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Learning and Expertise with Scientific External Representations: An Embodied and Extended Cognition Model

Abstract

This paper takes an embodied and extended cognition perspective to ER integration – a cognitive process through which a learner integrates external representations (ERs) in a domain, with her internal (mental) schema, as she interacts with, uses, understands and transforms between those ERs. In the paper, I argue for a theoretical as well as empirical shift in future investigations of ER integration, by proposing a model of cognitive mechanisms underlying the process, based on recent advances in distributed, extended and embodied cognition. I present this new model in contrast to the still dominant classical cognitivist (information processing) approaches to ER integration, and various educational technology intervention designs such approaches inspire. I then exemplify the contrast between the information processing model and the new model through a case of arithmetic problem solving, in the light of corroborative neuroscience evidence which demonstrates the criticalilty of action/sensorimotor mechanisms to ER integration and learning. Finally, as educational implications of the new model, I demonstrate the need for: (i) re-viewing the development of ER integration and expertise also as fine-tuning of the cognitive agent's sensorimotor (action) system, and (ii) a shift of focus in new-media intervention design principles based on the newer understanding of ER integration in science and mathematics.

Keywords: Representations, Extended cognition, Embodied cognition, Sensorimotor, Constitutivity, Expertise.

Introduction

External representations (ERs; e.g. equations, graphs, diagrams, etc.) can be broadly defined as 'symbolic elements', external to the brain and body, that stand in for actual entities and phenomena of study (e.g. oscillation, arithmetic operations). ERs are central to the learning and practice of science and mathematics; they allow access to the complex and often imperceptible entities and phenomena, and the concepts related to them. In other words, ERs make these entities and phenomena tangible, manipulable, and available for thought, action and imagination. The learning and practicing of science are thus impossible without developing expertise over ERs (Johnstone, 1991; Lesh, Post, & Behr, 1987; Tsui & Treagust, 2013). This expertise over ERs is formally termed as representational competence (RC, Kozma & Russell, 1997 & 2005). RC presents a fundamental cognitive difficulty, cutting across different domains such as science and mathematics, and there is consensus in the education literature that many learning difficulties students face in these domains are attributable to difficulties in achieving RC (Chi, Feltovich & Glaser, 1981; Johnstone, 1991 & 2000; Johri, Roth & Olds, 2013; Larkin et al., 1980; Stieff, 2011; Tsui & Treagust, 2013). Further, RC studies in different STEM disciplines have demonstrated significant differences between experts and novices, in terms of one or several of the following abilities: understanding individual representations; integrating ERs; using and generating ERs for conceptual understanding, discovery and problem solving (Chi et al., 1981; Larkin et al., 1980; Kohl & Finkelstein, 2008; Kozma, 2003; Kozma & Russell, 1997; Majumdar et al., 2014).

Broadly compiled, RC is primarily thought to comprise of the following non-exclusive interrelated set of abilities (Ainsworth, 2006; Kozma & Russell, 1997; Kozma & Russell, 2005; Madden et al., 2011):

- integrating internal and external representations, as well as different external representations;
- generating ERs appropriate to the situation or problem (e.g. through sketching);
- communication through and about ERs (e.g. in a scientific community);
- reasoning through and about ERs
- choosing ERs (from a pool) that most appropriately address the need of the situation;
- understanding and describing the different properties of ERs in relation to other ERs (e.g. complementarity, their social nature)
- critiquing ERs in terms of their strengths and shortcomings, and other properties.

Figure 1 below situates this paper in relation to these facets of RC.

--Figure 1--

The scope of this paper is limited to ER integration – a process in a learner's mind, of integrating ERs with her internal (mental) schema, as she uses, understands and transforms between those ERs (Kozma & Russell, 1997; Pape & Tchoshanov, 2001). ER integration could also be described as a cognitive process where different ERs in a domain, and a learner's interaction with them, are the input, and an integrated mental schema corresponding to a wholistic understanding of the dynamics of the represented concept(s) is the output. The output is both a product and a (dynamic) process, as integration is constantly ongoing.

With considerable support from recent advances in cognitive science, neuroscience, and science and mathematics education – the paper argues for a theoretical as well as empirical shift in approaches to ER integration, RC and domain expertise. First, I outline an information processing model of ER integration, abstracted out largely from the currently dominant studies of RC and expertise in science and mathematics education. The model also captures how many of the current intervention design approaches are grounded in the principle of optimising and/or lowering (primarily extraneous) cognitive load. I then contrast this currently dominant approach with an emerging understanding of ER integration that is inspired by some of the recent theoretical and empirical research in the broad domains of extended, distributed and embodied cognition (increasingly referred to as 'field' theories of cognition; Gooding, 2006). These newer approaches theorizing relationships between the mind and external structures hold importance in science and mathematics education because external structures, particularly symbol-based systems, are integral to and indivisible from the practice and learning of science and mathematics. Most phenomena and entities that the external structures represent in these domains are not available to direct perception and action. I capture this emerging understanding in a newly synthesized model of the ER integration process. I then exemplify the contrast between the information processing model and the new model through a case of arithmetic problem solving, and some corroborative neuroscience evidence in relation to this case.

