

## The dirac equation with light-cone data

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The Dirac Equation with Light-Cone Data.

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ABSTRACT

The Dirac equation with light-cone data ;

$$(i\gamma^0 \frac{\partial}{\partial t} + i\gamma \cdot \nabla - m\mathbb{1})u(t,x) = 0.$$

$$u(-|x|,x) = f(x), x \in \mathbb{R}^3$$

is considered and explicitly solved in terms of a "Light-cone  
Fourier-transform" under appropriate conditions on  $f$ .

## 1. Introduction.

We shall consider the characteristic Cauchy problem for the free Dirac equation

$$(i\gamma^0 \frac{\partial}{\partial t} + i\gamma^j \nabla_j - m\mathbb{1})u(t,x) = 0 \quad (1.1)$$

$$u(-|x|,x) = f(x) \quad (1.2)$$

under suitable conditions on  $f$ , in four space-time dimensions.

The analogous problem for the Klein-Gordon equation was considered in [1]. The wave-equation has been considered by Riesz [2] and Strichartz [3], but in a very different way. For some general results on characteristic Cauchy-problems see Hörmander [4].

We shall show that there exist a Hilbert-space  $\mathcal{H}$  of light-cone data such that (1.1) and (1.2) have a unique weak solution  $u(t,x)$ , with  $u(t,\cdot) \in L^2(\mathbb{R}^3, \mathbb{C}^4)$  for  $f$  in  $\mathcal{H}$  and so that all " $L^2$ -solutions" of (1.1) have the property that  $u(t-|\cdot|, \cdot)$  is in  $\mathcal{H}$ .

The Dirac equation (1.1) will be rewritten, as an evolution

equation  $i \frac{\partial u}{\partial t} = Hu$  in  $\mathcal{H}$  and the generator  $H$  will be diagonalized

by a "light-cone Fourier transform", which then allows us to give an explicit formula for  $u(t,x)$  in terms of  $f$ .

## 2. The Dirac Equation and Associated Evolution Equations.

The Dirac equation is a hyperbolic system of linear partial differential equations which can be written as

$$(i\gamma^0 \frac{\partial}{\partial t} + i\gamma \cdot \nabla - m\mathbb{1})u(t,x) = 0 \quad (2.1)$$

where  $m \neq 0$  is the mass of the particle,  $u: \mathbb{R} \times \mathbb{R}^3 \rightarrow \mathbb{C}^4$ ,  $\gamma^0$  and the three components of the vector  $\gamma = (\gamma^1, \gamma^2, \gamma^3)$  are  $4 \times 4$  complex matrices fulfilling the anti-commutation relation

$$\gamma^\mu \gamma^\nu + \gamma^\nu \gamma^\mu = 2g^{\mu\nu} \mathbb{1} \quad (2.2)$$

where  $g^{\mu\nu} = 0$  for  $\mu \neq \nu$ ,  $g^{00} = -g^{ii} = 1$ , for  $i = 1, 2, 3$ , and  $\mathbb{1}$  is the  $4 \times 4$  unit matrix. An explicit representation of the  $\gamma^\mu$ 's is given by

$$\gamma^0 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad \text{and} \quad \gamma^j = \begin{bmatrix} 0 & -\sigma^j \\ \sigma^j & 0 \end{bmatrix}, \quad \text{where } \mathbb{1} \text{ in } \gamma^0 \text{ is the}$$

$2 \times 2$  unit matrix and the  $\sigma^j$ 's are the familiar Pauli-matrices given by

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \quad \text{We shall use this representation}$$

in the following.

The Dirac equation (2.1) is often considered as an evolution equation with initial data on a space-like hyperplane, consider for example a  $t = \text{constant}$  hyperplane and write the equation as

$$i \frac{\partial u}{\partial t} = \gamma^0 (-i \gamma \cdot \nabla + m \mathbb{1}) u, \quad \text{which can be considered as an evolution}$$

equation in the Hilbert-space  $L^2(\mathbb{R}^3, \mathbb{C}^4)$ . The generator

$D = \gamma^0 (-i \gamma \cdot \nabla + m \mathbb{1})$  defines a self-adjoint operator  $D$  with domain

$\mathcal{D}(D) = H^1(\mathbb{R}^3, \mathbb{C}^4)$ , the direct sum of the four identical Sobolev

spaces  $H^1(\mathbb{R}^3)$ . The associated spectral representation of a solution

$e^{-iDt} f$  can be written

$$u(t, x) = (2\pi)^{-3/2} \sum_{\pm} \sum_{s=1,2} \int_{\mathbb{R}^3} \frac{d^3 k}{\omega_k} e^{\mp i \omega_k t + i k \cdot x} u_{\pm}(k, s) \hat{f}_{\pm}(k, s) \quad (2.3)$$

for  $f(\cdot) \in \mathcal{S}(\mathbb{R}^3, \mathbb{C}^4)$ , the direct sum of the four identical Schwartz-spaces  $\mathcal{S}(\mathbb{R}^3)$ , where  $u_{\pm}(k,s)$ ,  $s = 1, 2$ , are four orthogonal eigenvectors of  $\hat{D} = \gamma^0(\gamma \cdot k + m\mathbb{1})$  normalized such that  $u_{\pm}^* u_{\pm} = \omega_k = (|k|^2 + m^2)^{1/2}$ . The "Fourier-components"  $\hat{f}_{\pm}(\cdot, s)$  is essentially given by the ordinary Fourier-Plancherel transformation,

since  $L^2(\mathbb{R}^3, \mathbb{C}^4) = \bigoplus_{n=1}^4 L^2(\mathbb{R}^3)$  it follows that the ordinary Fourier-

