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Warm-Glow Investment and the Underperformance of Green Stocks

by

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Abstract

In this paper we develop a novel theory to explain why green stocks should underperform relative to conventional stocks. We assume that investors derive utility from investing in green stocks – what we call "warm-glow" investment. We derive the theoretical implications of these preferences in a model that is an extension of the Consumption-based Capital Asset Pricing Model. We estimate the model using the Generalized Method of Moments. Our estimates of the strength of the preference for warm glow before the financial crisis are statistically significant but economically insignificant; our estimates of it after the crisis are significant both statistically and economically.

(*JEL* G1, G2)

Keywords: warm glow, ESG investing, asset pricing, green preferences, green stocks

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"*Boomers see doing good as separate from investing; whereas millennials don't see how you could possibly separate the two.*"

Julia Balandina Jaquier, a family adviser in Zurich, quoted in the *Economist* (2017)

"*'Socially responsible investing' is an extreme form of tastes for assets as consumption goods that are unrelated to returns.*"

Eugene Fama and Kenneth French (2007)

I. Introduction

Green investing has become increasingly popular over the last couple of decades. As the preceding quote from Ms. Jaquier in the *Economist* suggests, it is only likely to become more so. This reality raises a contentious question: How do "green" assets – or more generally "socially responsible investments" (SRIs) – perform relative to "conventional assets"? There is a large, inconclusive empirical literature addressing this question. A few authors observe a negative alpha in returns on green investments when compared to other assets. However, this is not obvious. Most of the literature actually reports no significance or even sometimes a positive alpha.

In this paper we develop and empirically estimate a model that explains why green assets should perform poorly relative to conventional non-green assets. We postulate that investors may not only be concerned with the returns on their investments but also derive some psychic benefit from making them. In other words, some financial assets may be consumption goods. This is not new: We will presently survey a small but compelling literature [Beal, Goyen, and Phillips (2005), Fama and French (2007), Hong and Kacperczyk (2009), Pástor, Stambaugh, and Taylor (2020), Statman (2010), Keloharju, Knüpfer, and Linnainmaa (2012), and Luo and Subrahmanyam (2019)] that has explored this idea. Indeed, Fama and French (2007) assert flatly that it is "unrealistic" to assume that assets are *not* consumption goods. Our contribution is to flesh out this insight and apply it to a specific case: We develop a theoretical explanation for how such preferences affect pricing in a more general model of portfolio choice, and empirically test its implications for the pricing of green assets.

If investors derive pleasure from holding assets, we say that they have "warm-glow" preferences. This terminology is inspired by the notion of "warm-glow" giving introduced by Andreoni (1989, 1990), and is now a staple in Public Economics: In traditional public good models, people donate because they are altruistic; that is, they derive utility from their own consumption C and the *total* provision of the public good $D,$ which is the sum of the contributions from all of the individuals involved, $D = \sum_i d_i$. In other words, the utility of person i is $U(C, D)$. Andreoni considered an alternative form of "impure altruism" where people derive utility not from the total provision of the good but only from their own contribution to it, so $U(c, d_i)$. In other words, people get a "warm glow" from the act of giving; they feel good about themselves for contributing.

In our model, investors feel good about themselves for investing in green assets, so that utility is $U(C, investment\ in\ green)$. An investor derives pleasure from the act of going green. We embed such preferences into the canonical, dynamic portfolio problem and study their effects on excess returns. We then develop and elaborate upon this theory and estimate it empirically. The basic intuition is quite simple: Warm-glow preferences create an extra, psychic marginal benefit to investing in a green asset. This increases its demand, causing its price to increase and lowering its rate of return. Paradoxically, trying to support sustainability by investing in green firms may intrinsically tend to make their stocks underperform in the market!

The structure of the paper is as follows. In Section II we review the existing literature. In Section III we develop the theory of warm-glow investment. We construct a model of consumption and portfolio choice where an investor can hold a green asset, a conventional non-green asset, or a risk-free asset. The investor has warm-glow preferences but may also exhibit a spirit of capitalism [Bakshi and Chen (1996b), Smith (2001)] where wealth itself appears in the utility function. Asset returns follow general discrete-time diffusion processes. The resulting Euler equations differ from those of the standard model in two ways:

- The Euler equation for the green asset includes an extra term capturing the extra marginal utility conferred by the warm glow of holding it.
- The stochastic discount factor now includes not only the growth of consumption but also the growth of the portfolio share invested in the green asset. Just as conventional investors practice consumption smoothing, so too do green investors practice *asset smoothing*.

We take the continuous-time limit to derive exact expressions for the excess returns of both the green and the non-green assets. The presence of asset smoothing introduces a new covariance term – between the return of the asset and the growth of the share invested in the green asset – in both excess returns. The warm glow also adds a negative "alpha" to the excess return on the green asset, which tends to make it lower *ceteris paribus* than the excess return to the non-green asset.

Section IV constructs an example that permits closed-form solutions for the optimal portfolio and consumption policies in the presence of warm-glow investment. Time is continuous, and the underlying stochastic processes are geometric Brownian motions. We first revisit Merton's (1969, 1971) classic partial equilibrium model of portfolio choice, but allow the investor to have warm-glow preferences. Warm-glow preferences alter both risk preferences and preferences for intertemporal substitution, so they affect the optimal consumption and portfolio policies in complicated ways. Provided that a transversality condition is satisfied, however, the demand for a risky asset always increases with the strength of warm-glow preferences for it. We then embed these policies into a general equilibrium asset pricing model, à la Lucas (1978). The price of a green stock always increases – again, given a transversality condition – as the strength of the taste for green increases; the excess return on the green asset always decreases with the taste for green.

In Section V we describe our data – taken from the list of Environmental, Social, and Governance (ESG) companies reported by Morgan Stanley Capital International (MSCI) over the period from 1998 to 2015 – and define what we mean by "green." We then establish two stylized facts:

- The proportion of wealth invested in green stocks has increased over the entire sample period, dramatically so since the financial crisis. We argue that this may be explained by methodological changes in the way the MSCI index has been calculated, as well as by an increase in the number of green firms.
- Over the entire period 1998–2015 green stocks consistently had lower returns than non-green stocks. This is true even controlling for size or value factors with Fama-French (1992) three-factor alphas. In other words, we find evidence supporting the hypothesis that green underperforms.

Warm-glow investing relates these two observations: *Ceteris paribus*, an exogenous increase in the supply of green stocks should lower their prices and raise their expected returns, causing them to outperform. The observed underperformance is explained by the growth of warm-glow preferences, which raises the demand for green stocks and lowers their expected returns.

Section VI estimates the discrete-time model using the Generalized Method of Moments (GMM). The results differ pre- and post-crisis. The estimates for the parameter governing the strength of preference for warm glow before the crisis are statistically significant but economically insignificant. After the crisis, however, the estimates are still statistically significant but much larger. Our estimates also provide evidence for an operative spirit of capitalism [Bakshi and Chen (1996b)]. This is true both before and after the crisis. Zhang (2006) had previously rejected the hypothesis of the spirit of capitalism for the period from 1959 to 2001.

Section VII offers some concluding thoughts and suggests other applications of the framework.

II. The literature

A. Does green underperform? The extant empirical literature

There is an extensive empirical literature investigating the financial performance of green firms – or, more broadly, firms that practice SRI – relative to conventional ones. One strand of the literature defends the proposition that green or SRI firms do worse than non-green ones. An early example is White (1995). He finds that environmental funds in the United States produced inferior results compared to the market and SRI funds proxied by the Domini Social Index (DSI), a widely used benchmark for most socially responsible funds (a socially responsible version of the S&P 500). However, the performance of German environmental funds was not significantly different from the overall German stock exchange. Brammer, Brooks, and Pavelin (2006) study the impact of SRI on firms in the UK. By disaggregating Corporate Social Responsibility (CSR) into environmental, community, and employment indicators, they find that while the first two are negatively related to returns, the last is positively related. Renneboog, Ter Horst, and Zhang (2008) find that SRI funds in several European, North American, and Asia-Pacific countries underperform their benchmark. For countries like Ireland, France, Sweden, and Japan, the alpha generated by SRI funds is lower than that of conventional funds by 4 to 7 percent, implying that investors are paying a price for being ethical. Climent and Soriano (2011) compare the performance of SRI, green, and conventional funds in the United States. They find that between 1987 and 2009 environmental funds performed poorly when compared to conventional funds with similar characteristics. However, from 2001 to 2009 the performance of green funds was not different from other SRI and conventional funds. Ibikunle and Steffen (2017) compare the financial performance of green, black, and conventional European mutual funds over the 1991–2014 period. They find that green mutual funds significantly underperform compared to their conventional counterparts. Like Climent and Soriano (2011) they find that the performance of green mutual funds improves in recent times. Finally, Bauer, Koedijk and Otten (2005) find that SRI funds deliver lower alphas compared to conventional funds in the early 1990s but afterwards catch up with them.

Another strand of the literature contests the hypothesis that green and SRI firms underperform. One of the earlier examples is Hamilton, Jo, and Statman (1993). They find that the performance of SRI mutual funds is not statistically different from that of conventional mutual funds. Statman (2000) compares the performance of the S&P 500 to the DSI between 1990 and 1998 and finds that the latter outperformed. However, the risk-adjusted returns of the two indices are comparable and the difference between them is not statistically significant. Moreover, they find no significant differences between the performance of conventional funds and socially responsible funds. Derwall, Guenster, Bauer, and Koedijk (2004) compare the performance of two portfolios that differed in environmental impact over the period 1995–2003 and find that the one with higher environmental scores outperforms the other. Bauer, Otten, and Tourani-Rad (2006) compare Australian SRI and conventional funds and find that the former has lower alphas over the period 1992–1996. However, these differences become insignificant during more recent periods. Bauer, Derwall, and Otten (2007) and Gregory and Whittaker (2007) find no significant differences between the alphas generated by SRI and conventional funds in Canada and the UK, respectively. Cai and He (2014) show that a long-run, environmentally responsible portfolio earned an annual alpha in excess of industry benchmarks. The excess returns persist for more than four years. Nofsinger and Varma (2014) find that socially responsible mutual funds outperform during periods of market crisis, compared to matching conventional mutual funds. The conventional funds perform better during non-crisis periods, but overall across all periods the difference in performance is not significant.

The studies on the performance of SRI, green, and conventional firms and portfolios therefore present no conclusive evidence. However, studies of their effects on the cost of capital, especially the cost of equity, provide a more concrete negative relationship for SRI and green firms. The cost of equity is usually calculated from analysts' expectations of future returns. Ghoul, Guedhami, Kwok, and Mishra (2011) examine the effect of CSR on the cost of equity capital for US firms and find that firms with higher CSR scores are associated with lower costs of equity. Chava (2014) finds that firms that have environmental concerns have a higher cost of both equity and debt. Ghoul, Guedhami, Kim, and Park (2018) extend the cost of capital study internationally for manufacturing firms across 30 countries and find that firms that rank higher on environmental responsibility have lower costs of equity capital.

B. Other related research

As mentioned in the Introduction, there is a small literature built on the premise that there may be psychological costs or benefits associated with investing in financial assets. Beal, Goyen, and Phillips (2005) suggest that ethical investing may yield psychic benefits. Fama and French (2007) argue that deviations from the efficient frontier may be explained by investors having tastes for assets. They specifically mention socially responsible investing as "an extreme form of tastes for assets as consumption goods that are unrelated to returns." They also consider other applications, such as deriving utility from holding the stock of one's employer [Cohen (2009)], the home bias puzzle [French and Poterba (1991), Karolyi and Stulz (2003)], and getting pleasure from investing in growth stocks or displeasure from investing in distressed stocks [Daniel and Titman (1997)]. Hong and Kacperczyk (2009) show that stocks in "sinful" business – such as guns, alcohol, tobacco, and gambling – outperform the market; they attribute this to the social stigma of investing in sinful activities, what might be called "cold-glow" preferences. Statman (2010) thinks that socially responsible investors may "sacrifice investment profits for human rights." Keloharju, Knüpfer, and Linnainmaa (2012) and Frieder and Subrahmanyam (2005) hypothesize that people develop preferences for the stocks of companies that produce the goods with which they are most familiar. These studies are empirical exercises; none of them develop any theory of how preferences for assets might affect asset prices. We will both develop this theory and test it in the context of green assets.

We have recently discovered two papers that also explore the theoretical implications of such preferences. Like us, these papers introduce stock holdings as an argument in the utility function; like us, they demonstrate that if investors feel good about themselves for buying a stock its price will increase and the rate of return will decrease. However, our models differ substantially and are designed for different purposes.

Luo and Subrahmanyam (2019) consider how stock-in-the-utility-function preferences affect the informational efficiency of the stock market; they also develop predictions about its cross-sectional implications across investors with different preferences and show that it may contribute to booms and busts. Pástor, Stambaugh, and Taylor (2021) develop an equilibrium model of the pricing of green stocks when firms differ in the sustainability of their behavior, while investors differ in their tastes for sustainability. Investors with stronger preferences for sustainability choose portfolios that are more "tilted" toward green assets.

Both papers focus on how heterogeneity of preferences affects the pricing of green assets. To incorporate heterogeneity into a general equilibrium setting, both use a version of the classic Grossman and Stiglitz (1980) model: there are two periods, normally distributed shocks, constant-absolute-risk-aversion (CARA) preferences, and perfect substitutability (linear utility) between consumption and stock holdings. These are classic, but highly restrictive assumptions. Both papers also develop potentially testable implications for what we call warm-glow preferences; Pástor, Stambaugh, and Taylor (2021) calibrate their model to get a sense of quantitative magnitudes. However, neither paper attempts to estimate its model empirically. Our model, on the other hand, is designed to explain the ways in which warm-glow investment affects the portfolio decision of a representative agent in a very general setting. We map out how it should affect the returns on green assets, derive an Euler equation that can be brought to the data, and estimate it. To this end we use an infinite-horizon model, with shocks that follow general diffusion processes, and with a more general class of preferences that allows richer interactions between consumption and stock holdings and that nests the canonical constant-relative-risk-aversion (CRRA) version of the Consumption-based Capital Asset Pricing Model (CCAPM) as a special case. It is only in this general, dynamic setting that asset smoothing reveals itself. In a nutshell, Luo and Subrahmanyam (2019) and Pástor, Stambaugh, and Taylor (2021) develop more restrictive theoretical models to explore interesting theoretical questions;

we develop a more general theoretical model that can be econometrically tested in the specific context of green assets.

We should emphasize that our concern here is with how investor preferences affect financial performance, not with how the behavior of firms affects their financial performance. There are two papers that do endogenize the behavior of firms in the presence of sustainable investment. In Heinkel, Kraus, and Zechner (2001) firms alter their investment when investors refuse to purchase the stock of polluting firms. In Albuquerque, Koskinen, and Zhang (2019) socially responsible behavior raises customer loyalty to a firm, and so raises its market power.