Finally, I discuss the following two implications for future research in science and mathematics education:

- With empirical support from literature on expert-novice differences, I conjecture that expertise development could be accompanied by changes in the learner's sensorimotor or action system[1].
- I advocate for a shift of focus from information processing and offloading (e.g. through external visualizations) as core intervention design principles, to field theory-based principles of interactivity (Malinverni & Peres, 2014) and integrative coupling.

Information processing model of ER integration

ER integration involves performing multiple cognitive tasks such as;

- simultaneously perceiving ERs (Chi, et al., 1981; Johnstone, 1997),
- holding them in the working memory (Johnstone, 1997; Ozogul, Johnson, Moreno & Reisslein, 2012),
- mapping the relationships between those ERs (Çelik & Sağlam-Arslan, 2012; Hinton & Nakhleh, 1999; Sanger, 2005),
- interpreting the represented content, and the meaning of their relationships with each other (Cook, Wiebe & Carter, 2008; Ainsworth, 2006),
- reasoning symbolically about ERs in relation to the represented phenomena (Battista, 2003; Kamii & Kysh, 2006), and in relation to the learner's prior knowledge (Cook et al.,

[1]The action-perception-expertise loop is not new in cognitive science (e.g. see Glenberg, Witt & Metcalfe, 2013 for a detailed review largely in the context of language reading and comprehension). However, reviews reveal that it is yet to percolate in science and mathematics education, specifically in the context of ER integration and conceptual learning. This paper is an attempt to adoptively revise the models of ER integration in science and mathematics.

2008; Nitz, Nerdel & Prechtl, 2012; Nitz & Tippett, 2012) as well as social surroundings (Kozma, 2003), and

- translating between such different spatial dimensions or levels (e.g. 2-dimensional, 3-dimensional; Bodner & Domin, 2000; Wu & Shah, 2004).

Extensive cross-disciplinary reviews of work, related to ER integration, RC, and expertise development in science and mathematics, reveal that most accounts of these processes either implicitly assume, or explicitly subscribe to the classical information processing theory of cognition. Although not limited to, at surface-level, this theory broadly establishes structural and functional analogies between computers and the human brain (e.g. both as information processors, both having information input sources and actuators; Gooding, 2006; Paivio, 2006). The learner's mind, according to the information processing theory-based accounts, engages in information extraction on encounter with an external representation (e.g. perceptual input). Correspondences between different ERs, or between different areas of the same ER, are established through (symbolic) 'translation' of the extracted information in the mind. The translation processes are also responsible for coordinating between ERs and the phenomena they represent, as well as between the learner's prior knowledge (in other words, mental models or internal representations – i.e.. IRs) and the ERs (Paivio, 2006; Wu, Krajcik & Soloway, 2001). Learner's prior knowledge (e.g. through information chunking and other top-down processes; Larkin, 1979) significantly influences and/or constrains information extraction (Chi et al., 1981; Pretz, Naples & Sternberg, 2003). Information extraction processes may be mediated, for instance, through various (attentional) filtering mechanisms (see Baddeley, Allen & Hitch, 2011; Johnstone, 1997).

The simultaneity and interdependence of perception, information extraction, filtration and translation processes, which is further constrained by fully internal processing, modality-independent (i.e. amodal) symbolic transformation at multiple levels, and most of all such limited mental capacities as imagery and working memory (Reid, 2007; Newell & Simon, 1972; Sweller, Merrienboer & Paas, 1998), collectively lead to an increased load on the learner's cognitive system, eventually resulting in learning difficulties (Johnstone 1982 & 1991; Kohl & Finkelstein, 2008; Larkin et al., 1980).

Figure 2 below attempts to captures this in a diagrammatic information processing theory-based model of ER integration.

--Figure 2--

ERs, in this view, act as 'vehicles', tools or transmission media, that simply carry the information – the key element which the cognitive agent (learner) uses to 'extract' meaning and understanding through amodal syntactic processing. Further, the sole purpose of externalization (as an activity) and/or tools such as external visualizations, is to help offload the cognitive load generated during these symbol-based processes.

The emerging 'field' theory-based model of ER integration

Recent field theories of cognition, such as distributed and extended cognition, and embodied cognition[2], strongly attack information processing accounts, by questioning the boundar-

[2]Note that each of the above mentioned field theory in itself has a spectrum of perspectives. The field theory-based model of ER integration presented in this paper utilizes: (i) the extended cognition view, which grants that the mind extends beyond the brain/body to the external structures in the environment; as opposed to distributed cognition, which is more of a systemic perspective examining cognition as emerging out of complex interactions between distributed parts of a system (e.g. socio-material-technical ecologies such as ships). The extended cognition view is an individual (cognitive agent)-centric approach, more concerned with the

ies of the mind (e.g. mind-body dualism), the nature of cognition (e.g. nature of internal representations, or if they exist at all!), and the relationships between mind, body and the external world (Chandrasekharan & Nersessian, 2015; Glenberg, Witt & Metcalfe, 2013; Hutchins, 1995; Kirsh, 2010; Kirsh & Maglio, 1994; Landy, Allen, & Zednik, 2014; White & Pea, 2011). Collectively, these perspectives assert, with neurological evidences, that the mind and cognition are not limited to the brain but extend beyond the physical-biological borders (Clark & Chalmers, 1998, Kirsh, 2010; Kirsh & Maglio, 1994), and that all of cognition is body-based (e.g. Barsalou, 2010; Clark, 1999; Chandrasekharan, 2009; Landy, Allen & Zednik, 2014; Shubotz, 2007). Considered collectively, the new field theories emphasize an agent's sensorimotor interaction, with external structures in her environment, as the central process driving meaning and understanding.

