Universal low temperature ac conductivity of macroscopically disordered non-metals

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UNIVERSAL LOW TEMPERATURE AC CONDUCTIVITY OF MACROSCOPICALLY

DISORDERED NON-METALS.

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Abstract:

This paper discusses a macroscopic model for AC conduction in electronically or ionically conducting disordered solids. The model considers AC conduction in a macroscopically inhomogeneous solid that is characterized by a spatially varying (frequency-independent) conductivity. It is assumed that the conductivity is thermally activated. Discretizing Maxwell's equations leads to an electrical equivalent circuit where each pair of nodes are linked by a resistor in parallel with a capacitor. It is shown that the capacitor currents are Maxwell's displacement currents. Assuming uncorrelated admittances, the model is solved in the effective medium approximation (EMA) and in a new "percolation path approximation" (PPA). Both approximations predict universality of the AC response at low temperatures where the frequency dependent conductivity becomes independent of the activation energy probability distribution. The prediction is confirmed by computer simulutions of 200x200 lattices in 2-D and of 50x50x50 lattices in 3-D.

1. INTRODUCTION

AC conduction in disordered solids has been studied during the last 40 years [1-7]. Numerous papers have appeared, especially after 1970, reporting the frequency and temperature dependence of the electrical conductivity in electronically or ionically conducting disordered solids like glasses or various forms of non-perfect crystals. With modern frequency analyzers the measurements are fast and fairly straightforward. A considerable amount of work has gone into developing theories for AC conduction, with main focus on hopping models [8-10]. Despite this, it is still not clear what is the correct model for AC conduction (in particular, whether macroscopic or microscopic inhomogeneities are responsible), it is unknown when and if Coulomb interactions are important [11,12], etc. As a consequence, the interpretation of data is highly subjective and few examples exist of AC measurements yielding reasonably certain and unambiguous [nontrivial] information about a bulk disordered solid.

Besides the lacking understanding of AC phenomena, there is another problem with the application of "impedance spectroscopy" to disordered solids: These solids show remarkably similar behavior as regards their frequency dependent conductivity and its temperature dependence [1-3,13]. Thus, ionically as well as electronically disordered solids all have an AC conductivity which depends on frequency as an approximate power law ω^n where n is less than but close to one and goes to one as the temperature goes to zero. Furthermore, one observes in all cases a much less

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pronounced temperature dependence of the AC conductivity than that of the DC conductivity.

The present paper investigates a macroscopic model for AC conduction. The model, which is conceptually somewhat simpler than the presently popular hopping models, focusses on evaluating the AC consequences of a spatially varying electrical conductivity. The model is based on the well-known fact that inhomogeneities give rise to a frequency dependence of the conductivity because charge carriers accumulate at the boundaries to less conducting regions, thereby creating dipolar polarization (the socalled Maxwell-Wagner effect). A number of papers have discussed the DC conductivity of disordered solids with macroscopic inhomogeneities, but little work has gone into the studying the AC aspects. In this paper, that extends and details a recent Rapid Publication [14], the model is derived from Maxwell's equations, assuming the local conductivity is thermally activated with a spatially varying activation energy. It is shown how to discretize the model and two approximations are applied, focussing on the low temperature range of the model. Both approximations predict universality of the AC response as T goes to zero. This prediction is confirmed by computer simulations in two and three dimensions.

The paper is organized as follows. In Sec. 2 a brief review is given of the experimental observations and of the models hitherto studied. In Sec. 3 the macroscopic model is formulated and discretized, and in Sec. 4 two approximations are applied to the model, the effective medium approximation (EMA) as well as a naive percolation path analysis. In Sec. 5 the results from

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computer simulations are reported, and finally Sec. 6 contains a discussion.

2. THE PHYSICS OF AC CONDUCTION IN DISORDERED SOLIDS

The first systematic work on AC conduction in disordered solids were the "dielectric" studies of ionic conductive oxide glasses that appeared toward the end of the 1950's [15-17]. In 1961 Pollak and Geballe [18] reported AC measurements on n-type crystalline silicon at helium temperatures (where the disorder is due to the random substitution of the dopants, this disorder becomes important when one enters the hopping regime at helium temperatures). Only much later was the similarity between the AC response of ionic glasses and of electronically conducting systems noted [1-3,13]. During the last 20 years a large number of publications have reported AC measurements on disordered solids like amorphous semiconductors [4,19], fast ionic conductors [5,7], nonstoichiometric- or polycrystals [20,21], ionic or electronically conducting polymers [22,23], metal-cluster compounds [24], polaronically conducting transition metal oxides [25], organic semiconductors [26], or high temperature superconductors above T_c [27].

Experimental data are usually reported in terms of the real part $\sigma'(\omega)$ of the frequency dependent conductivity $\sigma(\omega) = \sigma'(\omega) + i\sigma''(\omega)$; however alternatives to this are available. Early publications dealing with ionic glasses presented data in terms of the

negative imaginary part $\epsilon''(\omega)$ of the complex dielectric constant $\epsilon(\omega) = \epsilon'(\omega) - i\epsilon''(\omega)$ defined by

$$\epsilon(\omega) = \frac{\sigma(\omega) - \sigma(0)}{i\omega} \quad .\tag{1}$$

Presently it is common to present data for ionic systems in terms of the socalled electric modulus $M(\omega)$ defined [28] by $M(\omega) = i\omega/\sigma(\omega)$ (however it has been argued that this is not a good means of presenting data [29,30]). There is also the possibility of using the complex resistivity $\rho(\omega) = 1/\sigma(\omega)$, but this is mainly used for data on inhomogeneous systems like boundary layers etc [31].

As mentioned above, all disordered solids exhibit the same qualitative AC behavior: Around the dielectric loss peak frequency, ω_m , [marking the maximum of $\epsilon''(\omega)$] $\sigma'(\omega)$ starts to increase, and for $\omega > \omega_m$ $\sigma'(\omega)$ follows an approximate power law: $\sigma'(\omega) \propto \omega^{n'}$. This behavior continues right up to phonon frequencies where the conductivity around $\omega = 10^{12}$ Hz is of order $1(\Omega cm)^{-1}$ [32]. The signature of a power law is a straight line in a log-log plot. There is some controversy as to whether the observed power laws are truly fundamental [1,33-35] or just an approximate description [8,30]. In any case, the exponent n' is always between 0.7 and 1.0 (the only exception seems to be one-dimensional conductors [36,37]), and one always finds that n' goes to one as the temperature goes to zero. As regards the temperature dependence of $\sigma'(\omega)$,the AC conductivity is always less temperature dependent than the DC conductivity (when viewed

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in the usual log-log plot), and for $T \to 0$ the AC conductivity becomes almost temperature independent. The DC conductivity usually follows an Arrhenius law, but a weaker temperature dependence is occasionally observed, e. g., for group IV amorphous semiconductors [6]. An important observation which is apparently always valid is the Barton-Nakajima-Namikawa (BNN) relation [38-42],

$$\sigma(0) = p \, \Delta\epsilon \, \omega_m \, , \qquad (2)$$

where $\Delta\epsilon$ is the dielectric loss strength: $\Delta\epsilon = \epsilon'(0) - \epsilon'(\infty)$, and p is a numerical constant of order one. Since $\Delta\epsilon$ depends only weakly on temperature, the BNN relation implies that the activation energy of $\sigma(0)$ is equal to that of ω_m [17]. In the majority of disordered solids $\sigma'(\omega)$ obeys the time-temperature superposition principle, i. e., the fact that at different temperatures one observes the same function $\sigma'(\omega)$ just scaled (displaced in the log-log plot). This, in conjunction with the BNN relation and the Debye law $\Delta\epsilon \propto T^{-1}$, shows that the dimensionless conductivity $\sigma'(\omega)/\sigma(0)$ is a function of $\omega/[T\sigma(0)]$ [43-45].

A convenient name for the fact that all disordered solids show the same qualitative AC behavior is to refer to it as "quasi-universality". This term was coined by Summerfield in 1985 in a paper [44] dealing not with experiments but with approximate solutions of hopping models; however it seems to be a very good name for the experimental situation as well.

The early experiments on AC properties of ionic glasses were interpreted in terms of a distribution of relaxation times for associated Debye processes, as is common for dielectric relaxation in liquids [46]. In order to understand the coupling between DC and AC properties, Stevels [15] and Taylor [16] proposed a model assuming the ions (typically Na^+ or K^+) feel a randomly varying potential energy deriving from the random network structure of the glass. At that time no useful analytic approximations or powerful computers were available and the model remained qualitative (for some time the model was generally but erroneously [45] assumed to be inconsistent with experiment).

Workers in semiconductor physics in the 1960's proposed the pair approximation as a model for AC loss [47,48]. This model assumes the loss derives from independent pairs of sites in the solid, where each pair provides two possible positions for a localized electron. Mathematically, this corresponds to the description in terms of Debye processes in parallel that was used in the early work on ionic glasses. The problem with this approach is that there is no DC conduction (which has to be assumed to derive from a completely different process whereby the BNN relation becomes very hard to understand). Furthermore, the pair approximation cannot explain the fact that the exponent n' goes to one as $T \rightarrow 0$. Recently, a version of the pair approximation, the correlated barrier hopping model [6,49], has become popular; this model is able to explain the low temperature behavior of s as a consequence of Coulomb force controlled variable range hopping and does have a non-zero DC conductivity

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because the pairs are not isolated from each other.

Throughout the years a number of authors have considered models based on some kind of networks composed of resistors and capacitors [11,21,28,34,50-55]. This approach is phenomenological in the sense that the networks are regarded as an intuitive means of picturing the conduction and the networks are not justified from fundamental equations. In the present paper, following Springett, Webmann et al, Sinkkonen, and Fichshuk [56-59], we arrive at a resistor-capacitor network which is justified from Maxwell's equations. Since the circuit has not been discussed in detail in the literature, some care is taken to derive it in detail (Sec. 3). It turns out that the physical interpretation of the network is not completely straightforward.

In the last 15 years a number of models have appeared [1,33-35,60-62], emphasizing the power law behavior of $\sigma'(\omega)$ which is taken as a fundamental experimental fact, much like the power laws found close to T_c of a second order phase transition. Thus, fractal aspects of the conduction process is emphasized [35,63]. Power laws also result if it is assumed the phase difference between field and current is frequency independent; this is sometimes taken as the starting point for modelling AC conduction [64].

The most thoroughly studied models for AC conduction in disordered solids are the socalled hopping models [8-10]. Hopping models consider the stochastic motion of (usually) independent charge carriers in disordered structures. If the

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charge carrier sites are marked s and $\Gamma(s \rightarrow s')$ denotes the rate for jumps from site s to site s', a hopping model is characterized by the following equation for the probability of finding a charge carrier at site s, $P(s, t)$,

$$\frac{\partial P(s, t)}{\partial t} = -\gamma_s P(s, t) + \sum_{s'} \Gamma(s' \rightarrow s) P(s', t) \qquad (3)$$

where $\gamma_s = \sum_{s'} \Gamma(s \rightarrow s')$. The sites are often taken to lie on a cubic lattice. The jump rates are usually assumed to vary randomly and to be non-zero only for nearest neighbor jumps. The frequency dependent conductivity is calculated from the Kubo formula [65] that in D dimensions is

$$\sigma(\omega) = \lim_{V \rightarrow \infty} \frac{1}{Dk_B TV} \int_0^\infty \langle J(0) \cdot J(t) \rangle e^{-i\omega t} dt \qquad (4)$$

where J is the total current in the volume V and T is the temperature. In hopping models the current-current autocorrelation function is negative for $t > 0$, reflecting a negative correlation in the jump-jump directions; besides the negative part the autocorrelation function contains a term $\propto K\delta(t)$ where K determines the high frequency limit of the conductivity, $\sigma(\infty)$.

It can be shown that in hopping models $\sigma'(\omega)$ is always an increasing function of frequency [66], as in experiments. In order to get a conductivity that increases several decades when the frequency is increased, the jump rates themselves must vary

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several decades. This is realistic; the jump rates are usually taken to be an exponential function of an activation energy and/or a tunnelling distance.

Hopping models are complex and cannot be solved analytically. To evaluate $\sigma(\omega)$ either one has to computer simulate, or some analytical approximation must be evoked . An early approximation was the continuous time random walk (CTRW) approximation derived by Scher and Lax [43]. Today the CTRW is regarded as the simplest available approximation, a mean-field Hartree type approximation [67] (the original derivation that converted the hopping model to a non-markovian random walk was inconsistent [30,68]). The standard approximation usually applied to disordered systems, which is often used for hopping models, is the effective medium approximation (EMA) [9,67,69-71]. A related approach is the extended pair approximation (EPA) of Summerfield and Butcher [72].

While hopping models are reasonably successful, they usually assume non-interacting charge carriers. Thus, the self-exclusion effect (allowing at most one particle at each site [73]) is ignored, as well as are Coulomb interactions. Recent work includes these effects [12], but at the price that the model becomes very complex and has to be studied by means computer simulations. The macroscopic model considered in the next section does include Coulomb interactions via Gauss' law without becoming extremely complex.

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3. THE MACROSCOPIC MODEL

This section deals with setting up and discretizing the equations governing AC conduction in a solid with a spatially varying [frequency independent] conductivity [14,56-59]. It is assumed that the solid has free charge carriers that are characterized by a local conductivity denoted by $g(\mathbf{r})$. The solid also has bound charges that are described by the frequency independent dielectric constant ϵ_∞ (equal to the $\omega \to \infty$ limit of $\epsilon(\omega)$ in Eq. (1)). It is not entirely unproblematic to assume distinguishability between free and bound charge carriers [74], but the assumption will be made here without further justification.

The quantity of interest is the macroscopic free charge carrier conductivity, defined as the ratio between the spatially averaged free charge current density and the spatially averaged electric field (the latter is given by the potential drop across the entire sample). If $\mathbf{D}$ denotes the displacement vector, $\mathbf{J}$ the free charge carrier current density, and ϕ the electrostatic potential, the basic constitutive equations are

$$\mathbf{D}(\mathbf{r}, t) = -\epsilon_\infty \nabla \phi(\mathbf{r}, t)$$
$$\mathbf{J}(\mathbf{r}, t) = -g(\mathbf{r}) \nabla \phi(\mathbf{r}, t) \quad .$$

$$(5)$$

These equations should be combined with Gauss' law

$$\nabla \cdot \mathbf{D}(\mathbf{r}, t) = \rho(\mathbf{r}, t) \tag{6}$$

(where ρ is the free charge carrier density) and the continuity equation

$$\dot{\rho}(\boldsymbol{r}, t) + \nabla \cdot \boldsymbol{J}(\boldsymbol{r}, t) = 0 \quad . \tag{7}$$

In a periodically varying field all quantities are written as a factor $e^{i\omega t}$ times a function of space. Thus the continuity equation becomes $i\omega\rho + \nabla \cdot \boldsymbol{J} = 0$. Substituting Eqs. (5) and (6) into this expression and introducing the "Laplace frequency"

$$s = i\omega\epsilon_\infty \quad , \tag{8}$$

one arrives at the following equation for the electrostatic potential:

$$\nabla \cdot [\ (s + g(\boldsymbol{r})) \ \nabla\phi(\boldsymbol{r}, s)] = 0 \quad . \tag{9}$$

This is an elliptic differential equation and the solution is uniquely determined from a knowledge of $\phi(\boldsymbol{r}, s)$ at the boundaries. Once the solution has been found, the average current density is given by (where V is the volume of the solid)

$$\boldsymbol{J}(s) = \frac{1}{V} \int_V g(\boldsymbol{r}) \ (-\nabla\phi(\boldsymbol{r}, s)) \ d\boldsymbol{r} \quad . \tag{10}$$

We now turn to the discretization of Eq. (9) [14,59]. A discretization is necessary for solving the equation numerically, but it is also useful for developing an intuition about the problem and arriving at approximate analytical solutions. The discretization will be performed in D dimensions. It is assumed that the function $\phi(\boldsymbol{r}, s)$ is known only at the points of a simple cubic lattice with lattice constant a . In principle the lattice constant is to be taken to zero; the determination of a will be returned to below. If Eq. (9) is considered at a lattice point with coordinates $(n_1 a, \ldots, n_D a)$, the first of the D terms

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on the left hand side becomes upon discretization (for simplicity only the first coordinate is written out explicitly, the remaining unchanged coordinates are $n_2 a, \ldots, n_D a$):

$$\frac{\partial}{\partial x_1} [(s+g) \frac{\partial \phi}{\partial x_1}] (n_1 a, s) =$$

$$= a^{-2} \left[[s+g((n_1+\frac{1}{2}) a, s)] [\phi((n_1+1) a, s) - \phi(n_1 a, s)] \right] \qquad (11)$$

$$- a^{-2} \left[[s+g((n_1-\frac{1}{2}) a, s)] [\phi(n_1 a, s) - \phi((n_1-1) a, s)] \right]$$

There are $D-1$ other terms like this, and Eq. (9) becomes the condition that the sum of all D terms is zero. Remembering the definition of S (Eq. (8)), this zero sum requirement is recognized as the Kirchhoff current conservation law for a lattice where each link is a resistor in parallel to a capacitor (Fig. 1). Clearly it is natural to think of the local conductivities as defined on the **links** of the lattice and not, e. g., on the points of the lattice. Each link admittance y , determined by a resistor in parallel to a capacitor, is given by $y=K(s+g)$ where K is a constant that is determined from requiring the correct continuum limit of the free charge current density J: If the resistor current is I, J is numerically given by $J=I/a^{D-1}$. On the other hand, if the potential drop across a link is denoted by $\Delta \phi$ one has $I=Kg\Delta\phi$ and $J=g\Delta\phi/a$. Combining these equations we find $K=a^{D-2}$, so the link admittance is given by

$$y = a^{D-2} (s+g) \qquad . \qquad (12)$$

The circuit of Fig. 1 is not a direct physical representa-

tion of the solid. This is because, while the resistor currents are indeed the true free charge currents, the capacitor currents are "ghost" currents that are **not** just the currents due to the dielectric displacement of the bound charge carriers. For instance, if $\epsilon_\infty = \epsilon_0$ (the vacuum permittivity) there are no bound charges but the capacitors are still important in the circuit. The correct interpretation of Fig. 1 is the following: In an external AC field the circuit determines the electrostatic potential. This potential in turn determines the free charge currents as those running through the resistors. Obviously the capacitors give rise to a frequency dependence of the overall circuit admittance, but this is **not** the effect we are looking for. The frequency dependence of the free charge currents comes about only as an **indirect** effect of the capacitors because of their influence on the node potentials.

