1067), *At-tadkira bi-uṣūl al-ḥisāb wa l'farā'iḍ* (*Buch über die Grundlagen der Arithmetik und der Erbteilung*). (Islamic Mathematics and Astronomy, 107). Frankfurt a.M.: Institute for the History of Arabic-Islamic Science at the Johann Wolfgang Goethe University.

§78 Die Grundlage für ›Verkauf und Kauf‹ kann man dem siebten Buch Ūqlīdis' **(93)** über die Art der Zahl entnehmen. Er erwähnt dort, daß von diesen vier Benennungen drei bekannt und eine unbekannt (*ġair maʿlūma*) sind. Das entspricht Deiner Formulierung: »Soviel, was bekannt ist, für soviel, was bekannt ist; wie groß ist der Preis von soviel, was auch bekannt ist«? Deine Frage richtet sich auf das Wieviel. Das ist so, als ob Du sagst: »Wieviel für soviel«?, woraus sich dann das, dessen Preis gesucht ist (*muṯamman*), als bekannt ergibt. Der Preis (*ṯaman*)[114] ist [auch] bekannt und das Gesuchte (*maṭlūb*) ist ein Teil (*ǧuz'*) des Preises.

Beispiel: 5 *ra*[115] – das ist das, dessen Preis bekannt ist (*muṯamman maʿlūm*) – für 1 *din* – das ist das *muṯamman* und [auch] bekannt (*maʿlūm*); wieviel [bekommt man davon] für 1 *qīrāṭ*[116]? Der gesuchte Teil am gesamten *muṯamman* ist unbekannt. Sein Preis, 1 *qi*, ist bekannt.

§79 Wir haben alle Abschnitte des ›Verkaufs und Kaufs‹ und was man dabei herausbekommt im *Kitāb al-Maʿūna* erwähnt; [auch dazu,] wenn Dir gesagt wird: »Wieviel ist soviel für soviel [und] wieviel ist der Preis für soviel«[117]? Hier nun erwähnen wir nur die Größen (*maqādīr*) und Gewichte (*auzān*), derer man bei der Arbeit mit den Erbteilen, mit dem Recht (*fiqh*) und mit den Testamenten (*waṣāyā*) bedarf. Bei der Besprechung der Art der Zahl, bringt Ūqlīdis [folgendes] vor: »Wenn Du die erste von vier Zahlen, die in einem Verhältnis zueinander stehen (*aʿdād mutanāsiba*), mit der vierten multiplizierst, dann ist das [Produkt] gleich **(94)** [dem Produkt] der Multiplikation der zweiten mit der dritten«.

Beispiel: 2, 3, 6 und 9; setze ins Verhältnis: $2 : 3 = 6 : 9$; $2 \cdot 9 = 18 = 3 \cdot 6 = 18$. Das ist die Formulierung (*kalām*) von Ūqlīdis. Wir haben einiges dazu an Beispielen in *al-Muwallad* angeführt.

Boncompagni, Baldassare (ed.), 1857. *Scritti* di Leonardo Pisano matematico del secolo decimoterzo. I. Il *Liber abbaci* di Leonardo Pisano. Roma: Tipografia delle Scienze Matematiche e Fisiche.

Incipit capitulum octauum de reperiendis preciis mercium per maiorem guisam.