A note about terminology: Luo and Subrahmanyam (2019) call the presence of asset holdings in the utility function the *affect heuristic*. This applies the terminology of psychology [Finucane, Alhakami, Slovic, and Johnson (2000), p. 31], where an affect heuristic means that "images, marked by positive and negative affective feelings, guide judgement and decision making." While we recognize that *affect heuristic* may be a more clinically correct way of describing the behavior we are postulating, we will adhere to *warm glow* in this paper, on the grounds that it is more evocative and will be more familiar to economists.

Incorporating things other than consumption into the utility function, of course, is a long tradition in economics. This makes our specification of preferences a first cousin to those in several literatures:

- For example, Abel (1990), Campbell and Cochrane (1999), and Campbell and Kogan (2002), *inter alia*, model external habit formation by making aggregate consumption an argument in the utility function.
- Zou (1994), Bakshi and Chen (1996b), and Smith (2001), *inter alia*, incorporate wealth – individual or relative to aggregate – in the utility function as a measure of status.
- Sidrauski (1967) and Brock (1974), *inter alia*, studied long-run growth in monetary economies by adding real balances to the utility function. The presence of money in the utility function is designed to capture the value of the transactions services it provides.

- The Sidrauski (1967) setup is stylized since there is in fact a spectrum of monetary assets that provides transaction services. Barnett (1980) applied index theory and aggregation theory to devise an aggregate measure of such transaction services defined over a vector on monetary assets (different types of transaction accounts in different financial institutions); this aggregator function then appears in the utility function. A rich literature on monetary aggregation has flowed from his seminal paper. For example, Barnett and Liu (2019) incorporate both monetary assets and credit card transactions into a CCAPM with risk aversion and intertemporal non-separability.

Our work is also related to three other literatures. First, there is a single paper by Nelling and Webb (2009) that tries to disentangle the causal relationship between CSR and the financial performance of the firm. They find little support for CSR driving performance and weak support for financial performance driving CSR; they conclude that CSR is driven by unobservable firm characteristics. We will simply assume that "greenness" is an exogenous characteristic of a firm. Indeed, we go one step further: For our purposes it does not matter whether a firm is actually green; what matters is whether investors *perceive* it to be green. We take investor perceptions of the environmental behavior of firms to be exogenous.

Second, there is a small literature [D'Amato, Mancinelli, and Zoli (2014) and Abbot, Nandeibam, and O'Shea (2013)] that applies warm-glow preferences to problems in environmental policy. "Extrinsic" incentives (such as social norms) do not seem to be effective in inducing prosocial behavior such as recycling [Viscusi, Huber, and Bell (2011)]. Warm-glow preferences constitute a kind of "intrinsic" incentive that may work better. In this case the warm-glow preferences are defined directly over environmental outcomes; in our model they are defined over the assets of firms associated with desirable environmental outcomes.

Third, Dietz, Gollier, and Kessler (2018) have recently derived a "climate β," which they use to calculate a risk-adjusted rate of return for climate mitigation projects. Since they also use the CCAPM, they generate an expression that looks similar to the excess returns we derive.

However, our model differs from theirs in two ways. First, they do not incorporate warm-glow preferences. Second, theirs is a normative exercise in deriving the appropriate discount factor for climate change policies; ours is a positive exercise in explaining how warm-glow preferences affect asset prices.

III. The model

We now develop a theoretical model that explains the underperformance of green assets. Consider a consumer who can invest in a risk-free asset, a conventional "non-green" asset, and a "green" asset. For simplicity we ignore the possibility of a "black" asset. Denote the cum dividend prices[1] of the non-green and green assets at time t by $P_{n,t}$ and $P_{g,t}$, respectively. Following Grossman and Shiller (1982), Bakshi and Chen (1996a, 1996b), and Smith (2001), suppose that these prices follow a discrete-time vector diffusion:

$$\frac{\Delta P_{i,t}}{P_{i,t}} = r_{i,t}\Delta t + \sigma_{i,t}\Delta\omega_{i,t}, i = n, g. \quad (1)$$

$r_{i,t}$ and σ_{it} are the expectation and standard deviation of the rate of return to asset i; $\omega_{i,t}$ is a Wiener process. The rate of return to the risk-free asset is constant and equal to r_f. We will be taking the continuous-time limit as $\Delta t \to 0$.

Suppose that the shares of wealth invested in the non-green and green assets in time t are λ_{nt} and λ_{gt}. Given consumption C_t in period t the consumer's wealth W_t then evolves according to

$$W_{t+\Delta t} = (W_t - C_t\Delta t)\left[1 + \left(1 - \lambda_{n,t} - \lambda_{g,t}\right)r_f\Delta t + \lambda_{n,t}\frac{\Delta P_{n,t}}{P_{n,t}} + \lambda_{g,t}\frac{\Delta P_{g,t}}{P_{g,t}}\right]. \quad (2)$$

[1] Consider a discrete-time model where a stock pays out a dividend starting at t = 0. Cum dividend means that the dividend in t = 0 has already been paid out; on the contrary, ex dividend means that it has not. In the former case the discounted present value then starts at t = 0. In our continuous-time setting this amounts to starting the integral over the present value of dividends at zero.

The consumer has an infinite planning horizon, time-separable preferences, and a constant rate of time preference θ. In each period the consumer derives utility both from consumption and from their investment in the green asset. However, it is not immediately clear how to specify the consumer's warm-glow preferences: Do they care about the share of their wealth invested in the green asset $U(C_t, \lambda_{g,t})$, or are they concerned about the total amount they invest in it, $U(C_t, \lambda_{g,t} W_t)$? We call the former specification *proportional warm glow*, and the latter *total warm glow*. *A priori* we do not want to assume one or the other. Furthermore, the presence of wealth in total warm glow is suggestive of models of the spirit of capitalism [Bakshi and Chen (1996b) and Smith (2001)], where wealth appears independently in the utility function, $U(C_t, W_t)$. We therefore begin with a general specification $U(C_t, \lambda_{g,t}, W_t)$ that imposes no restrictions on the interaction of warm glow and wealth. We assume only that this period utility function is increasing and concave in all three arguments.

The investor's lifetime expected utility is thus

$$E \sum_{t=0}^{\infty} e^{-\theta \Delta t} U(C_t, \lambda_{g,t}, W_t) \Delta t. \quad (3)$$

For empirical estimation we will use the particular utility function

$$U(C_t, \lambda_{gt}, W_t) = \frac{\left(C_t^a \lambda_{g,t}^b W_t^d\right)^{1-\gamma}}{1-\gamma}, \quad (4)$$

where $a, b, d \in (0,1)$.[2] This subsumes four different models:

- setting $b = d = 0$ recovers the standard CCAPM model;
- setting $b > 0, d = 0$ yields proportional warm glow;
- $b = d > 0$ produces total warm glow;
- $b = 0, d > 0$ gives a "pure" spirit of capitalism.[3]

[2] When $\gamma = 1$ we assume $U(C_t, \lambda_{g,t}, W_t) = a \ln C_t + b \ln \lambda_{g,t} + d \ln W_t$.

[3] It is also possible to incorporate warm glow into recursive generalized iso-elastic (GIE) preferences [Duffie and Epstein (1992a, 1992b), Epstein and Zin (1989, 1991), Svensson (1989), *inter alia*]. This would permit warm-glow

The consumer chooses consumption and portfolio shares $\{C_t, \lambda_{n,t}, \lambda_{g,t}; t \in (0,\infty)\}$ to maximize Equation (4) subject to Equation (5), given initial wealth W_0. In Appendix A we solve this problem using methods similar to Grossman and Shiller (1982), Bakshi and Chen (1996a, 1996b), and Smith (2001). For now, we denote consumption and the portfolio shares of the two assets along the optimal path by C_t^*, $\lambda_{n,t}^*$, and $\lambda_{g,t}^*$, $t \in (0,\infty)$. To reduce clutter we define the rate of return on the market portfolio along the optimal path by

$$\frac{\Delta P_{m,t}^*}{P_{m,t}^*} = \left(1 - \lambda_{n,t}^* - \lambda_{g,t}^*\right) r_f \Delta t + \lambda_{n,t}^* \frac{\Delta P_{n,t}}{P_{n,t}} + \lambda_{g,t}^* \frac{\Delta P_{g,t}}{P_{g,t}} \,. \qquad (5)$$

A. Euler equations with warm glow

The first-order conditions for this problem then are:

$$U_C\left(C_t^*, \lambda_{g,t}^*, W_t\right) = e^{-\theta \Delta t} E\left[U_C\left(C_{t+\Delta t}, \lambda_{g,t+\Delta t}^*, W_{t+\Delta t}\right) + U_W\left(C_{t+\Delta t}, \lambda_{g,t+\Delta t}^*, W_{t+\Delta t}\right)\Delta t\right]\left(1 + \frac{\Delta P_{m,t}}{P_{m,t}}\right). \qquad (6)$$

$$E\left[U_C\left(C_{t+\Delta t}, \lambda_{g,t+\Delta t}^*, W_{t+\Delta t}\right) + U_W\left(C_{t+\Delta t}, \lambda_{g,t+\Delta t}^*, W_{t+\Delta t}\right)\Delta t\right]\left(\frac{\Delta P_{n,t}}{P_{n,t}} - r_f \Delta t\right) = 0 \qquad (7)$$

$$U_{\lambda_g}\left(C_t^*, \lambda_{g,t}^*, W_t\right) + e^{-\theta \Delta t}\left(W_t - C_t \Delta t\right) E\left[U_C\left(C_{t+\Delta t}, \lambda_{g,t+\Delta t}^*, W_{t+\Delta t}\right) + U_W\left(C_{t+\Delta t}, \lambda_{g,t+\Delta t}^*, W_{t+\Delta t}\right)\Delta t\right]\left(\frac{\Delta P_{g,t}}{P_{g,t}} - r_f \Delta t\right) = 0. \qquad (8)$$

For future reference note that for the preferences in Equation (4) these reduce to

$$1 = e^{-\theta \Delta t} E\left(\frac{C_{t+\Delta t}^*}{C_t^*}\right)^{a(1-\gamma)-1}\left(\frac{\lambda_{g,t+\Delta t}^*}{\lambda_{g,t}^*}\right)^{b(1-\gamma)}\left(\frac{W_{t+\Delta t}}{W_t}\right)^{d(1-\gamma)}\left[1 + \frac{d}{a}\frac{C_{t+1}^*}{W_{t+1}}\Delta t\right]\left(1 + \frac{\Delta P_{m,t}}{P_{m,t}}\right). \qquad (9)$$

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preferences to be disentangled from risk preferences and tastes for intertemporal substitution, along the lines of Smith (2001). However, the non-stationarity of the resulting Euler equations raises econometric issues that we will address in another paper.

$$E\left(\frac{C^*_{t+\Delta t}}{C^*_t}\right)^{a(1-\gamma)-1}\left(\frac{\lambda^*_{g,t+\Delta t}}{\lambda^*_{g,t}}\right)^{b(1-\gamma)}\left(\frac{W_{t+\Delta t}}{W_t}\right)^{d(1-\gamma)}\left[1+\frac{d}{a}\frac{C^*_{t+1}}{W_{t+1}}\Delta t\right]\left(\frac{\Delta P_{n,t}}{P_{n,t}}-r_f\Delta t\right)=0 \quad (10)$$

$$\frac{b}{a}\frac{C^*_t}{W_t-C^*_t\Delta t}\frac{1}{\lambda^*_{g,t}}+e^{-\theta\Delta t}E\left(\frac{C^*_{t+\Delta t}}{C^*_t}\right)^{a(1-\gamma)-1}\left(\frac{\lambda^*_{g,t+\Delta t}}{\lambda^*_{g,t}}\right)^{b(1-\gamma)}\left(\frac{W_{t+\Delta t}}{W_t}\right)^{d(1-\gamma)}\left[1+\frac{d}{a}\frac{C^*_{t+1}}{W_{t+1}}\Delta t\right]\left(\frac{\Delta P_{g,t}}{P_{g,t}}-\right.$$

$$\left.r_f\Delta t\right)=0. \quad (11)$$

When $b=0$ this system of equations reduces to the Euler equations in Bakshi and Chen's (1996b) model of the spirit of capitalism; when $b=0$ and $d=0$ they reduce to the canonical CCAPM. Our Euler equations differ from the norm in two important ways:

First, the Euler equation for the green asset in Equation (8) has an extra positive factor U_{λ_g}, reflecting the additional, psychic marginal benefit conferred by warm-glow investment.

Second, the stochastic discount factor in both Euler equations depends not only on the growth of consumption but also on the *growth of the demand for the green asset.* In other words, there is "asset smoothing" as well as "consumption smoothing."

B. Warm-glow and excess returns

Let us explore how these properties affect the pricing of non-green as well as green assets. Appendix B derives the following two propositions, dealing with the excess returns of non-green and green assets in the continuous-time limit, as $\Delta t \to 0$.

Proposition 1. In the continuous-time limit the excess return to the non-green asset is

$$r_{n,t}-r_f=-\frac{U_{CC}(C^*_t,\lambda^*_{g,t},W_t)C^*_t}{U_C(C^*_t,\lambda^*_{g,t},W_t)}cov\left(\frac{dC^*_t}{C^*_t},\frac{dP_{n,t}}{P_{n,t}}\right)-\frac{U_{C\lambda_g}(C^*_t,\lambda^*_{g,t},W_t)\lambda^*_{g,t}}{U_C(C^*_t,\lambda^*_{g,t},W_t)}cov\left(\frac{d\lambda^*_{g,t}}{\lambda^*_{g,t}},\frac{dP_{n,t}}{P_{n,t}}\right)-$$

$$\frac{U_{CW}(C^*_t,\lambda^*_{g,t},W_t)W_t}{U_C(C^*_t,\lambda^*_{g,t},W_t)}cov\left(\frac{dP_{n,t}}{P_{n,t}},\frac{dW_t}{W_t}\right). \quad (12)$$

To make this concrete, we will rewrite this using the utility function in Equation (5):

$$r_{n,t} - r_f = [1 - a(1-\gamma)]cov\left(\frac{dC_t^*}{C_t^*}, \frac{dP_{n,t}}{P_{n,t}}\right) - b(1-\gamma)cov\left(\frac{d\lambda_{g,t}^*}{\lambda_{g,t}^*}, \frac{dP_{n,t}}{P_{n,t}}\right) - d(1-\gamma)cov\left(\frac{dP_{n,t}}{P_{n,t}}, \frac{dW_t}{W_t}\right). \quad (13)$$

There are now three factors driving the excess return. First, as in the conventional CCAPM, the excess return to an asset depends upon the covariance of its rate of return with consumption growth, $cov(dC_t^*/C_t^*, dP_{n,t}/P_{n,t})$.

Second, the presence of wealth also generates the covariance of the return with the growth rate of wealth $cov(dW_t/W_t, dP_{n,t}/P_{n,t})$ just as it does in models of the spirit of capitalism [Bakshi and Chen (1996b), Smith (2001)]. Indeed, Equation (12) reduces *mutatis mutandis* to Equation (12) in Proposition 1 of Bakshi and Chen (1996b) when $b = 0$.