Landy et al. (2014) articulated a distinction between *syntactic/semantic* (i.e. information processing theory) approaches and *constitutive* approaches towards symbolic (representational) reasoning in science and mathematics. In the former, as discussed earlier, symbols (or precisely, objective symbolic information) in ERs are internalized by the cognitive system, and then processed fully inside (Gilmartin, Newell & Simon, 1976), i.e. just using neural processes. In the constitutive account, however, the external symbols, the external (real-world) operations of those symbolic structures, and the sensorimotor system-based interaction processes (such as perception, physical manipulation, etc.) involved in these operations, all are part of the cognition process. The external structures here, thus, play a two-fold constitutive role in connecting with the world; they stand-in for the perceptible as well as imperceptible entities and phenomena in the

relationships between the internal (mental) and the external structures (Hutchins, 2014). (ii) a representationalist embodied cognition view which considers sensorimotor (action system-based) interaction as necessary and fundamental to cognition, and to a coupling between the internal and the external structures (Chandrasekharan & Osbeck, 2010; van Dijk, van der Lugt, & Hummels, 2014).

world, and they act as structures that help constitute concepts related to those entities and phenomena. Since ERs are external structures, operations done on them are critical to understanding the entities and processes represented by ERs. It can thus be said that, a learner's interactions with ERs constitute her understanding of the ERs as well as the entity or phenomenon they represent (Glenberg et al., 2013).

Landy and Goldstone's (2007) experiment demonstrated constitutivity in the context of arithmetic problems; they showed that visual cues, such as spacing the elements in an arithmetic equation differently, or adding lines and circles around equations, influence problem solvers' symbolic reasoning abilities, such as following (or not following) the operator-precedence rule in arithmetic problems. This influence, the authors argue, is a result of perceptual grouping, cued by the structural elements added to the equation, suggesting that external structures, and the perceptual as well as sensorimotor mechanisms involved in a problem solver's experiences with those external structures, constitute the processing and overall understanding (internal representations) of the symbols (Kirshner & Awtry, 2004; Landy et al., 2014).

Another fascinating case is the use of 'mental abacus' - an internal representation of the physical calculation device called abacus. Expert abacus users have been found to develop the ability to use an imagined internal abacus (Stigler, 1984) for rapid arithmetic calculations, especially with larger numbers. However, mental abacus has been investigated broadly as an aspect of visual working memory skill (Barner et al., 2016), characterized by experts' ability to imagine manipulations (visual and motor operations) on external structures in the device (e.g. beads in columns) that represent different numbers (Frank & Barner, 2012). In contrast, students who are not familiar with the abacus imagine the standard written arithmetic algorithms (learned through paper and pencil operations) while solving the same arithmetic tasks.

Pande and Chandrasekharan (2017), through a synthesis of the extended mind, constitutivity and common coding perspectives, argue that constitution and ER integration, are built on sensorimotor integration[3] – the ability or dynamic process of the nervous system to integrate different sensory inputs, and in parallel, transform these inputs into motor schemas, in coordination with object affordances, perception, proprioceptive feedback, forward models, etc. (Machado et al., 2010). As interaction with ERs (such as the abacus) is based on the sensorimotor system (physical manipulation of abacus), and such interactions exploit cognitive/brain mechanisms similar to those involved in sensorimotor integration (Pande & Chandrasekharan, 2017). ER integration (e.g. mental abacus) is a manifestation of two-way coupling between the ERs and their affordances, and the learner's mind. It is facilitated, the authors propose, by: (i) an 'incorporation' process driven by sensorimotor actions or manipulations (Maravita & Iriki, 2004) performed by the learner on the ERs, where ERs become part of, and thus extend, the cognitive system, while also forming and extending the internal model of the represented scientific domain (Chandrasekharan, 2014), and (ii) simulations of ERs and the sensorimotor interactions with

[3]For instance, to perform an action as simple as picking up an object, the sensorimotor system integrates: current state or posture of the body, spatial relations between the body and the object, previous experiences of the object, expected feedback or 'consequences' of touching the object or the action of picking it up, control and regulation of the body movement, etc., through the information coming from the skin, muscles and joints, vestibular system, the motor plan of the action, anticipatory adjustments of the posture and body parts/movements in relation to the motor plan and the position of the object, etc. (Machado et al., 2010).

them, overt as well as covert activations of the sensorimotor system in relation to different ER affordances, that help the learner 'capture' and 'unfold' different states of those ERs at will, thus strengthening the integrative coupling.

In figure 3 below, I synthesize these ideas in, what I call, a field theory model of ER integration.

--Figure 3--

ERs, in this model, are not passive information carriers, but are actively coupled with the cognitive agent's action-perception systems via their affordances. The internal representations corresponding to ERs are considered to encode the sensorimotor aspects of the interactions (as neural activation patterns). The mind, here, *is* this coupled system of internal and external elements, as well as the sensorimotor interactions coupling them together. Meaning is constituted in bodily (or sensorimotor) interaction with the ERs.

