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THE CALDERÓN PROJECTOR FOR OPERATORS WITH SPLITTING ELLIPTIC SYMBOLS

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Summary.

We shall investigate a class of "splitting" elliptic non-classical pseudo-differential operators of symbol class $S^{1,0}$, i.e. elliptic operators which take the form $A = G_A(y,t)(\partial_t + B_t)$ near the boundary Y of a smooth oriented Riemannian compact manifold X, where t denotes the normal coordinate, B_t is an elliptic (classical) pseudo-differential operator of order 1 over Y, and G_A is a bundle isomorphism over Y. We shall show the existence of a precise pseudo-differential projection, the Calderón projector, onto the space of Cauchy data of the kernel of A over Y.

0 Introduction.

More than 20 years ago A.P. Calderón observed that certain subspaces of the kernel of an elliptic differential operator A over a compact manifold X with boundary Y admit a pseudo-differential projection (see the short announcement of Calderón [5] and his beautiful lectures [6]). The construction of the Calderón projector was worked out in detail in Seeley [14] (for the case of an invertible elliptic pseudo-differential operator which is equal to a differential operator near Y), and in Hörmander [9]; and it was later extended in Grubb [7] to differential systems elliptic in the sense of Douglis and Nirenberg. In Hörmander [11] the construction of Calderón was generalized to a class of pseudo-differential operators with what we call splitting elliptic symbols (using a symbol class from Grubb [8]). These works, however, approach the Calderón projector only on the symbol level, i.e. the construction leads only to an approximate projection (modulo smoothing operators).

In this note we shall show that one obtains a true pseudo-differential Calderón projector for a certain class of elliptic operators of first order with "splitting symbol" near the boundary Y. There are different methods of proving this fact. We can follow the argumentation of the Calderón Lectures; this was done earlier by the authors in [4] in connection with a systematic reexamination of elliptic boundary value problems using the machinery of "elliptic towers" taken from Birman-Solomyak [1].

The present construction uses some ideas of Palais [12] and the "doubling construction" (continuation of an elliptic symbol splitting near the boundary to the double) of [18] and [4] and gives a more direct approach to the Calderón projector.

To some extent, the ease of this construction and the vigour of the Calderón projector in treating elliptic boundary value problems allows us to dispense with the elaborate machinery of the Boutet de Monvel calculus (see [15], [2]). Moreover it turns out that our way of looking at the Calderón projector and at elliptic boundary value problems extends to elliptic problems on manifolds with singularities. This will be explained separately.

The plan of this paper is as follows: In Section 1 we introduce the class $\mathrm{Ell}^1(X,Y)$ of operators with splitting elliptic symbols over a compact smooth oriented manifold X with smooth boundary Y. Section 2 contains two versions of our Continuation Lemma. The second leads immediately to the construction of the Calderón projector in Section 3. Section 4 presents some concluding remarks and a non-trivial example of an elliptic differential equation of second order which leads naturally to a (non-classical) pseudo-differential operator of first order with splitting elliptic symbol.

1 Operators With Splitting Elliptic Symbol.

Let X be a smooth compact oriented manifold with boundary Y. Let E and F be smooth complex vector bundles over X. We fix a Riemannian structure on X and a Hermitian structure on E and F. We use the Riemannian structure to construct a collar neighbourhood $N = Y \times I$ of the boundary Y, and we denote the (inward) normal coordinate by $t \in I = [0,1]$. We identify Y with $Y \times \{0\}$ and $E|_N$ and $F|_N$ with the liftings of $E|_Y$ and $F|_Y$ to N. We denote the strip $Y \times [0,t]$ by X_t for $t \in I$.

1.1 DEFINITION. (a) A linear map $A : C^\infty(X;E) \to C^\infty(X;F)$ belongs to the class $\mathrm{Pdiff}^1(X,Y)$ of (non-classical) pseudo-differential operators with ***splitting symbol***, if it can be written in the form $A = A^b + A^i$ where A^i is a pseudo-differential operator of order 1 over X with distributional kernel $K^i(x,x')$ such that

$$\operatorname{supp} K^i \subset (X \setminus X_{1/2}) \times (X \setminus X_{1/2}).$$

Moreover we shall assume that the operator A^b can be written in the form

$$A^b = \phi(t) G_A(y,t)(\partial_t + B_t),$$

where $B_t : C^\infty(Y;E|_Y) \to C^\infty(Y;F|_Y)$ is a pseudo-differential operator of first order and $G_A(.,t) : E|_Y \to F|_Y$ is a bundle isomorphism for any $t \in I$, and where $\phi(t)$ is a smooth function of t vanishing for $t > 2/3$. If necessary, we can reparametrize the collar. Without loss of generality we may assume that $\phi(t) = 1$ for small t.

(b) In the following we assume that the families $\{G_A(.,t)\}_{t\in I}$ and $\{B_t\}_{t\in I}$ depend smoothly on t. This assumption can be weakened somewhat, see Polking [13], but we will not go into the details in this paper.

Moreover, we will assume that the families extend to smooth families $\{B_t\}_{t\in[-\epsilon,1]}$ and $\{G_A(.,t)\}_{t\in[-\epsilon,1]}$ for some $\epsilon > 0$. Extending $\{G_A(.,t)\}$ is trivial. As shown in Seeley [15], a family $\{B_t\}_{t\in I}$ of differential operators can always be extended smoothly. However, the continuation of a family of pseudo-differential operators requires a more refined analysis which our assumption helps us avoid.

(c) The ***principal symbol*** σ_A of A is (by definition) the sum of the principal symbols of A^i and A^b. Close to Y it has the form

$$\sigma(x,\xi) = \phi(t) G_A(y,t)(i\tau + b(y,\eta)).$$

where $x = (y,t) \in X_{1/2}$, $y \in Y$, $0 \le 1/2$, $\xi = (\eta, \tau) \in T^*(X)_x$, and $b(y,\eta)$ is the principal symbol of the operator B. Hence the ***total symbol*** $p(x,\xi)$ (which is the principal symbol + lower order terms) will locally satisfy estimates of the form

$$|D_\xi^\alpha D_x^\beta p(x,\xi)| \le C_{\alpha\beta}(1+|\xi|)^{m-\alpha''}(1+|\eta|)^{m'-|\alpha'|}$$

with $m = 1$ and $m' = 0$, where α, β are multiindices, $\alpha = (\alpha_1, .., \alpha_n)$, $\alpha' = (\alpha_1, .., \alpha_{n-1})$, $|\alpha'| = \alpha_1 + .. + \alpha_{n-1}$, and $\alpha'' = \alpha_n$. So our operator A has a symbol which belongs to the class $S^{m,m'}$, for $m = 1$ and $m' = 0$, which is defined in Grubb [8] and further investigated in Hörmander [11, Section 20.1, pp 242 ff].

(d) We say that A is ***elliptic*** , $A \in \mathrm{Ell}^1(X,Y)$, if the principal symbol σ_A is an isomorphism.