In omnibus itaque negotiationibus quattuor numeri proportionales semper reperiuntur, ex quibus tres sunt noti, reliquus uero est ignotus: primus quidem illorum trium notorum numerorum est numerus uenditionis cuiuslibet mercis, siue constet numero, siue pondere, siue mensura. Numero quidem ut centum coria, uel centum beccune et similium: pondera quoque ut cantarum, uel centum, uel libre, aut unce et similium. Mensura quidem ut metra olei, sextaria frumenti, et canne panni et similium. Secundum autem est pretium illius uenditionis, hoc est illius primi numeri, siue sit quantitas quorumlibet denariorum, siue bizantiorum, siue tarenorum, uel alicuius alie currentis monete. Tertius uero quandoque erit aliqua eiusdem uendite mercis quantitas, cuius pretium, scilicet quartus numerus, ignoratur; et quandoque erit aliqua similis quantitas secundi pretii, cuius merces, scilicet quartus ignotus numerus, iterum ignorabitur. Quare, ut ignotus numerus per notos reperiatur, talem in omnibus tradimus regulam uniuersalem, uidelicet ut in capite tabule, in dextera parte scribas primum numerum, scilicet mercem; retro in eadem linea ponas pretium ipsius mercis, uidelicet secundum numerum ; tertium quoque si fuerit mercis, scribe eum sub merce, scilicet sub primo; et si fuerit pretium, scribe eum sub pretio, uidelicet sub secundo; ita tamen, ut sicut fuit ex genere ipsius, sub quo scribendum est, ita etiam sit ex qualitate uel ex quantitate ipsius in numero, uel in pondere, uel in mensura; hoc est si superior numerus, sub quo scribendus est, fuerit numerus ipsorum, et ipse similiter fiat rotulorum; si librarum, librarum; si uncearum, uncearum; si cannarum, cannarum. Et si fuerit numerus soldorum, et ipse sit numerus soldorum; si denariorum, denariorum; si tarenorum, tarenorum; et si bizantiorum, bizantiorum. Quibus ita descriptis, euidentissime apparebit, quod duo illorum positi erunt semper ex aduerso, que insimul multiplicentur, et summa multiplicationis eorum, si per reliquum tertium numerum diuidatur, quartus ignotus nimirum inuenietur: et ut hoc apertius intelligatur, cum diuersis mercibus et pretiis, in sequentibus explanabimus. Sed primum ostendam, vnde hic modus procedit: sunt enim, ut dixi, in negotiationibus IIII^{or} numeri proportionales, scilicet, ut sicut primus est ad secundum, ita tertius ad quartum, hoc est, sicut numerus alicuius quantitatis mercis est ad numerum quantitatis sui pretii, ita numerus cuiusuis quantitatis eiusdem mercis ad numerum sui pretii: uel sicut aliqua quantitas cuiusuis mercis est ad quamuis quantitatem eiusdem mercis , ea est pretii unius ad pretii alterius: et cum ita IIII^{or} quantitates proportionales sunt, erit multiplicatio secunde in tertiam equa multiplicacioni prime in quartam, ut in arismetricis, et geometria probatum est : quare si quarta quantitas est ignota tantum , ex multiplicatione quidem secunde quantitatis in tertia diuisa per primam, nimirum ex diuisione, quarta quantitas prouenit: quare cum diuiditur aliquis numerus per aliquem numerum, et ex diuisione aliquid proueniat; si proueniens in diuisorem | multiplicaueris, nimirum diuisus numerus inde proueniet. Similiter si tertia quantitas ignoratur, diuidenda est per tertiam multiplicatio prime in quartam: et ut ea, que ad negotiationes pertinent, perfecte in hoc libro habeantur, hoc capitulum in quattuor partes diuidimus ; quarum prima erit in uenditione cantarium, et earum rerum, que ad pondus uel numerum uenduntur; secunda in eis que ad toloneum seu ad cambium pertinent, ut soldus, libra, uel marca argenti, uncia auri et similia; tertia in uenditione cannarum, ballarum, torscelli et similium; quarta pars erit in reductione Rotulorum unius cantaris ad Rotulos cuiuslibet alterius cantarium, secundum eius diuersitatem.

Jacopo da Firenze, *Tractatus algorismi* (1307)

If some computation should be given to us in which three things were proposed, then we should always multiply the thing that we want to know against that which is not similar, and divide in the third thing, that is, in the other that remains.

Arrighi, Gino (ed.), 1989. "Maestro Umbro (sec. XIII), *Livero de l'abbecho*. (Cod. 2404 della Biblioteca Riccardiana di Firenze)". *Bollettino della Deputazione di Storia Patria per l'Umbria* **86**, 5–140.

Lo primo chapitolo ène de le regole de le tre chose.

Se ce fosse dicta alchuna ragione ella quale se proponesse tre chose, sì devemo moltiplicare quilla chosa che noie volemo sapere con quella che non è de quilla medessma, a partire nell'altra.

Regole senza nome.