Finally, in the presence of warm-glow investment there is an extra covariance term between the rate of return of the non-green asset and the growth rate of the share of the portfolio invested in the *green* asset, $cov(d\lambda_{g,t}^*/\lambda_{g,t}^*, dP_{n,t}/P_{n,t})$. In other words, warm-glow investment induces *asset smoothing* in addition to consumption smoothing. This is a unique feature of warm-glow preferences. Notice that the correlation of the non-green return with the growth of the portfolio share of the *green* asset affects the excess return of the *non-green* asset. In other words, warm-glow investment has implications for the equity premium puzzle [Mehra and Prescott (1985), *inter alia*] on non-green assets.

Proposition 2. In the continuous-time limit as $\Delta t \to 0$ ***the excess return to the green asset is***

$$r_{g,t} - r_f = -\frac{U_{\lambda_g}(C_t^*, \lambda_{g,t}^*, W_t)}{U_C(C_t^*, \lambda_{g,t}^*)}\frac{1}{W_t} - \frac{U_{CC}(C_t^*, \lambda_{g,t}^*, W_t)C_t^*}{U_C(C_t^*, \lambda_{g,t}^*, W_t)}cov\left(\frac{dC_t^*}{C_t^*}, \frac{dP_{g,t}}{P_{g,t}}\right) - \frac{U_{C\lambda_g}(C_t^*, \lambda_{g,t}^*)\lambda_{g,t}^*}{U_C(C_t^*, \lambda_{g,t}^*)}cov\left(\frac{d\lambda_{g,t}^*}{\lambda_{g,t}^*}, \frac{dP_{g,t}}{P_{g,t}}\right) - \frac{U_{CW}(C_t^*, \lambda_{g,t}^*, W_t)W_t}{U_C(C_t^*, \lambda_{g,t}^*, W_t)}cov\left(\frac{dW_t}{W_t}, \frac{dP_{g,t}}{P_{g,t}}\right). \quad (14)$$

For our example, this reduces to

$$r_{g,t} - r_f = -\frac{b}{a}\frac{1}{\lambda^*_{g,t}}\frac{C^*_t}{W_t} + [1 - a(1-\gamma)]cov\left(\frac{dC^*_t}{C^*_t}, \frac{dP_{g,t}}{P_{g,t}}\right) - b(1-\gamma)cov\left(\frac{d\lambda^*_{g,t}}{\lambda^*_{g,t}}, \frac{dP_{g,t}}{P_{g,t}}\right) - d(1-\gamma)cov\left(\frac{dP_{g,t}}{P_{g,t}}, \frac{dW_t}{W_t}\right). \quad (15)$$

There are two effects at work here, both of which may help explain why green assets should have low excess returns: First, warm-glow investment induces a second covariance term, $cov\left(d\lambda^*_{g,t}/\lambda^*_{g,t}\, , dP_{g,t}/P_{g,t}\right)$ into the excess return. If the rate of return of the green asset covaries negatively with the growth rate of its own portfolio share, it would tend to reduce the excess return. We call this the "asset-smoothing" effect.

Most importantly, notice the negative "intercept" term, $-\frac{b}{a}\frac{1}{\lambda^*_{g,t}}\frac{C^*_t}{W_t}$ in Equation (15). This says that warm-glow investment will *always* tend to reduce the excess return to green assets. The intuition is simple: The preference for green assets raises the marginal benefit of investing in them. This raises demand for them, and so increases their prices. In turn this reduces their rates of return, even ignoring the asset-smoothing effect. Notice also that the strength of this effect is determined by the marginal rate of substitution between consumption and the warm-glow, U_{λ_g}/U_c; for our iso-elastic preferences the marginal rate of substitution is proportional to the ratio of consumption to the total investment in the green asset.

IV. An i.i.d. example

We now develop a simple i.i.d. (independent and identically distributed) example to help elucidate how warm-glow preferences affect investor behavior and asset prices. We assume that time is continuous and that the exogenous processes are geometric Brownian motions. For simplicity we also shut down the non-green asset entirely $\left(\lambda_{n,t} = 0\right)$, so that the investor can hold only either a green stock or the risk-free asset. These assumptions permit us to derive closed-form solutions for the optimal consumption and portfolio policies, as well as the equilibrium price of the green stock.

The model is a continuous-time version of a Lucas (1978) tree model, similar to that in Smith (2001).[4] The green stock pays dividends D_t that follow a geometric Brownian motion:

$$\frac{dD_t}{D_t} = \nu dt + \sigma d\omega_t, \tag{16}$$

where ω_t is a Wiener process. The constants ν and σ are the instantaneous drift and the standard deviation of the growth rate of dividends.

Now let P_t denote the ex-dividend price of the stock. Assume that in equilibrium it also follows a geometric Brownian motion

$$\frac{dP_t}{P_t} = \pi dt + \sigma_p d\omega_{p,t}, \tag{17}$$

where $\omega_{p,t}$ is Wiener and π and σ_p are the instantaneous drift and the standard deviation of capital gains. The cum-dividend rate of return is then

$$\frac{dP_t}{P_t} + \frac{D_t}{P_t} dt = \left(\pi + \frac{D_t}{P_t}\right) dt + \sigma_p d\omega_{p,t}. \tag{18}$$

In equilibrium the price:dividend ratio will be constant, so to simplify notation we define $r = \pi + D_t/P_t$.

Since the green stock is the only risky asset, we denote its portfolio share by λ_t. The investor's flow budget constraint is then

$$dW_t = \{[(1 - \lambda_t)r_f + \lambda_t r]W_t - C_t\}dt + \lambda_t \sigma_p d\omega_{p,t}. \tag{19}$$

The investor is endowed with the continuous-time version of the preferences described in Equations (3) and (4). To simplify the algebra, however, we restrict the preferences in two ways. First, we assume that the investor has "total warm-glow" preferences, where $b = d$; second, we

[4] It would be interesting to keep a second, "conventional" tree in the model. However, as Cochrane, Longstaff, and Santa-Clara (2008) show, dealing with two trees is so complicated, even with conventional preferences, that we set it aside in this paper.

impose the homogeneity restriction $a = 1 - b$. With these assumptions the period utility function becomes

$$U\left(C_t, \lambda_{gt}, W_t\right) = \frac{\left[C_t^{1-b}(\lambda_t W_t)^b\right]^{1-\gamma}}{1-\gamma}. \quad (20)$$

In addition to tractability, this specification has the added virtue of disentangling risk aversion from "ordinal" preferences in a multivariate utility function: The homogeneity of the aggregator inside the parentheses implies that, following Khilstrom and Mirman (1974, 1981), we can interpret γ as the coefficient of relative risk aversion, while the parameter b can be interpreted as governing the strength of preference for total warm-glow investment relative to consumption.[5]

Given these assumptions, the investor's expected lifetime utility is then

$$E \int_0^\infty e^{-\theta t} \frac{\left[C_t^{1-b}(\lambda_t W_t)^b\right]^{1-\gamma}}{1-\gamma} dt. \quad (21)$$

The investor chooses policies for C_t and λ_t to maximize Equation (21) subject to Equation (19), given initial wealth $W_0 > 0$.

A. Warm-glow investment and the optimal consumption and portfolio policies

First consider the partial equilibrium problem of characterizing the optimal consumption and portfolio policies. We show in Appendix C that the optimal portfolio policy is to set a constant share equal to

$$\lambda^*\left(b, r - r_f\right) = \frac{1}{2} \frac{r - r_f}{\left[\gamma + \frac{b}{2}(1-\gamma)\right]\sigma_p^2} + \frac{1}{2}\sqrt{\left\{\frac{r - r_f}{\left[\gamma + \frac{b}{2}(1-\gamma)\right]\sigma_p^2}\right\}^2 + 4b\,\frac{\theta - (1-\gamma) r_f}{\gamma + \frac{b}{2}(1-\gamma)}\frac{1}{\gamma\sigma_p^2}}. \quad (22)$$

[5] See Laffont (1989) for a concise discussion of this literature.

We write the portfolio function as an explicit function of $r - r_f$ since we will be interested in its comparative statics with respect to the excess return. Observe that in the absence of warm-glow preferences $(b = 0)$ this reduces to the standard solution of Merton (1969, 1971),

$$\lambda_M = \frac{r-r_f}{\gamma\sigma_p^2}. \tag{23}$$

The "M" stands for "Merton."

The associated consumption policy is

$$C_t^*(W_t, b) = \frac{1-b}{1-(1-b)(1-\gamma)}\left\{\theta - (1-\gamma)\left[r_f + \left(r - r_f\right)\lambda^*\left(b, r - r_f\right) - \gamma\lambda^*\left(b, r - r_f\right)^2 \frac{\sigma_p^2}{2}\right]\right\} W_t. \tag{24}$$

When $b = 0$ we recover the consumption policy in Merton (1969, 1971):

$$C_t^*(W_t, 0) = C_M(W_t) = \frac{\theta-(1-\gamma)\left[r_f+\frac{\left(r-r_f\right)^2}{2\gamma\sigma_p^2}\right]}{\gamma} W_t. \tag{25}$$

Since preferences are time separable, $C_t^*(W_t, b)/W_t > 0$ is both a feasibility condition and a transversality condition [Merton (1969, 1971), Smith (1996)]. Luckily, this condition reduces to something simple and intuitive in our model: For $b > 0$, $C_t^*(W_t, b)/W_t > 0$ if and only if

$$\theta - (1-\gamma)\left[r_f + \frac{\left(r-r_f\right)^2}{2\gamma\sigma_p^2}\right] > 0. \tag{26}$$

This is exactly the transversality condition from Merton (1969).

How do warm-glow preferences affect the demand for the green asset? The answer is complicated because there are both direct and indirect effects: An increase in b has the direct effect of raising the marginal benefit of holding the green asset, so, *ceteris paribus*, λ increases. The change in the portfolio share then has the indirect effect of changing consumption. This feeds back to change the marginal benefit of the warm-glow investment. How λ will change is not immediately obvious. Nonetheless, it turns out that, provided the transversality condition is satisfied, the net effect is unambiguous. Appendix D derives the following theorem.

Proposition 3. The share invested in the green asset λ^* has the following properties. If the transversality condition is satisfied, then for all $b > 0$:

- ***a.*** $\lambda^* > \lambda_m$
- ***b.*** $\partial\lambda^*/\partial b > 0$.

The first part of the proposition establishes that the portfolio demand with warm glow is always strictly greater than in the canonical model. The second part says that the portfolio demand is monotonically increasing in the strength of warm glow.

Finally, we consider one more result that distinguishes the warm-glow portfolio demand from those in the canonical model: It is *nonlinear* in the excess return.

Proposition 4. If the transversality condition is satisfied, then

- **a.** $\partial\lambda^*/\partial(r - r_f) > 0$
- **b.** $\partial^2\lambda^*/\partial(r - r_f)^2 > 0$.

In the standard model, the portfolio demand is a linear function of the excess return [see Equation (22)]**.** With warm-glow preferences, however, the portfolio demand is an increasing, convex function of the excess return. Kraft and Weiss (2019) generate a similar nonlinear effect by allowing preferences to depend upon real balances, as in Sidrauski (1967). There, as here, the optimal portfolio is a solution to a polynomial equation; in the standard model it is the solution to a linear equation.

B. Warm-glow investment and stock prices

Now we embed this consumer in a general equilibrium setting, à la Lucas (1978). There is a representative consumer with the preferences and budget constraint described above. The green stock is supplied inelastically, with the supply normalized to one share. Its dividends evolve according to the geometric Brownian motion in Equation (16). The formal definition of an equilibrium in this economy is as follows:

Definition: An equilibrium consists of an asset-pricing function $\boldsymbol{P_t = F(D_t)}$ for $\boldsymbol{t \in [0, \infty)}$ and a risk-free rate $\boldsymbol{r_f}$ such that

- ***a. the consumer implements optimal portfolio and consumption policies given by Equations (22) and (24) given $\boldsymbol{F(D_t)}$ and $\boldsymbol{r_f}$;***
- ***b. in each period all dividends are consumed, $\boldsymbol{C_t^*(W_t, b) = D_t}$;***
- ***c. the risk-free asset is in zero net supply, so that $\boldsymbol{\lambda_t = 1}$.***

In Appendix E we show that the pricing function satisfies the following second-order differential equation:

$$\frac{D_t}{F(D_t)} = (1-b)\left[\frac{\theta}{1-\gamma} - \frac{F^{/}(D_t)D_t}{F(D_t)}\nu + \frac{F^{//}(D_t)D_t^2}{F(D_t)}\frac{\sigma^2}{2} + \gamma\left(\frac{F^{/}(D_t)D_t}{F(D_t)}\right)^2\frac{\sigma^2}{2}\right]. \quad (27)$$

The solution to this differential equation is[6]

$$F(D_t) = \frac{D_t}{(1-b)\left[\theta-(1-\gamma)\left(\nu-\frac{\sigma^2}{2}\right)\right]}. \quad (28)$$

The term in brackets is just the expression on the left-hand side of Equation (26) evaluated in equilibrium. Therefore, the transversality condition guarantees a positive stock price.

Two notes on interpretation: First, in the absence of warm glow $(b = 0)$ this reduces to the standard solution to the Lucas (1978) model in continuous time.[7] Second, the expression $\nu - \sigma^2/2$ is the growth rate of the certainty-equivalent dividend process. Following the reasoning in Smith (2001), it can be shown that $F(D_t)$ is the present value of certainty-equivalent dividends.[8]

[6] We ignore bubble solutions.

[7] See Equation (21) in Smith (2001), *mutatis mutandis*.

[8] This is a variation on analogous results by Rubinstein (1976), Lucas (1978), Huang (1987), Duffie and Zame (1989), and Duffie and Epstein (1992a).

An immediate implication of Equation (28) is

Proposition 5. The price of the stock is increasing and convex in $\boldsymbol{b}$, $\boldsymbol{dF/db > 0}$, $\boldsymbol{d^2F/db^2 > 0}$.

Intuitively, warm-glow preferences raise the demand for the green asset, so its price increases with the strength of green preferences at an increasing rate.

This has immediate implications for the excess return in equilibrium:

$$r - r_f = \gamma\sigma^2 - b\left[\theta - (1-\gamma)\left(\nu - \frac{\sigma^2}{2}\right)\right]. \quad (29)$$

Without warm glow the excess return reduces to the familiar $\gamma\sigma^2$. Thus we have

Proposition 6. Warm-glow preferences reduce the excess return to the green asset.

Intuitively, the psychic benefits of going green raise the price of the green asset, which must reduce its rate of return. However, since the preferences over warm glow and consumption are non-separable, the magnitude of this effect depends upon how much the investor is consuming.

V. The data

In this section we introduce a data set to address the question of how green assets perform relative to non-green assets. We use it to document the demand for green stocks and their performance relative to non-green stocks over the last two decades.

A. Data

We collect monthly data from April 1998 to September 2015. The source for the macroeconomic data is the Federal Reserve (Fed). Stock returns and market capitalization data are collected from the Center for Research in Security Prices (CRSP), and the environmental profiles of the companies, which we use to classify them into green or non-green, come from the MSCI's ESG database (previously known as the KLD database).