(e) In the same way we define the operator classes $\mathrm{Pdiff}^1(M,Y)$ and $\mathrm{Ell}^1(M,Y)$ for any closed manifold M which contains Y as a submanifold of codimension 1 such that Y has a bicollar neighbourhood in M.

(f) Finally we introduce the classes $\mathrm{Pdiff}^{-1}(X,Y)$ and $\mathrm{Ell}^{-1}(X,Y)$ by assuming that the total symbol $p(x,\xi)$ belongs to the splitting symbol class $S^{-1,0}$, i.e. $p(x,\xi)$ satisfies the inequality

$$|D_\xi^\alpha D_x^\beta p(x,\xi)| \le C_{\alpha\beta}(1+|\xi|)^{m-\alpha''}(1+|\eta|)^{m'-|\alpha'|}$$

for $m = -1$ and $m' = 0$, and the principal symbol σ_A is invertible.

Note that all differential operators of first order belong to our class Pdiff^1 and that an operator A of class Pdiff^1 is a pseudo-differential operator of first order in the classical sense, i.e. it fulfills the symbol estimates

$$|D_\xi^\alpha D_x^\beta p(x,\xi)| \le C_{\alpha\beta}(1+|\xi|)^{1-|\alpha|}$$

if and only if A^b is a differential operator for a suitable reparametrization of the collar of Y.

We recall the following technical result without proof: it is a special case of Grubb [8, Théorème 2.1] and Hörmander [11, Proposition 20.1.11]:

12 THE CALCULUS OF SPLITTING ELLIPTIC SYMBOLS. *Let M be a closed manifold with submanifold Y of codimension 1, let E and F be vector bundles over M , and let $A : C^\infty(M;E) \to C^\infty(M;F)$ belong to the class $\mathrm{Ell}^1(M,Y)$. Then there exists a parametrix T of A which belongs to the class $\mathrm{Ell}^{-1}(M,Y)$, i.e. an operator $T : C^\infty(M;F) \to C^\infty(M;E)$ with splitting elliptic symbol of order -1 such that $TA = Id - K_1$ and $AT = Id - K_2$ where $K_1, K_2 \in \mathrm{Pdiff}^{-1}(M,Y)$.*

Following the fundamental tenents of numerical analysis we build our analysis upon Green's Formula and the concept of the spaces of Cauchy data.

13 THEOREM (GREEN'S FORMULA). *For any $A \in \mathrm{Pdiff}^1(X,Y)$ with $A = G_A(y,t)(\partial t + B)$ near Y and for any $u \in C^\infty(X;E)$ and $v \in C^\infty(X;F)$ we have*

$$< Au, v >_F - < u, A^* v >_E = -\int_Y (G_A(y,0)u(y), v(y))_{Fy}$$

where A^* denotes the formal-adjoint of A and $< .,. >_E$ and $< .,. >_F$ denote the inner products in the section spaces defined by the Hermitian structure of the bundles and the volume form on X given by the Riemannian metric.

PROOF: We can just follow the arguments given in Palais [12, Theorem 17.1], Seeley [14, pp. 793 ff], or Hörmander [17, p. 183]. □

14 DEFINITION. For $A \in \mathrm{Ell}^1(X,Y)$ we define
(a) the space of *interior solutions* of the equation $Au = 0$ by

$$\ker_X A := \{u \in C^1(X;E) \mid Au = 0 \quad \text{and} \quad u|_Y = 0\};$$

(b) the space of *Cauchy data* (along Y) by

$$H(A) := \{u|_Y \mid u \in C^\infty(X;E) \quad \text{and} \quad Au = 0\}.$$

151 REMARK. "In general" , namely when A satisfies the unique continuation property, the space of interior solutions consists only of the zero section, cf. Taylor [16, Chapter XIV].

152 REMARK. From Green's Formula it follows immediately that

$$H(A) \perp (G_A)^{-1}(H(A^*)).$$

Actually, the spaces are orthogonal complementary subspaces in $C^\infty(Y;E|_Y)$ as shown in by the authors in [4, Proposition 3.2] and - differently - in [19, Theorem 1].

2 "Symmetrization" and Continuation of Operators.

Operators with splitting elliptic symbols can be "symmetrized" near the boundary in the sense of the following Lemma. This was noted already in our [4, Lemma 3.1] from a topological perspective. Now we give an explicit deformation on the operator level:

21 LEMMA. *Let X, Y, E, and F be as in Section 1 and let $A : C^\infty(X;E) \to C^\infty(X;F)$ belong to the class $\mathrm{Ell}^1(X,Y)$ such that A takes the form $G_A(y,t)(\partial_t + B_t)$ on the collar $N = Y \times I$ where t denotes the (inward) normal coordinate, $B = \{B_t\}_{t\in I}$ is a family of elliptic pseudo-differential operators of first order $B_t : C^\infty(Y;E|_Y) \to C^\infty(Y;F|_Y)$, and $G_A(.,t) : E|_Y \to F|_Y$ is a bundle isomorphism for any $t \in I$. As in 11(b) we assume that $\{B_t\}$ and $\{G_A(.,t)\}$ depend smoothly on t and extend to small negative t. Then any such A extends to an $A' \in \mathrm{Ell}^1(X',Y')$ such that A' takes the form $G_A(y,0)(\partial_t + B')$ on the new collar $Y \times [-1,-1+\epsilon]$ for some $\epsilon > 0$ where $B' : C^\infty(Y;E|_Y) \to C^\infty(Y;F|_Y)$ is an elliptic self-adjoint operator (which does not depend on t) and $X' = (X \setminus N) \cup (Y \times [-1,1])$ denotes a larger manifold (diffeomorphic to X and with boundary $Y' = Y \times \{-1\}$; moreover we assume that E and F are defined on the whole of X').*

PROOF: Since A is elliptic, the principal symbols $b_t(y,\eta)$ of B_t, $t \in I$, have no purely imaginary eigenvalues. Moreover, because of 11(b) we can extend this family to a family $\{B_t\}_{t\in[-\epsilon,1]}$ for some small $\epsilon > 0$ such that the principal symbols of B_t for negative t have no purely imaginary eigenvalues either. Now we further deform this family on the interval $[-\epsilon, -\epsilon/2]$ such that $B_t = B_{-\epsilon}$ in a neighbourhood of $-\epsilon$. We denote the principal symbol of $B_{-\epsilon}$ by b.

Now, let $y \in Y$, $(y,\eta) \in (T^*Y)_y$, $\eta \neq 0$, and let $p_+(y,\eta) : E_y \to E_y$ denote the projection onto the direct sum of the eigenspaces of the linear transformation $b(y,\eta) : E_y \to E_y$ corresponding to the eigenvalues with a positive real part. We may assume that p is a family of orthogonal projections. This can be obtained by a suitable deformation of the Hermitian structure of $E|_{Y\times[-\epsilon,-\epsilon/2]}$.