Famme quista ragione: se 2/3 7 fosse 2/5 9, che serìa 1/2 6? Quiste sonno ragione che none agiono nome, che la prima dèie essere partedore, donqua dèie tu dire: 2/5 9 via 1/2 6 chedè 1/2 6, fanne 1/2 che sonno 13 e 2/5 fanne quinte che sonno 47 e dì 13 via 47 che fanno 611 gle quagle sonno 1/2 de 1/5 çioè dicine; donqua dèie tu partire 611 en 10 ched e' viene 1/10 61 e cotanto fa 2/5 9 via 1/2 6, la quale somma si dèie partire en 2/3 7. Se tu vuogle partire 1/10 61 in 2/3 7 si menna 30 fiade ennante tutte le 2 parte e di': 1/10 61 via 30 che fa 1833 di 2/3 7 via 30 che fa 230, parte 1833 in 230 che ne viene 223/230 7, | per le 3 cose de d. e cotanto si n'à li 6 1/2.

Se 1/3 5 valese 1/5 7, que ne venna 1/7 9? Sì devemo sapere en que numero se truoveno amendoro le parte denançe çioè 1/3 1/5, che se truova en 15. Or devemo multiplicare amedoro quiste parte per 15 çioè 5 che fa 75 e pigla el 1/3 de 15 ch'è 5 e giogne sopre 75 e farà 80. Or devemo fare 7 via 15 che fa 105, el 1/5 è 3 lo quale giogne sopra al 105 e farà 108. Or aremo rechate amedoro le parte denançe a sano e podemo dire: 80 vale 108, che ne varrà 1/7 9? Sì devemo fare 9 via 108 che fa 972 e piglare el 1/7 de 108 ch'è 3/7 15 lo quale giogne sopra a 972 e farà 987 e 3/7, partire per 80 che ne viene 197/460 12.

Liber habaci (c. 1310)

Arrighi, Gino (ed.), 1987a. Paolo Gherardi, *Opera mathematica: Libro di ragioni – Liber habaci.* Codici Magliabechiani Classe XI, nn. 87 e 88 (sec. XIV) della Biblioteca Nazionale di Firenze. Lucca: Pacini-Fazzi.

Incipit liber habaci.

Se ci fosse detta alchuna ragione nella quale si proponesse tre chose, sì dobbiamo multiprichare quella chosa che nnoj volglamo sapere chontr'a quella che nnonn è di quella medesima et partire nell'altra.

Paolo Gherardi, *Libro di ragioni* (1327)

Arrighi, Gino (ed.), 1987a. Paolo Gherardi, *Opera mathematica: Libro di ragioni – Liber habaci.* Codici Magliabechiani Classe XI, nn. 87 e 88 (sec. XIV) della Biblioteca Nazionale di Firenze. Lucca: Pacini-Fazzi.

Se ne fosse dicta alcuna ragione la quale si proponesse in 3 cose, sì dovemo multiplicare la cosa che noi volemo sapere contra quella che non è di quella medesima e partire nell'altra.

Arrighi, Gino (ed.), 1987b. Giovanni de' Danti Aretino, *Tractato de l'algorisimo*. Dal Cod. Plut. 30. 26 (sec. XIV) della Biblioteca Medicea Laurenziana di Firenze. *Atti e Memorie della Accademia Petrarca di Lettere, Arti e Scienze*, nuova serie **47**, 3–91.

Se cci fosse decto alcuna rasgione nella quale si proponessero 3 cose, sì dobiamo multipricare quella cosa che noi volgliamo sapere contra da quella che non è de quella medesima e partire nell'altra. Pongote l'asempro a la decta regola e

Ibero-Provençal books of abbacus-type

Caunedo del Potro, Betsabé, & Ricardo Córdoba de la Llave (eds), 2000. *El arte del alguarismo*. Un libro castellano de aritmética comercial y de ensayo de moneda del siglo XIV. (Ms. 46 de la Real Colegiato de San Isidoro de León). Salamanca: Junta de Castilla y León, Consejeria de Educación y Cultura.

Sy quisieres saber que sy el 1/3 de 3 menos 1/3 fuese el 4° de 4 qué sería el 1/5 de 5 menos 5, deves primeramente saber qué es el tercio e qué es el 4° de 4 menos 1/4 // es 15/16, el 1/5 de 5 menos 1/5 son 2lr 24/25, agora viene aquí la regla de sy tanto fuese tanto qué sería tanto, primeramente pon los 8/9 al comience segund aquí está, adelante dellos los 15/16 e adelante destos 15/16, pon los 24/25, 815/916, 24/25 e toma los 15 e multiplícalos por los 24 que son las nominaciones de encima e serán 360 e después multiplicarás las nominaciones de ayuso, la una con la otra, que son 16 e 25 e fallarás que monta 400, estos 400 ponlos de yuso de los 360 e será 360/400 aos de uno y estos

Traicté de la praticque

Lamassé, Stéphane, 2007. "Les problèmes dans les arithmétiques commerciales en langue française et occitane de la fin du Moyen Âge". *Thèse*, Université de Paris 1 Panthéon-Sorbonne, novembre 2007.