In the real solid the free charges accumulate at certain places; there is no corresponding charge accumulation in the circuit, where in fact the role of the capacitors is to exactly compensate the free charge accumulation. It follows from Eq. (12) that the continuous analogue of the capacitor currents is nothing but the well-known Maxwell displacement current $\boldsymbol{J_D} = \dot{\boldsymbol{D}}$.

At first sight this may seem surprising since the displacement current is usually introduced in connection with completing Maxwell's equations to ensure that $\nabla \cdot (\nabla \times \boldsymbol{H}) = 0$. But this is done by adding to the free charge current $\boldsymbol{J}$ the term $\boldsymbol{J_D} = \dot{\boldsymbol{D}}$ constructed so that the divergence of $\boldsymbol{J} + \boldsymbol{J_D}$ is zero; in an AC field this con-

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dition is nothing but Eq. (9).

The macroscopic frequency dependent free charge conductivity may be calculated from the overall circuit admittance $Y(s)$. Here and henceforth the free charge conductivity will be denoted by $\sigma(s)$, despite the risk of confusing it with the total conductivity appearing in Eq. (1). The latter quantity differs from the former by the factor $i\omega(\epsilon_\infty - \epsilon_0)$; in most experiments one looks for the real part of the conductivity only, and in any case it turns out that the $i\omega(\epsilon_\infty - \epsilon_0)$ term is insignificant in the present model at low temperatures and moderately low frequencies, which is the area of main focus below.

Working in D dimensions, the solid is discretized into N^D points of a cubic lattice with sidelength $L=Na$. Two opposing faces of the cube are identified with the electrodes and short-circuited. If the electrodes are subjected to a potential drop $\Delta\phi(s)$ (with s indicating that a periodic field is considered), the resulting current between the eletrodes is given by (by definition of the overall circuit admittance $Y(s)$): $I(s) = Y(s)\Delta\phi(s)$. In order to calculate the free charge conductivity from $Y(s)$ one has to subtract from $Y(s)$ the contribution due to the capacitor currents. Between the electrodes there are $N-1$ layers of parallel RC-elements. The total current $I(s)$ is the same in each layer. Therefore, the sum of all resistor currents and all capacitor currents in the direction perpendicular to the electodes is given by (with obvious notation)

$$\sum I_C(s) + \sum I_R(s) = (N-1) \, I(s) \quad . \tag{13}$$

The sum of all capacitor currents is easily evaluated. $\sum I_C(s)$

is rewritten as a sum of N^{D-1} terms where each term is the "one-

dimensional" sum in the field direction (with obvious notation)

$\sum a^{D-2} s \Delta \phi \, (i \rightarrow i+1) = a^{D-2} s \Delta \phi$.   Thus Eq. (13) becomes

$$\sum I_R(s) = (N-1) \, I(s) - N^{D-1} a^{D-2} s \Delta \phi \, (s)$$
$$= [\, (N-1) \, Y(s) - N^{D-1} a^{D-2} s \,] \;\; \Delta \phi \, (s) \tag{14}$$

The macroscopic free charge conductivity is defined as the ratio

between average free charge current density and average electric

field.   The former quantity is $\sum I_R(s) / [\, a^{D-1} N^{D-1} (N-1) \,]$ and the

latter is $\Delta \phi (s) / Na$.   Using Eq. (14) we finally find (remember-

ing $L = Na$)

$$\sigma (s) = \frac{1}{N-1} \frac{1}{L^{D-2}} \sum I_R(s) / \Delta \phi (s)$$
$$= \frac{Y(s)}{L^{D-2}} - \frac{N}{N-1} s \quad . \tag{15}$$

For $N \rightarrow \infty$ Eq. (15) reduces to

$$\sigma (s) = \frac{Y(s)}{L^{D-2}} - s \quad . \tag{16}$$

Equation (16) expresses the fact that the average capacitor

current density, to be subtracted to get the free charge current

density, is equal to S times the electric field.

For a given continuously varying local conductivity $g(\mathbf{r})$ the discretization becomes exact for $a \to 0$. A few further assumptions are now made. First, it is assumed that the local conductivity is thermally activated and that the spatial variation in conductivity is due to a varying activation energy, $E(\mathbf{r})$:

$$g(\mathbf{r}) = g_0 e^{-\beta E(\mathbf{r})} \quad . \tag{17}$$

Here $\beta = 1/k_B T$. The activation energy is assumed to vary because the local structure of the solid varies, leading to a varying mobility or perhaps to a varying electrostatic potential [59,75,76]. In most cases one expects the activation energy to vary relatively little; however our main focus here and below is the low temperature limit where the local conductivity eventually varies several orders of magnitude.

It is realistic to assume a finite correlation length, ξ, for $E(\mathbf{r})$, where ξ as usual is defined by

$$\langle E(\mathbf{r}) E(\mathbf{r}') \rangle \simeq E_0^2 \, e^{-|\mathbf{r}-\mathbf{r}'|/\xi} \; , \quad |\mathbf{r}-\mathbf{r}'| \to \infty \quad . \tag{18}$$

We now make an assumption which is very useful both from an analytical point of view (Sec. 4) and a numerical point of view (Sec. 5): It is assumed that, by putting the lattice constant a equal to the correlation length, correlations beyond a may be ignored [57,77]. The values of g are thus assumed to be uncorrelated from link to link. In this approximation the problem is fully specified by the local activation energy

probability distribution, $p(E)$; details concerning how the activation energy varies in space are ignored.

Let us consider the low and high frequency limits of $\sigma(s)$ in the model. For $s \to 0$ the capacitors play no role and all circuit currents are free charge currents. Effectively, the circuit reduces to an ordinary resistor circuit. Such resistor circuits have been investigated extensively in the low temperature limit [78-80]. In this limit the currents mainly follow the percolation paths giving the "easiest" ways between the electrodes. This picture is arrived at as follows. Imagine the resistors being removed from the lattice and then reintroduced in order of decreasing admittance. At a certain filling rate, the socalled link percolation threshold, infinitely large connected clusters appear, creating a connection between the electrodes. In two dimensions the link percolation threshold is given by $p_c=1/2$ exactly [81] while simulations in three dimensions have shown that $p_c=0.2488$ [82]. At low temperatures, adding further admittances beyond the percolation threshold does not change the overall circuit admittance significantly since the added admittances are much smaller than the admittance of the percolation cluster. Therefore, the total admittance is dominated by the admittance of the percolation cluster that in turn is dominated by the largest resistors on the cluster. This idea which is now more than 20 years old [78-82] has been proved rigorously [83]. At low temperatures one finds

$$\sigma(0) \propto e^{-\beta E_c} \tag{19}$$

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where the percolation energy E_c is defined by

$$\int_{-\infty}^{E_c} p(E) \, dE = p_c \quad . \tag{20}$$

The high frequency limit of the conductivity is straightforward to evaluate. For $S \to \infty$ the capacitors completely dominate the circuit. As a result the potential drop perpendicular to the electrodes is everywhere $\Delta\phi/(N-1)$ and the average resistor current is $a^{D-2}\langle g\rangle \Delta\phi/(N-1)$, implying that the macroscopic conductivity is given by

$$\sigma(\infty) = \langle g\rangle \quad . \tag{21}$$

In one dimension the circuit becomes particularly simple. Since the total circuit impedance is a sum of the impedances of RC-elements, one finds (where $g(E) = g_0 e^{-\beta E}$)

$$\frac{1}{Y(s)} = (N-1) \int_{-\infty}^{\infty} \frac{p(E)}{a^{-1}(g(E)+s)} \, dE \quad . \tag{22}$$

Substituted into Eq. (16) this implies for $N \to \infty$ the following equation for $\sigma(s)$

$$\frac{1}{\sigma(s)+s} = \int_{-\infty}^{\infty} \frac{p(E)}{g(E)+s} \, dE \quad . \tag{23}$$

As a simple example consider the "box model", i. e., the case where $p(E) = 1/E_0$ $(0 < E < E_0)$. In that case the distribution of local admittances is (compare Eq. (A.7) in Appendix 2)

$$p(g) = p(E) \left| \frac{dE}{dg} \right| = \frac{1}{\beta E_0} \frac{1}{g} \quad (g_0 e^{-\beta E_0} < g < g_0) \quad . \tag{24}$$

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Equation (23) thus becomes

$$\frac{1}{\sigma(s)+s} = \frac{1}{\beta E_0} \int_{g_0 e^{-\beta E_0}}^{g_0} \frac{1}{g(g+s)} dg$$

$$= \frac{1}{\beta E_0} \frac{1}{s} \ln\left(\frac{1+s/g_0 e^{-\beta E_0}}{1+s/g_0}\right) .$$

(25)

At low Laplace frequencies, $s \ll g_0$, this reduces to

$$\sigma(s)+s = \beta E_0 \frac{s}{\ln(1+s/g_0 e^{-\beta E_0})} .$$

(26)

Letting s go to zero one finds that the DC conductivity is given by

$$\sigma(0) = \beta E_0 g_0 e^{-\beta E_0} .$$

(27)

Thus, the DC conductivity activation energy is equal to the largest activation energy met on the one-dimensional path between the electrodes. Clearly, this is the 1-D analogoue of Eq. (19).

According to Eq. (26) the conductivity becomes frequency dependent when s is of order $g_0 e^{-\beta E_0} = \sigma(0)/(\beta E_0)$. Thus, at low temperatures the frequency dependence sets in already for $s \ll \sigma(0)$. At these low temperatures and moderate frequencies s may be ignored in $\sigma(s)+s$, meaning that the capacitor currents are very small. Introducing $s_0 = g_0 e^{-\beta E_0}$ Eq. (26) may thus be written

$$\sigma(s) = \sigma(0) \frac{s/s_0}{\ln(1+s/s_0)} .$$

(28)

In terms of the real frequency ω and the characteristic time

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$\tau = \varepsilon_\infty e^{\beta E_0}/g_0$ Eq. (28) becomes

$$\sigma(\omega) = \sigma(0) \frac{i\omega\tau}{\ln(1+i\omega\tau)} . \qquad (29)$$

Note that, even though the capacitor currents for $\beta \to \infty$ are negligible for a range of low frequencies, the capacitors may not be ignored from the circuit. If the capacitors are removed, there is no frequency dependence left. Thus, while the free charge currents run through the resistors and while the capacitor currents are in some cases extremely small, the latter still have a very important effect on the magnitude of the average free charge currents, resulting in the dramatic frequency dependence of Eq. (29). In the numerical simulations in two and in three dimensions the same effect was found at low temperatures (Sec. 5).

4. TWO APPROXIMATE ANALYTICAL SOLUTIONS

This section develops two analytical approximations for calculating $\sigma(s)$. In both approximations the problem becomes quite simple in the $\beta \to \infty$ limit where a universality appears and [except for scaling] the shape of $\sigma(s)$ becomes independent of the activation energy probability distribution $p(E)$.

The standard approximation for treating disordered systems analytically is the effective medium approximation (EMA) [77,84]. In other contexts, e. g., in quantum mechanics, this approach is

referred to as the coherent potential approximation (CPA) [85,86]. It has a number of desirable analyticity properties and seems to offer the best available compromise between being simple and being realistic. The present problem is to calculate the overall admittance of a large network whose admittances are independent random variables. The basic idea of the EMA is to focus on one particular admittance of the network, regarding it as placed in an "effective medium" with equal admittances y_m.

The effective medium is constructed to best possibly mimic the average surroundings of the particular admittance that is in focus. This is done by requiring that on average the electric field around the particular admittance in focus is equal to the homogeneous field of the surrounding effective medium. This leads to the following equation for determining y_m in D dimensions [77,87]

$$\left\langle \frac{y - y_m}{y + (D-1) y_m} \right\rangle_y = 0 \ . \tag{30}$$

The total network admittance Y is found from y_m [for $N \to \infty$] via the obvious identity

$$Y = N^{D-2} y_m \ . \tag{31}$$

The EMA is exact in one dimension and it becomes exact for $D \to \infty$ [88]. In the high frequency limit the EMA is correct in all dimensions. In two dimensions the EMA is believed to be quite reliable [89]. Here it gives the correct percolation threshold $p_c = 1/2$ and in a recent weak disorder perturbation calculation it was

shown that the EMA is correct up to and including the fourth order terms [90]. These results in two dimensions are both consequences of the EMA satisfying the duality symmetry of the square lattice [91], requiring that if all y's are replaced by y^{-1}, the total admittance of the network is Y^{-1}. In three dimensions the EMA is less reliable [84,90]; thus the EMA predicts $p_c=1/3$ whereas simulations yield $p_c=0.2488$. Various improvements of the EMA exist [92] but they are rather involved and will not be used here.

Combining Eqs. (12), (16) and (31) yields $y_m=a^{D-2}(\sigma+s)$. When this is substituted into Eq. (30) the equation for the conductivity becomes [59]

$$\left\langle \frac{g-\sigma}{g+(D-1)\sigma+Ds} \right\rangle_g = 0 \quad . \tag{32}$$

For $s \rightarrow \infty$ Eq. (32) implies $\sigma=\langle g \rangle$ (Eq. (21)) because the denominator becomes almost constant and may be ignored. Equation (32) may be solved numerically (Appendix 1). The predictions of Eq. (32) at finite temperatures are compared to the results of simulations in 2-D in the next section. Here we proceed to investigate the $T \rightarrow 0$ limit where Eq. (32) implies a universal frequency dependence given as the solution of a simple transcendental equation.

Equation (32) may be rewritten as an average over the activation energy probability distribution as

$$\frac{1}{D(\sigma+s)} = \left\langle \frac{1}{g(E) + (D-1)\sigma + Ds} \right\rangle_E \quad . \quad (33)$$

Equation (33) simplifies considerably in the $\beta \to \infty$ limit. In this limit $g(E)$ varies rapidly and for given σ and s there are basically just two extreme possibilities, depending on E, $g(E) \ll (D-1)\sigma + Ds$ or $g(E) \gg (D-1)\sigma + Ds$. In the former case $g(E)$ may be ignored while in the latter case the denominator becomes very large and there is little contribution to the right hand side. The energy separating the two cases, $E_g(s)$, is given by

$$E_g(s) = -\frac{1}{\beta} \ln\left(\frac{(D-1)\sigma + Ds}{g_0} \right) \quad . \quad (34)$$

For large β Eq. (33) thus becomes

$$\frac{1}{D(\sigma+s)} = \frac{1}{(D-1)\sigma + Ds} \int_{E_g(s)}^{\infty} p(E) \, dE \quad , \quad (35)$$

or

$$\frac{D-1}{D} + \frac{s}{D(\sigma+s)} = \int_{E_g(s)}^{\infty} p(E) \, dE \quad . \quad (36)$$

Subtracting from Eq. (36) the $s=0$ case of Eq. (36) itself leads to

$$\frac{s}{D(\sigma+s)} = \int_{E_g(s)}^{E_g(0)} p(E) \, dE \quad . \quad (37)$$

For large β $E_g(s)$ is close to $E_g(0)$ and the integral may be replaced by $p(E_g(0))[E_g(0) - E_g(s)]$; thus

$$\frac{s}{D(\sigma+s)} = -\frac{p(E_g(0))}{\beta}\left[\ln\left(\frac{(D-1)\sigma(0)}{g_0}\right) - \ln\left(\frac{(D-1)\sigma+Ds}{g_0}\right)\right]$$

$$= \frac{p(E_g(0))}{\beta}\ln\left(\frac{\sigma}{\sigma(0)} + \frac{Ds}{(D-1)\sigma(0)}\right) \quad .$$

(38)

Introducing the dimensionless "reduced" variables

$$\tilde{\sigma} = \frac{\sigma}{\sigma(0)} \quad ; \quad \tilde{s} = \frac{\beta}{D\,p(E_g(0))\,\sigma(0)}\,s \quad , \qquad (39)$$

it is easy to see that Eq. (38) for $\beta \to \infty$ reduces to

$$\tilde{\sigma}\ln(\tilde{\sigma}) = \tilde{s} \quad . \qquad (40)$$

Equation (40) first appeared in 1980 in a paper by Bryksin who derived it for a hopping model with electrons tunnelling between randomly placed sites [69]. Equation (40) has also been derived for a hopping model with a box type distribution of activation energies [45]. In the present context of macroscopically inhomogeneous solids, Eq. (40) was derived by Fishchuk for the box distribution of activation energies [59]. The importance of Eq. (40), as emphasized in the above derivation, lies in the fact that the equation is completely independent of the activation energy probability distribution (an implicit assumption made above is that $p(E)$ is smooth around E_c). Note that Eq. (40) is only valid for $D > 1$; for $D = 1$ one has $E_g(0) = \infty$ and the step leading from Eq. (37) to Eq. (38) is invalid.