The case of mental abacus seems particularly interesting and useful in illustrating this field new model of ER integration, as neurological (e.g. f-MRI) evidence strongly suggests that the two operations – mental abacus and paper/pencil algorithms in imagination (essentially operations based on two different kinds of external structures) – 'run' in different areas of the brain (Chen et al., 2006; Hanakawa et al., 2003). In the case of mental abacus, predominantly visuomotor areas of the brain are activated, whereas imagination of the paper/pencil-based algorithms mostly activates frontal areas of the brain.

How can one explain this result using the classical cognition model? According to the classical information processing model, information in both the abacus as well as paper/pencil-based problem-solving cases would be extracted in a symbolic form, and processed inside the brain amodally (Larkin, 1979; Paivio, 2006). As there is no visual or motor activity components to internally processing the amodal symbolic operations, there should be no activation in the visuomotor areas of the brain in either of the cases.

In contrast, the field theory approaches suggest that, as the mental abacus operations are learned with, and thus rely heavily on, visuomotor operations, imagination based on the 'stored' abacus-based operations would activate visuomotor areas of the brain significantly differently than in the case of imagination of written algorithms (Barsalou, 2008). Similarly, in the latter, operations are based on generating and manipulating text-based images in working memory, so these operations would activate the frontal areas more. This view accounts well for the fMRI data and suggests that internal representations a state of the mind where ERs as well as a learner's interactions on them are coupled to each other. During imagination (e.g. mental abacus), the actions associated with ERs are covertly or overtly activated (Rahaman et al, 2018), resulting in a dynamic (sensorimotor) simulation of the coupled states of the mind. If overt, I suppose, these simulations are instantiated as different actions. This is further supported by evidences for perception-action-imagination coupling in such diverse yet highly relevant areas to ER integration as mental rotation (e.g. Schwartz & Holton, 2000) and language (e.g. Glenberg et al., 2013).

Figure 4 illustrates how abacus-based and paper-pencil-based interactions through arithmetic problem-solving practices are stored and instantiated as qualitatively different behaviors (or simulations).

--Figure 4--

In the light of the above-cited evidence, expert abacus users could be said to develop an internal abacus learned through sensorimotor interaction with the physical abacus. This mental abacus is used to solve arithmetic problems mentally (i.e. in imagination), by 'running' the same sensorimotor interactions internally. Some of this covert sensorimotor process 'leaks' into overt action, leading to gestures similar to the actions on the abacus (top panel in figure 4). Problem-solvers not familiar with the abacus imagine written arithmetic algorithms, learned through paper-pencil-based interactions with the symbols and operations, sometimes resulting in pointing or writing-like finger-movements (on a related note, see Kirsh & Maglio, 1994 for cognitive roles of epistemic actions).

The reader should note that the new field theory-based model does not deny symbols or symbolic relations (Gooding, 2006; Van Dijk, Van Der Lugt & Hummels, 2014). In the above example, bead positions in the abacus have a symbolic value (more broadly, it is a representation) that stand in for numbers. However, focusing on this symbolic nature directs the analysis away from how mental abacus (a thinking process) is generated from manipulating physical abacus (a doing process), as the symbolic view would consider both as based on symbols (Gooding, 2006 – representations as captured patterns of processes – arrested process).

Educational Implications

Considering the involvement of sensorimotor mechanisms in the ER integration and coupling processes, a promising testable conjecture naturally follows from the field theory-based model of ER integration: As a manifestation of integrative coupling, and hence a reorganization of her cognitive system (mind), the development of expertise should result in a fine-tuning of the learner’s sensorimotor system. This is a basic research implication of the new model.

As a corollary, providing novice learners with the 'right', diverse and rich sensorimotor experiences with ERs would lead to integrative coupling and expertise development. This sets the ground for a new direction in intervention development to support ER integration based on the new model.

I discuss these two direct implications to science and mathematics education in the following two sub-sections.

Expertise and the fine-tuning of the sensorimotor system

It is well known that expertise is marked by specific changes in cognition (Glaser & Chi, 1988; Ericsson & Ward, 2014; Sella & Kadosh, 2018) and perception, particularly related to problem-solving (e.g. response times, visual attention, etc.; NRC, 2000). These changes have been documented across multiple domains (e.g. chess, science, mathematics, medicine, etc.; NRC, 2000). de Groot (1978), for instance, was among the first to demonstrate how expert chess players could almost instantaneously *see* problems, as well as possible moves to address those problems, when presented with different configurations of pieces on a chess board. Prevalence of quickened high-quality responses among experts made de Groot conclude that training in chess gradually reduced the time and efforts required to abstract patterns, thus marking the replacement of thought by perception (de Groot, 1978).

Similar reports have been documented in science and mathematics education research domains, especially in terms of how experts differ from novices in the way they pick up information during a problem situation, as a result of their visuomotor (e.g. perceptual and hands-on) experiences with the symbolic structures involved in that problem (Brathwaite et al., 2016; De Wolf et al., 2017; Kellman et al., 2010; Landy & Goldstone, 2007; Rivera & Garrigan, 2016).