Then we have

$$b = (2p_+ - Id)^2 b = (2p_+ - Id)q_0$$

where $(2p_+ - Id)$ is an elliptic symbol of order 0 and $q_0 := (2p_+ - Id)b$ has the following property:

There exists a constant $C > 0$ such that

$$\Re(\langle q_0(y,\eta)v, v\rangle_{E_y}) \quad \geq \quad C\|v\|^2$$

for any $y \in Y$, $(y,\eta) \in S_yY$ and $v \in E_y$, where $\Re$ denotes the real part.

Now let f be a smooth non-negative function on $\mathbf{R}$ equal to 0 in a neighbourhood of 0 and $f(t) = 1$ for $t > 1 - \delta$ with $\delta > 0$. Then the family of symbols

$$q_t(y,\eta) \quad = \quad f(t)f(|\eta|)|\eta| \quad + \quad (1 - f(y))q_0(y,\eta)$$

gives a smooth homotopy of elliptic symbols between $q_0 = (2p_+ - Id)b$ and $f(|\eta|)|\eta|$. Therefore we obtain a homotopy $r_t := (2p_+ - Id)q_t$ between b and the self-adjoint symbol $(2p_+ - Id)f(|\eta|)|\eta|$. We lift $\{r_t\}$ to a smooth homotopy $\{R_t\}$ of pseudo-differential operators (see e.g. Palais [12, Chapter XVI]) with $\sigma_L(R_t) = r_t$.

Clearly we may assume that $R_t = R_i$ in some neighbourhoods of $i \in \{0,1\}$. We also may assume that $R_1^* = R_1$. In fact: We can replace $\{R_t\}$ near 1 (where it is the constant family equal to R_1) by a new family $\tilde{R}_t := (1 - f(t))R_1 + f(t)\frac{1}{2}(R_1 + R_1^*)$.

Now we make the deformation on the other end where $R_t = R_0$. Since $r_0 = b$, the operator $R_0 - B_{-\epsilon}$ is of order 0. So, we may connect R_0 and $B_{-\epsilon}$ by a family $\tilde{R}_t := (1 - f(t))B_{-\epsilon} + f(t)R_0$

As a result of all these deformations we get (after reparametrization) a family of operators $\{\tilde{R}_t\}$ with principal symbols without purely imaginary eigenvalues which connects $B_{-\epsilon}$ with a self-adjoint operator $B' := R_1$. Moreover, this family is constant near $i = 0$, hence glueing is possible. So, after further reparametrization, we finally get a family which joins $B = \{B_t\}$ with the self- adjoint B'. This ends the proof. □

The following Proposition provides the main tool for our approach to the Calderón projector.

22 PROPOSITION. *Let X, Y, N, E, and F be as in Section 1. Then any $A \in \mathrm{Ell}^1(X,Y)$ extends to an $\tilde{A} \in \mathrm{Ell}^1(\tilde{X},Y)$ such that*

$$\tilde{A}|_{X\cup(Y\times[-1/4,0])} = A|_{X\cup(Y\times[-1/4,0])}.$$

Here $\tilde{X}$ denotes the closed double of X. Moreover, we assume that $X = (X \setminus N) \cup (Y \times [0,1])$, that $X' = (X \setminus N) \cup (Y \times [-1,1])$ is a larger manifold (diffeomorphic to X and with boundary $Y' = Y \times \{-1\}$), and that E, F and A are defined on the whole of X'.

PROOF: First, we apply Lemma 21 and deform A into an $A' \in \mathrm{Ell}^1(X', Y')$ with

$$A'|_{X \cup (Y \times [-1/4, 0])} = A|_{X \cup (Y \times [-1/4, 0])}$$

and with a self-adjoint split summand B' near Y'. We denote the principal symbol of B' by b'.

Then we apply the glueing construction as it is described by the authors in [18] and [4]: Over the double $\tilde{X}' = (X')_+ \cup (X')_-$ with $(X')_+ = (X')_- = X'$ we have a well defined operator

$$\tilde{A} = A' \cup (A')^* := \begin{cases} A' & \text{on} \quad (X')_+ \\ (A')^* & \text{on} \quad (X')_- \end{cases}$$

acting from C^∞ sections of the bundle $E \cup_G F$ to C^∞ sections of the bundle $F \cup_{G^{-1}} E$ (with $G := (G_A)|_Y$). Note that $(A')^* = (-\partial_t + B')(G_A)^{-1}$ in a collar neighbourhood of $Y \times \{-1\}$, hence the operators A' and $(A')^*$ and the symbols

$$\sigma_{\tilde{A}}(-1, y; \tau, \eta) = \begin{cases} \sigma_{A'}(-1, y_+; \tau, \eta) = G_A(i\tau + b'(y_+, \eta)) \\ \sigma_{(A')^*}(-1, y_-; \tau, \eta) = (-i\tau + b'(y_-, \eta))(G_A)^{-1} \end{cases}$$

fit together under the identification $(-1, y_+; \tau, \eta) \sim (-1, y_-; -\tau, \eta)$ of the cotangent sphere bundles near $Y' = Y \times \{-1\}$ and yield an elliptic symbol $\sigma_{\tilde{A}} = \sigma_{A'} \cup \sigma_{(A')^*}$.

The operator $\tilde{A}$ thus becomes the operator we are looking for, if we identify $\tilde{X}'$, the double of X', with $\tilde{X}$, the double of X, in such a way that $(X')_+$ goes over in X' and $(X')_- \setminus Y'$ goes over in $\tilde{X} \setminus X'$. □

23 COROLLARY. *Let A be an operator of class $\mathrm{Ell}^1(X, Y)$ where X is a smooth oriented compact manifold with boundary Y. Then the space $\ker_X A$ is finite-dimensional and consists of smooth sections.*

PROOF: By the preceding Proposition we can assume that A is the restriction of an elliptic operator $\tilde{A}$ on some closed manifold M containing X. We embed $\ker_X A$ in $\ker \tilde{A}$ by extending any interior solution over the whole of M by 0. Since the operators A and $\tilde{A}$ are of first order, we obtain a true solution of the homogeneous equation over M by the continuation (see Palais [12, Lemma 17.A]). The space $\ker \tilde{A}$ is finite-dimensional and consists of smooth sections. □