Figure 2a shows the dimensionless conductivity of Eq. (40) (full curve) in a log-log plot for real reduced Laplace frequencies (the numerical solution of Eq. (40) is discussed in Appendix 1 that also gives an analytical approximation to $\tilde{\sigma}(\tilde{s})$).

For large $\tilde{s}$ the conductivity follows an approximate power law $\tilde{\sigma} \propto \tilde{s}^{u}$ where u is about 0.9 in a large region. For large Laplace frequencies Eq. (40) implies the following rough estimate

$$\tilde{\sigma}_{EMA} = \frac{\tilde{s}}{\ln(\tilde{\sigma}_{EMA})} \simeq \frac{\tilde{s}}{\ln(\tilde{s})} \quad , \tag{41}$$

which implies

$$u = \frac{d \ln(\tilde{\sigma})}{d \ln(\tilde{s})} \simeq 1 - \frac{1}{\ln(\tilde{s})} \quad (EMA, \tilde{s} \gg 1) \quad . \tag{42}$$

At real frequencies $\tilde{s}$ is imaginary. Writing $\tilde{s} = i\tilde{\omega}$, Fig. 2b shows the real part $\tilde{\sigma}'_{EMA}(\tilde{\omega})$ (full curve) and the imaginary part $\tilde{\sigma}''_{EMA}(\tilde{\omega})$ (dashed curve) of the conductivity. At large frequencies these functions both follow approximate power laws. From the approximate expression

$$\tilde{\sigma}_{EMA} \simeq \frac{i\tilde{\omega}}{\ln(i\tilde{\omega})} = \frac{i\tilde{\omega}}{\ln(\tilde{\omega}) + i\frac{\pi}{2}} \quad (\tilde{\omega} \gg 1) \tag{43}$$

one finds

$$\sigma'_{EMA} \simeq \frac{\pi}{2} \frac{\tilde{\omega}}{\ln^2(\tilde{\omega})} \quad ; \quad \sigma''_{EMA} \simeq \frac{\tilde{\omega}}{\ln(\tilde{\omega})} \quad (\tilde{\omega} \gg 1) \quad . \tag{44}$$

This implies for the exponents defined by $\tilde{\sigma}' \sim \tilde{\omega}^{n'}$ and $\tilde{\sigma}'' \sim \tilde{\omega}^{n''}$

$$n' = 1 - \frac{2}{\ln(\tilde{\omega})} \quad ; \quad n'' = 1 - \frac{1}{\ln(\tilde{\omega})} \quad (EMA, \tilde{\omega} \gg 1) \quad . \tag{45}$$

The conductivity at real Laplace frequencies, as well as both its real and imaginary parts taken at real frequencies, all become

almost proportional to frequency as it goes to infinity. This is not just a trivial effect reflecting conduction in the capacitors since, as noted already, the capacitor currents do **not** contribute to the free charge conductivity. The latter quantity, in fact, at any given temperature stabilizes and becomes frequency independent at sufficiently large frequencies (the range of validity of Eq. (40) is only finite but becomes very large at low temperatures).

It is possible to throw light on the EMA solution by adopting a phenomenological point of view that makes sense in any dimension $D > 1$ at sufficiently low temperatures. In this regime, the admittances of the network vary many orders of magnitude and the currents primarily follow paths of least resistance, the "critical" or "percolation" paths [78,79]. This is the idea leading to Eq. (19). We now propose an approximation referred to as the percolation path approximation (PPA) that assumes that not only the DC currents but also the low frequency AC currents mainly follow the percolation paths. The solid is regarded as consisting of several independently conducting "channels", each channel corresponding to a percolation path. This approximation ignores the complicated fractal nature of the percolation cluster [93]. The problem of calculating the conductivity now becomes one-dimensional and one finds (where K is an unknown numerical constant)

$$\frac{1}{\sigma(s) + s} = K \int_{-\infty}^{E_c} \frac{p(E)}{g(E) + s} dE \quad . \tag{46}$$

For a definite range of frequencies around the transition

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frequency like, e. g., the 9 decades of Fig. 2, the dominant contribution to Eq.(46) at low temperatures comes from energies close to E_c. Therefore $p(E)$ may be replaced by $p(E_c)$ and the conductivity is the same as that of the one-dimensional box model already solved in Sec.3. Defining the dimensionless Laplace frequency $\tilde{s}=s/s_0$, the PPA thus predicts (compare Eq. (28))

$$\tilde{\sigma}_{PPA}(\tilde{s}) = \frac{\tilde{s}}{\ln(1+\tilde{s})} \quad . \tag{47}$$

This function is plotted in Fig. 2a for real $\tilde{s}$ where s_0 has been adjusted to fit to the EMA solution at low frequencies. The two solutions are very similar. As for the EMA solution one finds from Eq. (47) at real Laplace frequencies $\tilde{\sigma} \sim \tilde{s}^u$ where

$$u = 1 - \frac{1}{\ln(\tilde{s})} \quad (PPA, \tilde{s} \gg 1) \quad . \tag{48}$$

At real frequencies $\tilde{\omega}=\tilde{s}/i$ one finds since $\ln(1+i\tilde{\omega})=\ln(1+\tilde{\omega}^2)/2 + i\, Arctan(\tilde{\omega})$

$$\tilde{\sigma}'_{PPA}(\tilde{\omega}) = \frac{\tilde{\omega}\, Arctan(\tilde{\omega})}{\ln^2(1+\tilde{\omega}^2)/4 + Arctan^2(\tilde{\omega})}$$

$$\tilde{\sigma}''_{PPA}(\tilde{\omega}) = \frac{\tilde{\omega}\, \ln(1+\tilde{\omega}^2)}{\ln^2(1+\tilde{\omega}^2)/2 + 2\, Arctan^2(\tilde{\omega})} \quad . \tag{49}$$

For $\tilde{\omega} \to \infty$ one has $\tilde{\sigma}'_{PPA} \sim \frac{\pi}{2}\tilde{\omega}/\ln^2(\tilde{\omega})$ and $\tilde{\sigma}''_{PPA} \sim \tilde{\omega}/\ln(\tilde{\omega})$ leading to the exponents

$$n' = 1 - \frac{2}{\ln(\tilde{\omega})} \quad ; \quad n'' = 1 - \frac{1}{\ln(\tilde{\omega})} \quad (PPA,\ \tilde{\omega} \gg 1) \quad . \tag{50}$$

These exponents are identical to those of the EMA (Eq. (45).

Both the EMA and the PPA predict a universal frequency dependence of the low temperature conductivity, independent of the activation energy probability distribution [assuming $p(E)$ is smooth around E_c]. It is interesting that the two approximations yield similar predictions (Fig. 2), despite being derived from completely different points of view. The EMA, which is usually believed to be best for systems with **weak** disorder [88,90,94], has here been applied in the limit of extreme disorder. The PPA, on the other hand, only makes sense for the extreme disorder found at low temperatures. The similarity between the two approximations indicates that the EMA may be reliable even for systems of extreme disorder. On the other hand, the quantitative EMA prediction for the DC conductivity is known to be wrong in 3-D because the percolation threshold is wrongly predicted, and thus one can at most expect the **shape** of the conductivity curve to be correct in the EMA. Only computer simulations can give reliable information as to whether universality really exists and, if it exists, whether it is well described by the approximate theories developed here.

5. COMPUTER SIMULATIONS

This section reports the results of computer simulations of the model in two and three dimensions. In each case a lattice of admittances is generated where each impedance is determined by an activation energy randomly chosen according to some probability

distribution $p(E)$. Several different probability distributions were used; how the activation energies were generated is explained in Appendix 2. At low temperatures large lattices are needed to obtain reasonable statistics. Even for relatively large lattices the system is not self-averaging and it was necessary to average over several lattices to obtain reproducible results. For simplicity all simulations were carried out at **real** Laplace frequencies; this is enough when one wants to compare the simulations to an analytical expression for the conductivity.

The calculation of the frequency dependent conductivity may be performed by several methods. One possibility, the "brute force method", is to solve Kirchhoff's equations for the potential via some sparse matrix algorithm. Another possibility is to calculate the overall circuit admittance between short-circuited electrodes by "elimination methods". These methods eliminate nodes of the lattice by introducing new admittances without changing the overall admittance, a process that is continued until one is left with only one admittance. The most common elimination method was introduced by Fogelholm in 1980 [95]; it works as follows. Whenever a node is eliminated with n neighbors and the admittances $Y_1, .., Y_n$ to its neighbors, all possible connections between the neighbors are introduced such that the i'th and the j'th neighbor are given the [additional] admittance $Y_i Y_j / (Y_1 + ... + Y_n)$. For a full lattice the algorithm becomes very inefficient, but it works well for calculations close to the percolation threshold where many admittances are zero [82]. In two dimensions Frank and Lobb have developed a

useful algorithm that eliminates nodes by working each of them towards the lower right corner via consecutive star-triangle transformations [96]. By means of this algorithm it is possible to calculate the admittance of a 200x200 lattice in a few minutes on a modern PC.

Unfortunately no similarly efficient algorithm is available in 3-D. Here it was found most efficient to use brute force methods. It is a rather complex numerical problem to solve large sparse matrix problems when the coefficients vary several decades. The standard Gauss-Seidel, as well as overrelaxation methods [97], converge too slowly. Fortunately, an algorithm has been developed, the algebraic multigrid [98,99], that was made precisely for problems of this type. The algorithm is an algebraic generalization of the standard geometric multigrid method used for solving elliptic differential equations. An excellent introduction to the multigrid idea has been given by Goodman and Sokal [100]. The algebraic multigrid (AMG) solves a problem in a time only proportional to the number of equations. For a NxN lattice in 2-D the computing time varies as N^2, for a NxNxN lattice in 3-D the time varies as N^3. The Frank-Lobb algorithm in 2-D has a computing time varying as N^3; however in practical applications it is still far superior to the AMG because it avoids overflow problems and because the prefactors are clearly in its favor. It should be mentioned that other methods are also available. The transfer matrix method [101] is an elimination method that works in all dimensions, and the Fourier acceleration brute force method [102] is an alternative

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to algebraic multigrid for speeding up the Gauss-Seidel relaxation scheme. None of these methods were used in the present work, since it was estimated that they are not competitive.

In two dimensions results of simulations of 100x100 lattices have been reported elsewhere [14] for 5 activation energy probability distributions for the values $\beta=4,16,64$. In Fig. 3 these results are supplemented by simulations of 200x200 lattices for the following activation energy distributions at $\beta=5,10,20,40,80,160$ (see Appendix 2): a) Asymmetric Gaussian, b) Cauchy, c) Symmetric Exponential, d) Powerlaw with exponent -4, e) Box, f) Triangle. For each temperature and activation energy distribution the average of 10 simulations are shown in Fig. 3. The full curve is the EMA prediction found by solving Eq. (32) numerically (Appendix 1), where the reduced variables are given by Eq. (39). Clearly the EMA works well in 2-D at all temperatures. In particular, one finds that universality is approached as $T \to 0$. Some of the low temperature data deviate from the EMA prediction. However, given the fact that the frequency in some cases (at low temperatures) have been shifted more than 50 decades according to Eq. (39), the fit must be said to be satisfactory. It is unclear whether the deviations from the EMA predictions are real or due to insufficient statistics. The shape of $\tilde{\sigma}(\tilde{s})$ at low temperatures is studied in Fig. 4 where the $\beta=80,160$ data from Fig. 3 have been replotted, this time allowing an empirical scaling of the Laplace frequency to fit the EMA universal prediction (Eq. (40)) best possible.

In three dimensions results for 50x50x50 lattices are shown in Fig. 5 for the values $\beta = 10, 30, 60$ for the activation energy distributions: a) Asymmetric Gaussian, b) Cauchy, c) Exponential, d) Box, e) Triangle. The conductivity was found by solving Kirchhoff's equations to determine the potential at each node and subsequently averaging all resistor currents. A potential $\phi = 0$ was imposed at one electrode and $\phi = 1$ at the other electrode; the remaining 4 faces of the cube were joined by imposing periodic boundary conditions. The 120.000 equations for the node potentials were solved by means of the AMG1R5 Fortran algebraic multigrid subroutine available from the Yale multigrid library [103]. This subroutine was found to be efficient, well documented and providing useful error statements and warnings. The subroutine was previously successfully used for large resistor network calculations by Edwards et al [104]. Still, the present problem is very complex because the area of main interest is at low temperatures where the coefficients of the linear equations vary several orders of magnitude. To avoid overflow and "division by zero" problems, the equations must be "massaged" a bit. This was done by increasing the lowest g's to a standard low value, a procedure that is easily justified since these weak links carry little current anyway. The highest g's were also changed to a standard value by lowering them; this is allowed because they more or less short-circuit their nodes anyway: they are certainly not "bottlenecks" for the conduction process. In both cases, it was carefully checked that the calculated conductivity is independent of the cut-off's introduced,

indicating that the procedure is permissible. The calculations are rather demanding; the present work was carried out on a modern workstation with 128 MB of memory (the AMG1R5 is quite memory consuming). The results are shown in Fig. 4 where they are compared to the EMA prediction (full curve) only as regards the predicted **shape** of the conductivity curve (the scaling involved in defining the reduced frequency was here regarded as a fitting parameter). Again one finds at low temperatures that the universal conductivity curve is approached. Unfortunately, it was not possible to go to lower temperatures without serious overflow problems. At present it is therefore not possible to conclude whether the universal conductivity curve in 3-D is slightly less steep than in 2-D.

Finally, it should be mentioned that in both the 2-D and the 3-D simulations it was found that the capacitor currents at low temperatures are very small compared to the resistor currents in a wide range of frequencies around the transition frequency, thus confirming the result for 1-D box model reported at the end of Sec.3.

6. DISCUSSION

This paper dealt with the AC consequences of having a spatially varying electrical conductivity, focussing on the low temperature limit and the case when the local conductivity is thermally activated. The fact that spatial inhomogeneities give rise to polarization phenomena and therefore to a frequency

dependence of the conductivity has been known since the beginning of the century. Simultaneously, numerous experiments have appeared on AC properties of in homogeneous media. It is therefore surprising that little work has gone into studying AC aspects of the "macroscopic" model dealing with a range of local conductivities. For instance, no paper were given dealing with AC properties in this model of inhomogeneous media at the two "Conferences on Electrical Transport and Optical Properties of Inhomogeneous Media" held in 1978 and 1988 [105]. Usually, AC properties are modelled by means of hopping models where the disorder is assumed to be on the atomic scale [8-10,35,43-45]. Hopping models usually do not include Coulomb interactions, a problem of recent concern [12]. Via Gauss' law the macroscopic model does include the effect of Coulomb interactions.

The idea of a spatially randomly varying activation energy for the conductivity has been discussed for some time in connection with particular systems like heavily doped or compensated semiconductors [59,75] or granular materials [76]. For any disordered system one expects the various forms of inhomogeneities to influence the exact value of the activation energy of the local conductivity. The question is not whether or not this effect is present, but whether it is so pronounced that the concept of a local conductivity stops making sense, in which case the relevant model to use is a hopping model. It is an open question which disordered solids are best described by the macroscopic model and for which hopping models are best. It is far from obvious that the macroscopic model in general is to be preferred to hopping models. The purpose of the present work was

to investigate a simple approach to the modelling of AC conduction in disordered solids, an approach that may be appropriate for some classes of disordered solids.

For a solid with a spatially varying conductivity, the relevant Maxwell's equations in a periodic external field boils down to the single equation (9) for the electrostatic potential. This equation is discretized by putting it on a cubic lattice. If the lattice constant a is taken to zero, the discretization becomes exact; in Sec. 3, however, the model was simplified further by choosing a equal to the activation energy correlation length and thus ignoring correlations beyond a. Thereby one ignores details of how the conductivity varies in space and the model becomes uniquely defined by the activation energy probability distribution.