We use the quarterly non-seasonally adjusted Consumer Price Index (CPI) series from the Fed (Consumer Price Index: Total All Items for the United States, Growth Rate Previous Period, CPALTT01USQ657N) to transform our nominal data into real terms. To calculate per capita variables, we use the non-seasonally adjusted monthly series of Total Population: All Ages including Armed Forces Overseas (code: POP). Population in the first month of each quarter is used to proxy the quarterly population. Our measure of consumption also comes from the Fed on consumption of non-durables, services, and clothing and shoes. This annual data series is reported on a quarterly frequency. We follow the literature [among others Dreyer, Schneider, and Smith (2013)] and adjust our consumption series by subtracting the consumption of clothing and shoes from non-durable consumption and adding services. We then divide by population to convert our series into per capita terms and multiply it by the inflation discount factor to convert it to real terms. We modify household wealth, measured as non-seasonally adjusted Households and Nonprofit Organizations Net Worth from the Fed (code: HNONWRQ027S), in a similar manner. We divide household wealth by the population to put the data in per capita terms and multiply it by the inflation discount factor to calculate real per capita household wealth.

Considering Mehra's (2008) critique of the correct proxy for the risk-free asset, we follow Dreyer, Schneider, and Smith (2013) to use the rates of return of long duration T-Bills as the risk-free rate of return. Thus, we collect quarterly yields T-Bills of 30-year maturity. We get this data from the Fed's Table H.15, Selected Interest Rates, which provides us with the daily market yield on US Treasury securities for different maturities in annual terms. We then calculate the quarterly yield as the average of the daily yield during the quarter and adjust it for inflation to get the real risk-free rate of return.

B. Classifying stocks as green

MSCI's KLD ESG database is widely used by both the industry and academia. The database provides firm-level information on ESG profiles for a large sample of firms spanning a longer time frame than most other data sources. The KLD database is particularly suited for our study – where we are trying to infer whether a firm is green in the eyes of investors – because it is widely accepted in the institutional money-manager community. The KLD Social Index has been the de facto benchmark for SRI investments and money managers in the past two decades. Thus, investors worldwide use KLD data to incorporate ESG factors into their investment decisions.

The ESG database analyzes the quality of the environmental attributes of each company by breaking them down into different individual categories of strengths and concerns (weaknesses). Categories of environmental strengths include: environmental opportunities, waste management, packaging materials and waste, climate change, environmental management systems, water stress, biodiversity and land use, raw material sourcing, and other strengths. On the other hand, categories of environmental concerns include: regulatory compliance, toxic spills and releases, climate change, impact of products and services, biodiversity and land use, operational waste, supply chain management, water management, and other concerns. For each individual strength and concern, a company receives from MSCI the score of one when the characteristic is observed and zero when not.

Following Ghoul, Guedhami, Kwok, and Mishra (2011) who use the MSCI KLD ESG database to calculate a net strength measure for CSR, and Chava (2014) who calculates a net concerns measure focusing solely on the environmental information in the same database, we calculate a net strengths measure from the environmental profile of a company. It is the difference between the total number of environmental strengths and the total number of environmental concerns. A company is classified as green if the net strength number is positive and as non-green otherwise.

We collect the quarterly returns and market capitalization for all US stocks from CRSP. To calculate the returns of our green portfolio, we consider all stocks that classify as green and calculate market capitalization weighted returns. We then adjust this nominal return of the green portfolio by the inflation factor and arrive at real returns for the green portfolio. We calculate the real return on the non-green portfolio in an analogous way by calculating the market capitalization weighted returns of all non-green stocks and adjusting them for inflation. An important variable for both our theory and our empirical estimations is the share of wealth invested in green assets, lambda $(\lambda_{g,t})$. It is the per capita market capitalization of green stocks divided by wealth per capita. It is essentially the average percentage of household wealth invested in green stocks.

C. Data summary and limitations

One of the challenges of working with the MSCI ESG database is that its coverage changes three times during the duration of our study. The coverage increases from 650 firms to 1100 firms in 2001 and again to 2400 firms in 2003. Differences in the number of firms covered would have a direct effect on the total market capitalization of green stocks and consequently on the size of our lambdas. Thus, we decided to consider static lists of green and non-green shares from 1998 to 2003 based on the last year's MSCI scores. After 2003 the lists are dynamic, since the coverage remains stable after that.

Another problem of the MSCI database is that its methodology changes from time to time, when it adds and removes green indicators. According to MSCI, its methodology is reviewed every November to incorporate "emerging issues" and remove or reweight those that became "less significant" in each ESG category. This explains why some categories of strengths and concerns are only available in later periods, while others disappear. There was a major change in MSCI's methodology in 2010, when it introduced the industry-based ratings model which impacts the number of companies classified as green or non-green. As a consequence of these changes, there could be a mechanical increase in the number of firms that are classified as green.

The MSCI KLD rating has been discontinued in 2016, such that it is not possible to update our data. Since then, MSCI only continues to publish the MSCI IVA ratings where companies are rated in a range from AAA to CCC, instead of the traditional strengths and concerns method. Since the methodologies are not equivalent, it is impossible for us to compare post- and pre-2016 MSCI ratings.

As a response to these data limitations, we looked at other data sources that provide environmental indicators for US stocks. We considered the Thomson Reuters database and Bloomberg. The Bloomberg data coverage only begins in 2005 and limits our ability to run estimations. The Thomson Reuters database begins in 2002 and has limited coverage, especially for the environmental pillar score in all years prior to 2010. For example, there are only 12 firms that get classified as green (environment A rating) in 2002.

In summary, on the one hand, MSCI KLD provides us with a longer time period and a more complete data set, although it stops in 2015. On the other hand, Reuters would allow us to update the data up to 2020, but suffers from serious data limitations prior to 2010. These challenges led us to stick with the MSCI KLD ESG database, despite its shortcomings.

The left side of Figure 1 shows the evolution of firms that are classified as green and non-green over time. The number of green companies increases from 126 to 498, while the number of non-green companies falls from 2001 to 1602. The two lines in the first figure partly mirror each other and suggest that companies migrate from the classification of non-green to green. Moreover, we can notice a significant increase in the number of green companies (and a decline in non-green companies) in 2010. The right side of the figure shows the average strengths, concerns, and net strengths. Notice that average concerns decline over time, while average strengths and average net strengths increase. Again, the strongest movement happens in 2010, exactly when a significant change in methodology happens in MSCI.

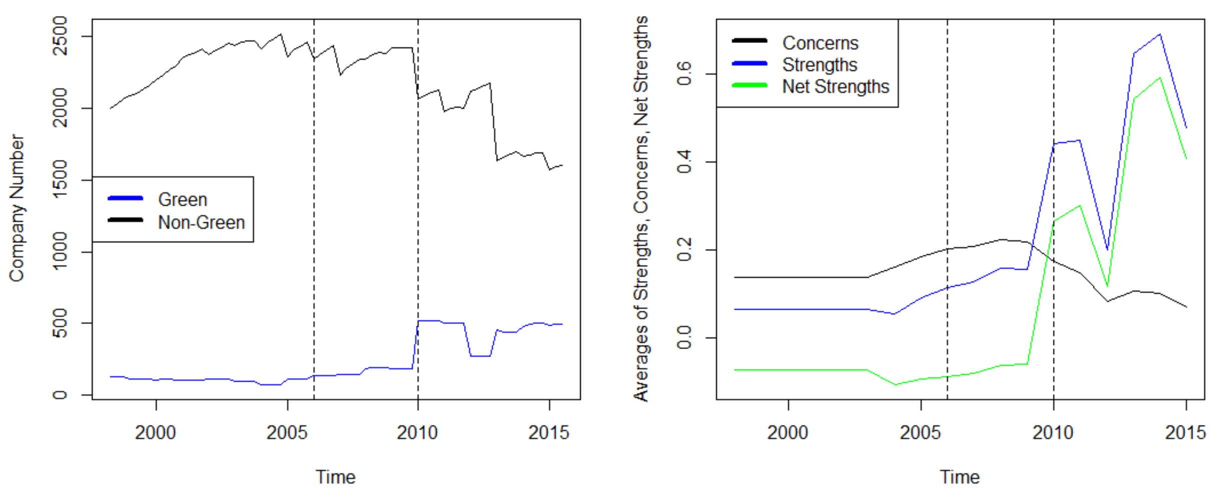

Figure 1: Green and non-green sample

Next consider Figure 2, which plots lambda $(\lambda_{g,t})$, the average percentage of wealth invested in green stocks over time. The vertical dashed lines mark the periods before, during, and after the financial crisis ("during" means the years 2007 to 2010). Notice that in the post-crisis period there is a significant increase in the percentage of the individual's wealth invested in green stocks; it stays high until the end of the sample period.

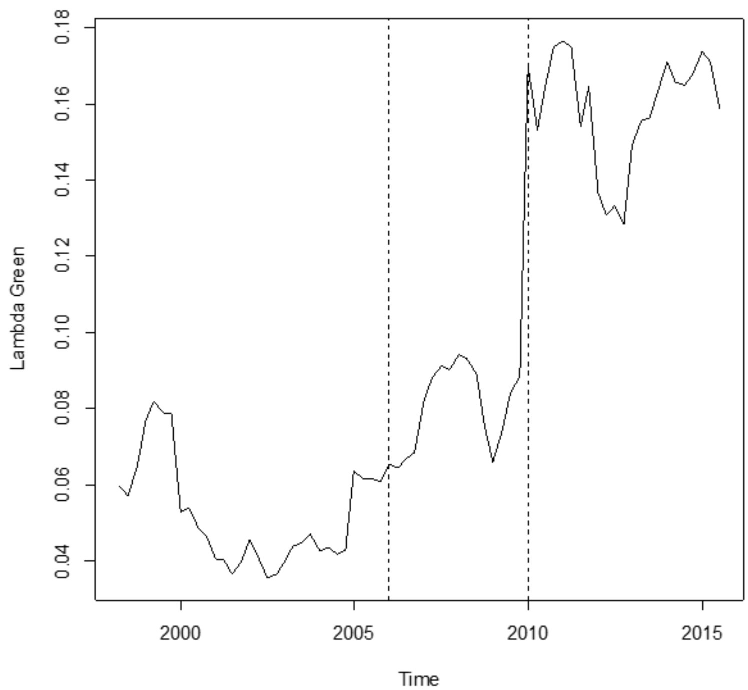

Figure 2: Lambda green along time (average percentage of wealth invested in green stocks)

Notice that lambda increases over time from 0.059 to 0.158, a change of 0.099 along the entire period. Besides, there is a drastic change in lambda from 2009 to 2010, from 0.088 to 0.170, a change of 0.082. One might be tempted to infer that the supply of green companies is increasing as a consequence of companies going green. However, as discussed above, the evidence suggests that at least a big part of the increase in lambda is associated with mechanical changes in methodology, which may have nothing to do with investors' preferences. This is especially true for 2010. This leads us to question the robustness of our measurement for lambda and our results.

We argue that these problems of the database do not invalidate our study for two different reasons:

- Our assumption is that investors take the environmental ratings of MSCI seriously. What matters for warm-glow investment is the *perception* of greenness, which we assume is related to the environmental ratings reported by MSCI year after year; they act as a "green label," especially for institutional and retail investors that base their investment decisions on such indicators. Thus, a change in either strengths or concerns for a company in the database will also translate into a change in investors' perception of the greenness of this company. If the investors use environmental indicators from MSCI to take their investment decisions, they will find that the sample of green firms increases, just as it does in our findings.
- As shown in Figures 1 and 2, one of the consequences of the changes in methodology is that the *supply* of green stocks has been increasing over time. Thus, it should be natural to expect a negative impact on prices of green stocks, which should increase their expected returns: C*eteris paribus*, an increase in the supply of green firms should cause our green portfolio to *outperform*. As result, a methodological bias in the calculation of the index should play against the warm-glow theory because an increase in lambda would be associated with even higher relative green returns. Let's see if this is true empirically.

Consider Figure 3, which plots the real returns on both green and non-green portfolios over time in annual terms. Again, the vertical dashed lines mark the periods before, during, and after the crisis. Notice that returns on the non-green portfolio are generally higher than those on the green portfolio, lending support to the hypothesis that the increase in supply of green stocks over time is not associated with higher green returns. This indicates that the green preference is strong enough to shift the *demand* curve enough to offset the increase in green supply.

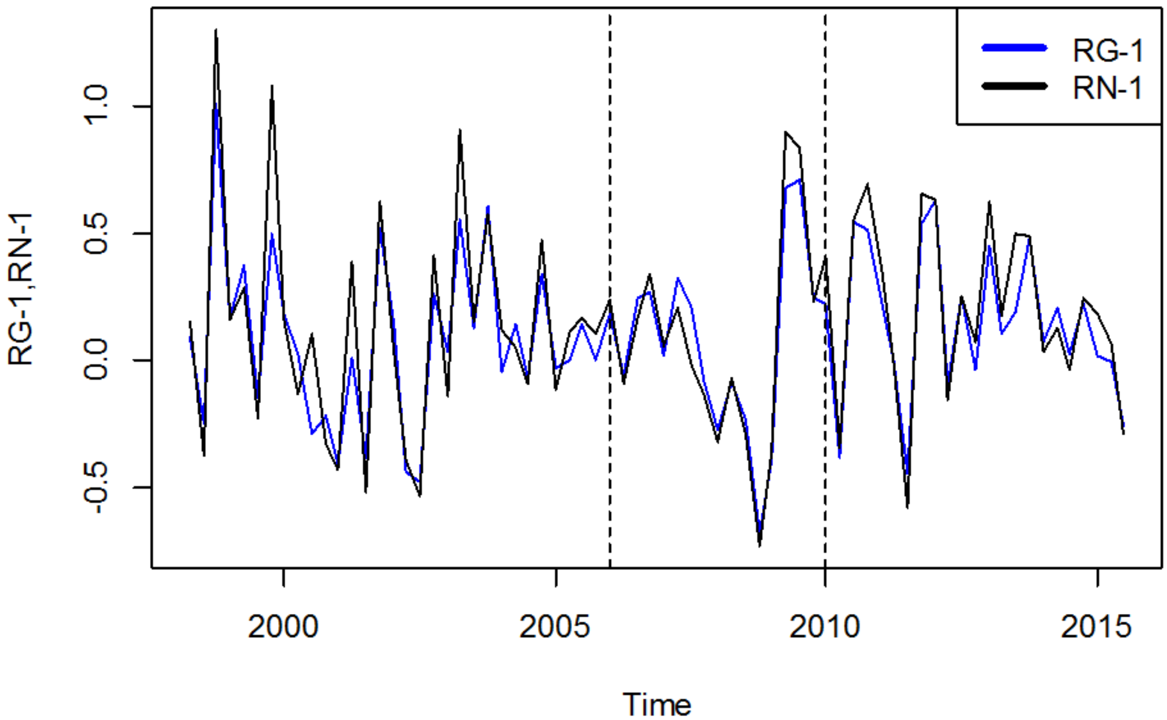

Figure 3: Quarterly green and non-green returns (in annual terms)

In Table 1 we calculate the mean returns, their standard deviations, the inverse of the coefficient of variation, and the Fama-French three-factor alphas for the green and non-green portfolios in the different periods analyzed. Notice that, no matter the period, green on average pays less, although with a lower level of risk. Using the inverse of the coefficient of variation to compare the relations between returns and risk confirms that the green portfolio generally pays less even after accounting for total risk. We also calculate the Fama-French three-factor alphas for both portfolios and find that the green portfolio underperforms the non-green portfolio for the various sub-sample periods. This shows that the underperformance is not driven by size or value factors.