Several studies specific to ER integration have popularly investigated eye movements and fixations. As eye-behavior during a task is mostly implicit, (i.e. driven by task demands and not completely in the control of the agent), it is anchored closely to the perceptual-cognitive processes (Henderson & Ferreira, 2013; Irwin, 2004), and can be considered a good sensorimotor marker of those processes (in this case ER integration). During an ER integration task that required imagining diffusion phenomena through its static macro and molecular graphs, Cook et al. (2008) observed the frequency of eye-movements between the different ERs exhibited by students with different domain expertise, and found that students with low expertise made more eye-movements than those with higher expertise. Similar patterns of eye behavior are reported by Kohl and Finkelstein (2008) in a study examining the generation of ERs by three groups of participants – experts, weak novices and strong novices – while solving sets of ER integration problems on electrostatics. Unfortunately, many of these studies are grounded in the information processing account of ER integration, and tend to regard the distinct visuomotor behavior among experts as acquired strategies to efficiently extract relevant information from ERs, and to counter or optimise cognitive load by utilizing such techniques as chunking (e.g. the ‘top-down’, ‘bottom-up’ explanations, etc.; Gegenfurtner, Lehtinen, & Säljö, 2011; Goldstone, 1998; Kundel, Nodine, Conant, & Weinstein, 2007; Lowe, 2015).

Moreover, although it is well known within scientific communities that experts 'act' on scientific concepts, procedures and ERs in a significantly different manner than novices, very few intersubjective scientific evidences have been documented, apart from expert-novice differences in patterns of attention and/or navigation over visual stimuli.

A promising direction for future research, grounded within the new field theory model of ER integration and expertise development, could employ a combination of different biometric technology tools (e.g. eye-tracking, gesture-control, haptic devices) to help us better understand the relationships between ER integration, expertise development and interactivity. Findings from such research could provide important insights into if and how this knowledge could be used to design novel sensorimotor interaction-based educational interventions in science and mathematics education.

Designing to support ER integration

Shifting design focus from cognitive offloading to sensorimotor interaction. With the advent of different new media interfaces, not only has it become inevitable but also advantageous to appropriate these interfaces in science and mathematics education for more effective learning. Especially in the context of ER integration, these technology interfaces provide a plethora of ways to access and interact with ERs and their affordances.

However, although there is a general agreement in the learning sciences and educational technology communities over the importance of interactivity, not all interactivity-based learning designs reflect the criticality of sensorimotor interaction to learning. Information processing and cognitive load-based frameworks are still relatively dominant even in recent work (Malinverni & Peres, 2014). The leading instructional design frameworks provided by cognitive load theory and cognitive theory of multimedia learning (Mayer, 1999 & 2005; Sweller 1998), for instance, grant learning as the formation of internal representations as a result of encoding of features observed by a learner in her environment. Central to these frameworks is the principle that the construction of deeper knowledge and formation of richer mental models can be better supported through multiple media (e.g. texts and pictures) as opposed to single or limited media, as learners have greater processing abilities as the processing is distributed/complemented through different media. While this principle is not entirely incommensurable, or in conflict, with the field theory approaches, the fact that these frameworks further regard ERs as mental scaffolds that facilitate information extraction and cognitive offloading mediated by interaction is vexed. Ascribing only a mediating (and not a constitutive) role to interactivity (and to ERs), these frameworks and inspired interventions (including those incorporating haptic feedback-like recent technologies) tend to overemphasize the structure and features of ERs (e.g. their visual nature, multimodal appeal, assistance in offloading; Lindner, Eitel, Strobel & Koller, 2017; Strobel, Lindner, Sass & Koller, 2018). As the emphasis on symbols and symbolic processing cannot account for how 'doing' (e.g. interaction with ERs) is related to mental processes (e.g. integrative coupling), such frameworks grounded in the information processing approach can lead to design principles only peripheral to cognitive mechanisms (e.g. only offloading as the purpose of interactivity and manipulation, multiple media as facilitators of multiple perspectives to concepts).

Alternatively, constructionism for instance (Papert & Harel, 1991), where the central role of interactivity is the support it provides for individual or collaborative building of mathematical objects and complex systems has inspired largely successful interactive computer simulation interfaces such as NetLogo for science learning (e.g. Wilensky & Reisman, 2006), through manipulation-based programming. Recent research, under the broad umbrella of embodied design, also shows positive results in terms of effectiveness of technology designs focusing on interactivity

(Abrahamson & Sánchez-García, 2016; Borar et al., 2017), especially new media-based manipulation. De Freitas and Sinclair (2014), and Sfard (1991 & 2000) demonstrated how learning through manipulative gestures in new computational media are similar to the process of gestures during the mathematical discovery process, which are hypothesized to be part of the mechanism that helps shift body-based intuitions (about possible results) into external symbolic proofs built using known and accepted mathematical structures. Kothiyal et al. (2014) argued, through testing of a fully manipulable multi-representational simulation interface that focuses on the enactivity/ manipulability of abstract ERs (e.g equations) related to the concept of oscillation, that the enactivity is critical for ER integration and imagination.

Of recent, the emerging 4E (embodied, embedded, extended and enactive) cognition approaches to learning do seem to contribute concretely in this regard; however, they vary greatly from each other in terms of the nature and degrees of interactivity (Weisberg & Newcombe, 2017), theorized relationships between sensorimotor interaction and body-based learning (Lindgren & Johnson-Glenberg, 2013; Skulmowski & Rey, 2018), and the current computer simulation interface designs (Malinverni & Peres, 2014). Also, they do not specifically apply to ER integration in science and mathematics.