The discretization of Eq. (9) corresponds to the electric network of Fig. 1. Similar networks have been used many times before as models for AC properties of disordered solids [11,21,28,34,50-55] and the one-dimensional version of the network lies behind the electric modulus formalism [28]. However, the network is traditionally used just as a suggestive picture of the solid and is not justified from basic principles. When this is done (Sec. 3) it turns out that the interpretation of the circuit is rather subtle. The network is not just to be thought of as a straightforward representation of the solid. This is because the capacitor currents are Maxwell's displacement currents, parts of which do not involve charge transport. Focussing on the free charge contribution to the conductivity, this quantity depends on frequency only as an indirect effect of

the capacitors: The electrostatic potential is determined from Kirchhoff's equations for the network. Thereby the node potentials become frequency dependent, making the local free charge currents that run through the resistors frequency dependent. A simple model which may be solved exactly is the one-dimensional box model. In this model, as $\beta \to \infty$, there is a strongly frequency dependent conductivity. It follows from Eq. (29) (compare Eq. (44)) that well above the transition frequency, the phase difference between field and **resistor** current is close to $\pi/2$ [for any non-zero temperature Eq. (29) is only valid in a finite range of frequencies, but this range becomes very large at low temperatures]. This happens while at the same time the capacitor currents are very small compared to the resistor currents (!). The same peculiar effect of a strong frequency dispersion simultaneously with small capacitor currents was observed in the computer simulations in 2-D and 3-D. In a sense then, the network of Fig. 2 may after all be said to give a genuine representation of the solid, but **only** at low temperatures and moderate frequencies where the capacitor currents are small.

In the computer simulations one observes an approach towards low temperature universality both in 2-D and 3-D. In 2-D the results were fitted to the EMA prediction of Eq. (32). In 3-D it is necessary to regard the scaling involved in the definition of the reduced Laplace frequency as a fitting parameter to obtain a fit. There are two conclusions to be drawn from the simulations. First, the simulations strongly indicate that the frequency dependence of the conductivity **is** universal at low temperatures, and this is the major result reported here. As far as is known

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to the author, this is the first time that universality has been demonstrated either analytically or from simulations of any model for AC conduction. In 1985 Summerfield [44] reported that several hopping models involving tunnelling electrons solved in the extended pair approximation (EPA) yield almost identical predictions for the frequency dependence of the conductivity. This phenomenon was referred to as "quasi-universality". Since no simulations were reported it was not clear whether the effect is real or due to the approximation used, and there was no study of the temperature dependence of the conductivity for activated jump rates indicating an approach to true universality as $T \to 0$.

The second conclusion to be drawn from the simulations is that the EMA works very well in 2-D at all temperatures, while in 3-D the predicted universal conductivity is approached as $T \to 0$. These results are highly non-trivial since doubt has often been expressed as to how reliable the EMA is for systems with a broad distribution of admittances [88,90,94]. In one dimension the EMA universality prediction is not valid. But since the EMA becomes exact for $D \to \infty$ [88], there is reason to believe in Eq. (40) for all $D > 1$ as a good approximation to the universal conductivity.

The percolation path approximation is based on an insight gained from percolation theory, that the percolation paths are preferred at low temperatures. These paths contain activation energies ranging up to a sharp cut-off at the E_c given by Eq. (20). Furthermore, it is clear that in a fixed frequency range around the transition frequency at sufficiently low temperatures, only a narrow interval of activation energies are important

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(namely those around the percolation energy determining the smallest admittances on the percolation paths). Therefore, the solid admittance is expected to correspond to that of a one-dimensional solid with a sharp activation energy cut-off; this is why the PPA predicts the same frequency response as the one-dimensional box model. The fact that the PPA solution is very similar to the EMA solution shows that, while the world is not one-dimensional and seldomly has a sharp activation energy cut-off, at sufficiently low temperatures a disordered solid **looks** like this as a consequence of the underlying percolation.

The PPA is not quite as straightforward as it seems at first sight, though. For very small S the currents do follow the percolation paths. As long as $S \ll y_c$, where y_c is the smallest admittance on percolation path, the conductivity is clearly frequency independent. The conductivity starts to increase only for $S = S_m$ where $S_m \sim y_c$. But this is exactly the regime where the percolation picture starts to break down because all surrounding admittances also become of order y_c! In this range of frequencies the currents still mainly follow percolation paths; however when the current meets one of the poorest admittances on a percolation path, it may as well pass through the surrounding admittances. This happens seldomly, though, and one still expects the percolation path calculation to be roughly valid.

Since the DC conductivity is determined by the poorest admittances on a percolation path (of order y_c), the frequency for onset of AC conduction, S_m, is of order $\sigma(0)$. In particular

one expects $\sigma(0)$ and s_m to have the same temperature dependence. This is consistent with the well-known BNN-relation (Eq. (2)) which is seen in all experiments. The EMA has the prediction that s_m scales with $T\sigma(0)$ rather than with just $\sigma(0)$ (Eq. (39)). This is sometimes referred to as "Summerfield scaling" [44] though it was apparently first discussed by Scher and Lax in their important papers on the continuous time random walk approximation [43]. It is interesting to note that the extra factor T in the Summerfield scaling of **hopping models** [43-45] derive from the $1/T$ factor of the fluctuation-dissipation theorem Eq. (4) that is not used in the macroscopic model.

What are the consequences of the presently reported results for the interpretation of experiments? First, it is to be noted that the EMA (or the PPA) universality prediction does give a good fit to many experiments. This has been shown in detail elsewhere [45] where a box-type hopping model yielding the same frequency dependent conductivity (Eqs. (40) and (47)) was discussed in detail. Thus, like in experiments [45]: 1) Eqs. (40) or (47) imply for $\sigma'(\omega)$ an approximate power law with an exponent n' less that but close to one, where $n'(\omega)$ is weakly increasing and $n' \to 1$ as $T \to 0$ (the latter is because, in a fixed frequency range, as the temperature goes to zero one in effect measures further and further out on the universal conductivity curve). 2) When there is no detectable DC conductivity, the exponent is very close to one. 3) The BNN-relation (Eq. (2)) is satisfied with $p=0.59$ in the EMA and $p=0.42$ in the PPA. 4)

The time-temperature superposition principle is satisfied. 5) The AC conductivity is less temperature dependent than the DC conductivity and for n' very close to one the AC conductivity becomes almost temperature independent; in particular this always happens as $T \to 0$. 6) While $\sigma(0)$ may vary several orders of magnitude for different solids at different temperatures, the AC conductivity varies only relatively little (a consequence of the fact that $\Delta\varepsilon$ is usually of order ε_0).

The fact that the universal conductivity gives a good fit to many experiments implies that little can be learnt from AC experiments. For instance, observing a power law dependence for the real part of the conductivity with an exponent of 0.8 a few decades above the transition frequency provides no useful information about the solid under study. This is completely contrary to ones intuition and is perhaps the single most important conclusion to be drawn from the present work. In 1972 Pollak and Pike [106] suggested that the microscopic details of a solid are reflected in deviations from $n'=1$. But it now appears that the situation is more complex. Apparently, details are to be deduced from deviations from the **universal conductivity curve**, that itself has a non-trivial structure.

The universality seen in the macroscopic model at low temperatures is also present at low temperatures in hopping models, with an identical predicted universal frequency dependence [107]. At first sight this is surprising, since in many ways hopping models are complementary to the macroscopic model: 1) Hopping models are microscopic, not macroscopic, 2)

they usually involve non-interacting charge carriers and ignore Coulomb interactions, 3) in hopping models one controls the local electric field while in the macroscopic model the overall potential difference is controlled, and finally 4) hopping models are stochastic while the macroscopic model is deterministic. Still, both types of models lead to large systems of sparse linear equations with coefficients that at low temperatures vary several orders of magnitude . Apparently this is enough to produce the same universality. Several mathematical connections exist between hopping models and resistor networks [10,108], but it is not possible to transform a hopping model into the network of Fig. 2: the Miller-Abrahams equivalent circuit for a hopping model has capacitors from each node to the ground [47]. In conclusion, AC measurements alone cannot determine what is the relevant model for conduction in a given disordered solid. To this end other measurements have to be performed [30] like, e. g., transient current experiments [30,109,110], non-linear conductivity measurements (the macroscopic model becomes non-linear much earlier than hopping models), 1/f noise measurements [111], or Hall effect measurements [112].

Some open questions remain. Is the universality seen in computer simulations as $T \to 0$ a mathematically exact fact (as believed by the author) or is there just "quasi-universality"? It is clear that the universality is closely linked to the percolation phenomenon, but around the percolation threshold it has recently been shown [113,114] (albeit in a different context than the present) that there are **non-universal** critical exponents when a broad distribution of admittances is involved. If the

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universality is confirmed, what is the exact form of the universal frequency dependence? From the ubiquitous existence of long time tails one expects the universal conductivity curve to be non-analytic at $\tilde{s}=0$. However, since the universal conductivity exists as a limit only, it is possible that the function is analytic and that one of Eq. (40) or Eq. (47) is exact.

ACKNOWLEDGEMENT

APPENDIX 1: Solving the EMA equation

Equation (32) may easily be solved at any temperature and real Laplace frequency. The full curves of Fig. 3 were obtained by discretizing the equation into 3000 terms using regularly spaced energies; then the conductivity was determined at fixed S and β by the bisection method [97]. Before this is done, however, it is convenient to rewrite Eq. (32) in terms of the reduced variables $\tilde{\sigma}$ and $\tilde{S}$ defined in Eq. (39).

The universal conductivity given by Eq. (40) may be determined by means of the Newton-Raphson method. Consider first the case of real reduced Laplace frequencies $\tilde{S}$. Introducing $\lambda = \ln(\tilde{\sigma})$, Eq. (40) is rewritten by taking ln on both sides:

$$\lambda + \ln(\lambda) = \ln(\tilde{S}) \quad . \tag{A.1}$$

The Newton-Raphson method [97] for solving the equation $f(\lambda) = 0$ consists in iterating after the recipe

$$\lambda_{n+1} = \lambda_n - \frac{f(\lambda_n)}{f'(\lambda_n)} \quad . \tag{A.2}$$

In the case $f(\lambda) = \lambda + \ln(\lambda) - \ln(\tilde{S})$, Eq. (A.2) becomes

$$\lambda_{n+1} = \lambda_n \frac{1 - \ln(\lambda_n) + \ln(\tilde{S})}{1 + \lambda_n} \quad . \tag{A.3}$$

Equation (A.3) is iterated until convergence by starting with $\lambda_1 = \tilde{S}$ if $0 < \tilde{S} < 1$ (utilizing the fact that $\lambda = \ln(\tilde{\sigma}) \approx \tilde{S}$ for small frequencies) and starting with e. g. $\lambda_1 = 1$ if $\tilde{S} > 1$.

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An analytic fit to $\tilde{\sigma}(\tilde{S})$ is the following expression

$$\tilde{\sigma}(\tilde{S}) = \frac{\tilde{S}}{LN1(\tilde{S}) + b*LN2(\tilde{S}) + c*LN3(\tilde{S}) - (b+c)*LN4(\tilde{S})} \quad , \quad \text{(A.4)}$$

where $LN1(\tilde{S}) = \ln(1+\tilde{S})$ and one recursively defines $LN(i+1)(\tilde{S}) = \ln(1+LNi(\tilde{S}))$. The case $b=c=0$ corresponds to the PPA solution Eq. (47). For $b=-2.2$ and $c=3.5$ Eq. (A.4) provides a fit whose logarithm (base 10) for all $\tilde{S}$ is within 0.01 of the logarithm of the true solution.

At real frequencies $\tilde{\omega}=\tilde{S}/i$ the EMA equation becomes complex, but it can still be solved by the Newton-Raphson method. Writing $\lambda=x+iy$ Eq. (A.3) becomes

$$x_{n+1} = x_n - \frac{A1(n)*B1(n) - A2(n)*B2(n)}{DN(n)}$$

$$y_{n+1} = y_n - \frac{A1(n)*B2(n) + A2(n)*B1(n)}{DN(n)} \quad \text{(A.5)}$$

where, if $L1(n) = \ln(x_n^2+y_n^2)/2$ and $L2(n) = Arctan(y_n/x_n)$, the following abbreviations have been introduced:

$$A1(n) = x_n^2 - y_n^2 + x_n[L1(n) - \ln(\tilde{\omega})] + y_n[\frac{\pi}{2} - L2(n)]$$

$$A2(n) = 2x_n y_n + x_n[L2(n) - \frac{\pi}{2}] + y_n[L1(n) - \ln(\tilde{\omega})]$$

$$\text{(A.6)}$$

$$B1(n) = 1+x_n$$

$$B2(n) = y_n$$

$$DN(n) = B1^2(n) + B2^2(n) \quad .$$

Equation (A.5) is iterated until convergence starting with e. g. $x_1=0.01$

and $y_1 = \tilde{\omega}$ if $0 < \tilde{\omega} < 1$ (reflecting $\ln(\tilde{\sigma}) \cong i\tilde{\omega}$ at low frequencies) and with e.g. $x_1 = 1$ and $y_1 = 0$ if $\tilde{\omega} > 1$.

Substituting $\tilde{s} = i\tilde{\omega}$ into Eq. (A.4) provides an analytic approximation both $\tilde{\sigma}'(\tilde{\omega})$ and $\tilde{\sigma}''(\tilde{\omega})$. For the real part the fit (again with $b = -2.2$ and $c = 3.5$) has logarithm (base 10) within 0.03 of the exact solution; for the imaginary part the logarithm of the fit is within 0.05 of the exact value for $\tilde{\omega} > 10^{-2}$ (for smaller frequencies the fit becomes poor).

APPENDIX 2: Activation energy probability distributions

In the computer simulations dimensionless activation energies were used. With the exceptions of the two Gaussian distributions (see below) the activation energies for a given probability distribution $p(E)$ were generated by utilizing the following well-known fact. If x is a random number between 0 and 1 and $E(x)$ is some function of x, E is distributed according to $p(E)$ given by $p(E)|dE| = dp = p(x)|dx|$. Since $p(x) = 1$ one thus finds

$$p(E) = \frac{1}{\left|\dfrac{dE}{dx}\right|} \quad . \tag{A.7}$$

To avoid spurious effects due to subtle correlations in system supplied random numbers, the random numbers were generated

using the function RANO [97]. This function starts by setting up an array RAN[i] of 97 system-supplied random numbers. Reliable random numbers X are supplied from RANO by using the given random pointer to an index i_y between 1 and 97: RANO then returns $X=RAN[i_y]$ and a new system-supplied random number is filled into the RAN-array to replace X. The same system-supplied random number is used to generate the new pointer index i_y which is used to point out the next random number, etc.

The percolation threshold p_c is given by $p_c=0.5000$ (exactly) in 2-D and $p_c=0.2488$ in 3-D [81,82]. The percolation energy E_c that determines the DC conductivity activation energy (Eq. (19)) is found from

$$\int_{-\infty}^{E_c} p(E)\, dE = p_c \quad . \tag{A.8}$$

If $E(X)$ is an increasing function of X one finds from Eqs. (A.7) and (A.8)

$$E_c = E(X=p_c) \quad . \tag{A.9}$$

Some of the distributions used involve only positive activation energies, others involve also negative activation energies. The latter is justified since one should think of $p(E)$ as centered around some energy $E_0>0$ [which is left out here because it just gives an extra multiplicative factor $e^{-\beta E_0}$ to the conductivity while simultaneously displacing the range of frequencies so that

both $\tilde{s}$ and $\tilde{\sigma}$ are unchanged].