C[183]este regle est appellee regle de troys pource que es raisons qui se font par ceste regle sont tousiours requis troys nombres desquelz le premier et le tiers doivent tousiours estre semblants en nombrant une chose. Et diceulx troys nombres en resulte ung autre qui est la raison et conclusion de ce que l'on veult savoir. Et est tousiours semblable au second nombre des troys. Ceste regle selon aucuns est appellee regle doree et selon autres regle des proportions. Les raisons et questions de ceste regle se forment en ceste maniere. Si tant vault tant que vauldra tant ? Comme par exemple : se 6 valent 18 que vauldront 9 ? Pour faire telles raisons il en est une telle regle.

Multiplie ce que veulz savoir par son contraire et puis partiz par son semblant. Ou multiplie le tiers nombre par le second et puis partiz par le premier.

[**89**] Exemple de la question devant dicte : Se 6 valent 18 que vauldront 9 ?

DE LA REGLA DE TRES

Segueix-se la sisena espècie, que s'anomena regla de tres. Diu-se pròpiament per tant regla de tres per quant dins la dita espècie se contenen 3 coses, de les quals les dues són semblants i l'una és dissemblant. La qual espècie és general en tota mercaderia, car no és ninguna raó ni qüestió per fort que sia que per aquesta espècie essent bé reduïda no sia absolta.

I comença la dita espècie en nostre [parlar] vulgar: si tant val tant, què valdrà tant.

L'absolució d'aquesta regla comunament se diu: multiplica per son contrari i parteix per son semblant.

Per declaració de la dita regla havem d'entendre que en aquell «si tant val tant què valdrà tant», que hi ha 3 coses. La primera, que ja és certa de sa valor, que val tant. Per ço la primera és l'un semblant. Aquella quantitat o preu que la primera costa, que és

De la regla de tres.

S Egueix se la sisena specia: ques nomena regla de tres. Diu se ppriament per tant regla de tres: per quant dins la dita specia se contenen .3. coses. de les q̄ls les dues son semblāts e la vna es desēblāt. La q̄l specia es geñal en tota mercaderia. Car no es nēgūa rao ne questio per fort que sia: que per aq̄sta specia esset be reduida no sia absolta

E comēça la dita specia en nostre vulgar si tant val tant: que valra tant.

La absolucio de aquesta regla que comunament se diu multiplica per son contrari e parteix per son semblant

Per declaracio de la dita regla havem de ētēdre: q̄ en aq̄ll si tāt val tant: q̄ valra tāt que y ha .3. coses. la primera: que ja es certa de sa valor: que val tant. perço la primera es lo vn semblant. Aquella quantitat o preu que la primera costa: que es:

val tant: es lo contrari. E laltra quantitat que spera a saber que valdra: es laltre semblant.

E perque millor se entena: que non lexē confusament, com sta en la regla damūt dita: que diu: multiplica per son contrari. e parteix per son semblāt es necessari dar conexença: per qual se te de multiplicar / e per qual se te de partir.

E sta la difficultat en la dita regla en trobar lo partidor. E perque may lo pugues perdre: te done aquesta regla. çoes: que quāt te sia donada alguna questio: guarda fort aquella quantitat: que ja es certa lo que costa o val: que aquella tal tostēps es lo ptidor. E apres les dues quātitats çoes lo preu o valor ab aquell: q̄ desija saber que costara / o lo que valdra la vna per laltra sia multiplicada.

val tant, és lo contrari. I l'altra quantitat, que espera a saber que valdrà, és l'altre semblant.

I perquè millor s'entenga, que no ho lleixem confusament com està en la regla damunt dita, que diu multiplica per son contrari i parteix per son semblant, és necessari dar coneixença per qual se té de multiplicar i per qual se té de partir.

Està la dificultat en la dita regla en trobar lo partidor. I perquè mai lo pugues perdre, te done aquesta regla. Ço és, que quan te sia donada alguna qüestió, guarda fort aquella quantitat que ja és certa lo que costa o val, que aquella tal tostemps és lo partidor. I aprés les dues quantitats, ço és lo preu o valor amb aquell que desitja saber què costarà o lo que valdrà, l'una per l'altra sia multiplicada.