Table 1: Mean returns and total risk (in annualized terms)

Period	Portfolio	Mean return (%)	Standard deviation (%)	Mean return / standard deviation (%)	FF 3-factor alpha
Total	Green	10.11	32.81	30.81	4.03
	Non-green	13.91	40.40	42.41	6.54
Pre-crisis	Green	8.98	32.13	27.95	2.90
	Non-green	13.57	41.93	42.26	6.65
Post-crisis	Green	15.16	29.35	51.68	-5.28
	Non-green	20.29	34.31	69.13	-2.51

Thus, although we witness a sharp increase in the share of green stocks since the crisis, the returns on green stocks remain consistently lower than those on non-green stocks, across all three periods. This leads us to conclude that the preference for investing in green had to be strong enough to allow a shift in the demand of green stocks of a magnitude that could compensate for the higher green supply, allowing higher lambdas to be associated with lower relative green returns as observed in our data.

In summary, the possible bias in lambda – because of an increase in the number of green, and a decrease in non-green, companies due to changes in methodology – should play against the warm-glow theory and make its empirical verification less likely in this study. If investors accept the green ratings, and are indifferent to, or unaware of, the methodological changes, then there has been an exogenous, artificial increase in green supply. We are not arguing that warm-glow preferences explain increase in lambda; on the contrary, we are arguing that they explain the underperformance of green in equilibrium despite the increase in its supply. An increase in the taste for green generates an increase in green demand that offsets the increased supply of green, thereby reducing the relative rate of green return. Thus, the increase in green supply together with lower green returns observed in our data only strengthens the argument in favor of warm-glow preferences.

VI. Empirical estimation

To implement the model empirically we rewrite the discrete-time Euler equations. Let $R_{n,t+1}$, $R_{g,t+1}$, and $R_{f,t}$ denote the returns to the non-green, green, and risk-free assets, respectively. We will also need the market return, $R_{m,t+1} = (1 - \lambda_{n,t} - \lambda_{g,t})R_{f,t+1} + \lambda_{n,t}R_{n,t+1} + \lambda_{g,t}R_{g,t+1}$. We define Z_t as the information set conditional upon information at time t. The risk-free rate is now permitted to be time-varying.[9] Now evaluate Euler Equations (9), (10), and (11) at $\Delta t = 1$, and define $\beta = e^{-\theta}$. In Appendix F we show that the resulting system of equations has the alternative representation.

$$E\left\{\beta E\left(\frac{C_{t+1}}{C_t}\right)^{a(1-\gamma)-1}\left(\frac{\lambda_{g,t+1}}{\lambda_{g,t}}\right)^{b(1-\gamma)}\left(\frac{W_{t+1}}{W_t}\right)^{d(1-\gamma)}\left[1+\frac{d}{a}\frac{C_{t+1}}{W_{t+1}}\right]R_{f,t+1}\middle|Z_t\right\} = 1+\frac{b}{a}\frac{C_t}{W_t-C_t}. \quad (30)$$

$$E\left\{\beta E\left(\frac{C_{t+1}}{C_t}\right)^{a(1-\gamma)-1}\left(\frac{\lambda_{g,t+1}}{\lambda_{g,t}}\right)^{b(1-\gamma)}\left(\frac{W_{t+1}}{W_t}\right)^{d(1-\gamma)}\left[1+\frac{d}{a}\frac{C_{t+1}}{W_{t+1}}\right]R_{n,t+1}\middle|Z_t\right\} = 1+\frac{b}{a}\frac{C_t}{W_t-C_t} \quad (31)$$

$$E\left\{\beta E\left(\frac{C_{t+1}}{C_t}\right)^{a(1-\gamma)-1}\left(\frac{\lambda_{g,t+1}}{\lambda_{g,t}}\right)^{b(1-\gamma)}\left(\frac{W_{t+1}}{W_t}\right)^{d(1-\gamma)}\left[1+\frac{d}{a}\frac{C_{t+1}}{W_{t+1}}\right]R_{g,t+1}\middle|Z_t\right\} = 1-\frac{1-\lambda_{g,t}}{\lambda_{g,t}}\frac{b}{a}\frac{C_t}{W_t-C_t}. \quad (32)$$

Notice that these reduce to the familiar Euler equations when b = 0. We will estimate this system of equations using the GMM.

[9] Our theoretical model employs mathematical methods used by Bakshi and Chen (1996b). Similarly, the specification of the Euler equations in Equations (30)–(32) used to implement our model empirically is similar to the discrete-time specification they use to test their model.

Since this system is nonlinear, we decided to create boundaries for our parameter estimations following Dreyer, Schneider, and Smith (2013). The definition of these boundaries will help us find the local minimum that is economically relevant to us, which may not necessarily equal the global minimum of the GMM algorithm. A global minimum with only one single negative parameter would not make sense to us because it would indicate a negative taste for green or wealth or even a negative risk aversion parameter gamma (a risk-loving investor). Thus, we define the restrictions for our parameters as follows:

$$b = [0; \quad 100]$$

$$d = [0; \quad 100]$$

$$\gamma = [0; \quad 100]$$

For our estimations we normalize the taste for consumption to one, so that $a = 1$. This is the convention in other models of asset pricing with multivariate utility functions [for example, Bakshi and Chen (1996b), Zhang (2006), and Smoluk and Voyer (2014) for the spirit of capitalism; Uhlig (2007) for multiplicative habit formation; and Dreyer, Schneider, and Smith (2013) for saving-based preferences]. We adhere to common practice by fixing β. There is some variation about what value to pick, however. Some have set it at $\beta = .95$ [Collard, Fève, and Ghattassi (2006), Malloy, Mosokowitz, and Vissing-Jørgensen (2009), Rangvid, Schmeling, and Schrimpf (2010), Savov (2011), and Kolve (2013)]. Others advocate larger values, from $\beta = .95$ or $\beta = .96$ to $\beta = .99$ [Zhang (1997)], or even larger [Hansen, Heaton, and Li (2008), Bansal, Kiku, and Yaron (2012), and Andries (2013)]. We opt to use $\beta = .95$.

To estimate Equations (30)–(32) using GMM, we need to define the instrumental vector to be used. We initially postulated the following candidate variables that could be used as instruments:

$$X_t = \left[k, \frac{c_t}{c_{t-1}}, \frac{c_{t-1}}{c_{t-2}}, R_{g,t}, R_{g,t-1}, R_{n,t}, R_{n,t-1}, \frac{\lambda_t}{\lambda_{t-1}}, \frac{\lambda_{t-1}}{\lambda_{t-2}}, \frac{w_t}{w_{t-1}}, \frac{w_{t-1}}{w_{t-2}}, R_{f,t}, R_{f,t-1} \right]$$

The instrumental vector X_t includes a constant k and up to two lags for all the other variables listed. We followed the same procedure as Dreyer, Schneider, and Smith (2013) to construct the different alternatives of instrumental vectors, starting with a minimum of one constant and four lags of variables. We then increased the number of these variables slowly so that we respected the identification condition of the GMM.

As discussed by Dreyer, Schneider, and Smith (2013), many instruments do not necessarily lead to good estimations. If a GMM estimation is rejected, it is important to identify the cause of this rejection. If we start with many variables, it could be hard to identify the one causing the rejection problem. Besides, it is also important to select instruments so that the identification condition is respected, and estimations are improved.

Thus, we start with only one constant k and four variables: the first lags of the risk-free rate and green returns and the second lags of the non-green returns and wealth growth. We follow by adding variables to the vector of instruments in a systematic way: If the estimation quality increases when adding a variable, this variable is used again in other vectors of instruments. On the other hand, if the addition of this variable increases the correlation between the vector of instruments and the GMM residuals, causing GMM to violate its identification conditions (rejection of the J-test), or if we find corner results for our parameters, the variable is then removed. Since our Euler equations are nonlinear, we also chose to use a Parzen kernel according to Smith (2004).

Before running any GMM estimation on Equations (30)–(32) we first tested our data series for stationarity. We ran augmented Dickey–Fuller (ADF) tests according to the procedure of Dolado, Jenkinson, and Sosvilla-Rivera (1990) by starting with the most unrestricted model, which includes drift, time trend, and five lags. Panel A of Table 2 shows that the null hypothesis of the unit root is rejected at the 10 percent significance level in all cases except for two variables: 1) the relationship of consumption with the difference between wealth and consumption, and 2) wealth growth. Given the economic meaning of these variables and their visual characteristics, we follow the advice of Dolado, Jenkinson, and Sosvilla-Rivera (1990) and test for the existence of a trend using linear regressions on the two variables.

In the case of no trend, Dolado, Jenkinson, and Sosvilla-Rivera (1990) advocate repeating a more restrictive ADF test where no time trend is imposed, which will consequently raise the power of the test. As the regression results in this case indicated that no time trend could be verified in these two variables,[10] we therefore repeated the ADF tests on them, but now without accounting for trend. Under these conditions, the hypothesis of a unit root was rejected also for these variables (see Panel B of Table 2).

Table 2: Unit root test on model variables (five lags)

	ADF test
Panel A: Drift and trend	
$\frac{c_t}{w_t - c_t}$	-2.53
$\frac{w_{t+1}}{w_t}$	-2.80
$\frac{c_{t+1}}{c_t}$	-3.32*
$R_{g,t+1}$	-3.75**
$R_{n,t+1}$	-3.30*
$R_{m,t+1}$	-3.42*
$R_{f,t+1}$	-4.49***
$\frac{\lambda_{t+1}}{\lambda_t}$	-3.71**
Panel B: Drift and no trend	
$\frac{c_t}{w_t - c_t}$	-2.63*
$\frac{w_{t+1}}{w_t}$	-2.83*

Note: ***, ** and * indicate statistical significance at 1%, 5% and 10% levels, respectively.

[10] See Appendix G for regression results on the time series of these variables and the lack of significance of time trends.

We initially planned to implement our estimations using the entire data sample. Our data ranges from 1998 to 2015 and we use quarter frequency. Thus, the period of the financial crisis constitutes a considerable part of our data sample (around 13 out of 71 quarters). However, this was an unusual time: During this crisis period the equity premium was negative, so investors would have had to have been risk lovers to accept holding stocks. That could imply low or even negative risk aversion (γ) during these quarters, which could bias our results. Because of this problem and to further check the robustness of our estimates, we also decided to split the data sample into two different periods: before and after the financial crisis of 2007.

A. Using the entire data period

We run estimations of Equations (30)–(32) for the entire data sample using the following alternatives for vector of instruments enumerated in Table 3:

Table 3: Instruments used for entire period estimations

Instrument number	Instrument vector
1.1	$X_t = \left[k, R_{g,t}, R_{n,t-1}, R_{f,t}, \frac{W_{t-1}}{W_{t-2}}\right]$
1.2	$X_t = \left[k, R_{g,t}, R_{g,t-1}, R_{c,t-1}, \frac{W_t}{W_{t-1}}\right]$
1.3	$X_t = \left[k, \frac{C_t}{C_{t-1}}, \frac{C_{t-1}}{C_{t-2}}, R_{g,t-1}, R_{n,t-1}, R_{f,t-1}\right]$
1.4	$X_t = \left[k, \frac{C_{t-1}}{C_{t-2}}, R_{g,t-1}, R_{n,t-1}, R_{f,t}, \frac{\lambda_t}{\lambda_{t-1}}, \frac{W_{t-1}}{W_{t-2}}\right]$
1.5	$X_t = \left[k, \frac{C_t}{C_{t-1}}, \frac{C_{t-1}}{C_{t-2}}, R_{g,t-1}, R_{n,t-1}, R_{f,t-1}, \frac{\lambda_{t-1}}{\lambda_{t-2}}\right]$
1.6	$X_t = \left[k, \frac{C_t}{C_{t-1}}, \frac{C_{t-1}}{C_{t-2}}, R_{g,t-1}, R_{n,t-1}, R_{f,t-1}, \frac{\lambda_t}{\lambda_{t-1}}, \frac{W_{t-1}}{W_{t-2}}\right]$

Table 4 offers the GMM estimation results for the entire period. As explained earlier, none of the instruments selected were rejected by the test of over-identifying the restrictions of the GMM (J-test) at the 5 percent significance level (except for the initial alternative 1.1). If the identification condition of the GMM were not satisfied, we would not be able to trust the estimation results associated with these alternatives.

Table 4: GMM estimation results for the entire period (starting point 1, 1, 1)

Instrument number	γ	b	d	J-test
	Estimate	Estimate	Estimate	Qui sq.
1.1	1.2303*** (0.3045)	0.0046 (0.0053)	0.0742*** (0.0149)	21.397**
1.2	0.9845*** (0.2612)	0.0061 (0.0045)	0.0972*** (0.0131)	15.499
1.3	0.9689*** (0.2209)	0.0065 (0.0040)	0.0616*** (0.0119)	23.996*
1.4	1.0125*** (0.1470)	0.0032 (0.0044)	0.0584*** (0.0082)	27.294*
1.5	1.3144*** (0.1384)	0.0052 (0.0045)	0.0687*** (0.0079)	23.193
1.6	1.2892*** (0.1289)	0.0053 (0.0046)	0.0652*** (0.0079)	24.844
Averages	1.1333	0.0052	0.0709	

Note: ***, ** and * indicate statistical significance at 1%, 5%, and 10% levels, respectively.

The estimate for the green taste parameter "b" is not significant in any of the different instrument alternatives at the 10 percent level of significance. Thus, we can say that there is no evidence that investors had a strong green preference over this whole period. On the other hand, the taste for wealth "d" is significant in all alternatives and has an average of 7.08 percent. This is evidence for the spirit of capitalism [Bakshi and Chen (1996b)], although Zhang (2006) has rejected that hypothesis using data for the period 1959–2001. Alternatively, it might be taken as evidence for saving-based preferences [Dreyer, Schneider, and Smith (2013)], where it is the growth of wealth – but not the level – that enters the utility function.

The estimates for γ are also significant and have an average of 1.13, which might be considered rather low compared to the literature. However, this low gamma could be a consequence of the inclusion of the crisis period in our data sample, which could be biasing our estimates, as discussed earlier. Thus, in what follows we re-estimate the model using the periods before and after the crisis separately.