The following activation energy distributions were used in the simulations of Ref. 14 and this paper:

Gaussian

From 12 random numbers X_i (between 0 and 1) : $E = \sum_{i=1}^{12} X_i - 6$ gives a

nice Gaussian distribution of E with variance 1 [97]:

$$p(E) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{E^2}{2}\right) \quad (-\infty < E < \infty) \quad . \qquad (A.10)$$

The percolation energies are determined from Eq. (A.8) that implies $erf(-E_c/\sqrt{2}) = 1 - 2p_c$. Denoting the percolation energy in 2-D by $E_c^{(2)}$ and in 3-D by $E_c^{(3)}$ one finds

$$E_c^{(2)} = 0 \quad ; \quad E_c^{(3)} = -0.678 \quad . \qquad (A.11)$$

Asymmetric Gaussian

For 12 random numbers one calculates $E = \left| \sum_{i=1}^{12} X_i - 6 \right|$; this generates

the distribution

$$p(E) = \sqrt{\frac{2}{\pi}} \exp\left(-\frac{E^2}{2}\right) \quad (0 < E < \infty) \quad . \qquad (A.12)$$

Equation (A.8) implies for $erf(E_c/\sqrt{2}) = p_c$; thus

$$E_c^{(2)} = 0.674 \quad ; \quad E_c^{(3)} = 0.317 \quad . \tag{A.13}$$

Cauchy

Writing $E=\tan(\frac{\pi}{2}x)$ where x is random, produces according to Eq. (A.7)

$$p(E) = \frac{2}{\pi}\frac{1}{1+E^2} \quad (0<E<\infty) \quad . \tag{A.14}$$

Equation (A.9) implies $E_c=\tan(\frac{\pi}{2}p_c)$; thus

$$E_c^{(2)} = 1 \quad ; \quad E_c^{(3)} = 0.412 \quad . \tag{A.15}$$

Exponential

If $E=-\ln(1-x)$ one finds

$$p(E) = e^{-E} \quad (0<E<\infty) \quad . \tag{A.16}$$

Equation (A.9) implies $E_c=-\ln(1-p_c)$; thus

$$E_c^{(2)} = 0.693 \quad ; \quad E_c^{(3)} = 0.286 \quad . \tag{A.17}$$

Symmetric exponential

If $E=\pm\ln(1-x)$ with a random sign one finds

$$p(E) = \frac{1}{2}e^{-|E|} \quad (-\infty < E < \infty) \quad . \tag{A.18}$$

Equation (A.8) implies $E_c = \ln(2p_c)$; thus

$$E_c^{(2)} = 0 \quad ; \quad E_c^{(3)} = -0.698 \quad . \tag{A.19}$$

Power law with exponent -4

If $E = x^{-\frac{1}{3}} - 1$ one finds

$$p(E) = 3(1+E)^{-4} \quad (0 < E < \infty) \quad . \tag{A.20}$$

Equation (A.8) implies $E_c = (1-p_c)^{-\frac{1}{3}} - 1$; thus

$$E_c^{(2)} = 0.260 \quad ; \quad E_c^{(3)} = 0.100 \quad . \tag{A.21}$$

Box

If $E = x$ one finds

$$p(E) = 1 \quad (0 < E < 1) \quad . \tag{A.22}$$

and

$$E_c^{(2)} = 0.500 \quad ; \quad E_c^{(3)} = 0.249 \quad . \tag{A.23}$$

Triangle

If $E = x^{\frac{1}{2}}$ one finds

$$p(E) = 2E \quad (0 < E < 1) \quad .$$ (A.24)

Equation (A.9) implies $E_c = p_c^{\frac{1}{2}}$; thus

$$E_c^{(2)} = 0.707 \quad ; \quad E_c^{(3)} = 0.499 \quad .$$ (A.25)

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FIGURE CAPTIONS

Fig. 1: Electrical equivalent circuit of Maxwell's equations discretized in 2-D for an inhomogeneous conductor. Similar circuits exist in higher dimensions. All capacitors are equal while the resistors vary, reflecting the spatially varying conductivity of the solid. In the model studied here the resistors were assumed to vary randomly (and uncorrelated) according to some activation energy distribution. In any external field the electrostatic potential defined at each node is found from Kirchhoff's equations. The currents through the resistors are the free charge currents while the capacitor currents are Maxwell's displacement currents. The latter are non-zero in an AC field, thus allowing free charge accumulation (the Maxwell-Wagner effect) without violating the Kirchhoff equation expressing that no "total charge" accumulation at a node is allowed.

Fig. 2: Comparison of the predictions of the effective medium approximation (EMA) and the percolation path approximation (PPA) for the low temperature universal frequency dependent conductivity (Appendix 1). Fig. 2a shows a log-log plot (base 10) of the function $\tilde{\sigma}_{EMA}(\tilde{s})$ (Eq. (40), full curve) and the function $\tilde{\sigma}_{PPA}(\tilde{s}')$ (Eq. (47), dots) where $\tilde{s}'=2\tilde{s}$ was chosen with a scaling of the Laplace frequency so that the Taylor expansion

of the two functions agree to first order at $\tilde{S}=0$. Similarly, Fig. 2b compares the real and imaginary parts of the two approximations where $\tilde{\sigma}'_{EMA}(\tilde{S})$ is the full curve, $\tilde{\sigma}''_{EMA}(\tilde{S})$ is the dashed curve, $\tilde{\sigma}'_{PPA}(\tilde{S}')$ is given by the circles, and $\tilde{\sigma}''_{PPA}(\tilde{S}')$ is given crosses. The two approximations yield very similar predictions for the universal conductivity. Analytically, one finds that their asymptotic behavior are identical for $\tilde{S}\rightarrow\infty$ (Eqs. (42) vs. (48) and (45) vs. (50)).

Fig. 3: Log-log plots (base 10) of results of computer simulations in two dimensions compared to the EMA predictions (full curves; Appendix 1). Each figure shows the results of averaging 10 simulations using the Frank-Lobb algorithm [96] of a 200x200 lattice at real Laplace frequencies. The following inverse temperatures are shown: $\beta=5$ (Δ), $\beta=10$ (o), $\beta=20$ (∇), $\beta=40$ ($\Diamond$), $\beta=80$ ($\square$), $\beta=160$ ($+$), for the following activation energy probability distributions (Appendix 2): (a) Asymmetric Gaussian, (b) Cauchy, (c) Symmetric exponential, (d) Power Law with exponent -4, (e) Box, (f) Triangle. The dimensionless conductivity and Laplace frequency are defined by Eq. (39). The EMA predictions were found by solving Eq. (32) numerically (Appendix 1). Given the fact that the EMA has no fitting parameters and that $\tilde{S}$ is constructed by scaling S by a factor which is in some cases larger than 10^{50}, the EMA provides

a good fit to the simulations. In particular, the frequency dependent conductivity at low temperatures becomes universal both in the EMA and in the simulations.

Fig. 4:   Test of the EMA prediction for the low temperature universal conductivity (full curve) in log-log plots (base 10). An empirical rescaling of $\tilde{s}$ has been allowed to facilitate a comparison to the EMA focussing only on the **shape** of $\tilde{\sigma}(\tilde{s})$. The figure shows data from the simulations reported in Fig. 3 at $\beta=80$ and $\beta=160$ for Asymmetric Gaussian (Δ), Cauchy (O), Symmetric Exponential ($\lozenge$), Power law with exponent -4 ($\square$), Box (+), Triangle (∇).

Fig.5:   Approach towards universality at low temperatures in three dimensions plotted in log-log plots (base 10). The five figures show results for $\beta=10$ (Δ), $\beta=30$ (O), and $\beta=60$ ($\bullet$), for the following distributions: Asymmetric Gaussian (a), Cauchy (b), Exponential (c), Box (d), and Triangle (e). Each point represents the average of 5 simulations of a 50x50x50 lattice where Kirchhoff's equations were solved by the algebraic multigrid algorithm using the AMG1R5 Fortran subroutine [103]. The full curve is the EMA prediction for the low temperature universal conductivity (Eq. (40)). No curves were drawn for the finite temperature EMA predictions [these are known to be inaccurate because, while $\sigma(s\to\infty)$ is accurate, $\sigma(0)$ is wrong

because the EMA has the percolation threshold wrong in 3-D]. As in Fig. 4 an empirical rescaling of the real dimensionless Laplace frequency was allowed, to focus on the **shape** of the conductivity curve only. It is clear that universality is approached at low temperatures. Unfortunately, it was not possible to go to low enough temperatures to allow a detailed study of the exact shape of the universal conductivity in 3-D.

Fig. 1
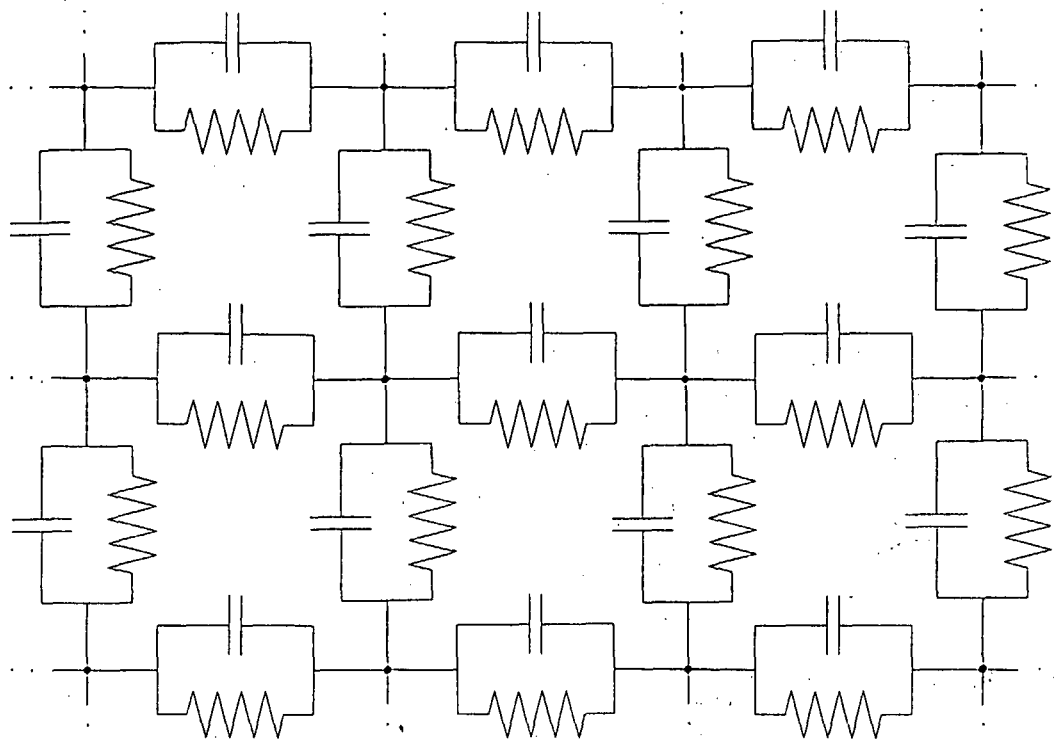
Fig 2b: EMA-PPA Comparison
(Real Frequencies)

Log(Conductivity) vs Log(Frequency)
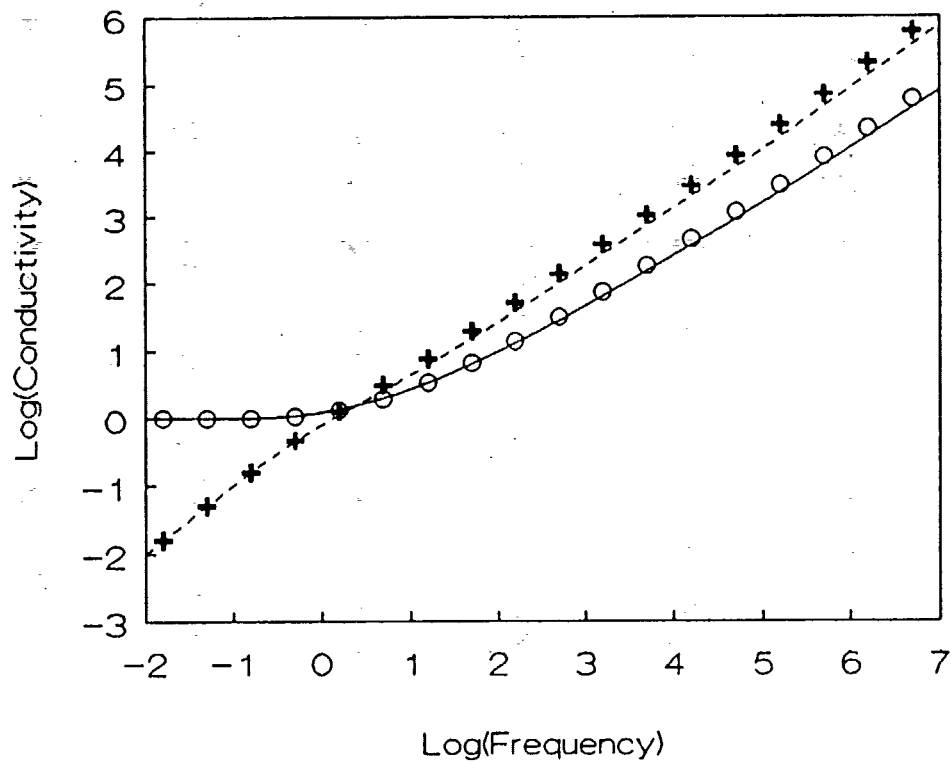
Fig 2a: EMA-PPA Comparison
(Real Laplace Frequencies)

Log(Conductivity) vs Log(Laplace Frequency)

Fig 3a: Asymmetric Gaussian
Dimension 2
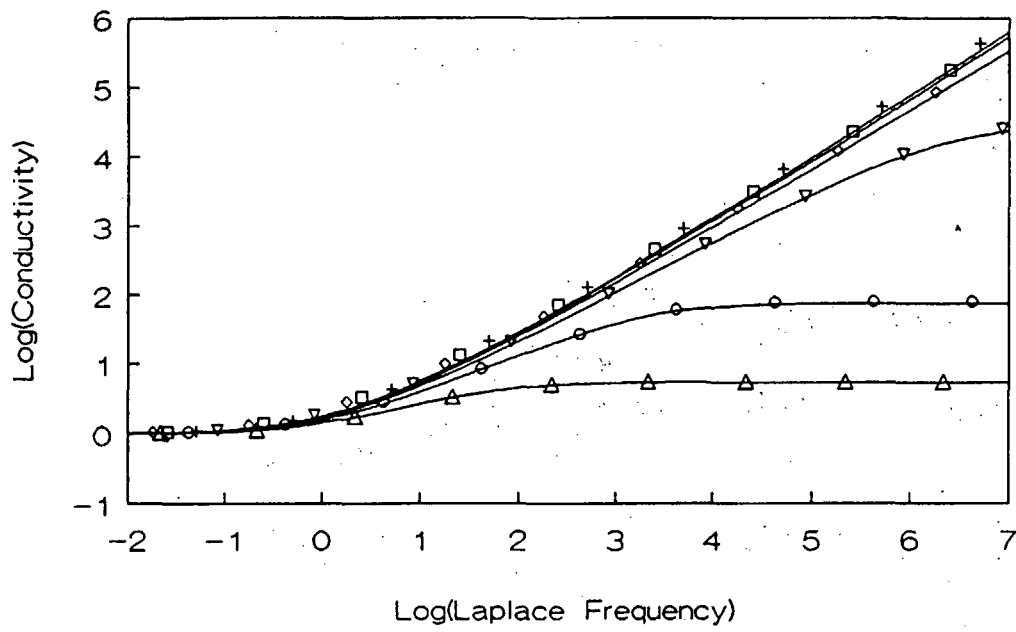
Fig 3b: Cauchy
Dimension 2
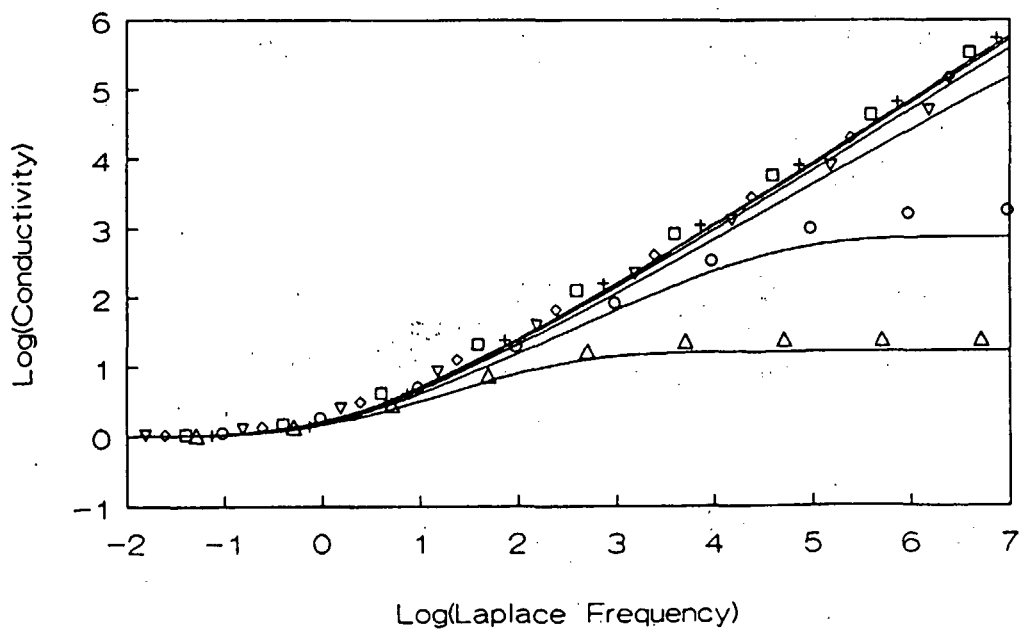
Fig 3c: Symmetric Exponential
Dimension 2