The field theory model helps explain most of these existing but distributed works in the context of ER integration and expertise development in science and mathematics, by accounting for the *processes* involved in the doing-to-thinking shift, as well as the cognitive and neural *mechanisms* involved (Rahaman et al., 2018). Actions performed on manipulable and enactive ERs, according to this new model, help in ER integration and learning because actions are inherently integrative in nature (Machado et al., 2010), as every action requires a complex integration process, bringing together objects, forward models and feedback from various channels (e.g. visual, tactile, proprioception).

This has serious implications for (educational technology) intervention design, especially for designs utilizing the potential of new media, as new media could allow the educational technology community to re-represent ERs (e.g. in immersive and interactive VR, run dynamic manipulable simulations of abstract ERs), and reveal and generate newer affordances of those ERs for learners to perceive, interact with, enact and embody those ERs (and the phenomena they represent) in a number of potential ways – thus ensuring richer sensorimotor interaction. Moreover, these technologies make possible learner manipulation of ERs and observation of effects of those manipulations in real time, thus facilitating external and internal coupling of static and dynamic states of ERs at will.

Designing beyond sensorimotor interaction. Although there is ample theoretical and empirical support that learners benefit from interactivity and enaction, recent research shows that it would be too premature to assume that interactivity alone is enough for effective learning. Kothiyal et al. (2017), for instance, reported a design-based research experiment conducted at an elementary school to test the effectiveness of a fully manipulable multi-representational computer simulation interface designed to support ER integration around the concept of oscillation. Through eye-tracking and interaction-tracking, the study revealed that students who were more successful in solving ER integration and transfer problems related to oscillation, always exhibited, although with considerable individual variations, both quantitatively and qualitatively rich patterns of visuomotor interactions with the interface. However, a few unsuccessful students also showed visuomotor interaction patterns similar to those of the successful ones, suggesting that high interactivity is necessary but may not always lead to ER integration. The individual variations also suggest that there could be multiple effective ways of interacting with ERs.

Similarly, a group of researchers incorporated socio-cultural contexts and inter-student collaboration possibilities with enactivity in the play-environments where a group of students interact with simulations (mixed-reality systems) with their full bodies by collectively and collaboratively enacting avatars representative of scientific concepts (Danish et al., 2015; Enyedy et al., 2012). Students, in this study, moved around in the classroom space, play-acting particulate matter (say water molecules), to arrive at an understanding of how particles in different states of matter (e.g. water, ice, vapor) would behave in a range of everyday situations such as a freezing cold day, heating or boiling water, etc.. The students could see their 'enaction' projected into a computer simulation where an avatar in the form of a particle is displayed. The authors found that although this type of enaction was useful to direct students' attention towards key concepts and help them make their own choices and decisions in arriving at a proper understanding of these concepts, mere embodiment and enaction were not sufficient to bring clarity to nuanced concepts such as energy and its relationships with states of matter, as some (groups of) students tended to confuse between concepts such as energy and matter. This suggested the authors to incorporate the element of 'reflection' in their intervention design, that helped students reflect on their own actions and observed effects of those actions on avatar behavior.

These two examples indicate that there is more to ER integration and learning than sesnorimotor interaction and integrative coupling. More research is needed to understand and better support the cognitive mechanisms involved in ER integration and learning in science and mathematics education.

Conclusion

Extended and embodied cognition (or the whole 4E package for that matter; Menary, 2010) are now considered mainstream theories of the mind in cognitive science (Fiore, 2019). While by and large the learning sciences and education research communities, and communities in allied fields also seem to increasingly sympathise with recent theoretical and intervention design approaches to learning based on these theories, they are still to be adopted as mainstream approaches to understanding and supporting learning (Fiore, 2019) in science and mathematics. This paper contributes to the growing body of work trying to convince various communities the strength of these approaches, and their potential intervention design applications.

In this paper, I abstracted out and attacked the currently dominant information processing approaches to ER integration in science and mathematics – one's ability to integrate scientific and mathematical ERs with her internal (mental) schema. In contrast to these approaches, I proposed a new model of cognitive mechanisms based on recent advances in extended, distributed and embodied cognition (i.e. field theories in cognitive science), and with empirical support from the fields of cognitive science, science and mathematics education, and neuroscience. As a major contribution to the field, this paper provides insights into how sensorimotor interactions relate to ER integration, and perhaps the associated conceptual learning in science and mathematics, as a manifestation of a coupling between the ERs and the learner's mind. Two major conjectures emerged from the new model: (i) development of ER integration and expertise in science and mathematics should result in fine-tuning of the sensorimotor system, as a result of integrative coupling, and (ii) richer sensorimotor interaction should facilitate ER integration development; hence, novel technology interfaces designed to support ER integration and learning should promote interactivity, enaction and embodiment. Empirical evidences discussed in support of the conjectures show promising direction for future research with respect to the field theory-based model. Specifically, the new view offers the possibility of providing critical direction to the design of new computational media for learning, where embodied controllers such as multi-touch devices, Leap Motion, Kinect, Real Sense and Virtual Reality are used to develop new learning experiences, i.e. constitute new ways of integrating ERs (Abrahamson & Sánchez-García, 2016; Borar et al., 2017; Dickes et al., 2016; Karnam et al., 2016; Ottmar et al., 2015, Sinclair & De Freitas, 2014).