241 REMARK. Note that

$$\ker(A' \cup A'^*) = \{(u, v) \mid u \in C^\infty(X'; E), \quad v \in C^\infty(X'; F), \quad G_A(u|_{Y \times \{-1\}}) = \\ = v|_{Y \times \{-1\}}, \text{ and } A'u = 0, \quad A'^* v = 0\}$$

consists of the direct sum $\ker_{X'} A' \oplus \ker_{X'} A'^*$ of the spaces of interior solutions. Because of the mutual orthogonality of the spaces $H(A')$ and $(G_A)^{-1}(H(A'^*))$, see Remark 152, there exists no solution of the equation $(A' \cup A'^*)w = 0$ with $w|_Y$ not vanishing identically. Now it is also easy to determine the range $\operatorname{ran}(A' \cup A'^*)$ and the $\operatorname{index}(A' \cup A'^*)$. Since $A' \cup A'^*$ has an elliptic symbol, the orthogonal complement of $\operatorname{ran}(A' \cup A'^*)$ in $L^2(\widetilde{X'}; F \cup_{G^{-1}} E)$ is identical with $\ker(A' \cup A'^*)^* \subset C^\infty(\widetilde{X'}; F \cup_{G^{-1}} E)$. We have $(A' \cup A'^*)^* = A'^* \cup A'$, so

$$\operatorname{coker}(A' \cup A'^*) \cong \ker_{X'} A'^* \oplus \ker_{X'} A' \overset{j}{\cong} \ker_{X'} A' \oplus \ker_{X'} A'^* \cong \ker(A' \cup A'^*),$$

hence $\operatorname{index}(A' \cup A'^*) = 0$. Here j denotes the involution of the double of X' which interchanges the two factors $(X')_+$ and $(X')_-$.

242 REMARK. Our construction differs from the usual doubling construction (see e.g. Hörmander [11, Section 20.3, pp. 257 f]) which simply reflects the bundles and the operator along the boundary Y in the same way as the manifold. This results in a singularity along Y. The "supersymmetry" idea of looking at A and A^* simultaneously was suggested by A. P. Calderón and exploited already by Seeley in [14, Section 8] with his corrections given in [15, Appendix]. He considers the continuation of the matrix

$$\begin{pmatrix} 0 & A^* \\ A & 0 \end{pmatrix}$$

over the double through the identity (on the symbol level). Our construction is topologically different from Seeley's: (1) The operator $A \cup A^*$ can be defined on the operator level as a true continuation of A, and (2) it is not self-adjoint in general. (Although more explicit and more subtle than Seeley's construction, it does not provide additional information about the index because its index also vanishes.)

243 REMARK. If A satisfies the unique continuation property, then there exists no non-trivial interior solution and the continuation $A' \cup A'^*$ of A over the double is invertible because of Remark 241. In general, we get an invertible "approximative" continuation of A as a second Corollary to Proposition 22:

25 COROLLARY. *Let X, Y, E, F be as above. For any $A \in \mathrm{Ell}^1(X, Y)$ there exists a bijective operator $A'' \in \mathrm{Ell}^1(\tilde{X}, Y)$ over the double $\tilde{X}$ with the following properties:*

(i) $A''|_X - A|_X$ is an operator of finite rank.

(ii) The Cauchy data space $H_+(A'')$ of A'' along Y is equal to the Cauchy data space $H(A)$ of A along Y.

(iii) The inverse of A'' belongs to the class $\mathrm{Ell}^{-1}(\tilde{X}, Y)$.

PROOF: To (i). From Remark 241 we recall that $A' \cup A'^*$ is a Fredholm operator with $\operatorname{index}(A' \cup A'^*) = 0$ and that the reflection j defines an isomorphism

$$\ker(A' \cup A'^*) \overset{j}{\cong} \ker_{X'} A' \oplus \ker_{X'} A'^* \cong \ker_{X'} A'^* \oplus \ker_{X'} A' \cong (\operatorname{ran} A' \cup A'^*)^\perp.$$

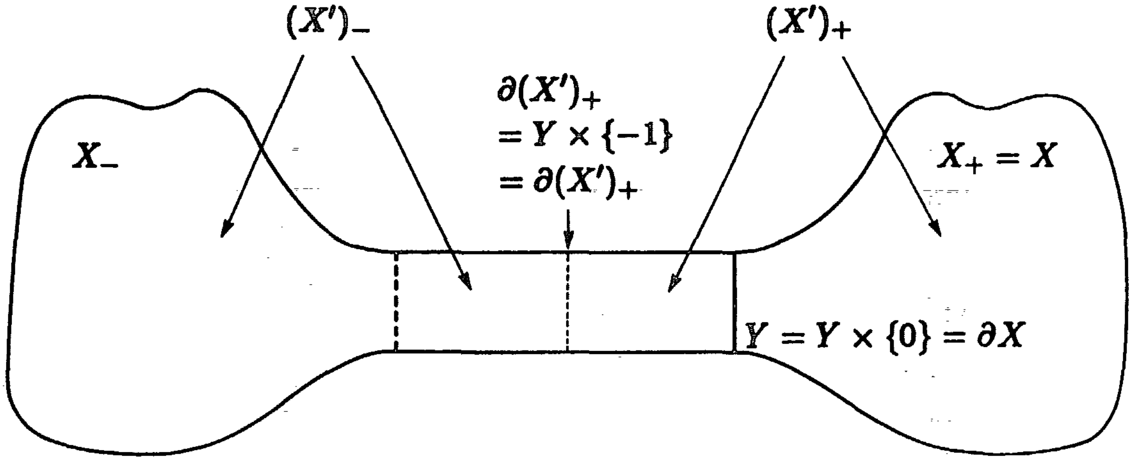

Fig. 1. The decomposition of $\tilde{X}$

The standard method of getting an invertible operator from a Fredholm operator with vanishing index consists in identifying kernel and cokernel and then adding the orthogonal projection onto the kernel to the operator. This is very easy in our situation. We define $A'' := A' \cup A'^* + jk$, where $k : C^\infty(\tilde{X}; E \cup_G F) \to \ker_{X'} A'$ is the orthogonal projection. More explicitly, we write

$$A''w = A''(u,v) := (A'u + jk_2 v, A'^* v + jk_1 u),$$

where $w = (u,v)$, $u \in C^\infty(X';E)$, $v \in C^\infty(X';F)$ with $G_A(u|_{Y\times\{-1\}}) = v|_{Y\times\{-1\}}$, and k_1 and k_2 are the orthogonal projections of $C^\infty(X';E)$ and $C^\infty(X';F)$ onto the finite-dimensional subspaces $\ker_{X'} A'$ and $\ker_{X'}(A')^*$ of interior solutions. The difference jk between the operators A'' and $A' \cup A'^*$ is of finite rank and hence a smoothing operator. Therefore A'' belongs to $\mathrm{Ell}^1(\tilde{X}, Y)$, too.