Fig 3d: Power Law (exponent −4)
Dimension 2
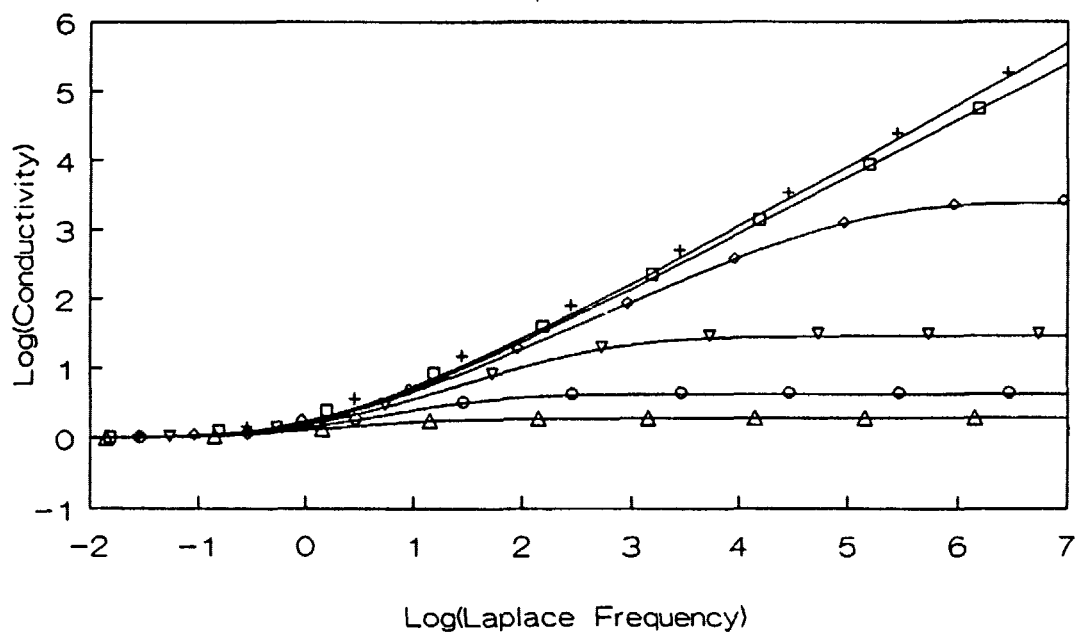
Fig 3e: Box
Dimension 2
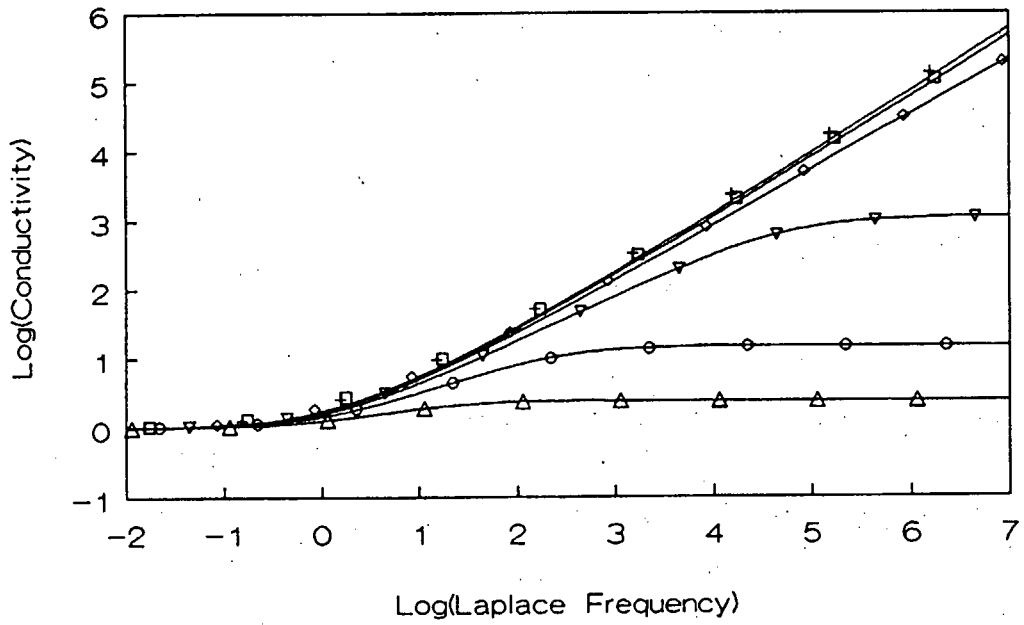
Fig 3f: Triangle
Dimension 2
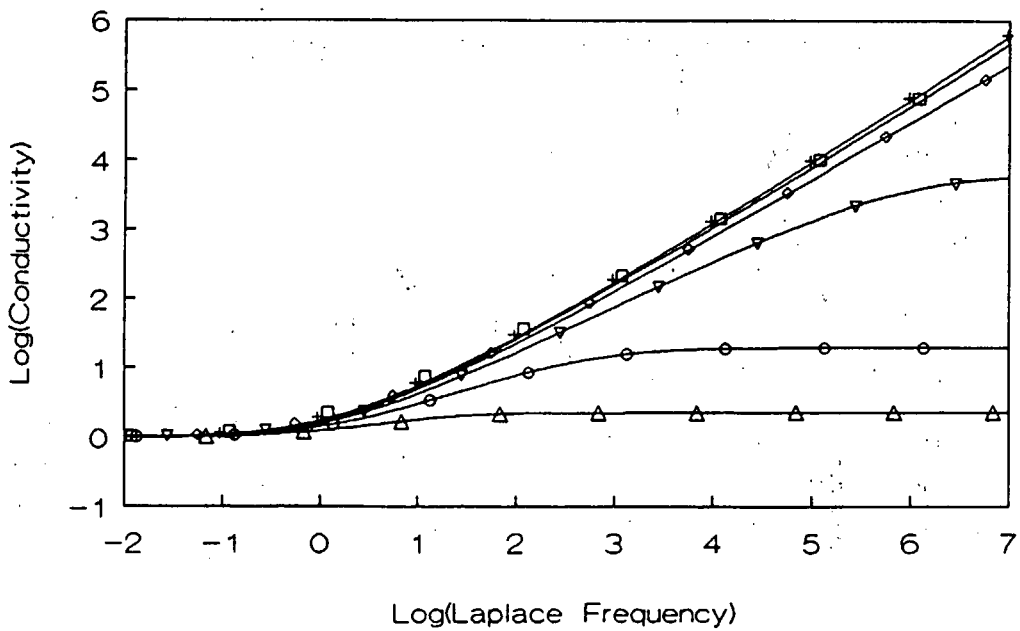
Fig 4a: Beta=80
Dimension 2

Δ Asym.   O Cauchy   ◇ Sym. E
□ Power I   + Box   ▽ Triangle

Log(Conductivity) vs Log(Laplace Frequency)
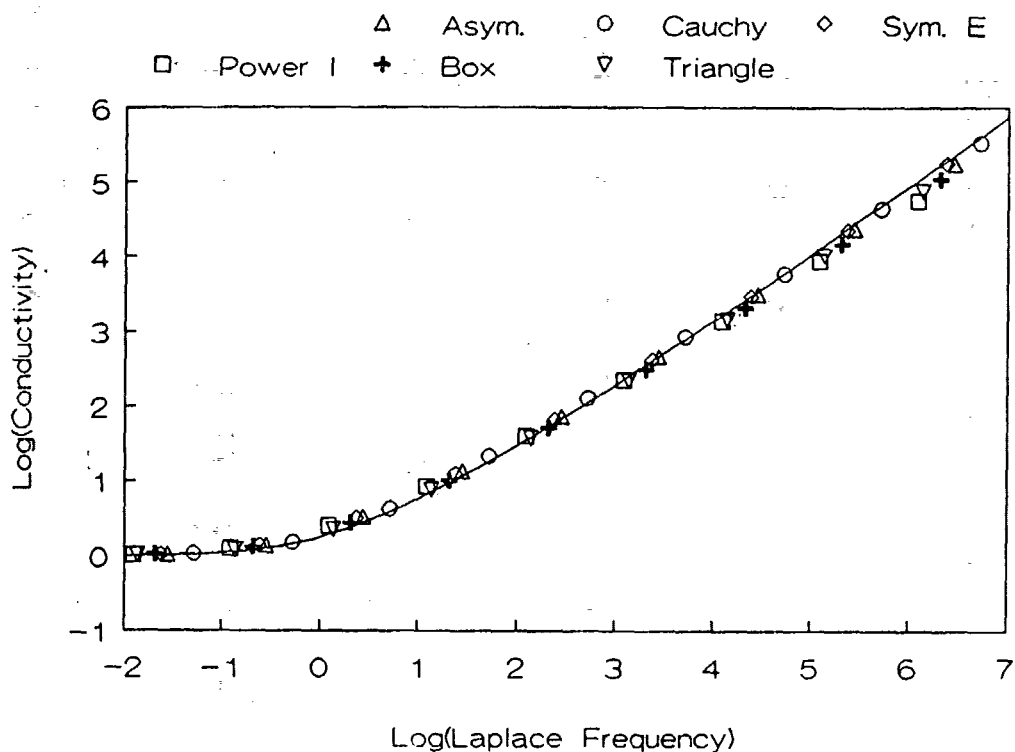
Fig 4b: Beta=160
Dimension 2

+ Box   O Cauchy   ◇ Sym. E
□ Power I   Δ Asym.   ▽ Triangle

Log(Conductivity) vs Log(Laplace Frequency)
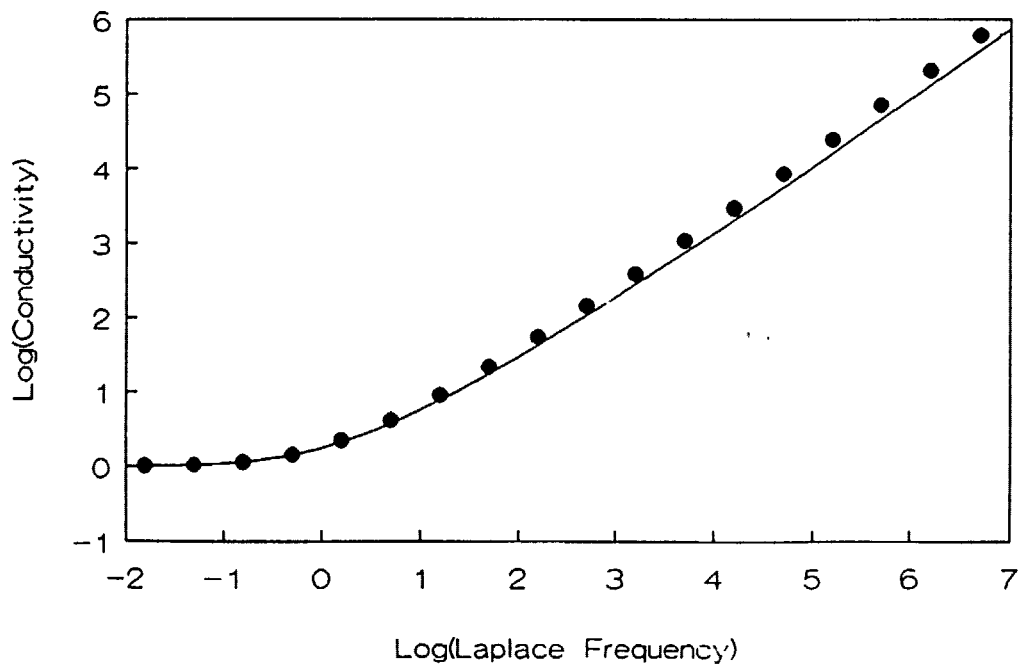
Fig 5a: Asymmetric Gaussian
Dimension 3