However, it is still too early, especially in relation to the design conjecture emerging from the model, to provide definite and tested intervention design principles as the field approaches to cognition and learning are relatively recent. Further, there are other complex aspects to learning and technology interface design for learning based on new media; for instance, it is not clear to what extent is interactivity beneficial, what is the relationship between interactivity and motivation, how can the field theories incorporate affordances of and phenomena (e.g. sense of immersion and presence, 3-dimensional manipulation) associated with such new technologies as VR and AR (Johnson-Glenberg, 2018).

Design-based research, I opine, would be the best way forward to explore these directions, and answer the questions they open.

References

Abrahamson, D., & Sánchez-García, R. (2016). Learning Is Moving in New Ways: The Ecological Dynamics of Mathematics Education. *Journal of the Learning Sciences*, *25*(2), 203–239.

Ainsworth, S. (2006). DeFT: A conceptual framework for considering learning with multiple representations. *Learning and instruction*, *16*(3), 183-198.

Baddeley, A., Allen, R., & Hitch, G. (2011). Binding in visual working memory: The role of the episodic buffer. *Neuropsychologia*, 49(6), 1393-1400.

Barner, D., Alvarez, G., Sullivan, J., Brooks, N., Srinivasan, M., & Frank, M. C. (2016). Learning mathematics in a visuospatial format: A randomized, controlled trial of mental abacus instruction. *Child Development*, *87*(4), 1146-1158.

Barsalou, L. (2008). Grounded Cognition. *Annual Review of Psychology*, 59(1), 617-645.

Barsalou, L. W. (2010). Introduction to 30th anniversary perspectives on cognitive science: Past, present, and future. *Topics in cognitive science*, *2*(3), 322-327.

Battista, M. T. (2003). Understanding students'. In *thinking about area and volume measurement. Learning and teaching measurement* (pp. 122–142).

Bodner, G. M., & Domin, D. S. (2000). Mental models: The role of representations in problem solving in chemistry. *University Chemistry Education*, *4*(1).

Borar, P., Karnam, D., Agrawal, H., & Chandrasekharan, S. (2017, September). Augmenting the Textbook for Enaction: Designing Media for Participatory Learning in Classrooms. In *IFIP Conference on Human-Computer Interaction* (pp. 336-339). Springer, Cham.

Braithwaite, D. W., Goldstone, R. L., van der Maas, H. L. J., & Landy, D. H. (2016). Non-formal mechanisms in mathematical cognitive development: The case of arithmetic. *Cognition*, *149*, 40–55.

Çelik, D., & Sağlam Arslan, A. (2012). The Analysis of Teacher Candidates' Translating Skills in Multiple Representations. *Elementary Education Online*, *11*(1), 239–250.

Chandrasekharan, S. (2009). Building to discover: a common coding model. *Cognitive Science*, *33*(6), 1059–1086.

Chandrasekharan, S. (2014). Becoming knowledge: Cognitive and neural mechanisms that support scientific intuition. *Rational Intuition: Philosophical Roots, Scientific Investigations*, 307–337.

Chandrasekharan, S., & Nersessian, N. J. (2015). Building Cognition: The Construction of Computational Representations for Scientific Discovery. *Cognitive Science*, 39(8), 1727–1763.

Chandrasekharan, S., & Osbeck, L. (2010). Rethinking Situatedness. *Theory & Psychology*, *20*(2), 171–207.

Chen, F., Hu, Z., Zhao, X., Wang, R., Yang, Z., Wang, X., & Tang, X. (2006). Neural correlates of serial abacus mental calculation in children: A functional MRI study. *Neuroscience Letters*, 403(1), 46–51.

Chi, M. T., Feltovich, P. J., & Glaser, R. (1981). Categorization and representation of physics problems by experts and novices. *Cognitive Science*, 5(2), 121–152.

Clark, A. (1999). An embodied cognitive science?. *Trends in cognitive sciences*, *3*(9), 345-351.

Clark, A., & Chalmers, D. (1998). The extended mind. *Analysis*, *58*(1), 7-19.

Cook, M., Wiebe, E. N., & Carter, G. (2008). The influence of prior knowledge on viewing and interpreting graphics with macroscopic and molecular representations. *Science Education*, *92*(5), 848–867.

Danish, J.A., Enyedy, N., Saleh, A., Lee, C. and Andrade, A. (2015). Science Through Technology Enhanced Play: Designing to Support Reflection Through Play and Embodiment. *Proceedings of the 11th International Conference on Computer Supported Collaborative Learning (CSCL2015)* (2015), 332–339.

De Freitas, E., & Sinclair, N. (2014). *Mathematics and the body: Material entanglements in the classroom*. Cambridge University Press.

De Jong, T., & Van Joolingen, W. R. (1998). Scientific Discovery Learning with Computer Simulations of Conceptual Domains. *Review of Educational Research*, 68(2), 179–201.

DeWolf, M., Son, J. Y., Bassok, M., & Holyoak, K. J. (2017). Relational Priming Based on a Multiplicative Schema for Whole Numbers and Fractions. *Cognitive Science*. https://doi.org/10.1111/cogs.12468

Dickes, A. C., Sengupta, P., Farris, A. V., & Basu, S. (2016). Development of Mechanistic Reasoning and Multilevel Explanations of Ecology in Third Grade Using Agent−Based Models. *Science Education*. Retrieved from http://onlinelibrary.wiley.com/doi/10.1002/sce.21217/full

Enyedy, N., Danish, J.A., Delacruz, G. and Kumar, M. (2012). Learning physics through play in an augmented reality environment. *International Journal of Computer- Supported Collaborative Learning*. 7, 3 (Jul. 2012), 347– 378.