To (ii). Next we compare the Cauchy data spaces

$$H_+(A'') = \{w|_Y \mid w \in C^\infty(\tilde{X}; E \cup_G F) \text{ and } (A''w)|_X = 0\}$$

and

$$H(A) = \{u|_Y \mid u \in C^\infty(X;E) \text{ and } Au = 0\}.$$

Let us denote the operator $A' \cup A'^*$ by $\tilde{A}$. We split $C^\infty(\tilde{X}; E \cup_G F)$ into its two components $\ker \tilde{A}$ and $(\ker \tilde{A})^\perp$. Then we have

$$A''w = \tilde{A}w + jkw = \begin{cases} \tilde{A}w & \text{for} \quad w \in (\ker \tilde{A})^\perp \\ jw & \text{for} \quad w \in \ker \tilde{A}. \end{cases}$$

Hence the contributions to the Cauchy data spaces $H_+(A'')$ and $H(A) = H_+(\tilde{A})$ coming from sections in $(\ker \tilde{A})^\perp$ are identical, namely equal to

$$\{w|_Y \mid w \in (\ker \tilde{A})^\perp \quad \text{and} \quad (\tilde{A}w)|_X = 0\}.$$

The contributions coming from $\ker\tilde{A}$ are also identical: The contribution to $H(A)$ consists of

$$\{w|_Y \mid w \in \ker\tilde{A}\} = \{w|_Y \mid w \in \ker_{(X')_+}\tilde{A}\}.$$

The contribution to $H_+(A'')$ consists of

$$\{w|_Y \mid w \in \ker\tilde{A} \quad \text{and} \quad (jw)|_X = 0\} = \{w|_Y \mid w \in \ker_{\tilde{X}\setminus X_-}\tilde{A}\}.$$

Since $\ker\tilde{A}$ decomposes into the sum

$$\ker\tilde{A} = \ker_{(X')_+} + \ker_{(X')_-}\tilde{A},$$

only elements in $\ker_{(X')_+}\tilde{A}$ contribute to $(\ker_{\tilde{X}\setminus X_-}\tilde{A})|_Y$. See Figures 1 and 2.

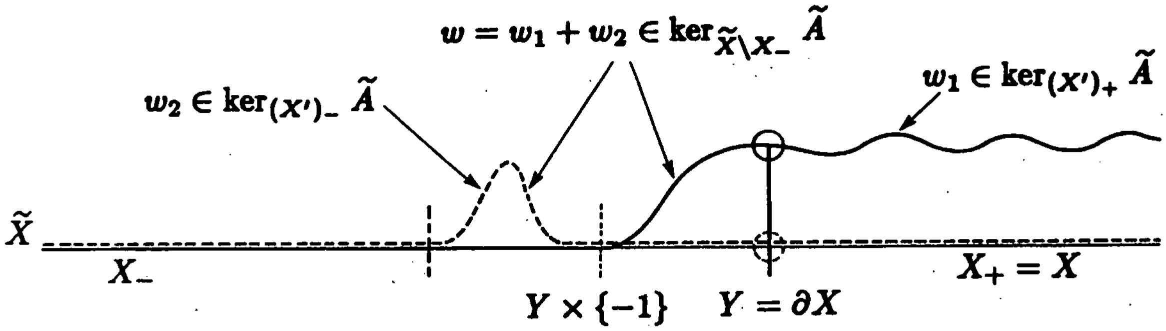

Fig. 2. The decomposition of $\ker_{\tilde{X}\setminus X_-}\tilde{A}$

This proves the equality.

To (iii). Let us denote the inverse of A'' by S. Now, exactly in the same way as in Palais [12, Lemma 17.B] we prove that $S \in \mathrm{Pdiff}^{-1}(\tilde{X}, Y)$. First we apply Proposition 12 and choose a parametrix $T \in \mathrm{Ell}^{-1}(\tilde{X}, Y)$ of A'', i.e.

$$TA'' = Id - K, \quad \text{where} \quad K \in \mathrm{Pdiff}^{-1}(\tilde{X}, Y).$$

Now we have the equality

$$(Id + K + K^2 + .. + K^m)TA'' = Id - K^{m+1}, m \geq 1,$$

and we put the operator S on both sides (from the right). We get

$$(Id + K + .. + K^m)T = S - K^{m+1}S.$$

Thus we have $S = T + KT + .. + K^mT + K^{m+1}S$. The operators K^jT, $j = 0, \ldots, m$, belong to the class $\mathrm{Pdiff}^{-j-1}(\tilde{X}, Y) \subset \mathrm{Pdiff}^{-1}(\tilde{X}, Y)$, and $K^{m+1}S$ is the composition of an operator of order -1 with an operator of order $-m-1$, hence it is an operator of order $-m-2$. So we are able to approximate a pseudolocal operator $S = (A'')^{-1}$ by operators belonging to the class $\mathrm{Pdiff}^{-1}(\tilde{X}, Y)$ up to any order. This proves that S belongs to $\mathrm{Pdiff}^{-1}(\tilde{X}, Y)$ by a similar argument as in Hörmander [10, Theorem 5.26]. The ellipticity of the symbol follows as usual from the Fredholm properties. □

26 REMARK. Note that the construction of $\tilde{A} = A' \cup A'^*$ does not necessarily take us out of the class of differential operators. For example $\tilde{A}$ is a well defined differential operator, if B is differential and self-adjoint. The same is not rtue for A''. In fact, if $A'' \neq \tilde{A}$, i.e. if there exist non-trivial interior solutions, then the operator A'' is not local and thus definitely not a differential operator - even though A and B may be differential. Then, Corollary 25 tells us, A'' still is pseudolocal and belongs to our class Pdiff^1.

3 The Calderón Projector - Revisited.

By the preceding Continuation Lemma 25 it is now very easy to carry out a slight modification of the constructions given in Palais [12], Seeley [14], and Hörmander [11]. The purpose of our modification is two-fold: (1) we get a ***true*** Calderón projector - not just an approximative one as in Hörmander [11]. And (2) our construction applies to ***arbitrary*** operators with splitting elliptic symbols. Thus the construction of the Calderón projector is no longer restricted to invertible operators differential over the whole manifold as in Palais [12] or to invertible operators differential near the boundary as in Seeley [14].

31 THEOREM. *Let X be a compact smooth oriented Riemannian manifold with smooth boundary Y and let E and F be smooth Hermitian vector bundles over X. Let $A : C^\infty(X;E) \to C^\infty(X;F)$ be an operator of class $\mathrm{Ell}^1(X,Y)$ with $A = G_A(y,t)(\partial_t + B_t)$ near Y where $y \in Y, t$ the (inward) normal coordinate, B_t an elliptic pseudo-differential operator of first order over Y, and $G_A(.,t)$ a bundle isomorphism from $E|_Y$ to $F|_Y$. Then there exists*
(i) a pseudo-differential operator of 0-th order $P(A) : C^\infty(Y{:}E|_Y) \to C^\infty(Y{:}E|_Y)$ such that
(ii) $P(A) = P(A)^* = P(A)^2$ and
(iii) $\operatorname{ran} P(A) = H(A)$, where $H(A)$ is the Cauchy data space of A over Y.
(iv) At each point (y,η) of the covariant sphere bundle SY the principal symbol of $P(A)$ is the projection $p(y,\eta) : E_y \to E_y$ onto the direct sum of the eigenspaces of the linear transformation $b(y,\eta) : E_y \to E_y$ corresponding to the eigenvalues with positive real part. Here b denotes the principal symbol of the operator B_0.