Plancherel transform lifts in a canonical way to a unitary operator  $F_0 : L^2(\mathbb{R}_x^3, \mathbb{C}^4) \rightarrow L^2(\mathbb{R}_k^3, \mathbb{C}^4)$  such that

$F_0 D F_0^* = \hat{D} = \gamma^0(\gamma \cdot k + m\mathbb{1})$  and  $\hat{f} = F_0 f$ . For  $f(\cdot) \in \mathcal{S}(\mathbb{R}^3, \mathbb{C}^4)$  we have the following explicit formula

$$(F_0 f)_{\pm}(k, s) = \langle v_{\pm}(\cdot, k, s), f(\cdot) \rangle_{L^2(\mathbb{R}_x^3, \mathbb{C}^4)} \quad (2.4)$$

Where

$$v_{\pm}(x, k, s) = (2\pi)^{-3/2} e^{ik \cdot x} u_{\pm}(k, s) \quad (2.5)$$

The associated Parseval's formula reads

$$\int_{\mathbb{R}^3} d^3x u^*(t,x)u(t,x) = \sum_{\pm} \sum_{s=1,2} \int_{\mathbb{R}^3} \frac{d^3k}{\omega_k} \left| \widehat{f}_{\pm}(k,s) \right|^2 \quad (2.6)$$

for every  $t \in \mathbb{R}$ . Consider a solution  $u(t,x)$  such that  $\widehat{f}_{\pm} \in \mathcal{S}(\mathbb{R}^3)$  and construct the current  $J^{\mu} = \bar{u} \gamma^{\mu} u$ , where  $\bar{u} = u^* \gamma^0$ . It follows from the Dirac equation (2.1) that  $J^{\mu}$  is conserved, i.e.

$$\nabla_{\mu} J^{\mu} \equiv \frac{\partial}{\partial t} J^0 + \nabla \cdot J = 0. \text{ Intergration of this conservation equation}$$

over the region in  $\mathbb{R} \times \mathbb{R}^3$  between a  $t=t_0$  hyperplane,  $t_0 \in \mathbb{R}$ , and the lighth-cone  $\mathcal{C} = \{ (t,x) : t=t'-|x|, t' < t_0 \}$  together with Gauss's theorem and a decay estimate, given in [1], gives

$$\begin{aligned} \int_{\mathbb{R}^3} d^3x \bar{u}(t'-|x|,x) (\gamma^0 + \gamma \cdot n) u(t'-|x|,x) \\ = \sum_{s=1,2} \int_{\mathbb{R}^3} \frac{d^3k}{\omega_k} ( \left| \widehat{f}_{+}(k,s) \right|^2 + \left| \widehat{f}_{-}(k,s) \right|^2 ) \end{aligned} \quad (2.7)$$



where  $n = x|x|^{-1}$ , equation (2.7) resembles a Parseval's formula.

This indicates that there might exist a Hilbert-space  $\mathcal{H}$  of lighth-cone data  $u(t-|\cdot|, \cdot)$  with the left-hand side of (2.7) as scalar product and such that the map  $u(-|\cdot|, \cdot) \rightarrow u(t-|\cdot|, \cdot)$  defines a unitary strongly continuous one-parameter group  $t \rightarrow U(t)$  on  $\mathcal{H}$ . This will be shown to be the case in the next section.

Let  $u(0,x)$  have support in the region  $|x| \leq R$  and consider  $t \geq R$ . It then follows from the finite propagation velocity that the support of  $u(t-|x|, x)$  is in the region  $|x| \leq (R+t)/2$ . This means that there is a subspace  $\mathcal{H}_0 \subset \mathcal{H}$  of lighth-cone data with compact support.

It follows again from the finite propagation velocity, that  $U(t)\mathcal{H}_0 \subset \mathcal{H}_0$ , for  $t \geq 0$ . This need not be the case for  $t < 0$ , which means that compact support of lighth-cone data, is in general, only a semi-group property.

### 3. The light-cone evolution equation.

Consider the Dirac equation (2.1) and make the coordinate exchange  $t \rightarrow t' = t + |x|$  and  $x \rightarrow x' = x$  such that a  $t' = \text{constant}$  is the backward light-cone with its apex at  $(t,0)$ . The new coordinates will be called light-cone (LC) coordinates. In the LC coordinates the Dirac

equation becomes  $i(1 + \alpha \cdot n') \frac{\partial u'}{\partial t'} = (-i\alpha \cdot \nabla' + m\gamma^0)u'$  where

$u'(t',x') = u(t' - |x'|, x')$ ,  $\alpha = \gamma^0 \gamma$  and  $n' = x' |x'|^{-1}$ , note that  $\nabla'$  is associated with the light-cone  $\mathfrak{C} = \{(t,x) : t = -|x|\}$  in the original coordinates and  $\mathfrak{C} = \{x' \in \mathbb{R}^3\}$  in the new coordinates, i.e. the light-cone can be identified with  $\mathbb{R}^3$  which will be done in the following. We shall only use the LC coordinates from now on and can therefore drop the primes. Hence the Dirac equation becomes

$$i(1 + \alpha \cdot n) \frac{\partial u}{\partial t} = (-i\alpha \cdot \nabla + m\gamma^0)u \quad (3.1)$$

where  $n = x |x|^{-1}$  and  $\alpha = \gamma^0 \gamma$ .