B. Pre-crisis estimations

We run estimations of Equations (30)–(32) for the period prior to the crisis using the alternative vectors of instruments reported in Table 5:

Table 5: Instruments used for pre-crisis period estimations

Instrument number	Instrument vector
2.1	$X_t = \left[k, \frac{C_{t-1}}{C_{t-2}}, R_{g,t-1}, R_{n,t-1}, R_{f,t}, \frac{W_{t-1}}{W_{t-2}}\right]$
2.2	$X_t = \left[k, \frac{C_t}{C_{t-1}}, R_{g.t-1}, R_{c,t}, R_{n,t-1}, \frac{W_{t-1}}{W_{t-2}}\right]$
2.3	$X_t = \left[k, \frac{C_{t-1}}{C_{t-2}}, R_{g,t-1}, R_{n,t-1}, R_{f,t}, \frac{\lambda_t}{\lambda_{t-1}}, \frac{W_{t-1}}{W_{t-2}}\right]$
2.4	$X_t = \left[k, \frac{C_{t-1}}{C_{t-2}}, R_{g,t-1}, R_{n,t-1}, R_{f,t}, \frac{\lambda_{t-1}}{\lambda_{t-2}}, \frac{W_{t-1}}{W_{t-2}}\right]$
2.5	$X_t = \left[k, \frac{C_{t-1}}{C_{t-2}}, R_{g,t}, R_{g,t-1}, R_{n,t-1}, R_{f,t}, \frac{\lambda_{t-1}}{\lambda_{t-2}}, \frac{W_{t-1}}{W_{t-2}}\right]$
2.6	$X_t = \left[k, \frac{C_t}{C_{t-1}}, \frac{C_{t-1}}{C_{t-2}}, R_{g,t-1}, R_{n,t-1}, R_{f,t}, \frac{\lambda_t}{\lambda_{t-1}}, \frac{W_{t-1}}{W_{t-2}}\right]$

The results for the pre-crisis period are reported in Table 6. The results show that, over the full time period, none of the alternative instruments are rejected by the J-test at the 5 percent significance level.

Table 6: GMM estimation results for the pre-crisis period (starting point 1, 1, 1)

Instrument number	γ	b	d	J-test
	Estimate	Estimate	Estimate	Qui sq.
2.1	1.2473*** (0.4003)	0.0019 (0.0027)	0.0389* (0.0219)	14.301
2.2	1.2396*** (0.1913)	0.0121*** (0.0027)	0.1014*** (0.0181)	16.169
2.3	1.5531*** (0.3327)	0.0034 (0.2656)	0.0534*** (0.0192)	13.672
2.4	1.2704*** (0.4240)	0.0017 (0.0025)	0.0388 (0.0240)	11.491
2.5	1.1002*** (0.3822)	0.0043* (0.0023)	0.0364* (0.0204)	14.665
2.6	1.5916*** (0.2502)	0.0050* (0.0027)	0.0593*** (0.0114)	11.107
Averages	1.3337	0.0047	0.0547	

Note: ***, ** and * indicate statistical significance at 1%, 5%, and 10% levels, respectively.

Estimates for the green taste parameter "b" are statistically significant at the 10 percent level in instrument numbers 2.2, 2.5, and 2.6. However, given the small magnitude of these coefficient estimates, we could claim that they are not economically significant. These tiny coefficients imply that investors essentially derived no utility from investing in green assets. Pre-crisis investors seemed not to care about going green.

The parameter associated with the taste for wealth "d" is statistically significant and has an average of around 5.46 percent. Here, too, there is evidence to support the spirit of capitalism.

Estimates for γ are again statistically significant and in the order of 1.33.

C. Post-crisis estimations

Finally, we run estimations of Equations (30)–(32) for the period that follows the financial crisis using the following alternatives for vector of instruments as shown by Table 7. The results for these estimations are provided in Table 8.

Table 7: Instruments used for post-crisis period estimations

Instrument number	Instrument vector
3.1	$X_t = \left[k, R_{g,t}, R_{n,t-1}, R_{f,t}, \frac{W_{t-1}}{W_{t-2}}\right]$
3.2	$X_t = \left[k, R_{g,t}, R_{g,t-1}, R_{n,t-1}, R_{f,t}\right]$
3.3	$X_t = \left[k, \frac{C_t}{C_{t-1}}, R_{g,t}, R_{n,t-1}, \frac{W_{t-1}}{W_{t-2}}\right]$
3.4	$X_t = \left[k, \frac{C_t}{C_{t-1}}, R_{g,t}, R_{n,t}, R_{n,t-1}, \frac{W_{t-1}}{W_{t-2}}\right]$
3.5	$X_t = \left[k, \frac{C_t}{C_{t-1}}, R_{g,t-1}, R_{n,t}, R_{n,t-1}, \frac{W_{t-1}}{W_{t-2}}\right]$
3.6	$X_t = \left[k, R_{g,t}, R_{g,t-1}, R_{n,t}, R_{n,t-1}, R_{f,t-1}, \frac{\lambda_{t-1}}{\lambda_{t-2}}\right]$

Table 8: GMM estimation results for the post-crisis period (starting point 1, 1, 1)

Instrument number	γ	b	d	J-test
	Estimate	Estimate	Estimate	Qui sq.
3.1	1.4272*** (0.2712)	0.0253* (0.0139)	0.1979*** (0.0504)	14.648
3.2	1.4379*** (0.3309)	0.0201 (0.0135)	0.2019*** (0.0416)	12.769
3.3	1.7668*** (0.3437)	0.0176* (0.0091)	0.1739*** (0.0488)	8.057
3.4	1.3707*** (0.1711)	0.0289** (0.0124)	0.1805*** (0.0500)	7.715
3.5	1.2378*** (0.1018)	0.0424*** (0.0161)	0.2092*** (0.0375)	15.983
3.6	1.2347*** (0.0870)	0.0843*** (0.0114)	0.3823*** (0.0512)	19.622
Averages	1.4125	0.0364	0.2243	

Note: ***, ** and * indicate statistical significance at 1%, 5%, and 10% levels, respectively.

Once again, none of the alternative instruments selected were rejected by the J-test at the 5 percent significance level. Estimates for the green taste parameter “b” are now statistically significant at low levels of significance for all alternatives of instruments (except for instrument number 3.2). The estimates for the parameters have an average of 3.64 percent. This is large enough to be economically significant. In other words, post-crisis investors developed a much stronger preference for green assets.

At the same time investors developed a stronger taste for wealth. As in the other sampled periods, the parameter “d” associated with the preference for wealth was significant in all alternatives. The average value increased sharply from around 5.46 percent to 20.2 percent. Perhaps investors came to attach greater importance to their wealth because of their unpleasant experiences in the crisis.

Finally, the estimates for γ are again stable and statistically significant. The average of this estimate equals 1.41, which is slightly higher than from previous periods.

A note of care is important here: Given the ESG data limitations in the MSCI KLD ratings for the period since 2016, the post-crisis period is short, spanning only five years after the financial crisis. On the one hand, we acknowledge that this could be interpreted as a limitation of the post-crisis estimations, and thus weaken our conclusions for this sample period. On the other hand, we ran all estimations with three simultaneous equations that use different asset classes (risk-free, green, and non-green), which increases the degrees of freedom in all of the estimations.[11]

VII. Conclusion

This paper introduces a novel theory that explains why green assets underperform: If investors derive a warm glow of self-satisfaction from holding green stocks, it should raise their demand,

[11] With five years (each with four quarters), we have 20 data points for each of the three equations in our estimations. This means that our estimations post-crisis, although for a short period, use 60 points each.

increase their prices, and lower their returns. Our empirical estimates suggest that this "taste for green" has increased since the financial crisis; at the same time, the market share of green stocks has increased. Paradoxically, growing concern for environmentally responsible investment might explain the underperformance of green stocks. If, as the quote from the *Economist* (2017) at the beginning of the paper suggests, millennials are much more socially and environmentally engaged than baby-boomers, then we might expect green stocks to do even worse in the future.

It would be interesting to estimate the model using different data sets, in other countries, and over other time periods. It could also be used to assess the performance of SRI and so-called "impact investments". More generally, the framework of warm-glow investment should apply to *any* scenario where investors derive utility from holding financial assets. For example, workers in a firm may purchase its stock because they are proud of it. However, this may impact the financial performance of the firm. The home bias puzzle in equities [French and Poterba (1991), Tesar and Werner (1995), Lewis (1999), Karolyi and Stulz (2003), Coeurdacier and Rey (2013), *inter alia*] offers another enticing application of the framework: Domestic investors hold so few foreign stocks because they derive utility from holding domestic assets. In this case, warm glow might be called "patriotism"; it would act like home bias defined over assets rather than goods.

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Appendix A: Derivation of the discrete-time Euler equations

Recall from the text that the prices of the non-green and green assets follow the vector diffusion

$$\frac{\Delta P_{n,t}}{P_{n,t}} = r_{n,t}\Delta t + \sigma_{n,t}\Delta\omega_{n,t}, \tag{A1}$$

$$\frac{\Delta P_{g,t}}{P_{g,t}} = r_{g,t}\Delta t + \sigma_{g,t}\Delta\omega_{g,t}. \tag{A2}$$

The investor's flow budget constraint is given by Equation (2) in the text:

$$W_{t+\Delta t} = (W_t - C_t\Delta t)\left[1 + \left(1 - \lambda_{n,t} - \lambda_{g,t}\right)r_f + \lambda_{n,t}\frac{\Delta P_{n,t}}{P_{n,t}} + \lambda_{g,t}\frac{\Delta P_{g,t}}{P_{g,t}}\right]. \tag{A3}$$

The Bellman equation for this problem is

$$V(w_t) = \max_{C_t,\lambda_{gt},\lambda_{n,t}} U\left(C_t, \lambda_{g,t}, W_t\right)\Delta t + e^{-\theta\Delta t} E\, V(W_{t+\Delta t}). \tag{A4}$$

Recall that consumption and the portfolio shares of the assets along the optimal path are denoted by C_t^*, $\lambda_{n,t}^*$, and $\lambda_{g,t}^*, t \in (0, \infty)$. Performing the indicated maximization then yields the following first-order conditions:

$$U_C\left(C_t^*, \lambda_{g,t}^*, W_t\right) = e^{-\theta\Delta t} EV^{/}(W_{t+\Delta t})\left[1 + \left(1 - \lambda_{n,t}^* - \lambda_{g,t}^*\right)r_f\Delta t + \lambda_{n,t}^*\frac{\Delta P_{n,t}}{P_{n,t}} + \lambda_{g,t}^*\frac{\Delta P_{g,t}}{P_{g,t}}\right], \tag{A5}$$

$$EV^{/}(W_{t+\Delta t})\left(\frac{\Delta P_{n,t}}{P_{n,t}} - r_f\Delta t\right) = 0, \tag{A6}$$

$$U_{\lambda_g}\left(C_t^*, \lambda_{g,t}^*, W_t\right) + e^{-\theta\Delta t} W_t EV^{/}(W_{t+\Delta t})\left(\frac{\Delta P_{g,t}}{P_{g,t}} - r_f\Delta t\right) = 0. \tag{A7}$$

The usual envelope argument implies that $V^{/}(W_t) = U_C\left(C_t^*, \lambda_{g,t}^*, W_t\right) + U_W\left(C_t^*, \lambda_{g,t}^*, W_t\right)\Delta t$. Therefore we have

$$U_C\left(C_t^*,\lambda_{g,t}^*,W_t\right)=e^{-\theta\Delta t}E\left[U_C\left(C_{t+\Delta t},\lambda_{g,t+\Delta t}^*,W_{t+\Delta t}\right)+U_W\left(C_{t+\Delta t},\lambda_{g,t+\Delta t}^*,W_{t+\Delta t}\right)\Delta t\right]\Big[1+$$

$$\left(1-\lambda_{n,t}^*-\lambda_{g,t}^*\right)r_f\Delta t+\lambda_{n,t}^*\frac{\Delta P_{n,t}}{P_{n,t}}+\lambda_{g,t}^*\frac{\Delta P_{g,t}}{P_{g,t}}\Big], \qquad \text{(A8)}$$

$$E\left[U_C\left(C_{t+\Delta t},\lambda_{g,t+\Delta t}^*,W_{t+\Delta t}\right)+U_W\left(C_{t+\Delta t},\lambda_{g,t+\Delta t}^*,W_{t+\Delta t}\right)\Delta t\right]\left(\frac{\Delta P_{n,t}}{P_{n,t}}-r_f\Delta t\right)=0 \qquad \text{(A9)}$$

$$U_{\lambda_g}\left(C_t^*,\lambda_{g,t}^*,W_t\right)+e^{-\theta\Delta t}(W_t-C_t\Delta t)E\left[U_C\left(C_{t+\Delta t},\lambda_{g,t+\Delta t}^*,W_{t+\Delta t}\right)+\right.$$

$$\left.U_W\left(C_{t+\Delta t},\lambda_{g,t+\Delta t}^*,W_{t+\Delta t}\right)\Delta t\right]\left(\frac{\Delta P_{g,t}}{P_{g,t}}-r_f\Delta t\right)=0. \qquad \text{(A10)}$$

These are Equations (6), (7), and (8) in the text. For our utility function in Equation (5) they can be expressed as

$$1=e^{-\theta\Delta t}E\left(\frac{C_{t+\Delta t}^*}{C_t^*}\right)^{a(1-\gamma)-1}\left(\frac{\lambda_{g,t+\Delta t}^*}{\lambda_{g,t}^*}\right)^{b(1-\gamma)}\left(\frac{W_{t+\Delta t}}{W_t}\right)^{d(1-\gamma)}\left[1+\frac{d}{a}\frac{C_{t+1}^*}{W_{t+1}}\Delta t\right]\Big[1+\big(1-\lambda_{n,t}^*-$$

$$\lambda_{g,t}^*\big)r_f\Delta t+\lambda_{n,t}^*\ \frac{\Delta P_{n,t}}{P_{n,t}}+\lambda_{g,t}^*\frac{\Delta P_{g,t}}{P_{g,t}}\Big], \qquad \text{(A11)}$$

$$E\left(\frac{C_{t+\Delta t}^*}{C_t^*}\right)^{a(1-\gamma)-1}\left(\frac{\lambda_{g,t+\Delta t}^*}{\lambda_{g,t}^*}\right)^{b(1-\gamma)}\left(\frac{W_{t+\Delta t}}{W_t}\right)^{d(1-\gamma)}\left[1+\frac{d}{a}\frac{C_{t+1}^*}{W_{t+1}}\Delta t\right]\left(\frac{\Delta P_{n,t}}{P_{n,t}}-r_f\Delta t\right)=0, \qquad \text{(A12)}$$

$$\frac{b}{a}\frac{C_t^*}{W_t-C_t^*\Delta t}\frac{1}{\lambda_{g,t}^*}+e^{-\theta\Delta t}E\left(\frac{C_{t+\Delta t}^*}{C_t^*}\right)^{a(1-\gamma)-1}\left(\frac{\lambda_{g,t+\Delta t}^*}{\lambda_{g,t}^*}\right)^{b(1-\gamma)}\left(\frac{W_{t+\Delta t}}{W_t}\right)^{d(1-\gamma)}\left[1+\frac{d}{a}\frac{C_{t+1}^*}{W_{t+1}}\Delta t\right]\Big(\frac{\Delta P_{g,t}}{P_{g,t}}-$$

$$r_f\Delta t\Big)=0. \qquad \text{(A13)}$$

These are Equations (9), (10), and (11).