PROOF: We apply Continuation Lemma 25. This yields an invertible operator $A'' \in \mathrm{Ell}^1(\tilde{X}.Y)$. We repeat the arguments of Hörmander [11, Section 20.1, pp 234-237 and pp 242-249] with the following construction: We define $P(A'') := \gamma S A''^c$. where $A''^c g := G_A(.,0) g \otimes \delta_Y$ for $g \in C^\infty(Y;E|_Y)$, δ_Y is the Dirac measure in the parameter t, $S = (A'')^{-1}$, and $\gamma : C^\infty(\tilde{X};E \cup_G F) \to C^\infty(Y;E|_Y)$ is the restriction. Note the simplification in the form of A''^c which is due to our restriction to operators of order 1. A second simplification is due to our choice of S where we do not take a parametrix, but, instead, take the inverse of A'', which exists and also has splitting elliptic symbol as shown in 25. Thus we obtain a pseudo-differential operator

$$P(A'') : C^\infty(Y;E|_Y) \to C^\infty(Y;E|_Y)$$

of 0-th order fulfilling (ii) and (iv) and with $\operatorname{ran} P(A'') = H_+(A'')$. Since $H_+(A'') = H(A)$ we can define $P(A) := P(A'')$. □

32 REMARK. Since our proof closely follows the arguments of Hörmander [11] one may ask whether one can deduce our result - the existence of a pseudo-differential projector $P(A)$ onto the space of Cauchy data - just from Hörmander's result, i.e. from the existence of an idempotent pseudo-differential symbol $q(y,\eta)$ with its leading part equal to $p(y,\eta)$, the principal symbol of $P(A)$. In fact, any appropriate pseudo-differential projection Q with closed range in $H(A)$ and with $\sigma_L(Q) = p$ can be deformed into the Calderón projector by a series of modifications with suitable operators of finite rank. However, this obviously is not true if the closedness of the range of Q is not assured.

33 REMARK. If $A \in \mathrm{Ell}^1(X,Y)$ takes the form $A = G_A(\partial_t + B)$ near Y with B self-adjoint, then the spectral projection $\mathrm{sP}(B)$ is a well-defined pseudo-differential operator of order zero. It maps $C^\infty(Y;E|_Y)$ onto the span$\{e_j\}_{j>0}$ where the $\{e_j\}$ are an orthonormal system of the eigenfunctions of B with positive eigenvalues, and the principal symbols of the spectral projection $\mathrm{sP}(B)$ and of the Calderón projector $P(A)$ coincide, see Taylor [16, Chapter V] and our [3, Section 2]. Note, that the Calderón projector $P(A)$ depends solely on the choice of a normal vector field on X near Y which provides the splitting of A. In contrast, the spectral projection $\mathrm{sP}(B)$ is additionally dependent on the Riemannian structure of Y and on the Hermitian structure of E. In fact, the spectral projections may belong to different connected components (in the L^2-operator topology) of the space of pseudo-differential projections with the given principal symbol. This will be explained in a subsequent paper.

4 Examples and Concluding Remarks.

The main tool for the analysis of elliptic boundary value problems for operators of the class $\mathrm{Ell}^1(X,Y)$ is provided by Theorem 31 (and Corollary 25). With this tool we can prove the same regularity theorems and the finiteness of the index as was done for differential operators in the beautiful lectures Seeley [15, Chapter 6]. The results of [15] concerning the realizations of the form A_R with (local or global) elliptic boundary condition R also remain valid for $A \in \mathrm{Ell}^1(X,Y)$.

It is not difficult to extend the results of Polking [13] so that they become valid for any manifold with boundary; and not just for the cylinder $M \times I$ where M is a closed manifold.

These are (up to now) the main applications of our results. We would like to end with two easy examples which show that the class $\mathrm{Ell}^1(X,Y)$ is really a useful class of operators. Operators of class $\mathrm{Ell}^1(X,Y)$ namely appear naturally as the result of a reduction of operators of higher order:

41 PROPOSITION. *Let A_i, $i = 1,2$, be differential operators of order i (scalar or acting on the sections of a vector bundle) over a closed smooth oriented manifold M. Then the second order differential operator $\mathcal{A} = \partial_t^2 + A_1\partial_t + A_2$ over the cylinder $X = M \times I$ with boundary $Y = M \times \{0,1\}$ can be reduced to a (non-classical) pseudo-differential operator $\mathcal{A}'$ with splitting symbol such that $\ker \mathcal{A} \cong \ker \mathcal{A}'$. Here ∂_t denotes the differentiation with respect to t, i.e. the I-direction. Moreover, $\mathcal{A}$ is elliptic if and only if $\mathcal{A}'$ belongs to the class $\mathrm{Ell}^1(X,Y)$.*

PROOF: We choose a positive self-adjoint first order elliptic operator Λ over M with principal symbol $\sigma_\Lambda(y,\eta) = |\eta|$. For example we may take $\Lambda = Q^*Q + 1$ with suitable Q with $\sigma_Q(y,\eta) = |\eta|^{1/2}$. Here y denotes the coordinate in M and $\eta \in T^*M_y$. Then we obtain the following reduction scheme (cf. Taylor [16, Chapter IV and Chapter V]):

Near $M \times \{0\}$ we have

$$\mathcal{A}u = 0 \Longleftrightarrow \mathcal{A}'w = 0,$$

where

$$\mathcal{A}' = \begin{pmatrix} Id & 0 \\ 0 & Id \end{pmatrix} \partial_t + \begin{pmatrix} 0 & -\Lambda \\ A_2\Lambda^{-1} & A_1 \end{pmatrix}$$

and

$$w = \begin{pmatrix} \Lambda u \\ \partial_t u \end{pmatrix}.$$

In fact

$$\mathcal{A}'\, w = \begin{pmatrix} \partial_t \Lambda u - \Lambda \partial_t u \\ \mathcal{A}u \end{pmatrix} = \begin{pmatrix} 0 \\ \mathcal{A}u \end{pmatrix}.$$

The total symbol $a(x,\xi)$ of $\mathcal{A}'$ has the following form:

$$a(x,\xi) = \begin{pmatrix} i\tau & -|\eta| \\ a_2(y,\eta)\,|\eta|^{-1} & i\tau + a_1(y,\eta) \end{pmatrix}$$
$$+ \text{ lower order purely pseudo-differential terms,}$$

where $x = (y,t)$, $\xi = (\eta,\tau) \in (T^*X)_x$, and a_i denotes the principal symbol of A_i. Thus we obtain

$$\left|D_\xi^\alpha D_x^\beta a(x,\xi)\right| \le C_{\alpha\beta}(1+|\xi|)^{1-\alpha''}(1+|\eta|)^{-|\alpha'|}$$

for all multiindices $\alpha = (\alpha_1,..,\alpha_{n-1},\ \alpha'')$, $\alpha' = (\alpha_1,..,\alpha_{n-1})$ and β, so $\mathcal{A}' \in \text{Pdiff}^1(X,Y)$. Moreover, for the principal symbols we have the relation

$$\sigma_{\mathcal{A}}(x,\xi) = -\tau^2 + i\tau a_1(y,\eta) + a_2(y,\eta) = \det \sigma_{\mathcal{A}'}(x,\xi),$$

hence $\mathcal{A}'$ belongs to the class $\text{Ell}^1(X,Y)$ if and only if $\mathcal{A}$ is elliptic. □