Definition 3.1. Let  $L^2 = L^2(\mathfrak{E}, \mathbb{C}^4)$  where the measure on  $\mathfrak{E}$  is the Lebesgue measure on  $\mathbb{R}^3$ . Put

$$Q = \mathbf{1} + \alpha \cdot n \quad \text{and} \quad D = -i\alpha \cdot \nabla + \gamma^0 m \quad (3.2)$$

It then follows that  $2^{-1}Q$  is an orthogonal projection in  $L^2$  and that the Dirac operator  $D$  defines a self-adjoint operator in  $L^2$  with spectrum  $\sigma(D) = (-\infty, -m] \cup [m, \infty)$ ,  $m > 0$ .

The Dirac equation (3.1) can now be considered as an equation in  $L^2$

$$iQ \frac{du}{dt}(t) = Du(t) \quad (3.3)$$

which can be written as

$$iD^{-1}Q \frac{du}{dt}(t) = u(t) \quad (3.4)$$

since  $0 \notin \sigma(D)$ . From equation (3.4) it follows that if  $\frac{du}{dt} \in L^2$

then  $u \in D^{-1}QL^2$ .

Definition 3.2. Let  $B = D^{-1}Q$  and  $\mathcal{H}_0 = BL^2$ . Furthermore put

$$\langle f, g \rangle_{\mathcal{H}_0} = \langle f, Qg \rangle_{L^2} \quad (3.5)$$

for  $f, g \in \mathcal{H}_0$ .

It will be proved that  $\mathcal{H}_0$  is a pre-Hilbert space, but before doing that we have to mention two lemmas, both proved in [1].

Lemma 3.3. Let  $S_0 = -i(n \cdot \nabla + \nabla \cdot n)$  with domain  $\mathcal{D}(S_0) = C_0^\infty(\mathbb{R}^3)$

in  $L^2(\mathbb{R}^3)$ , then the following inequality holds

$$\| S_0 f \|_{L^2(\mathbb{R}^3)} \leq 4 \int_{\mathbb{R}^3} d^3x \left| \nabla f(x) \right|^2 \quad (3.6)$$

for all  $f \in \mathcal{D}(S_0)$ .

Lemma 3.4. The range  $\mathcal{R}(S_0)$  of  $S_0$  is dense in  $L^2(\mathbb{R}^3)$ .

Due to the inequality (3.6)  $S_0$  can be extended continuously to the Sobolev space  $H^1(\mathbb{R}^3)$ , let this extension be denoted  $S$ , it also follows that  $S$  is symmetric in  $L^2(\mathbb{R}^3)$ , since  $S_0$  is. Due to lemma 3.4. also  $S$  have dense range  $\mathcal{R}(S_0)$  in  $L^2(\mathbb{R}^3)$ , and since  $S$  is symmetric in  $L^2(\mathbb{R}^3)$  it follows that  $S$  is one-to-one, i.e.  $S$  has a densely defined inverse in  $L^2(\mathbb{R}^3)$ , in fact it is easy to show that

$$(S_0^{-1}g)(x) = \frac{i}{2r} \int_{-\infty}^r \rho g(\rho, x|x|^{-1}) d\rho \quad (3.7)$$

for  $g \in \mathcal{R}(S_0)$ .

Proposition 3.5.  $\mathcal{H}_0$  is a pre-Hilbert space.

Proof. The non-trivial part is to prove that if  $\langle f, f \rangle_{\mathcal{H}_0} = 0$  then  $f = 0$

and since  $\langle f, f \rangle_{\mathcal{H}_0} = \langle f, Qf \rangle_{L^2}$  this is to prove that  $Qf = 0$  implies

$f = 0$  in  $L^2$ . Let  $f \in \mathcal{H}_0 \setminus \{0\}$ . From the anti-commutation relation (2.2)

it follows by a direct computation that

$$(\alpha \cdot n)(\alpha \cdot \nabla) + (\alpha \cdot \nabla)(\alpha \cdot n) = (n \cdot \nabla + \nabla n) \mathbf{1} \quad (3.8)$$

on  $C_0^\infty(\mathbb{R}^3, \mathbb{C}^4)$ . In section 2 it was shown that the Dirac operator  $D$  in