Fig 5b: Cauchy
Dimension 3

Fig 5c: Exponential
Dimension 3

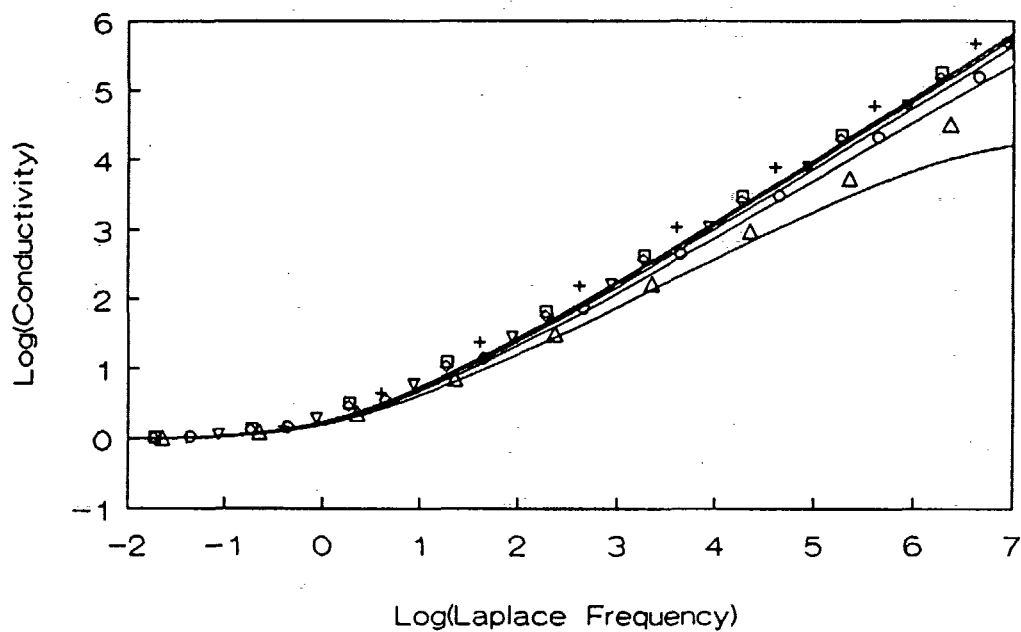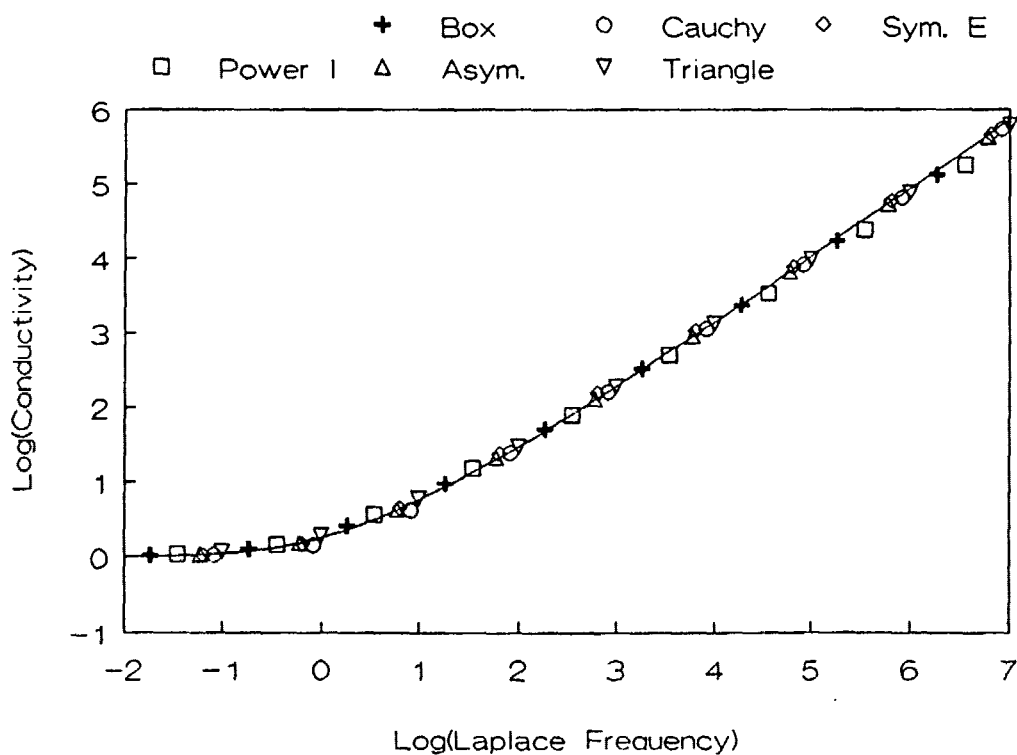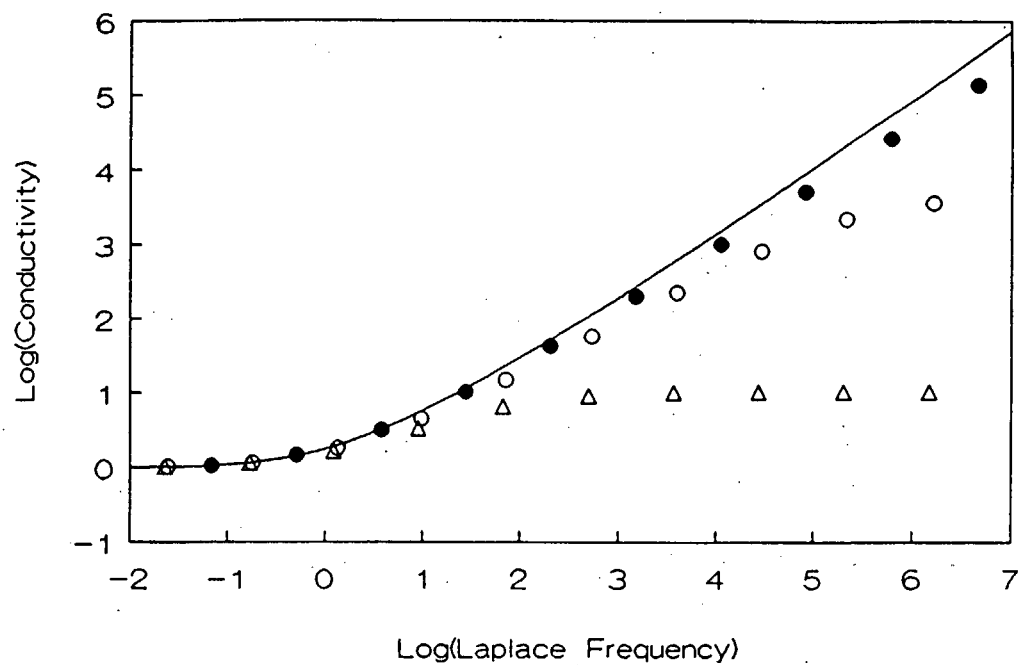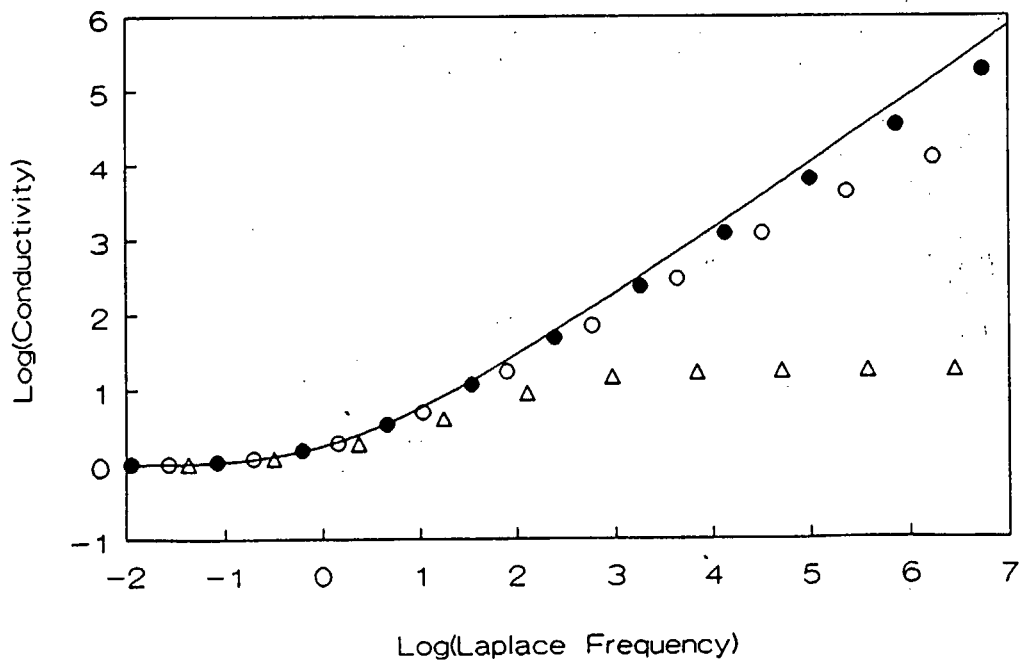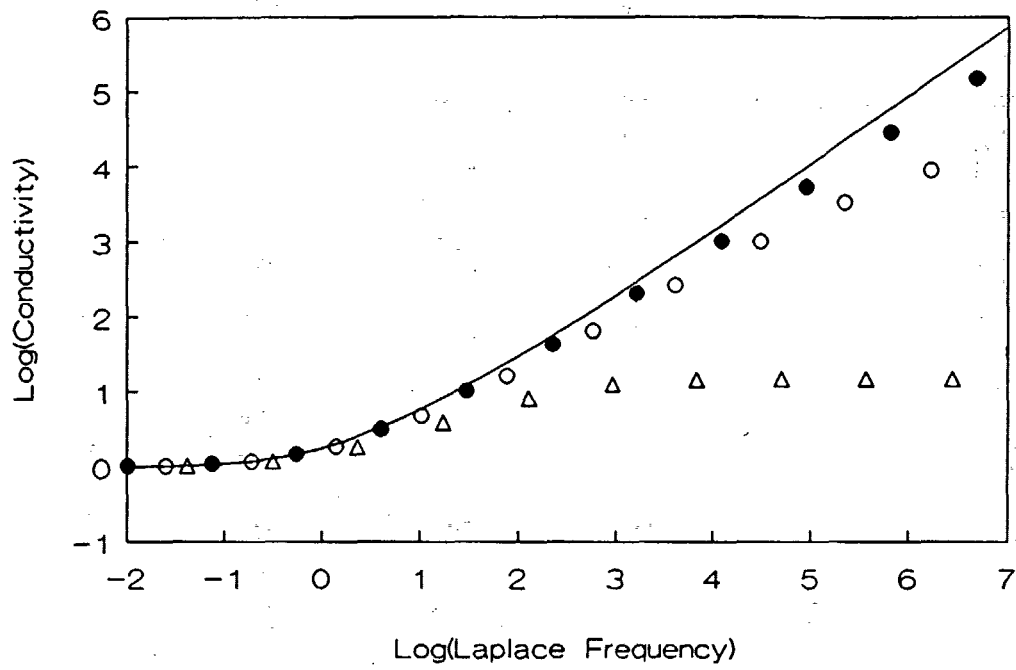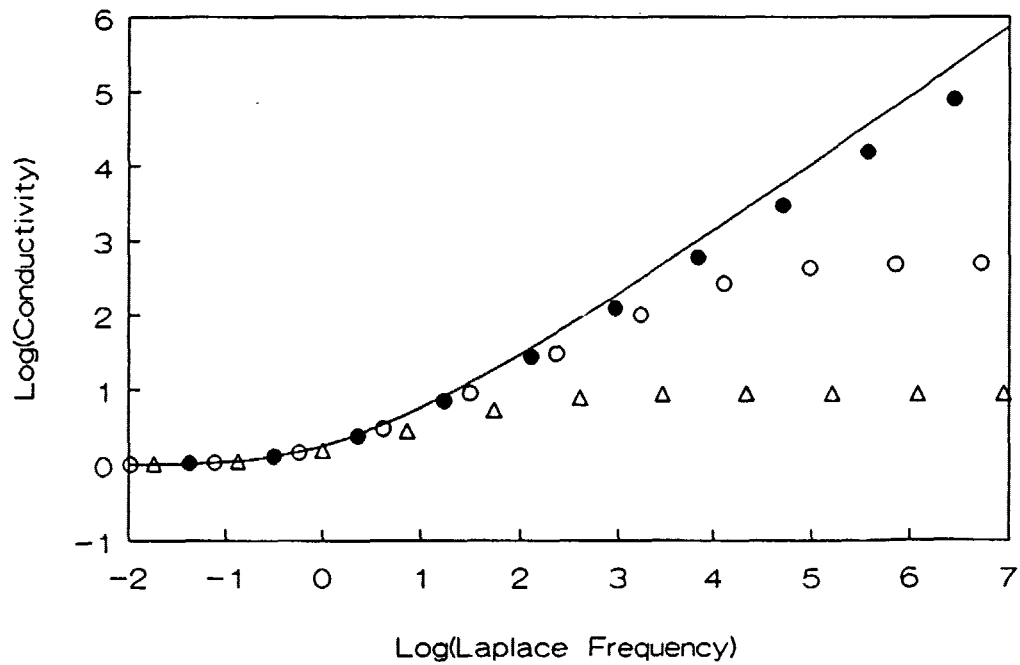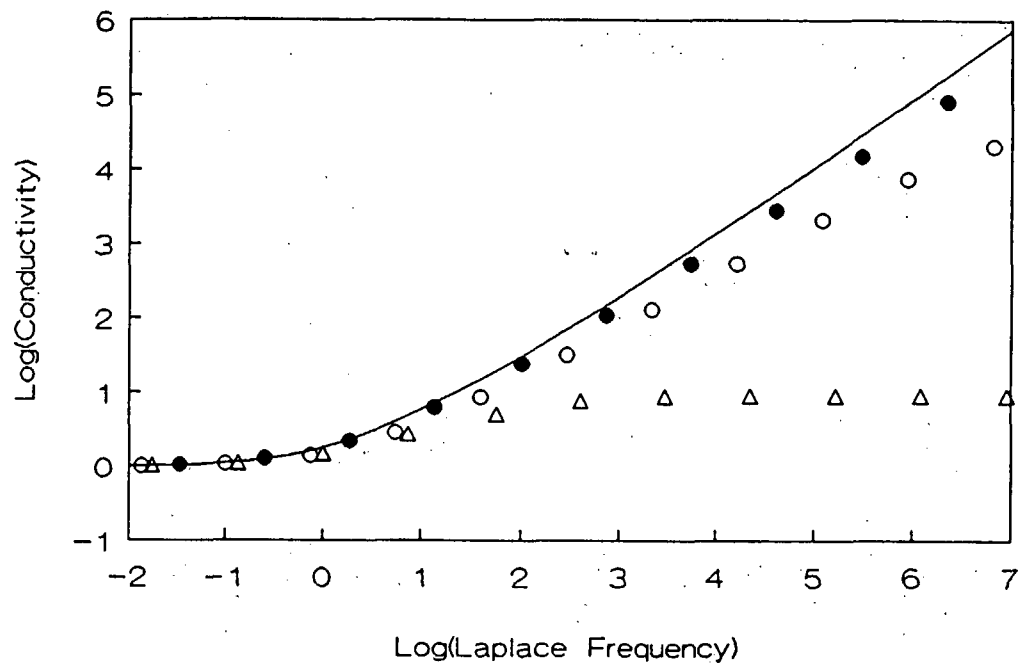Fig 5d: Box
Dimension 3

Fig 5e: Triangle
Dimension 3

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Vejleder: Mogens Niss.

112/85 "VIDENSKABELIGGØRELSE AF DANSK TEKNOLOGISK INNOVATION
FREM TIL 1950 – BELYST VED EKSEMPLER".
Projektrapport af: Erik Odgaard Gade, Hans Hedal,
Frank C. Ludvigsen, Annette Post Nielsen og Finn
Physant.
Vejleder: Claus Bryld og Bent C. Jørgensen.

113/85 "DESUSPENSION OF SPLITTING ELLIPTIC SYMBOLS 11".
Af: Bernhelm Booss og Krzysztof Wojciechowski.

114/85 "ANVENDELSE AF GRAFISKE METODER TIL ANALYSE
AF KONTIGENSTABELLER".
Projektrapport af: Lone Biilmann, Ole R. Jensen
og Anne-Lise von Moos.
Vejleder: Jørgen Larsen.

115/85 "MATEMATIKKENS UDVIKLING OP TIL RENÆSSANCEN".
Af: Mogens Niss.

116/85 "A PHENOMENOLOGICAL MODEL FOR THE MEYER-
NELDEL RULE".
Af: Jeppe C. Dyre.

117/85 "KRAFT & FJERNVARMEOPTIMERING"
Af: Jacob Mørch Pedersen.
Vejleder: Bent Sørensen

118/85 "TILFÆLDIGHEDEN OG NØDVENDIGHEDEN IFØLGE
PEIRCE OG FYSIKKEN".
Af: Peder Voetmann Christiansen

119/86 "DET ER GANSKE VIST – –. EUKLIDS FEMTE POSTULAT
KUNNE NOK SKABE RØRE I ANDEDAMMEN".
Af: Iben Maj Christiansen
Vejleder: Mogens Niss.

120/86 "ET ANTAL STATISTISKE STANDARDMODELLER".
Af: Jørgen Larsen

121/86 "SIMULATION I KONTINUERT TID".
Af: Peder Voetmann Christiansen.

122/86 "ON THE MECHANISM OF GLASS IONIC CONDUCTIVITY".
Af: Jeppe C. Dyre.

123/86 "GYMNASIEFYSIKKEN OG DEN STORE VERDEN".
Fysiklærerforeningen, IMFUFA, RUC.

124/86 "OPGAVESAMLING I MATEMATIK".
Samtlige opgaver stillet i tiden 1974-jan. 1986.

125/86 "UVBY,β – systemet – en effektiv fotometrisk spektral-
klassifikation af B-,A- og F-stjerner".
Projektrapport af: Birger Lundgren.

126/86 "OM UDVIKLINGEN AF DEN SPECIELLE RELATIVITETSTEORI".
Projektrapport af: Lise Odgaard & Linda Szkotak Jensen
Vejledere: Karin Beyer & Stig Andur Pedersen.

127/86 "GALOIS' BIDRAG TIL UDVIKLINGEN AF DEN ABSTRAKTE
ALGEBRA".
Projektrapport af: Pernille Sand, Heine Larsen &
Lars Frandsen.
Vejleder: Mogens Niss.

128/86 "SMÅKRYB" – om ikke-standard analyse.
Projektrapport af: Niels Jørgensen & Mikael Klintorp.
Vejleder: Jeppe Dyre.

129/86 "PHYSICS IN SOCIETY"
Lecture Notes 1983 (1986)
Af: Bent Sørensen

130/86 "Studies in Wind Power"
Af: Bent Sørensen

131/86 "FYSIK OG SAMFUND" – Et integreret fysik/historie-
projekt om naturanskuelsens historiske udvikling
og dens samfundsmæssige betingethed.
Projektrapport af: Jakob Heckscher, Søren Brønd,
Andy Wierød.
Vejledere: Jens Høyrup, Jørgen Vogelius,
Jens Højgaard Jensen.

132/86 "FYSIK OG DANNELSE"
Projektrapport af: Søren Brønd, Andy Wierød.
Vejledere: Karin Beyer, Jørgen Vogelius.

133/86 "CHERNOBYL ACCIDENT: ASSESSING THE DATA.
ENERGY SERIES NO. 15.
AF: Bent Sørensen.

134/87 "THE D.C. AND THE A.C. ELECTRICAL TRANSPORT IN AsSeTe SYSTEM"
Authors: M.B.El-Den, N.B.Olsen, Ib Høst Pedersen,
Petr Viscör

135/87 "INTUITIONISTISK MATEMATIKS METODER OG ERKENDELSES-
TEORETISKE FORUDSÆTNINGER"
MASTEMATIKSPECIALE: Claus Larsen
Vejledere: Anton Jensen og Stig Andur Pedersen

136/87 "Mystisk og naturlig filosofi: En skitse af kristendommens
første og andet møde med græsk filosofi"
Projektrapport af Frank Colding Ludvigsen
Vejledere: Historie: Ib Thiersen
Fysik: Jens Højgaard Jensen

137/87 "HOPMODELLER FOR ELEKTRISK LEDNING I UORDNEDE
FASTE STOFFER" – Resume af licentiatafhandling
Af: Jeppe Dyre
Vejledere: Niels Boye Olsen og
Peder Voetmann Christiansen.

138/87 "JOSEPHSON EFFECT AND CIRCLE MAP."
Paper presented at The International
Workshop on Teaching Nonlinear Phenomena
at Universities and Schools, "Chaos in
Education". Balaton, Hungary, 26 April-2 May 1987.

By: Peder Voetmann Christiansen

13 9/87 "Machbarkeit nichtbeherrschbarer Technik
durch Fortschritte in der Erkennbarkeit
der Natur"

Af: Bernhelm Booss-Bavnbek
Martin Bohle-Carbonell

140/87 "ON THE TOPOLOGY OF SPACES OF HOLOMORPHIC MAPS"

By: Jens Gravesen

141/87 "RADIOMETERS UDVIKLING AF BLODGASAPPARATUR –
ET TEKNOLOGIHISTORISK PROJEKT"

Projektrapport af Finn C. Physant
Vejleder: Ib Thiersen

142/87 "The Calderón Projektor for Operators With
Splitting Elliptic Symbols"

by: Bernhelm Booss-Bavnbek og
Krzysztof P. Wojciechowski

143/87 "Kursusmateriale til Matematik på NAT-BAS"

af: Mogens Brun Heefelt

144/87 "Context and Non-Locality - A Peircean Approach

Paper presented at the Symposium on the
Foundations of Modern Physics The Copenhagen
Interpretation 60 Years after the Como Lecture.
Joensuu, Finland, 6 - 8 august 1987.