42 REMARK. For elliptic $\mathcal{A}$ (and $\mathcal{A}'$) we find

$$H(\mathcal{A}') = \{(\Lambda u|_{\partial X}\,,\,(\partial_t u)|_{\partial X}) \mid u \in \ker \mathcal{A}\}.$$

So, the spaces of Cauchy data for the first order operator $\mathcal{A}'$ and for the second order operator $\mathcal{A}$ are isomorphic under the transformation $(r_1,r_2) \mapsto (\Lambda r_1, r_2)$, and we get for the Calderón projector

$$P(\mathcal{A}') : L^2(M \times \{0,1\}\,;E) \oplus L^2(M \times \{0,1\}\,;E) \longrightarrow H(\mathcal{A}')$$

the condition

$$P(\mathcal{A}')(r_1, r_2) = (r_1, r_2) \quad \Longleftrightarrow \quad u|_{M\times\{i\}} = \Lambda^{-1} r_{1i} \text{ and } (\partial_t u)|_{M\times\{i\}} = r_{2i},$$

for all $u \in C^\infty(M \times I; E)$ and $i \in \{0,1\}$ where $r_1 = (r_{10}, r_{11})$ and $r_2 = (r_{20}, r_{21})$.

43 REMARK. $\mathcal{A}$ is a differential operator. If additionally it is elliptic and scalar it then fulfills the unique continuation property. Hence the equivalent operator $\mathcal{A}'$ has no non-trivial interior solutions.

44 EXAMPLE. Let M be a closed smooth oriented Riemannian manifold. Let E be a Hermitian vector bundle over M, and let $B : C^\infty(M; E) \to C^\infty(M; E)$ be a self-adjoint elliptic differential operator of first order. Over the cylinder $X = M \times I$ we consider the operator

$$\mathcal{A} = \partial_t^2 - B^2 + \epsilon B \partial_t$$

where ϵ is a complex parameter. We want to make the following observations:

(i) As noticed above, we obtain

$$\sigma_{\mathcal{A}}(x, \xi) = -\tau^2 + i\tau\epsilon b(y, \eta) - b^2(y, \eta)$$

for $x = (y, t) \in X$ and $\xi = (\eta, \tau) \in (T^*X)_x$ where b denotes the principal symbol of B. Then a necessary and sufficient condition for the ellipticity of $\mathcal{A}$ is that the equation

$$-\tau^2 + i\tau\epsilon\lambda - \lambda^2 = 0$$

has no real solution τ for any eigenvalue λ of the endomorphism $b(y, \eta) : E_y \to E_y$. So, we have ellipticity for example for all real ϵ and for purely imaginary ϵ with $|\epsilon| \leq 2$. In the last case $\mathcal{A}$ becomes self-adjoint.

(ii) Now assume that $\mathcal{A}$ is elliptic. We apply Proposition 41 and obtain an equivalent first order operator with splitting symbol

$$\mathcal{A}' = \begin{pmatrix} Id & 0 \\ 0 & Id \end{pmatrix} \partial_t + \begin{pmatrix} 0 & -\Lambda \\ -B^2\Lambda^{-1} & \epsilon B \end{pmatrix}$$

where we choose $\Lambda := \sqrt{1 + B^2}$. Then we have

$$\mathcal{A}' = \begin{pmatrix} Id & 0 \\ 0 & Id \end{pmatrix} \partial_t + \mathcal{B}$$

with

$$\mathcal{B} = \begin{pmatrix} 0 & -\sqrt{1+B^2} \\ \frac{B^2}{\sqrt{1+B^2}} & \epsilon B \end{pmatrix} : C^\infty(E \oplus E) \to C^\infty(E \oplus E).$$

(iii) Recall that elliptic self-adjoint operators of positive order have a discrete spectrum of finite multiplicity $\{\lambda_k\}_{k\in\mathbf{Z}}$. Moreover, there exists no essential spectrum, and the eigenvectors $\{\phi_k\}$ span the whole L^2E. Let $\{\lambda_k, \phi_k\}_{k\in\mathbf{Z}}$ denote such a spectral decomposition of L^2E related to our self-adjoint operator B. Then

$\left\{\sqrt{1+\lambda_k^2}, \phi_k\right\}_{k\in\mathbf{Z}}$ is a spectral decomposition related to the operator $\sqrt{1+B^2}$. We want to construct a set of solutions of $\mathcal{A}u = 0$ from that information.

Let E_k denote the one-dimensional space spanned by ϕ_k. Then $\mathcal{B}$ maps $E_k \oplus E_j$ into $E_k \oplus E_j$ for all $k, j \in \mathbf{Z}$. We define $B_{kj} := \mathcal{B}|_{E_k \oplus E_j}$ and obtain $\mathcal{B} = \sum_{k,j} B_{kj}$.

For any $c, c' \in \mathbf{C}$ we have

$$\mathcal{B}(c\phi_k, c'\phi_k) = (\gamma\phi_k, \gamma'\phi_k)$$

with

$$\begin{pmatrix} \gamma \\ \gamma' \end{pmatrix} = \begin{pmatrix} 0 & -\sqrt{1+\lambda_k^2} \\ \frac{-\lambda_k^2}{\sqrt{1+\lambda_k^2}} & \epsilon\lambda_k \end{pmatrix} \begin{pmatrix} c \\ c' \end{pmatrix}.$$

So the operator B_{kk} takes the form

$$B_{kk} = \lambda_k \begin{pmatrix} 0 & \left(\frac{-\lambda_k}{\sqrt{1+\lambda_k^2}}\right)^{-1} \\ \frac{-\lambda_k^2}{\sqrt{1+\lambda_k^2}} & \epsilon \end{pmatrix}.$$

We determine the eigenvalues and eigenfunctions of B_{kk}. For $a \in \mathbf{R}$ we have

$$\begin{pmatrix} 0 & a^{-1} \\ a & \epsilon \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} a^{-1}v \\ au + \epsilon v \end{pmatrix},$$

hence

$$\mu u = a^{-1}v \quad \text{and} \quad \mu v = au + \epsilon v$$

if and only if $\mu^2 - \epsilon\mu - 1 = 0$, i.e.

$$\mu_\pm = \frac{\epsilon}{2} \pm \sqrt{1 + \frac{\epsilon^2}{4}}.$$

The corresponding eigenvectors are

$$\begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} 1 \\ a\left(\frac{\epsilon}{2} \pm \sqrt{1+\frac{\epsilon^2}{4}}\right) \end{pmatrix}.$$

Therefore, the eigenvalues of B_{kk} are $\mu_{kk} = \lambda_k\mu_\pm$ and the eigenfunctions are

$$\psi_{kk} = \left(\phi_k \, , \, -\frac{\lambda_k\mu_\pm}{\sqrt{1+\lambda_k^2}}\,\phi_k\right).$$

With similar - but more laborious - work we determine the eigenvalues and eigenfunctions of the partial operators B_{kj} for $j \neq k$.