$L^2$  is self-adjoint with domain  $\mathcal{D}(D) = H^1(\mathbb{R}^3, \mathbb{C}^4)$ . Now the above

defined operator  $S$  can be lifted in a canonical way to an operator  $S \cdot \mathbf{1}$

on  $\mathcal{D}(D)$ , where  $\mathbf{1}$  here denotes the  $4 \times 4$  unit matrix. Hence

$$DQ = (2-Q)D + S \cdot \mathbf{1} \quad (3.9)$$

on  $\mathcal{D}(D)$ , since the left-hand side and the right-hand side agree on the

dense subspace  $C_0^\infty(\mathbb{R}^3, \mathbb{C}^4)$ . Since  $0 \notin \sigma(D)$  then  $D$  has a bounded inverse

$D^{-1}$  defined on  $\mathcal{R}(D)$ , which is dense in  $L^2$ , hence  $D^{-1}$  can be extended

to a continuous operator on  $L^2$  also denoted by  $D^{-1}$ . Acting on (3.9) with  $D^{-1}$  from both left and right gives

$$QD^{-1} = D^{-1}(2-Q) + D^{-1}S \cdot 1D^{-1} \quad (3.10)$$

on  $\mathcal{R}(D)$ . By acting with  $Q$  from both left and right equation (3.10) gives

$$QB = D^{-1}S \cdot 1B \quad (3.11)$$

on  $\{f \in L^2 : Qf \in \mathcal{R}(D)\}$ , since  $Q^2 = 2Q$ . Now  $f = Bg \in \mathcal{H}_0 \setminus \{0\}$  so it follows from equation (3.11) that  $Qf = QBg = D^{-1}S \cdot 1Bg$  and since  $D^{-1}$  is a bounded operator and the kernel for  $S$  is trivial in  $L^2(\mathbb{R}^3)$  then  $D^{-1}S \cdot 1Bg \neq 0$ , i.e. if  $f \in \mathcal{H}_0 \setminus \{0\}$  then  $Qf \in L^2 \setminus \{0\}$ , or in other words if  $Qf = 0$  in  $L^2$  then  $f = 0$  in  $\mathcal{H}_0$ .

Definition 3.6. Let  $\mathcal{H}$  be the completion of  $\mathcal{H}_0$ .

It follows from the definition that  $2^{-1/2}Q$  defines an isometry from  $\mathcal{H}$  into  $L^2$ , and that  $B$  defines a bounded self-adjoint operator on  $\mathcal{H}$ .

Proposition 3.7. The range  $\mathcal{R}(B)$  of  $B : \mathcal{H} \rightarrow \mathcal{H}$  is dense in  $\mathcal{H}$ .

Proof. Let  $\psi \in (B\mathcal{H})^\perp$ , i.e.  $\psi$  is in the orthogonal complement of  $B\mathcal{H}$ ,

so  $\langle \psi, Bf \rangle_{\mathcal{H}} = 0$  for all  $f \in \mathcal{H}$ . Consider  $f \in \mathcal{H}_0$ , then  $f = Bg$  for some

$g \in L^2$ . Then  $\langle \psi, Bf \rangle = 0$  for all  $f \in \mathcal{H}$  implies that

$\langle \psi, QD^{-1}QD^{-1}Qg \rangle_{L^2} = 0$  for all  $g \in L^2$ . Since  $QD^{-1}QD^{-1}Q = (D^{-1}S \cdot 1D^{-1})^2Q$

and the commutator  $[Q, D^{-1}S \cdot 1D^{-1}] = 0$  then

$$\begin{aligned} \langle (D^{-1}S \cdot 1D^{-1})^2Q\psi, g \rangle_{L^2} &= \langle QD^{-1}QD^{-1}Q\psi, g \rangle_{L^2} \\ &= \langle \psi, QD^{-1}QD^{-1}Qg \rangle_{L^2} \\ &= \langle \psi, D^{-1}QD^{-1}Qg \rangle_{L^2} = 0 \end{aligned} \quad (3.12)$$

for all  $g \in L^2$ , i.e.  $(D^{-1}S \cdot 1D^{-1})^2Q\psi = 0$ . Hence  $Q\psi = 0$  in  $L^2$ , since  $D^{-1}$

is invertible and the kernel for  $S$  is trivial in  $L^2(\mathbb{R}^3)$ , i.e.  $\psi = 0$  in  $\mathcal{H}$

(which is proved in proposition 3.5.)

Proposition 3.7. shows that  $B$  is one-to-one since it is self-adjoint.



Definition 3.8. Put  $H = B^{-1}$  with  $\mathcal{D}(H) = \mathcal{R}(B)$  in  $\mathcal{H}$ .

Note that  $H$  is self-adjoint since  $B$  is. Equation (3.4) can then be written as

$$i \frac{du}{dt}(t) = Hu(t) \quad (3.13)$$

with the solution  $u(t) = e^{-iHt}u(0)$ ,  $u(0) \in \mathcal{D}(H)$ , i.e.  $U_t = e^{-iHt}$  is the time-evolution operator for light-cone data. From definition 3.8 it follows that

$$\langle f, Hf \rangle_{\mathcal{H}} = \langle f, Df \rangle_{L^2} \quad (3.14)$$

and

$$\langle Hf, Hf \rangle_{\mathcal{H}} = \langle Df, Df \rangle_{L^2} \quad (3.15)$$

for all  $f \in \mathcal{D}(H)$ , i.e.  $\mathcal{D}(H) \subset \mathcal{D}(D) = H^1(\mathbb{R}^3, \mathbb{C}^4)$ .