Commentaries in telegraphic style

NB: The "rule of three" is *a rule*, and to be kept apart from the kind of problems (problems of proportionality, "to *a* corresponds *b*, to *c* corresponds what) to which it is applied.

The rule can be identified through the order of operations to be performed: "first multiply *b* and *c*, then divide by *a*".

The intermediate result *bc* has no concrete meaning, whereas the intermediate results of the alternatives (division first) have a concrete interpretation; either "to 1 corresponds b/a" or "to *c* corresponds c/a times as much.

The earliest extant statement of the rule

In the *Vedāṅgajyotiṣa*, cautiously to be dated to c. 400 BCE.

In translation, this version of the rule runs

> The known result is to be multiplied by the quantity for which the result is wanted, and divided by the quantity for which the known result is given.

The reference to "the result that is wanted" has some similarity to what we find in the abbacus books – for instance, in Jacopo's *Tractatus*,

> If some computation should be given to us in which three things were proposed, then we should always multiply the thing that we want to know against that which is not similar, and divide in the third thing, that is, in the other that remains.

Vaguely suggestive, but probably an accident – central the the rule is in any case that there *is* a magnitude that is wanted.

Not clear whether the *Vedāṅgajyotisa* refers to a "[rule of] three things", but so do at least Āryabhaṭa, Brahmagupta, Mahāvīra and Bhāskara II.

All of these also refer to that which is wanted (*iccha*).

Āryabhaṭa's formulation is

> in the three magnitudes, after one has multiplied the magnitude *phala* ["fruit" / "outcome"] with the magnitude *icchā*, the intermediate outcome is divided by the *pramāṇa* ["measure"].

He has no reference to what is similar / not similar in kind, nor is any found in Bhāskara I's commentary to Āryabhaṭa.

However, this reference turns up as secondary information in the formulations of Brahmagupta, Mahāvīra and Bhāskara II – but in ways so different that direct descent between these can be excluded.

The rule in Arabic writings

The earliest extant Arabic reference: in al-Khwārizmī's algebra.

Al-Khwārizmī speaks of four quantities, not three.

Interpreters differ on the meaning of his words.

For four quantities in proportion $\frac{a}{b} = \frac{c}{d}$, Rosen takes al-Khwārizmī to claim that *a* is "inversely proportionate" to *d*, and *b* to *d*-

Rashed states that *a* is "not proportional" to *d* (etc.).

A slightly later passage states according to Rosen that among the three known quantities, two "must necessarily be inversely proportionate the one to the other".

According to Rashed there are two numbers, each of which is not proportionate to its associate; in both cases, these two numbers have to be multiplied.

None of this makes much sense mathematically, and the Latin translations of Gherardo da Cremona and Robert of Chester and Boris Rozenfeld's Russian translation agree that the essential adjective *mubāyn* means "opposite".

In the first statement, this "opposition" could refer to a graphical scheme (*our* scheme, and that of the 12th century; al-Khwārizmī has nothing of the kind).

The second passage, however, leaves only one possibility; that the term *mubāyn*, "différent", should be understood as *dissimilar* – in exact agreement with the secondary explanations of the Sanskrit mathematicians from Brahmagupta onwards.

Most Arabic treatments of the rule have as their primary examples problems confronting commodity and price, and designate the four terms accordingly.

They often present the rule after a short introduction of the proportion concept and the rule of cross multiplication.

Sometimes proportions and rule of three are linked, sometimes they are not – and often a formulation including the similar / non similar follows.

Al-Karajī's *Kāfī fi'l ḥisāb* does not link the rule with the preceding presentation of the proportion. His rule runs as follows:

> You find the unknown magnitude by multiplying one of the known magnitudes, for instance the sum or the quantity, by that which is not similar to it, namely the measure or the price, and dividing the outcome by the magnitude which is of the same kind.

Ibn al-Bannā integrates proportions and the rule of three, and gives the rule in this shape:

> You multiply the isolated given number, (that is, the one which is) dissimilar from the two others, by the one whose counterpart one does not know, and divide by the third known number.