By: Peder Voetmann Christiansen

145/87 "AIMS AND SCOPE OF APPLICATIONS AND
MODELLING IN MATHEMATICS CURRICULA"

Manuscript of a plenary lecture delivered at
ICMTA 3, Kassel, FRG 8.-11.9.1987

By: Mogens Niss

146/87 "BESTEMMELSE AF BULKRESISTIVITETEN I SILICIUM"

- en ny frekvensbaseret målemetode.
Fysikspeciale af Jan Vedde
Vejledere: Niels Boye Olsen & Petr Viščor

147/87 "Rapport om BIS på NAT-BAS"
redigeret af: Mogens Brun Heefelt

148/87 "Naturvidenskabsundervisning med
Samfundsperspektiv"

af: Peter Colding-Jørgensen DLH
Albert Chr. Paulsen

149/87 "In-Situ Measurements of the density of amorphous
germanium prepared in ultra high vacuum"

by: Petr Viščor

150/87 "Structure and the Existence of the first sharp
diffraction peak in amorphous germanium
prepared in UHV and measured in-situ"

by: Petr Viščor

151/87 "DYNAMISK PROGRAMMERING"

Matematikprojekt af:
Birgit Andresen, Keld Nielsen og Jimmy Staal

152/87 "PSEUDO-DIFFERENTIAL PROJECTIONS AND THE TOPOLOGY
OF CERTAIN SPACES OF ELLIPTIC BOUNDARY VALUE
PROBLEMS"

by: Bernhelm Booss-Bavnbek
Krzysztof P. Wojciechowski

153/88 "HALVLEDERTEKNOLOGIENS UDVIKLING MELLEM MILITÆRE
OG CIVILE KRÆFTER"

Et eksempel på humanistisk teknologihistorie
Historiespeciale

Af: Hans Hedal

Vejleder: Ib Thiersen

154/88 "MASTER EQUATION APPROACH TO VISCOUS LIQUIDS AND
THE GLASS TRANSITION"

By: Jeppe Dyre

155/88 "A NOTE ON THE ACTION OF THE POISSON SOLUTION
OPERATOR TO THE DIRICHLET PROBLEM FOR A FORMALLY
SELFADJOINT DIFFERENTIAL OPERATOR"

by: Michael Pedersen

156/88 "THE RANDOM FREE ENERGY BARRIER MODEL FOR AC
CONDUCTION IN DISORDERED SOLIDS"

by: Jeppe C. Dyre

157/88 " STABILIZATION OF PARTIAL DIFFERENTIAL EQUATIONS
BY FINITE DIMENSIONAL BOUNDARY FEEDBACK CONTROL:
A pseudo-differential approach."

by: Michael Pedersen

158/88 "UNIFIED FORMALISM FOR EXCESS CURRENT NOISE IN
RANDOM WALK MODELS"

by: Jeppe Dyre

159/88 "STUDIES IN SOLAR ENERGY"

by: Bent Sørensen

160/88 "LOOP GROUPS AND INSTANTONS IN DIMENSION TWO"

by: Jens Gravesen

161/88 "PSEUDO-DIFFERENTIAL PERTURBATIONS AND STABILIZATION
OF DISTRIBUTED PARAMETER SYSTEMS:

Dirichlet feedback control problems"

by: Michael Pedersen

162/88 "PIGER & FYSIK - OG MEGET MERE"

AF: Karin Beyer, Sussanne Blegaa, Birthe Olsen,

Jette Reich , Mette Vedelsby

163/88 "EN MATEMATISK MODEL TIL BESTEMMELSE AF
PERMEABILITETEN FOR BLOD-NETHINDE-BARRIEREN"

Af: Finn Langberg, Michael Jarden, Lars Frellesen

Vejleder: Jesper Larsen

164/88 "Vurdering af matematisk teknologi
Technology Assessment
Technikfolgenabschätzung"

Af: Bernhelm Booss-Bavnbek, Glen Pate med
Martin Bohle-Carbonell og Jens Højgaard Jensen

165/88 "COMPLEX STRUCTURES IN THE NASH-MOSER CATEGORY"

by: Jens Gravesen

166/88 "Grundbegreber i Sandsynligheds-
regningen"

   Af: Jørgen Larsen

167a/88 "BASISSTATISTIK 1. Diskrete modeller"

   Af: Jørgen Larsen

167b/88 "BASISSTATISTIK 2. Kontinuerte
modeller"

   Af: Jørgen Larsen

168/88 "OVERFLADEN AF PLANETEN MARS"
Laboratorie-simulering og MARS-analoger
undersøgt ved Mössbauerspektroskopi.

   Fysikspeciale af:

   Birger Lundgren

   Vejleder: Jens Martin Knudsen
                Fys.Lab./HCØ

169/88 "CHARLES S. PEIRCE: MURSTEN OG MØRTEL
TIL EN METAFYSIK."

   Fem artikler fra tidsskriftet "The Monist".
   1891-93.

   Introduktion og oversættelse:

   Peder Voetmann Christéansen

170/88 "OPGAVESAMLING I MATEMATIK"

   Samtlige opgaver stillet i tiden
   1974 - juni 1988

171/88 "The Dirac Equation with Light-Cone Data"

      af: Johnny Tom Ottesen

172/88 "FYSIK OG VIRKELIGHED"

   Kvantemekanikkens grundlagsproblem
   i gymnasiet.

   Fysikprojekt af:

   Erik Lund og Kurt Jensen

   Vejledere: Albert Chr. Paulsen og
   Peder Voetmann Christiansen

173/89 "NUMERISKE ALGORITMER"

      af: Mogens Brun Heefelt

174/89 " GRAFISK FREMSTILLING AF

      FRAKTALER OG KAOS"

      af: Peder Voetmann Christiansen

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DEPENDENT SPECTRUM OF THE NON-STATONARY
SOLUTION TO THE OPERATOR RICCATI EQUATION

      af: Michael Pedersen

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RESPONSE THEORY"

      af : Jeppe Dyre

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      af: Morten Andersen, Ulla Engström,
      Thomas Gravesen, Nanna Lund, Pia
      Madsen, Dina Rawat, Peter Torstensen
      Vejleder: Mogens Brun Heefelt

178/89 "BIOSYNTESEN AF PENICILLIN - en matematisk model"

      af: Ulla Eghave Rasmussen, Hans Oxvang Mortensen,
         Michael Jarden

      vejleder i matematik: Jesper Larsen
                    biologi: Erling Lauridsen

179a/89 "LÆRERVEJLEDNING M.M. til et eksperimentelt forløb
om kaos"

      af: Andy Wierød, Søren Brønd og Jimmy Staal

      Vejledere: Peder Voetmann Christiansen
                 Karin Beyer

179b/89 "ELEVHÆFTE: Noter til et eksperimentelt kursus om
kaos"

      af: Andy Wierød, Søren Brønd og Jimmy Staal

      Vejledere: Peder Voetmann Christiansen
                 Karin Beyer

180/89 "KAOS I FYSISKE SYSTEMER eksemplificeret ved
torsions- og dobbeltpendul".

      af: Andy Wierød, Søren Brønd og Jimmy Staal
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SHEAR VISCOELASTICITY"

      by: Jeppe Dyre

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   AND LINKS TO OTHER SUBJECTS - State. trends and

   issues in mathematics instruction

      by: WERNER BLUM, Kassel (FRG)   og
          MOGENS NISS, Roskilde (Denmark)

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   varmefylde af en underafkølet væske ved glasovergangen"

      af: Tage Emil Christensen

185/90 "EN NÆSTEN PERIODISK HISTORIE"
Et matematisk projekt
af: Steen Grode og Thomas Jessen
Vejleder: Jacob Jacobsen

186/90 "RITUAL OG RATIONALITET i videnskabers udvikling"
redigeret af Arne Jakobsen og Stig Andur Pedersen

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      af: Annemette Sofie Olufsen, Lars Frellesen
          og Ole Møller Nielsen

Vejledere: Michael Pedersen og Finn Munk

188/90 "FERMICONDENSATION - AN ALMOST IDEAL GLASS TRANSITION"
      by: Jeppe Dyre

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GYMNASIET OG HØJERE LÆREANSTALTER

      af: Finn Langberg

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APPROXIMATE NONLINEAR RESPONSE
THEORY"

by: Jeppe Dyre

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BUNDLES AND ACTIONS OF GROUPS
ON C*-ALGEBRAS"

by: Iain Raeburn and Dana P. Williams

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dynamics of endemic infectious diseases
and
SIR-models of the epidemiology and natural
selection of co-circulating influenza virus
with partial cross-immunity"

by: Viggo Andreasen

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Projektrapport af : Frank Olsen

194b/90 "DETERMINISTISK KAOS"
Kørselsrapport

Projektrapport af: Frank Olsen

195/90 "STADIER PÅ PARADIGMETS VEJ"
Et projekt om den videnskabelige udvikling
der førte til dannelse af kvantemekanikken.

Projektrapport for 1. modul på fysikuddan-
nelsen, skrevet af:

Anja Boisen, Thomas Hougård, Anders Gorm
Larsen, Nicolai Ryge.

Vejleder: Peder Voetmann Christiansen

196/90 "ER KAOS NØDVENDIGT?"

- en projektrapport om kaos' paradigmatiske
status i fysikken.

af: Johannes K. Nielsen, Jimmy Staal og
Peter Bøggild

Vejleder: Peder Voetmann Christiansen

197/90 "Kontrafaktiske konditionaler i HOL

af: Jesper Voetmann, Hans Oxvang Mortensen og
Aleksander Høst-Madsen

Vejleder: Stig Andur Pedersen

198/90 "Metal-Isolator-Metal systemer"
Speciale
af: Frank Olsen

199/90 "SPREDT FÆGTNING" Artikelsamling
af: Jens Højgaard Jensen

200/90 "LINEÆR ALGEBRA OG ANALYSE"
Noter til den naturvidenskabelige basis-
uddannelse.
af: Mogens Niss

201/90 "Undersøgelse af atomare korrelationer i
amorfe stoffer ved røntgendiffraktion"

af: Karen Birkelund og Klaus Dahl Jensen

Vejledere: Petr Viščor, Ole Bakander

202/90 "TEGN OG KVANTER"
Foredrag og artikler, 1971-90.

af: Peder Voetmann Christiansen

203/90 "OPGAVESAMLING I MATEMATIK" 1974-1990
afløser tekst 170/88

204/91 "ERKENDELSE OG KVANTEMEKANIK"
et Breddemodul Fysik Projekt
af: Thomas Jessen
Vejleder: Petr Viščor

205/91 "PEIRCE'S LOGIC OF VAGUENESS"
by: Claudine Engel-Tiercelin
Department of Philosophy
Université de Paris-1
(Panthéon-Sorbonne)

206a+b/91 "GERMANIUMBEAMANALYSE SAMT
A - GE TYNDFILMS ELEKTRISKE
EGENSKABER"
Eksperimentelt Fysikspeciale
af: Jeanne Linda Mortensen
og Annette Post Nielsen
Vejleder: Petr Viščor

207/91 "SOME REMARKS ON AC CONDUCTION
IN DISORDERED SOLIDS"

by: Jeppe C. Dyre

208/91 "LANGEVIN MODELS FOR SHEAR STRESS
FLUCTUATIONS IN FLOWS OF VISCO-
ELASTIC LIQUIDS"

by: Jeppe C. Dyre

209/91 "LORENZ GUIDE" Kompendium til den
danske fysiker Ludvig Lorenz,
1829-91.
af: Helge Kragh

210/91 "Global Dimension, Tower of Algebras,
and Jones Index of Split Seperable
Subalgebras with Unitality Condition.

by: Lars Kadison

211/91 "I SANDHEDENS TJENESTE"
- historien bag teorien for de komplekse tal.
af: Lise Arleth, Charlotte Gjerrild,
Jane Hansen, Linda Kyndlev, Anne
Charlotte Nilsson, Kamma Tulinius.
Vejledere: Jesper Larsen og Bernhelm
Booss-Bavnbek

212/91 "Cyclic Homology of Triangular Matrix
Algebras"
by: Lars Kadison

213/91 "Disease-induced natural selection in a
diploid host
by: Viggo Andreasen and Freddy B.Christiansen

214|91   "Halløj i æteren" - om
          elektromagnetisme. Oplæg
          til undervisningsmateriale
          i gymnasiet.

          Af: Nils Kruse, Peter Gastrup,
              Kristian Hoppe, Jeppe Guldager

          Vejledere: Petr Viscor, Hans Hedal

215|91   "Physics and Technology of Metal-
          Insulator-Metal thin film structures
          used as planar electron emitters

          by: A.Delong, M.Drsticka, K.Hladil,
          V.Kolarik, F.Olsen, P.Pavelka and
          Petr Viscor.

216|91   "Kvantemekanik på PC'eren"

          af: Thomas Jessen

217/92   "Two papers on APPLICATIONS AND MODELLING
          IN THE MATHEMATICS CURRICULUM"

          by: Mogens Niss

218/92   "A Three-Square Theorem"

          by: Lars Kadison

219/92   "RUPNOK - stationær strømning i elastiske rør"

          af: Anja Boisen, Karen Birkelund, Mette Olufsen

          Vejleder: Jesper Larsen

220/92   "Automatisk diagnosticering i digitale kredsløb"

          af: Bjørn Christensen, Ole Møller Nielsen

          Vejleder: Stig Andur Pedersen

221/92   "A BUNDLE VALUED RADON TRANSFORM, WITH
          APPLICATIONS TO INVARIANT WAVE EQUATIONS"

          by: Thomas P. Branson, Gestur Olafsson and
              Henrik Schlichtkrull

222/92   On the Representations of some Infinite Dimensional
          Groups and Algebras Related to Quantum Physics

          by: Johnny T. Ottesen

223/92   THE FUNCTIONAL DETERMINANT

          by: Thomas P. Branson .

224/92   UNIVERSAL AC CONDUCTIVITY OF NON-METALLIC SOLIDS AT
          LOW TEMPERATURES

          by: Jeppe C. Dyre   .

225/92   "HATMODELLEN" Impedansspektroskopi i ultrarent
          en-krystallinsk silicium

          af: Anja Boisen, Anders Gorm Larsen, Jesper Varmer,
              Johannes K. Nielsen, Kit R. Hansen, Peter Bøggild
              og Thomas Hougaard

          Vejleder: Petr Viscor

226/92   "METHODS AND MODELS FOR ESTIMATING THE GLOBAL
          CIRCULATION OF SELECTED EMISSIONS FROM ENERGY
          CONVERSION"

          by: Bent Sørensen

227/92   "Computersimulering og fysik"

          af: Per M.Hansen, Steffen Holm,
          Peter Maibom, Mads K. Dall Petersen,
          Pernille Postgaard, Thomas B.Schrøder,
          Ivar P. Zeck

          Vejleder: Peder Voetmann Christiansen

228/92   "Teknologi og historie"

          Fire artikler af:

          Mogens Niss, Jens Høyrup, Ib Thiersen,
          Hans Hedal

229/92   "Masser af information uden betydning"

          En diskussion af informationsteorien
          i Tor Nørretranders' "Mærk Verden" og
          en skitse til et alternativ basseret
          på andenordens kybernetik og semiotik.

          af: Søren Brier

230/92   "Vinklens tredeling - et klassisk
          problem"

          et matematisk projekt af

          Karen Birkelund, Bjørn Christensen

          Vejleder: Johnny Ottesen

231A/92   "Elektrondiffusion i silicium - en
          matematisk model"

          af: Jesper Voetmann, Karen Birkelund,
          Mette Olufsen, Ole Møller Nielsen

          Vejledere: Johnny Ottesen, H.B.Hansen

231B/92   "Elektrondiffusion i silicium - en
          matematisk model" Kildetekster

          af: Jesper Voetmann, Karen Birkelund,
          Mette Olufsen, Ole Møller Nielsen

          Vejledere: Johnny Ottesen, H.B.Hansen

232/92   "Undersøgelse om den simultane opdagelse
          af energiens bevarelse og isærdeles om
          de af Mayer, Colding, Joule og Helmholtz
          udførte arbejder"

          af: L.Arleth, G.I.Dybkjær, M.T.Østergård

          Vejleder: Dorthe Posselt

233/92   "The effect of age-dependent host
          mortality on the dynamics of an endemic
          disease   and
          Instability in an SIR-model with age-
          dependent susceptibility

          by: Viggo Andreasen

234/92   "THE FUNCTIONAL DETERMINANT OF A FOUR-DIMENSIONAL
          BOUNDARY VALUE PROBLEM"

          by: Thomas P. Branson and Peter B. Gilkey

235/92   OVERFLADESTRUKTUR OG POREUDVIKLING AF KOKS
          - Modul 3 fysik projekt -

          af: Thomas Jessen

236a/93 INTRODUKTION TIL KVANTE
HALL EFFEKTEN

af: Anja Boisen, Peter Bøggild

Vejleder: Peder Voetmann Christiansen
Erland Brun Hansen

236b/93 STRØMSSAMMENBRUD AF KVANTE
HALL EFFEKTEN

af: Anja Boisen, Peter Bøggild

Vejleder: Peder Voetmann Christiansen
Erland Brun Hansen

237/93 The Wedderburn principal theorem and
Shukla cohomology

af: Lars Kadison

238/93 SEMIOTIK OG SYSTEMEGENSKABER (2)

Vektorbånd og tensorer

af: Peder Voetmann Christiansen

239/93 Valgsystemer - Modelbygning og analyse

Matematik 2. modul

af: Charlotte Gjerrild, Jane Hansen,
Maria Hermannsson, Allan Jørgensen,
Ragna Clauson-Kaas, Poul Lützen

Vejleder: Mogens Niss

240/93 Patologiske eksempler.
Om sære matematiske fisks betydning for
den matematiske udvikling

af: Claus Dræby, Jørn Skov Hansen, Runa
Ulsøe Johansen, Peter Meibom, Johannes
Kristoffer Nielsen

Vejleder: Mogens Niss

241/93 FOTOVOLTAISK STATUSNOTAT 1

af: Bent Sørensen

242/93 Brovedligeholdelse - bevar mig vel

Analyse af Vejdirektoratets model for
optimering af broreparationer

af: Linda Kyndlev, Kare Fundal, Kamma
Tulinius, Ivar Zeck

Vejleder: Jesper Larsen

243/93 TANKEEKSPERIMENTER I FYSIKKEN

Et 1.modul fysikprojekt

af: Karen Birkelund, Stine Sofia Korremann

Vejleder: Dorthe Posselt

244/93 RADONTRANSFORMATIONEN og dens anvendelse
i CT-scanning

Projektrapport

af: Trine Andreasen, Tine Guldager Christiansen,
Nina Skov Hansen og Christine Iversen

Vejledere: Gestur Olafsson og Jesper Larsen

245a+b
/93 Time-Of-Flight målinger på krystallinske
halvledere
Specialerapport

af: Linda Szkotak Jensen og Lise Odgaard Gade

Vejledere: Petr Viscor og Niels Boye Olsen

246/93 HVERDAGSVIDEN OG MATEMATIK
- LÆREPROCESSER I SKOLEN

af: Lena Lindenskov, Statens Humanistiske

Forskningsråd, RUC, IMFUFA