#### 4. The light-cone Fourier transform.

The self-adjointness of  $H$  implies, due to the spectral theorem, that

there exist a spectral family  $\{ E(\lambda) \}_{\lambda \in \mathbb{R}}$  such that  $H = \int_{\mathbb{R}} \lambda dE(\lambda)$ .

The following well-known formula allows  $E(\lambda)$  to be expressed in terms of the resolvent  $R(z) = (H-z)^{-1}$ , for  $z$  in the resolvent set  $\rho(H)$

$$\begin{aligned} & \langle f, 1/2[(E(\beta)+E(\beta-0))-(E(\alpha)+E(\alpha-0))]f \rangle_{\mathcal{H}} \\ &= \lim_{\varepsilon \searrow 0} \frac{1}{2\pi i} \int_{\alpha}^{\beta} d\mu \langle f, [R(\mu+i\varepsilon)-R(\mu-i\varepsilon)]f \rangle_{\mathcal{H}} \end{aligned} \quad (4.1)$$

In the following the right-hand side of equation (4.1) will be evaluated.

Let  $f$  belong to  $C_0^{\infty}(\mathbb{R}^3, \mathbb{C}^4)$  and put  $g = R(z)f$  for  $\text{Im}(z) \neq 0$ .

It then follows that  $g$  belongs to  $\mathcal{D}(H)$  and fulfills the equation

$$(H-z)g = f \quad (4.2)$$

which also can be written as  $(B^{-1}-z)g = f$  . Now applying  $B$  to this equation gives  $(\mathbb{1}-zB)g = Bf$  . Consider this equation in  $L^2$ , then the fact that  $\mathcal{D}(H) \subset \mathcal{D}(D)$  allows us to act with  $D$ , which gives

$$(D-zQ)g = Qf \tag{4.3}$$

since  $B = D^{-1}Q$  .

Definition 4.1. Let  $V_z$  be defined in  $L^2$  by

$$(V_z\phi)(x) = e^{-iz|x|}\phi(x) \tag{4.4}$$

on its maximal domain.

Proposition 4.2. If  $\phi$  is in  $L^2$  such that  $V_z^{-1}\phi$  is in  $\mathcal{D}(D)$  then

$$(D-zQ)V_z^{-1}\phi = V_z^{-1}(D-z)\phi \tag{4.5}$$

Proof. Equation (4.5) follows from a straightforward computation.

Rewriting equation (4.3) as  $(D-zQ)V_z^{-1}V_zg = Qf$  and using

proposition 4.2 gives

$$V_z^{-1}(D-z)V_z g = f \quad (4.6)$$

for  $\text{Im}(z) < 0$  , such that  $V_z$  is bounded. Equation (4.6) can be solved for  $g$  , which gives

$$g = V_z^{-1}(D-z)^{-1}V_z Qf \quad (4.7)$$

and then

$$\langle f, R(z)f \rangle_{\mathfrak{H}} = \langle V_z^{-1}Qf, (D-z)^{-1}V_z Qf \rangle_{L^2} \quad (4.8)$$

for  $\text{Im}(z) < 0$  . The left-hand side of (4.8) is analytic for  $z$  not in the spectra  $\sigma(H)$  of  $H$  , which then shows that the right-hand side has an analytic continuation to  $\text{Im}(z) > 0$  .

As described in section 2 the spectral theory for the Dirac operator  $D$  in  $L^2$  , is given by a unitary operator  $F_0$  on  $L^2$  , such that

$F_0 D F_0^* = \widehat{D} = \alpha \cdot k + \gamma^0 m$ ,  $k \in \mathbb{R}^3$ , with eigenvalues  $\pm \omega_k$ . For  $f \in L^2$

put for short  $\widehat{f} = F_0 f$ . Equation (4.8) can then be written as

$$\langle f, R(z)f \rangle_{\mathfrak{H}} = \langle \widehat{V_{\bar{z}} Q f}, (\widehat{D} - z)^{-1} \widehat{V_z Q f} \rangle_{L^2} \quad (4.9)$$

where  $(\widehat{D} - z)^{-1}$  has the following explicit representation

$$(\widehat{D} - z)^{-1} = \omega_k^{-1} \sum_{\pm} \sum_{s=1,2} (\pm \omega_k - z)^{-1} u_{\pm}(k,s) u_{\pm}^*(k,s) \quad (4.10)$$

and  $u(\cdot, s)$  are four orthogonal eigenvectors of  $\widehat{D}$  normalized such

that  $u_{\pm}^*(k,s) u_{\pm}(k,s) = \omega_k$  also described in section 2.

Definition 4.3. If  $f \in C_0^{\infty}(\mathbb{R}^3, \mathbb{C}^4)$  then  $f(\cdot, s)$  is defined by

$$f_{\pm}(k,s) = u_{\pm}^*(k,s) \widehat{(V_{\pm \omega_k} Q f)}(k) \quad (4.11)$$

for  $s = 1, 2$ .