Appendix B: Derivation of Propositions 2 and 3

Assume consumption, wealth, and the portfolio share of the green asset along the optimal path are diffusion processes:

$$\frac{\Delta C_t^*}{C_t^*}=\frac{C_{t+\Delta t}^*-C_t^*}{C_t^*}=\ \mu_{C,t}\Delta t+\sigma_{C,t}\Delta\omega_{C,t}, \qquad \text{(B1)}$$

$$\frac{\Delta W_t}{W_t} = \frac{W_{t+\Delta t} - W_t}{W_t} = \mu_{W,t}\Delta t + \sigma_{W,t}\Delta\omega_{C,t}, \tag{B2}$$

$$\frac{\Delta\lambda^*_{g,t}}{\lambda^*_{g,t}} = \frac{\lambda^*_{g,t+\Delta t} - \lambda^*_{g,t}}{\lambda^*_{g,t}} = \mu_{\lambda_g,t}\Delta t + \sigma_{\lambda_g,t}\Delta\omega_{\lambda_g,t}. \tag{B3}$$

To exposit the method, begin with the simplest Euler equation – for the non-green asset – in Equation (A9). Take a Taylor series around $C^*_{t+\Delta t} = C^*_t$, $\lambda^*_{g,t+\Delta t} = \lambda^*_{g,t}$:

$$\begin{aligned}
&\left[U_C\left(C^*_t, \lambda^*_{g,t}, W_t\right) + U_W\left(C^*_t, \lambda^*_{g,t}, W_t\right)\Delta t\right] E\left(\frac{\Delta P_{n,t}}{P_{n,t}} - r_f\Delta t\right) \\
&\quad + E\left\{\left[U_{CC}\left(C^*_t, \lambda^*_{g,t}, W_t\right) + U_{WC}\left(C^*_t, \lambda^*_{g,t}, W_t\right)\Delta t\right]\frac{\Delta C^*_t}{C^*_t}\right. \\
&\quad + \left[U_{CW}\left(C^*_t, \lambda^*_{g,t}, W_t\right) + U_{WW}\left(C^*_t, \lambda^*_{g,t}, W_t\right)\Delta t\right]\frac{\Delta W_t}{W_t} \\
&\quad \left. + \left[U_{C\lambda_g}\left(C^*_t, \lambda^*_{g,t}, W_t\right) + U_{W\lambda_g}\left(C^*_t, \lambda^*_{g,t}, W_t\right)\Delta t\right]\frac{\Delta\lambda^*_{g,t}}{\lambda^*_{g,t}}\right\}\left(\frac{\Delta P_{n,t}}{P_{n,t}} - r_f\Delta t\right) = 0.
\end{aligned} \tag{B4}$$

Take the expectation in Equation (B4) and solve for the excess return. Now take the limit as $\Delta t \to 0$ and use the Ito multiplication rules to find

$$r_{n,t} - r_f = -\frac{U_{CC}\left(C^*_t, \lambda^*_{g,t}, W_t\right)C^*_t}{U_C\left(C^*_t, \lambda^*_{g,t}, W_t\right)} cov\left(\frac{dC^*_t}{C^*_t}, \frac{\Delta P_{n,t}}{P_{n,t}}\right) - \frac{U_{c\lambda_g}\left(C^*_t, \lambda^*_{g,t}, W_t\right)\lambda^*_{g,t}}{U_c\left(C^*_t, \lambda^*_{g,t}, W_t\right)} cov\left(\frac{d\lambda^*_{g,t}}{\lambda^*_{g,t}}, \frac{\Delta P_{n,t}}{P_{n,t}}\right) - \frac{U_{CW}\left(C^*_t, \lambda^*_{g,t}, W_t\right)W_t}{U_C\left(C^*_t, \lambda^*_{g,t}, W_t\right)} cov\left(\frac{dW_t}{W_t}, \frac{dP_{n,t}}{P_{n,t}}\right). \tag{B5}$$

This is Equation (12). For the preferences described by Equation (4), this reduces to

$$r_{n,t} - r_f = [1 - a(1-\gamma)]cov\left(\frac{dC^*_t}{C^*_t}, \frac{\Delta P_{n,t}}{P_{n,t}}\right) - b(1-\gamma)cov\left(\frac{d\lambda^*_{g,t}}{\lambda^*_{g,t}}, \frac{\Delta P_{n,t}}{P_{n,t}}\right) - d(1-\gamma)cov\left(\frac{dW_t}{W_t}, \frac{dP_{n,t}}{P_{n,t}}\right), \tag{B6}$$

which is Equation (13).

Next consider the Euler equation for the green asset in Equation (A10). The Taylor series in this case is

$$U_{\lambda_g}\left(C_t^*, \lambda_{g,t}^*, W_t\right) + e^{-\theta\Delta t}(W_t - C_t\Delta t)\left[U_C\left(C_t^*, \lambda_{g,t}^*, W_t\right) + U_W\left(C_t^*, \lambda_{g,t}^*, W_t\right)\Delta t\right]E\left(\frac{\Delta P_{g,t}}{P_{g,t}} - r_f\Delta t\right) + e^{-\theta\Delta t}(W_t - C_t\Delta t)E\left\{\left[U_{CC}\left(C_t^*, \lambda_{g,t}^*, W_t\right) + U_{WC}\left(C_t^*, \lambda_{g,t}^*, W_t\right)\Delta t\right]\frac{\Delta C_t^*}{C_t^*} + \left[U_{CW}\left(C_t^*, \lambda_{g,t}^*, W_t\right) + U_{WW}\left(C_t^*, \lambda_{g,t}^*, W_t\right)\Delta t\right]\frac{\Delta W_t}{W_t} + \left[U_{C\lambda_g}\left(C_t^*, \lambda_{g,t}^*, W_t\right) + U_{W\lambda_g}\left(C_t^*, \lambda_{g,t}^*, W_t\right)\Delta t\right]\frac{\Delta\lambda_{g,t}^*}{\lambda_{g,t}^*}\right\}\left(\frac{\Delta P_{g,t}}{P_{g,t}} - r_f\Delta t\right) = 0. \quad \text{(B7)}$$

As before, take the expectation, solve for the excess return, take the limit as $\Delta t \to 0$, and employ the Ito rules:

$$r_{g,t} - r_f = -\frac{U_{\lambda_g}\left(C_t^*, \lambda_{g,t}^*, W_t\right)}{U_C\left(C_t^*, \lambda_{g,t}^*\right)}\frac{1}{W_t} - \frac{U_{CC}\left(C_t^*, \lambda_{g,t}^*, W_t\right)C_t^*}{U_C\left(C_t^*, \lambda_{g,t}^*, W_t\right)}cov\left(\frac{dC_t^*}{C_t^*}, \frac{dP_{g,t}}{P_{g,t}}\right) - \frac{U_{c\lambda_g}\left(C_t^*, \lambda_{g,t}^*\right)\lambda_{g,t}^*}{U_c\left(C_t^*, \lambda_{g,t}^*\right)}cov\left(\frac{d\lambda_{g,t}^*}{\lambda_{g,t}^*}, \frac{dP_{g,t}}{P_{g,t}}\right) - \frac{U_{CW}\left(C_t^*, \lambda_{g,t}^*, W_t\right)W_t}{U_C\left(C_t^*, \lambda_{g,t}^*, W_t\right)}cov\left(\frac{dW_t}{W_t}, \frac{dP_{g,t}}{P_{g,t}}\right). \quad \text{(B8)}$$

This is Equation (14).

For our specification of preferences this becomes

$$r_{g,t} - r_f = -\frac{b}{a}\frac{1}{\lambda_{g,t}}\frac{c_t}{w_t} + [1 - a(1-\gamma)]cov\left(\frac{dC_t^*}{C_t^*}, \frac{dP_{g,t}}{P_{g,t}}\right) - b(1-\gamma)cov\left(\frac{d\lambda_{g,t}^*}{\lambda_{g,t}^*}, \frac{dP_{g,t}}{P_{g,t}}\right) - d(1-\gamma)cov\left(\frac{dW_t}{W_t}, \frac{dP_{g,t}}{P_{g,t}}\right). \quad \text{(B9)}$$

This is Equation (15).

Appendix C: Derivation of the consumption and portfolio policies in the i.i.d. case

The Bellman equation for this problem is

$$0 = \max_{C_t, W_t}\frac{\left[C_t^{1-b}(\lambda_t W_t)^b\right]^{1-\gamma}}{1-\gamma} - \theta J + J_W\left\{\left[r_f + \lambda_t\left(r - r_f\right)\right]W_t - C_t\right\} + J_{WW}\lambda_t^2 W_t^2\frac{\sigma_p^2}{2}. \quad \text{(C1)}$$

The resulting first-order conditions are

$$(1-b)C_t^{(1-b)(1-\gamma)-1}\lambda_t^{b(1-\gamma)}W_t^{b(1-\gamma)} = J_W, \tag{C2}$$

$$bC_t^{(1-b)(1-\gamma)}\lambda_t^{b(1-\gamma)-1}W_t^{b(1-\gamma)} + J_W(r-r_f)W_t + J_{WW}\lambda_t W_t^2\sigma_p^2 = 0. \tag{C3}$$

Divide Equation (C3) by Equation (C2) to find

$$\frac{b}{1-b}\frac{C_t}{\lambda_t} + (r-r_f)W_t + \frac{J_{WW}}{J_W}W_t^2\lambda_t\sigma_p^2 = 0. \tag{C4}$$

Substitute Equation (C2) into Equation (C1)

$$0 = \frac{1-(1-b)(1-\gamma)}{(1-b)(1-\gamma)}J_W C_t - \theta J + J_W\left[r_f + \lambda_t(r-r_f)\right]W_t + J_{WW}\lambda_t^2 W_t^2\frac{\sigma_p^2}{2}. \tag{C5}$$

Conjecture that

$$J(W_t) = A\frac{W_t^{1-\gamma}}{1-\gamma}, \tag{C6}$$

$$C_t(W_t) = BW_t. \tag{C7}$$

Use these conjectures to rewrite Equations (C4) and (C5), respectively, as

$$\frac{b}{1-b}\frac{B}{\lambda_t} + r - r_f - \gamma\,\sigma_p^2\lambda_t = 0, \tag{C8}$$

$$0 = \frac{1-(1-b)(1-\gamma)}{(1-b)(1-\gamma)}B - \frac{\theta}{1-\gamma} + r_f + (r-r_f)\lambda_t - \gamma\frac{\sigma_p^2}{2}\lambda_t^2. \tag{C9}$$

Since the optimal portfolio share is obviously constant, we will henceforth write it without a time subscript, so $\lambda_t = \lambda$. Substitute Equation (C9) into Equation (C8):

$$b\frac{\theta-(1-\gamma)\left[r_f+\lambda(r-r_f)-\gamma\lambda^2\frac{\sigma_p^2}{2}\right]}{1-(1-b)(1-\gamma)} + (r-r_f)\lambda - \gamma\sigma_p^2\lambda^2 = 0. \tag{C10}$$

Simplify and collect terms:

$$b\left[\theta-(1-\gamma)r_f\right]+\gamma\left(r-r_f\right)\lambda-\left[\gamma+\frac{b}{2}(1-\gamma)\right]\gamma\sigma_p^2\lambda^2=0. \tag{C11}$$

This may have two positive roots. However, if the transversality condition is satisfied [see below, in (C18)], the unique positive root of this equation is

$$\lambda^*\left(b,r-r_f\right)=\frac{1}{2}\frac{r-r_f}{\left[\gamma+\frac{b}{2}(1-\gamma)\right]\sigma_p^2}+\frac{1}{2}\sqrt{\left\{\frac{r-r_f}{\left[\gamma+\frac{b}{2}(1-\gamma)\right]\sigma_p^2}\right\}^2+4b\,\frac{\theta-(1-\gamma)r_f}{\gamma+\frac{b}{2}(1-\gamma)}\frac{1}{\gamma\sigma_p^2}} \tag{C12}$$

This is Equation (22). If $b=0$ it reduces to the standard portfolio demand with CRRA preferences

$$\lambda_M=\frac{r-r_f}{\gamma\sigma_p^2}. \tag{C13}$$

This is reproduced in Equation (23).

To find the marginal propensity to consume, $B(b)$, substitute Equation (C12) back into Equation (C9):

$$B(b)=\frac{1-b}{1-(1-b)(1-\gamma)}\left\{\theta-(1-\gamma)\left[r_f+\left(r-r_f\right)\lambda^*(b)-\gamma\lambda^*(b)^2\frac{\sigma_p^2}{2}\right]\right\}. \tag{C14}$$

This is Equation (24) in the text. When $b=0$ it reduces to the Merton solution:

$$B(0)=B_m=\frac{\theta-(1-\gamma)\left[r_f+\frac{\left(r-r_f\right)^2}{2\gamma\sigma_p^2}\right]}{\gamma}. \tag{C15}$$

This is Equation (25).

An alternative way of arriving at Equation (C14) is to substitute Equation (C13) into Equation (C8):

$$B(b)=-\frac{1-b}{b}\left[\lambda^*(b)\left(r-r_f\right)-\lambda^*(b)^2\gamma\sigma_p^2\right]=\frac{1-b}{b}\lambda^*\gamma\sigma_p^2\left[\lambda^*(b)-\lambda_M\right]. \tag{C16}$$

Combining this with Equation (C7) yields the consumption function in Equation (24). Taking the limit as $b \to 0$ yields an indeterminate form since $\lambda^*(0) = \lambda_M$. However, by using L'Hopitâl's rule we can recover the consumption policy in Merton:

$$\lim_{b\to 0} B(b) = B_m = \frac{\theta-(1-\gamma)\left[r_f+\frac{\left(r-r_f\right)^2}{2\gamma\sigma_p^2}\right]}{\gamma}. \tag{C17}$$

This also leads to Equation (25).

The feasibility/transversality condition guaranteeing $B(b) > 0$ follows directly from Equation (C14). For $b > 0$, $B(b) > 0$ if and only if $\lambda(b) > \lambda_m$. A little algebra in turn reveals that this holds if and only if

$$\theta - (1-\gamma)\left[r_f + \frac{\left(r-r_f\right)^2}{2\gamma\sigma_p^2}\right] > 0, \tag{C18}$$

which is inequality (27).

Finally, we need to derive the constant A in the conjectured value function in Equation (C6). To do this, evaluate Equation (C2) using the conjectures in Equations (C6) and (C7) and the solutions in Equations (C12) and (C14):

$$A(b) = (1-b)B(b)^{(1-b)(1-\gamma)-1}\lambda^*(b)^{b(1-\gamma)}. \tag{C19}$$

Appendix D: Derivation of Propositions 3 and 4

Part "a" of Proposition 3 follows by substituting Equation (C13) into Equation (C11) and simplifying.