(iv) As explained by the authors in [4, Example 3.5(a)], any solution of

$$\mathcal{A}'w = \left(\begin{pmatrix} Id & 0 \\ 0 & Id \end{pmatrix} \partial_t + \mathcal{B} \right) w = 0$$

has the form

$$w = \sum_{a_{kj\pm}} \exp(-t\mu_{kj\pm})\ \psi_{kj\pm},$$

and the space of Cauchy data $H(\mathcal{A}')$ is the subspace of $L^2(M \times \{0\}; E \oplus E) \oplus L^2(M \times \{1\}; E \oplus E)$ with the orthonormal base

$$\frac{1}{\sqrt{1+e^{2\mu_{kj\pm}}}} \Big(\underbrace{\psi_{kj\pm}}_{\text{over } M\times\{0\}} \quad ; \quad \underbrace{e^{\mu_{kj\pm}} \psi_{kj\pm}}_{\text{over } M\times\{1\}} \Big) .$$

(v) We finally obtain the solutions of $\mathcal{A}u = 0$ by back transformation following the rules given in the proof of Proposition 41. So, any $w \in \ker \mathcal{A}'$ provides a $u \in \ker \mathcal{A}$. In particular we obtain for any $k \in \mathbf{Z}$ a solution

$$u_k = \exp\left(-t\lambda_k \left(\epsilon/2 \pm \sqrt{1+\ \epsilon^2/4}\right)\right)\ (1+\lambda_k^2)^{-\frac{1}{2}}\ \phi_k .$$

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39/81 "TIL KRITIKKEN AF VÆKSTØKONOMIEN".
Af: Jens Højgaard Jensen.

40/81 "TELEKOMMUNIKATION I DANMARK - oplæg til en teknologivurdering".
Projektrapport af: Arne Jørgensen, Bruno Petersen og Jan Vedde.
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Af: Helge Kragh og Stig Andur Pedersen.

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Projektrapport af: Niels Thor Nielsen.
Vejleder: Bent C. Jørgensen.

45/82 Er aldrig udkommet.

46/82 "EKSEMPLARISK UNDERVISNING OG FYSISK ERKENDESE-
1+11 ILLUSTRERET VED TO EKSEMPLER".
Projektrapport af: Torben O.Olsen, Lasse Rasmussen og Niels Dreyer Sørensen.
Vejleder: Bent C. Jørgensen.

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ENERGY SERIES NO. 5.
Af: Bent Sørensen.

48/82 "EN UNDERSØGELSE AF MATEMATIKUNDERVISNINGEN PÅ ADGANGSKURSUS TIL KØBENHAVNS TEKNIKUM".
Projektrapport af: Lis Eilertzen, Jørgen Karrebæk, Troels Lange, Preben Nørregaard, Lissi Pedesen, Laust Rishøj, Lill Røn og Isac Showiki.
Vejleder: Mogens Niss.

49/82 "ANALYSE AF MULTISPEKTRALE SATELLITBILLEDER".
Projektrapport af: Preben Nørregaard.
Vejledere: Jørgen Larsen og Rasmus Ole Rasmussen.

50/82 "HERSLEV - MULIGHEDER FOR VEDVARENDE ENERGI I EN LANDSBY".
ENERGY SERIES NO. 6.
Rapport af: Bent Christensen, Bent Hove Jensen, Dennis B. Møller, Bjarne Laursen, Bjarne Lillethorup og Jacob Mørch Pedersen.
Vejleder: Bent Sørensen.

51/82 "HVAD KAN DER GØRES FOR AT AFHJÆLPE PIGERS BLOKERING OVERFOR MATEMATIK ?"
Projektrapport af: Lis Eilertzen, Lissi Pedersen, Lill Røn og Susanne Stender.

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Af: Else Høyrup.

Vedr. tekst nr. 55/82 se også tekst nr. 62/83.

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En undersøgelse af matematisk økologi.
Projektrapport af: Troels Lange.
Vejleder: Anders Madsen.

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Projektrapport af: Tom Juul Andersen.
Vejleder: Peder Voetmann Christiansen.
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Projektrapport af: Per Hammershøj Jensen og Lene Vagn Rasmussen.
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Projektrapport af: Annette Post Nielsen.
Vejledere: Jens Høyrup, Jens Højgaard Jensen og Jørgen Vogelius.

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Vejledere: Bernhelm Booss og Klaus Grünbaum.

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Specialerapport af: Jørgen Wind Petersen og Jan Christensen.
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Rapport fra et seminar afholdt i Hvidovre 25-27 april 1983.
Red.: Jens Højgaard Jensen, Bent C. Jørgensen og Mogens Niss.

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Af: Jens Højgaard Jensen, Mogens Niss m. fl.

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Projektrapport af: Lis Eilertzen, Kirsten Habekost, Lill Røn og Susanne Stender.
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Af: Jeppe C. Dyre.
Contributions to the Third International Conference on the Structure of Non - Crystalline Materials held in Grenoble July 1985.

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Fysiklærerforeningen, IMFUFA, RUC.

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Samtlige opgaver stillet i tiden 1974-jan. 1986.

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Projektrapport af: Birger Lundgren.

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Projektrapport af: Lise Odgaard & Linda Szkotak Jensen
Vejledere: Karin Beyer & Stig Andur Pedersen.

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Vejledere: Jens Høyrup, Jørgen Vogelius, Jens Højgaard Jensen.

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Authors: M.B.El-Den, N.B.Olsen, Ib Høst Pedersen, Petr Viščör

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Af: Jeppe Dyre
Vejledere: Niels Boye Olsen og Peder Voetmann Christiansen.

138/87 "JOSEPHSON EFFECT AND CIRCLE MAP."

Paper presented at The International Workshop on Teaching Nonlinear Phenomena at Universities and Schools, "Chaos in Education". Balaton, Hungary, 26 April-2 May 1987.

By: Peder Voetmann Christiansen

139/87 "Machbarkeit nichtbeherrschbarer Technik durch Fortschritte in der Erkennbarkeit der Natur"

Af: Bernhelm Booss-Bavnbek
Martin Bohle-Carbonell

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By: Jens Gravesen

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Projektrapport af Finn C. Physant

Vejleder: Ib Thiersen

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by: Bernhelm Booss-Bavnbek og
Krzysztof P. Wojciechowski