With proper interpretation formula (4.11) also can be written as

$$\widehat{f}_{\pm}(k,s) = \langle u_{\pm}(\cdot,k,s), f(\cdot) \rangle_{\mathfrak{H}} \quad (4.12)$$

where

$$u_{\pm}(x,k,s) = (2\pi)^{-3/2} e^{\pm i\omega_k |x| + ik \cdot x} u_{\pm}(k,s) \quad (4.13)$$

Now the right-hand side of equation (4.1) can be evaluated. Let

$f \in C_0^{\infty}(\mathbb{R}^3, \mathbb{C}^4)$  and put

$$g_{\pm}(k,s,z) = ( u_{\pm}^*(k,s) \widehat{(V_{\bar{z}} Q f)}(k) )^* ( u_{\pm}^*(k,s) \widehat{(V_z Q f)}(k) ) \quad (4.14)$$

then formula (4.9) becomes

$$\langle f, R(z)f \rangle_{\mathfrak{H}} = \sum_{\pm} \sum_{s=1,2} \int_{\mathbb{R}^3} \frac{d^3k}{\omega_k} (\pm\omega_k - z)^{-1} g_{\pm}(k,s,z) \quad (4.15)$$

Due to the fact that  $g_{\pm}$  is analytic in  $z$  and continuous in  $k$

Fubini's theorem can be used on equation (4.15) which gives

$$\begin{aligned} & \lim_{\epsilon \searrow 0} \frac{1}{2\pi i} \int_{\alpha}^{\beta} d\mu \langle f, [R(\mu+i\epsilon) - R(\mu-i\epsilon)] f \rangle_{\mathfrak{H}} \\ &= \sum_{\pm} \sum_{s=1,2} \lim_{\epsilon \searrow 0} \int_{\mathbb{R}^3} \frac{d^3 k}{\omega_k} \frac{1}{2\pi i} \int_{\alpha}^{\beta} d\mu h_{\pm}(k, s, \mu, \epsilon) \end{aligned} \quad (4.16)$$

where

$$\begin{aligned} h_{\pm}(k, s, \mu, \epsilon) &= (\pm\omega_k - (\mu+i\epsilon))^{-1} g_{\pm}(k, s, \mu+i\epsilon) \\ &\quad - (\pm\omega_k - (\mu-i\epsilon))^{-1} g_{\pm}(k, s, \mu-i\epsilon) \end{aligned} \quad (4.17)$$

Since  $g_{\pm}$  is analytic in  $z$  Taylor's formula can be used and together

with Lebesgue's dominated convergence theorem equation (4.16)

becomes

$$\begin{aligned} & \lim_{\epsilon \searrow 0} \frac{1}{2\pi i} \int_{\alpha}^{\beta} d\mu \langle f, [R(\mu+i\epsilon) - R(\mu-i\epsilon)] f \rangle_{\mathfrak{H}} \\ &= \sum_{\pm} \sum_{s=1,2} \int_{\mathbb{R}^3} \frac{d^3 k}{\omega_k} \pi^{-1} \lim_{\epsilon \searrow 0} \int_{\alpha}^{\beta} d\mu \Psi_{\pm}(k, s, \mu, \epsilon) \frac{\epsilon}{(\mu \mp \omega_k)^2 + \epsilon^2} \end{aligned} \quad (4.18)$$

where

$$\begin{aligned} \Psi_{\pm}(k,s,\mu,\epsilon) &= g_{\pm}(k,s,\mu) - g_{\pm}'(k,s,\mu)(\mu \mp \omega_k) \\ &\quad - \epsilon(1/2[\operatorname{Re}(g_{\pm}''(\xi_{\pm})) + \operatorname{Im}(g_{\pm}''(\eta_{\pm}))](\pm\omega_k - (\mu - i\epsilon)) \\ &\quad - 1/2[\operatorname{Re}(g_{\pm}''(\xi_{\pm}')) + \operatorname{Im}(g_{\pm}''(\eta_{\pm}'))](\pm\omega_k - (\mu - i\epsilon))) \end{aligned} \quad (4.19)$$

here  $g_{\pm}'$  denote the partial derivative of  $g_{\pm}$ ,  $g_{\pm}''$  the second partial derivative of  $g_{\pm}$  in the  $z$ -variable and  $0 \leq \xi_{\pm}, \xi_{\pm}', \eta_{\pm}, \eta_{\pm}' \leq \epsilon$ .

Finally since  $f_{\epsilon}(y) = \pi^{-1}\epsilon(y^2 + \epsilon^2)^{-1}$  is a delta family (as  $\epsilon \searrow 0$ ) equation (4.18) gives

$$\begin{aligned} \lim_{\epsilon \searrow 0} \frac{1}{2\pi i} \int_{\alpha}^{\beta} d\mu \langle f, [R(\mu + i\epsilon) - R(\mu - i\epsilon)] f \rangle_{\mathcal{H}} \\ = \sum_{\pm} \sum_{s=1,2} \int_{\alpha < \pm\omega_k < \beta} \frac{d^3k}{\omega_k} \Psi_{\pm}(k,s,\pm\omega_k,0) \end{aligned} \quad (4.20)$$



i.e.