Ibn Thabāt also integrates proportions and rule of three, and first gives rules based on the former. Then comes this rule – almost identical with the Italian abbacus formulation, always close to Jacopo:

> The fundament for all *muāmalāt*-computation is that you multiply a given magnitude by one which is not of the same kind, and divide the outcome by the one which is of the same kind.

Ibn Thabāt was active in Baghdad in the earlier thirteenth century, and primarily a legal scholar rather than a "mathematician" or "astronomer-mathematician".

That precisely *his* words should have been taken over by the abbacus school is not credible. We must rather assume that they reflect the formulation used by merchants in a wide area.

Fibonacci and Italy

If Fibonacci was actually taught for more than "some days" in Bejaïa (as he claims) he may even have encountered the same formulation there.

However, when introducing the rule in the *Liber abbaci* he does not speak of a "rule of three things" but (as common among Arabic mathematicians) of "four proportional numbers, of which three are known but the last unknown";

His rule prescribes the inscription of the numbers on a rectangular *tabula* (represented in the treatise by a rectangular frame) – probably a *lawha*, a clayboard for temporary writing. The marginal illustrations show a rectangular frame.

This method also used in the *Liber mahamaleth*, and thus familiar in 12th-c. Toledo. Likely to have inspired Robert's and Gherardo's understanding of *mubāyn* as "opposite".

The Italian abbacus school standard formulation is not adopted from Fibonacci but similar to that of Jacopo:

> If some computation should be given to us in which three things were proposed, then we should always multiply the thing that we want to know against that which is not similar, and divide in the third thing, that is, in the other that remains.

The reference of Italian abbacus authors as well as Sanskrit mathematicians to a "rule of three" suggests a circulation of this formulation not only along the shores of the Mediterranean but also along those of the Arabian Sea.

Enigmatic: the Columbia algorism and the Iberian tradition

The initial pages of the Columbia algorism (14th-c. copy of original from c. 1290) are missing; if the rule of three was presented here, we cannot known in which terms this was done.

However, all references to the rule within problems are through counterfactual questions, "if *a* were *b*, what would *c* be?".

Such questions are not absent from other Italian treatises. But they always occur as secondary examples of the rule of three, after problems confronting two different species of coin, or coin and commodity – or they are found wholly outside the presentation of the rule of three.

In the Ibero-Provençal treatises such counterfactual questions (or related abstract number questions like "if $4\frac{1}{2}$ are worth $7\frac{2}{3}$, what are $13\frac{3}{4}$ worth?") always provide the first and basic exemplification of the rule of three:

- the *Libro ... dicho alguarismo* (copy from 1393 of earlier original;
- the "Pamiers algorism" from c. 1430;
- the anonymous mid-fifteenth franco-Provençal *Traicté de la pratique*:
- Barthélemy de Romans' slightly later, equally Franco-Provençal *Compendy de la praticque des nombres*;
- Francesc Santcliment's Catalan *Summa de l'art d'Aritmètica* from 1482;
- Francés Pellos' *Compendion de l'abaco* from 1492 from Nizza.

Arabic standard treatises contain nothing similar.

There is us little doubt that the Columbia algorism depends on Iberian (or Ibero-Provençal) inspiration.

The origin of the Iberian recourse to counterfactual questions is not obvious.

It could represent a local development:
- the abstract number question is not difficult to produce by simple abstraction, al-Khwārizmī's example "ten for eight, how much for four" is not very different;
- nor would the step from the merely abstract to the explicitly counterfactual be more difficult to make in the Iberian world than elsewhere.

However, there is some reason to believe that at least the abstract formulation circulated in the Arabic commercial world.

Al-Khwārizmī's "ten for eight ..." is found in Rosen's, Rashed's and Robert of Chester's translations – but Gherardo has concrete numbers, "ten *cafficii* for six dragmas ...".

The abstract formulation may thus very well have crept into the manuscript tradition after al-Khwārizmī's time.

Moreover, ibn al-Khiḍr al-Qurašī, a little-known mid-eleventh-century author from Damascus, explains that the foundation for "sale and purchase" is *Elements* VII, and then goes on that "this corresponds to your formulation,

> 'So much, which is known, for so much, which is known; how much is the price for so much, which is also known?'".

Because of the possibility to identify specific markers in the formulations of the rule of three, scrutiny of a larger number of Sanskrit, Arabic and Christian-European presentations of the rule might yield more information about points of contact, transmission roads and communities.