To derive part "b," it is useful to define some new functions. Let

$$f(b) = \frac{\gamma}{\gamma+\frac{b}{2}(1-\gamma)}, \tag{D1}$$

$$g(b) = b\,\frac{\theta-(1-\gamma)r_f}{\gamma^2\sigma^2}. \tag{D2}$$

The portfolio demand in Equation (C12) can then be expressed as

$$\lambda^*\left(b, r-r_f\right) = \frac{1}{2}\lambda_m f(b) + \frac{1}{2}\sqrt{\lambda_m^2 f(b)^2 + 4g(b)f(b).} \qquad \text{(D3)}$$

Differentiating with respect to b yields

$$\frac{\partial\lambda^*}{\partial b} = \frac{1}{2}\lambda_m \frac{df(b)}{db} + \frac{1}{4}\frac{2\lambda_m^2 f(b)\frac{df(b)}{db} + 4\left[f(b)\frac{dg(b)}{db} + g(b)\frac{df(b)}{db}\right]}{\sqrt{\lambda_m^2 f(b)^2 + 4g(b)f(b).}}. \qquad \text{(D4)}$$

Some tedious algebra then shows that $\partial\lambda^*/\partial b > 0$ provided that the following inequality holds:

$$0 < g^{/}(b)^2 f(b)^2 + f^{/}(b)^2 g(b)^2 + \lambda_m^2 g^{/}(b) f^{/}(b) f(b)^2 + 2g^{/}(b) f^{/}(b) g(b) f(b). \qquad \text{(D5)}$$

However, the right-hand side of this inequality reduces to a positive expression multiplied by the left-hand side of inequality (C16). Therefore $\partial\lambda^*/\partial \mathrm{b} > 0$ if and only if the transversality condition is satisfied.

To derive Proposition 4, consider Equation (D3). First, note the non-green portfolio demand is linear in the excess return: $\partial\lambda_m/\partial\left(r-r_f\right) = 1/\gamma\sigma^2$. Part "a" follows directly by differentiating with respect to the excess return:

$$\frac{\partial\lambda^*}{\partial\left(r-r_f\right)} = \frac{f(b)}{2\gamma\sigma^2}\left[1 + \frac{\lambda_m f(b)}{\left(\lambda_m^2 f(b)^2 + 4g(b)f(b)\right)^{1/2}}\right] > 0. \qquad \text{(D6)}$$

To derive part "b" take the second derivative:

$$\frac{\partial^2\lambda^*}{\partial\left(r-r_f\right)^2} = 2\frac{f(b)^3}{\gamma^2\sigma^4}\left(\lambda_m^2 f(b)^2 + 4g(b)f(b)\right)^{-3/2} g(b). \qquad \text{(D7)}$$

Notice that the sign of this expression is governed by $g(b) = b\frac{\theta-(1-\gamma)r_f}{\gamma^2\sigma^2}$. If $\gamma \geq 0$ then $g(b) > 0$ automatically. If $\gamma < 0$ then we have to invoke the transversality condition. Notice that if $\gamma < 0$ then

$$\theta - (1-\gamma)r_f > \theta - (1-\gamma)\left[r_f + \frac{(r-r_f)^2}{2\gamma\sigma_p^2}\right] > 0, \tag{D8}$$

where the second inequality is the transversality condition in Equation (C18). Therefore $g(b) > 0$ if the transversality condition is satisfied.

Appendix E: Derivation of Propositions 5 and 6

First evaluate Equation (C12) in equilibrium, when $\lambda^*(b) = 1$. Solve for the equilibrium consumption/wealth ratio:

$$\frac{C_t^*(W_t)}{W_t} = B = \frac{(1-b)(1-\gamma)}{1-(1-b)(1-\gamma)}\left[\frac{\theta}{1-\gamma} - r + \gamma\sigma_p^2\right]. \tag{E1}$$

Conjecture that the equilibrium price is a function of the state variable, dividends D_t. Apply Ito's lemma to calculate the ex-dividend rate of return in equilibrium:

$$\frac{dP_t}{P_t} = \left[\frac{F^{/}(D_t)D_t}{F(D_t)}\nu + \frac{F^{//}(D_t)D_t^2}{F(D_t)}\frac{\sigma^2}{2}\right]dt + \frac{F^{/}(D_t)D_t}{F(D_t)}\sigma d\omega_t. \tag{E2}$$

This implies that the drift and the standard deviation of the growth of capital gains are

$$\pi = \left[\frac{F^{/}(D_t)D_t}{F(D_t)}\nu + \frac{F^{//}(D_t)D_t^2}{F(D_t)}\frac{\sigma^2}{2}\right]dt, \tag{E3}$$

$$\sigma_p^2 = \left(\frac{F^{/}(D_t)D_t}{F(D_t)}\right)^2\sigma^2 dt. \tag{E4}$$

Now substitute Equations (E3) and (E4) into Equation (E1). Use the fact that $r = \pi + D_t dt/F(D_t)$, remembering that in equilibrium all dividends are consumed [$C_t(W_t) = D_t$] and that all wealth consists of the value of the stock [$P_t = D_t$]. It follows that

$$\frac{D_t}{F(D_t)} = \frac{(1-b)(1-\gamma)}{1-(1-b)(1-\gamma)}\left[\frac{\theta}{1-\gamma} - \frac{F^{/}(D_t)D_t}{F(D_t)}\nu + \frac{F^{//}(D_t)D_t^2}{F(D_t)}\frac{\sigma^2}{2} + \frac{D_t}{F(D_t)} + \gamma\left(\frac{F^{/}(D_t)D_t}{F(D_t)}\right)^2\frac{\sigma^2}{2}\right]. \tag{E5}$$

This simplifies to Equation (27) in the text:

$$\frac{D_t}{F(D_t)} = (1-b)\left[\frac{\theta}{1-\gamma} - \frac{F^{/}(D_t)D_t}{F(D_t)}\nu + \frac{F^{//}(D_t)D_t^2}{F(D_t)}\frac{\sigma^2}{2} + \gamma\left(\frac{F^{/}(D_t)D_t}{F(D_t)}\right)^2\frac{\sigma^2}{2}\right]. \quad \text{(E6)}$$

Conjecture that $F(D_t) = AD_t$. This implies that $\pi = \nu$ and $\sigma_p^2 = \sigma^2$ so that Equation (E6) reduces to

$$\frac{1}{A} = (1-b)\left[\theta - (1-\gamma)\left(\nu - \frac{\sigma^2}{2}\right)\right]. \quad \text{(E7)}$$

This yields the pricing function in Equation (28).

$$F(D_t) = \frac{D_t}{(1-b)\left[\theta-(1-\gamma)\left(\nu-\frac{\sigma^2}{2}\right)\right]}. \quad \text{(E8)}$$

To derive the equilibrium risk-free rate, substitute Equation (E8) into Equation (C8):

$$r_f = \theta + \nu - \gamma\sigma^2 - (1-\gamma)\left(\nu - \frac{\sigma^2}{2}\right), \quad \text{(E9)}$$

which is independent of b. The excess return in Equation (29) and Proposition 6 then also follow from Equation (C8), remembering that $B = 1/A$.

Appendix F: Derivation of the Euler equations used for estimation

Here we derive Equations (30)–(32) in the text. Consider Equations (9)–(11) in the text. Define $\beta = e^{-\theta}$and set $\Delta t = 1$. Define the empirical analogues to the returns on the assets by $R_{n,t+1}$, $R_{g,t+1}$, and $R_{f,t}$ and denote the market return by $R_{m,t+1} = (1-\lambda_{n,t}-\lambda_{g,t})R_{f,t+1} + \lambda_{n,t}R_{n,t+1} + \lambda_{g,t}R_{g,t+1}$. Also, let the risk-free rate be time-varying.

Start with the consumption Euler equation in Equation (8) expressed in this notation:

$$\beta E\left(\frac{C_{t+1}}{C_t}\right)^{a(1-\gamma)-1}\left(\frac{\lambda_{g,t+1}}{\lambda_{g,t}}\right)^{b(1-\gamma)}\left(\frac{W_{t+1}}{W_t}\right)^{d(1-\gamma)}\left[1+\frac{d}{a}\frac{C_{t+1}}{W_{t+1}}\right]R_{m,t+1} = 1. \quad \text{(F1)}$$

The two asset Euler equations in Equations (10) and (11) are

$$E\left(\frac{C_{t+1}}{C_t}\right)^{a(1-\gamma)-1}\left(\frac{\lambda_{g,t+1}}{\lambda_{g,t}}\right)^{b(1-\gamma)}\left(\frac{W_{t+1}}{W_t}\right)^{d(1-\gamma)}\left[1+\frac{d}{a}\frac{C_{t+1}}{W_{t+1}}\right]\left(R_{n,t+1}-R_{f,t+1}\right)=0, \quad \text{(F2)}$$

$$\frac{b}{a}\frac{C_t}{W_t-C_t}\frac{1}{\lambda_{g,t}}+\beta E\left(\frac{C_{t+1}}{C_t}\right)^{a(1-\gamma)-1}\left(\frac{\lambda_{g,t+1}}{\lambda_{g,t}}\right)^{b(1-\gamma)}\left(\frac{W_{t+1}}{W_t}\right)^{d(1-\gamma)}\left[1+\frac{d}{a}\frac{C_{t+1}}{W_{t+1}}\right]\left(R_{g,t+1}-R_{f,t+1}\right)=0. \quad \text{(F3)}$$

Multiply (F2) and (F3) by their respective portfolio shares and add, then substitute back into Equation (F1) to find

$$\beta E\left(\frac{C_{t+1}}{C_t}\right)^{a(1-\gamma)-1}\left(\frac{\lambda_{g,t+1}}{\lambda_{g,t}}\right)^{b(1-\gamma)}\left(\frac{W_{t+1}}{W_t}\right)^{d(1-\gamma)}\left[1+\frac{d}{a}\frac{C_{t+1}}{W_{t+1}}\right]R_{f,t+1}=1+\frac{b}{a}\frac{C_t}{W_t-C_t}. \quad \text{(F4)}$$

Substitute Equation (F4) into Equation (F2) to find

$$\beta E\left(\frac{C_{t+1}}{C_t}\right)^{a(1-\gamma)-1}\left(\frac{\lambda_{g,t+1}}{\lambda_{g,t}}\right)^{b(1-\gamma)}\left(\frac{W_{t+1}}{W_t}\right)^{d(1-\gamma)}\left[1+\frac{d}{a}\frac{C_{t+1}}{W_{t+1}}\right]R_{n,t+1}=1+\frac{b}{a}\frac{C_t}{W_t-C_t} \quad \text{(F5)}$$

Next rewrite (F3) as

$$\beta E\left(\frac{C_{t+1}}{C_t}\right)^{a(1-\gamma)-1}\left(\frac{\lambda_{g,t+1}}{\lambda_{g,t}}\right)^{b(1-\gamma)}\left(\frac{W_{t+1}}{W_t}\right)^{d(1-\gamma)}\left[1+\frac{d}{a}\frac{C_{t+1}}{W_{t+1}}\right]R_{f,t+1}=$$

$$\beta E\left(\frac{C_{t+1}}{C_t}\right)^{a(1-\gamma)-1}\left(\frac{\lambda_{g,t+1}}{\lambda_{g,t}}\right)^{b(1-\gamma)}\left(\frac{W_{t+1}}{W_t}\right)^{d(1-\gamma)}\left[1+\frac{d}{a}\frac{C_{t+1}}{W_{t+1}}\right]R_{g,t+1}+\frac{b}{a}\frac{C_t}{W_t-C_t}\frac{1}{\lambda_{g,t}}. \quad \text{(F6)}$$

Equate (F6) to (F4):

$$1+\frac{b}{a}\frac{C_t}{W_t-C_t}=\beta E\left(\frac{C_{t+1}}{C_t}\right)^{a(1-\gamma)-1}\left(\frac{\lambda_{g,t+1}}{\lambda_{g,t}}\right)^{b(1-\gamma)}\left(\frac{W_{t+1}}{W_t}\right)^{d(1-\gamma)}\left[1+\frac{d}{a}\frac{C_{t+1}}{W_{t+1}}\right]R_{g,t+1}+\frac{b}{a}\frac{C_t}{W_t-C_t}\frac{1}{\lambda_{g,t}}. \quad \text{(F7)}$$

Solve this to find

$$\beta E\left(\frac{C_{t+1}}{C_t}\right)^{a(1-\gamma)-1}\left(\frac{\lambda_{g,t+1}}{\lambda_{g,t}}\right)^{b(1-\gamma)}\left(\frac{W_{t+1}}{W_t}\right)^{d(1-\gamma)}\left[1+\frac{d}{a}\frac{C_{t+1}}{W_{t+1}}\right]R_{g,t+1}=1-\frac{1-\lambda_{g,t}}{\lambda_{g,t}}\frac{b}{a}\frac{C_t}{W_t-C_t}. \quad \text{(F8)}$$

Equations (F4), (F5), and (F8) correspond to Equations (30), (31), and (32) in the text, except that they are expressed in terms of the information set at time t, Z_t.

Appendix G: Testing for time trends in the regression results

Table A1 documents the lack of a time trend in the regression results.

Table A1: Tests of time trends

Dependent variable: $\frac{c_t}{w_t - c_t}$					
Independent variables	Number of lags				
	1	2	3	4	5
Intercept	0.0076 (0.0063)	0.0098 (0.0063)	0.0123 (0.0064)	0.0151** (0.0066)	0.0174** (0.0069)
$\frac{c_{t-1}}{w_{t-1} - c_{t-1}}$	0.9507*** (0.0413)	1.1630*** (0.1188)	1.1161*** (0.1207)	1.0871*** (0.1210)	1.0624*** (0.1229)
$\frac{c_{t-2}}{w_{t-2} - c_{t-2}}$	x	-0.2278 (0.1198)	0.0040 (0.1841)	-0.0036 (0.1823)	0.0030 (0.1821)
$\frac{c_{t-3}}{w_{t-3} - c_{t-3}}$	x	x	-0.2026 (0.1232)	-0.0011 (0.1807)	-0.0073 (0.1805)
$\frac{c_{t-4}}{w_{t-4} - c_{t-4}}$	x	x	x	-0.1832 (0.1212)	-0.0381 (0.1786)
$\frac{c_{t-5}}{w_{t-5} - c_{t-5}}$	x	x	x	x	-0.1359 (0.1231)
Time Trend	-0.0000 (0.0000)	-0.0000 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)
Dependent variable: $\frac{w_{t+1}}{w_t}$					
Independent variables	Number of lags				
	1	2	3	4	5
Intercept	0.7991*** (0.1242)	0.6991*** (0.1570)	0.5790*** (0.1782)	0.5038** (0.1923)	0.5064** (0.2044)
$\frac{w_t}{w_{t-1}}$	0.2245 (0.1187)	0.1949 (0.1220)	0.1786 (0.1217)	0.1572 (0.1234)	0.1577 (0.1250)
$\frac{w_{t-1}}{w_{t-2}}$	x	0.1279 (0.1228)	0.0931 (0.1244)	0.0838 (0.1247)	0.0846 (0.1272)
$\frac{w_{t-2}}{w_{t-3}}$	x	x	0.1674 (0.1202)	0.1438 (0.1223)	0.1442 (0.1238)

$\frac{w_{t-3}}{w_{t-4}}$	x	x	x	0.1271 (0.1224)	0.1279 (0.1249)
$\frac{w_{t-4}}{w_{t-5}}$	x	x	x	x	-0.0050 (0.1247)
Time Trend	-0.0001 (0.0006)	-0.0001 (0.0006)	-0.0001 (0.0007)	-0.0001 (0.0006)	-0.0001 (0.0006)

Note: ***, ** and * indicate statistical significance at 1%, 5%, and 10% levels, respectively.