$$\lim_{\epsilon \searrow 0} \frac{1}{2\pi i} \int_{\alpha}^{\beta} d\bar{\mu} \langle f, [R(\bar{\mu} + i\epsilon) - R(\bar{\mu} - i\epsilon)] f \rangle_{\mathcal{H}}$$

$$= \left\{ \begin{array}{ll} \sum_{s=1,2} \int_{\alpha < \omega_k < \beta} \frac{d^3 k}{\omega_k} | \hat{f}_+(k,s) |^2, & m < \alpha < \beta \\ \sum_{s=1,2} \int_{-\beta < \omega_k < -\alpha} \frac{d^3 k}{\omega_k} | \hat{f}_-(k,s) |^2, & \alpha < \beta < -m \\ \sum_{s=1} \int_{\omega_k < \beta} \frac{d^3 k}{\omega_k} | \hat{f}_+(k,s) |^2 \\ + \sum_{s=1,2} \int_{\omega_k < -\alpha} \frac{d^3 k}{\omega_k} | \hat{f}_-(k,s) |^2, & \alpha < -m, \beta > m \\ 0 & , \text{ otherwise} \end{array} \right. \quad (4.21)$$

duing to the fact that  $\Psi_{\pm}(k,s,\pm\omega_k,0) = g_{\pm}(k,s,\pm\omega_k) = | \hat{f}_{\pm}(k,s) |^2$ .  
From equation (4.1) and (4.17) it then follows that

$$\langle f, [E(\beta) - E(\alpha)] f \rangle_{\mathfrak{H}}$$

$$= \left\{ \begin{array}{l} \sum_{s=1,2} \int_{\alpha < \omega_k < \beta} \frac{d^3k}{\omega_k} |\hat{f}_+(k,s)|^2, \quad m < \alpha < \beta \\ \sum_{s=1,2} \int_{-\beta < \omega_k < -\alpha} \frac{d^3k}{\omega_k} |\hat{f}_-(k,s)|^2, \quad \alpha < \beta < -m \\ \sum_{s=1,2} \int_{\omega_k < \beta} \frac{d^3k}{\omega_k} |\hat{f}_+(k,s)|^2 \\ + \sum_{s=1,2} \int_{\omega_k < -\alpha} \frac{d^3k}{\omega_k} |\hat{f}_-(k,s)|^2, \quad \alpha < -m, \beta > m \\ 0, \quad \text{otherwise} \end{array} \right. \quad (4.22)$$

Letting  $\alpha \rightarrow -\infty$  and  $\beta \rightarrow \infty$  formular (4.23) gives

$$\|f\|_{\mathfrak{H}}^2 = \sum_{\pm} \sum_{s=1,2} \int_{\mathbb{R}^3} \frac{d^3k}{\omega_k} |\hat{f}_{\pm}(k,s)|^2 \quad (4.24)$$

for  $f \in C_0^\infty(\mathbb{R}^3, \mathbb{C}^4)$ . Now the map defined in definition 4.3 can be extended by continuity to an isometry.

Definition 4.4. Let  $\widehat{\mathcal{H}} = \widehat{\mathcal{H}}_+ \oplus \widehat{\mathcal{H}}_-$ ,  $\widehat{\mathcal{H}}_\pm = L^2(\mathfrak{M}_\pm, \mathbb{C}^2)$  where  $\mathfrak{M}_\pm$  are the mass-hyperboloids and the measure is  $\frac{d^3k}{\omega_k}$ . Define

$$F : \mathcal{H} \rightarrow \widehat{\mathcal{H}} \text{ by } Ff = (\widehat{f}_+, \widehat{f}_-).$$

It follows from formula (2.7) that the map  $F : \mathcal{H} \rightarrow \widehat{\mathcal{H}}$  in fact is onto, i.e.  $F$  is unitary.

Theorem 4.5. The unitary map  $F : \mathfrak{H} \rightarrow \widehat{\mathfrak{H}}$ , called the "light-cone Fourier transform", diagonalizes  $H$ , i.e.

$$FHf = ( \omega_k \widehat{f}_+ , -\omega_k \widehat{f}_- ) \quad (4.25)$$

for  $f \in \mathfrak{D}(H)$ .

Proof. Let  $f \in C_0^\infty(\mathbb{R}^3, \mathbb{C}^4)$  then

$$\begin{aligned} \widehat{Hf}_\pm(k,s) &= \langle u_\pm(\cdot, k, s), Hf(\cdot) \rangle_{\mathfrak{H}} \\ &= \langle Du_\pm(\cdot, k, s), f(\cdot) \rangle_{L^2} \\ &= \pm \omega_k \langle u(\cdot, k, s), Qf(\cdot) \rangle_{L^2} \\ &= \pm \omega_k \widehat{f}_\pm(k, s) \end{aligned} \quad (4.26)$$

where  $Du_\pm$  is understood in weak sense. Since  $H$  is closed and  $F$  is continuous, formula (4.25) holds for all  $f \in \mathfrak{D}(H)$ .

The light-cone Fourier transform defines a kind of "duality" between the light-cone and the mass-hyperboloids.

Recall that formula (2.3) gives rise to a unitary map from  $\widehat{\mathcal{H}}$  onto  $L^2(\mathbb{R}^3, \mathbb{C}^4)$  of  $t = \text{constant}$  data, i.e. there is a one-to-one correspondance between weak solutions  $u(t, x)$  of the Dirac equation (2.1) with  $u(t, \cdot) \in L^2(\mathbb{R}^3, \mathbb{C}^4)$  and the solution  $e^{-iHt}f$ ,  $f \in \mathcal{H}$ , of equation (3.13).

## 5. Conclusion.

We have shown that the characteristic Cauchy problem for the Dirac equation

$$(i\gamma^0 \frac{\partial}{\partial t} + i\gamma \cdot \nabla - m\mathbb{1})u(t,x) = 0$$

$$u(-|x|,x) = f(x) \quad , \quad x \in \mathbb{R}^3 \quad (5.1)$$

has a unique weak solution with  $u(t,\cdot) \in L^2(\mathbb{R}^3, \mathbb{C}^4)$  for all  $f \in \mathcal{H}$  and that all "L<sup>2</sup>-solutions" are obtained in this way.

The Dirac equation was written as an evolution equation for light-cone data

$$i \frac{du}{dt} = Hu \quad (5.2)$$

and the spectral theory for  $H$  was developed including a "light-cone Fourier transform" which diagonalized  $H$ .

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References.

- [1] Lundberg, L.-E. : The Klein-Gordon Equation with  
Light-Cone Data. Comm. Math. Phys. 62 , 107 (1978).
- [2] Riesz, m.: L'intégrale de Riemann-Lionville et le problem de  
Cauchy. Acta Math. 81, 1 (1949).
- [3] Strichartz, R. S.: Harmonic analysis on hyperboloids. J. Funct.  
Anal. 12, 341 (1973).
- [4] Hörmander, L.: Linear partial differential operators, p.151.  
Berlin, Göttingen, Heidelberg, New York: Springer 1964.





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- 79/84 "MATEMATIK OG ALMENDANNELSE".  
Projekt rapport af: Henrik Ooster, Mikael Wennerberg Johansen, Povl Kattler, Birgitte Lydholm og Morten Overgaard Nielsen.  
Vejleder: Bernhelm Booss.
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Projekt rapport af: Niels Jørgensen og Mikael Klinton.  
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 Contributions to the Third International Conference  
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- 138/87 "JOSEPHSON EFFECT AND CIRCLE MAP."  
Paper presented at The International Workshop on Teaching Nonlinear Phenomena at Universities and Schools, "Chaos in Education". Balaton, Hungary, 26 April-2 May 1987.  
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- 144/87 "Context and Non-Locality - A Peircan Approach  
Paper presented at the Symposium on the Foundations of Modern Physics The Copenhagen Interpretation 60 Years after the Como Lecture. Joensuu, Finland, 6 - 8 august 1987.  
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redigeret af: Mogens Brun Heefelt
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- 150/87 "Structure and the Existence of the first sharp diffraction peak in amorphous germanium prepared in UHV and measured in-situ"  
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Matematikprojekt af:  
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Et eksempel på humanistisk teknologihistorie  
Historiespeciale  
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Vejleder: Ib Thiersen
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- 158/88 "UNIFIED FORMALISM FOR EXCESS CURRENT NOISE IN RANDOM WALK MODELS"  
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- 159/88 "STUDIES IN SOLAR ENERGY"  
by: Bent Sørensen
- 160/88 "LOOP GROUPS AND INSTANTONS IN DIMENSION TWO"  
by: Jens Gravesen
- 161/88 "PSEUDO-DIFFERENTIAL PERTURBATIONS AND STABILIZATION OF DISTRIBUTED PARAMETER SYSTEMS: Dirichlet feedback control problems"  
by: Michael Pedersen
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AF: Karin Beyer, Sussanne Blegaa, Birthe Olsen, Jette Reich, Mette Vedelsby
- 163/88 "EN MATEMATISK MODEL TIL BESTEMMELSE AF PERMEABILITETEN FOR BLOD-NETHINDE-BARRIEREN"  
Af: Finn Langberg, Michael Jarden, Lars Frellesen  
Vejleder: Jesper Larsen
- 164/88 "Vurdering af matematisk teknologi  
Technology Assessment  
Technikfolgenabschätzung"  
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Martin Bohle-Carbonell og Jens Højgaard Jensen
- 165/88 "COMPLEX STRUCTURES IN THE NASH-MOSER CATEGORY"  
by: Jens Gravesen

166/88 "Grundbegreber i Sandsynligheds-  
regningen"

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167a/88 "BASISSTATISTIK 1. Diskrete modeller"

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167b/88 "BASISSTATISTIK 2. Kontinuerte  
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168/88 "OVERFLADEN AF PLANETEN MARS"  
Laboratorie-simulering og MARS-analoger  
undersøgt ved Mössbauerspektroskopi.

Fysikspeciale af:

Birger Lundgren

Vejleder: Jens Martin Knudsen  
Fys.Lab./HCØ

169/88 "CHARLES S. PEIRCE: MURSTEN OG MØRTEL  
TIL EN METAFYSIK."

Fem artikler fra tidsskriftet "The Monist"  
1891-93.

Introduktion og oversættelse:

Peder Voetmann Christensen

170/88 "OPGAVESAMLING I MATEMATIK"

Samtlige opgaver stillet i tiden  
1974 - juni 1988