Ericsson, K. A., & Ward, P. (2014). Capturing the naturally occurring superior performance of experts in the laboratory: Toward a science of expert and exceptional performance. *Current Directions in Psychological Science*, 16, 346–350.

Fiore, S. (2019). Integrating Theorizing on Embodied, Enactive, Extended, and Embedded Cognition to Augment CSCL Research, Keynote Talk at the *13th International Conference on Computer-Supported Collaborative Learning (CSCL)*, International Society of the Learning Sciences (ISLS), Lyon, France.

Frank, M. C., & Barner, D. (2012). Representing exact number visually using mental abacus. *Journal of Experimental Psychology: General, 141*(1), 134-149. http://dx.doi.org/10.1037/a0024427

Gegenfurtner, A., Lehtinen, E., & Säljö, R. (2011). Expertise Differences in the Comprehension of Visualizations: a Meta-Analysis of Eye-Tracking Research in Professional Domains. *Educational Psychology Review*, *23*(4), 523–552.

Chi, Michelene T. H, Glaser, Robert, & Farr, Marshall J. (1988). *The nature of expertise*. Hillsdale, N.J: Lawrence Erlbaum.

Glenberg, A., Witt, J., & Metcalfe, J. (2013). From the Revolution to Embodiment: 25 Years of Cognitive Psychology. *Perspectives on Psychological Science*, 8(5), 573-585.

Goldstone, R. L. (1998). Perceptual learning. *Annual Review of Psychology*, *49*, 585–612.

Gooding, D. C. (2006). From Phenomenology to Field Theory: Faraday's Visual Reasoning. *Perspectives on Science: Historical, Philosophical, Social*, 14(1), 40–65.

Hanakawa, T., Honda, M., Okada, T., Fukuyama, H., & Shibasaki, H. (2003). Neural correlates underlying mental calculation in abacus experts: A functional magnetic resonance imaging study. *NeuroImage*, 19(2), 296–307.

Henderson, J. M., & Ferreira, F. (2004). Scene Perception for Psycholinguists. In J. M. Henderson & F. Ferreira (Eds.), *The interface of language, vision, and action: Eye movements and the visual world* (pp. 1–58). New York: Psychology Press.

Hinton, M. E., & Nakhleh, M. B. (1999). Students' microscopic, macroscopic, and symbolic representations of chemical reactions. *Chemical Educator*. Retrieved from http://link.springer.com/article/10.1007/s00897990325a

Hutchins, E. (1995). How a Cockpit Remembers Its Speeds. *Cognitive Science*, 19(3), 265–288.

Hutchins, E. (2014). The cultural ecosystem of human cognition. *Philosophical Psychology*, *27*(1), 34–49.

Irwin, D. E. (2004). Fixation location and fixation duration as indices of cognitive processing. *The Interface of Language, Vision, and Action: Eye Movements and the Visual World*, *217*, 105–133.

Johnson-Glenberg MC (2018) Immersive VR and Education: Embodied Design Principles That Include Gesture and Hand Controls. *Frontiers in Robotics and AI,* 5:81. doi: 10.3389/frobt.2018.00081

Johnstone, A. H. (1982). Macro and microchemistry. *School Science Review*, 64(227), 377–379.

Johnstone, A. H. (1991). Why is science difficult to learn? Things are seldom what they seem. *Journal of Computer Assisted Learning*, 7(2), 75–83.

Johnstone, A. H. (2000). TEACHING OF CHEMISTRY - LOGICAL OR PSYCHOLOGICAL? *Chemistry Education Research and Practice*, 1(1), 9–15.

Johnstone A.H., (1997), Chemistry teaching, science or alchemy?, *Journal of Chemical Education,* **7**, 262-268.

Johri, A., Roth, W., & Olds, B. (2013). The Role of Representations in Engineering Practices: Taking a Turn towards Inscriptions. *Journal of Engineering Education*, 102(1), 2-19.

Kamii, C., & Kysh, J. (2006). The difficulty of "length× width": Is a square the unit of measurement? *The Journal of Mathematical Behavior*, *25*(2), 105–115.

Karnam, D., Agrawal, H., Mishra, D., & Chandrasekharan, S. (2016). Interactive vectors for model-based reasoning. In W. Chen, T. Supnithi, A. F. Mohd Ayub, M. Mavinkurve, T. Kojiri, W. Chen, J. Yang, S. Murthy, S. L. Wong & S. Iyer. (Eds.), *Proceedings of the 24th international conference on computers in education* (pp. 401–406). Mumbai, India: Asia-Pacific Society for Computers in Education.

Kellman, P. J., & Garrigan, P. (2009). Perceptual learning and human expertise. *Physics of Life Reviews*, *6*(2), 53–84.

Kirsh, D. (2010). Thinking with external representations. *AI & SOCIETY*, 25(4), 441– 454.

Kirsh, D., & Maglio, P. (1994). On Distinguishing Epistemic from Pragmatic Action. *Cognitive Science*, 18(4), 513–549.

Kirshner, D., & Awtry, T. (2004). Visual Salience of Algebraic Transformations. *Journal for Research in Mathematics Education*, *35*(4), 224–257.

Kohl, P. B., & Finkelstein, N. D. (2008). Patterns of multiple representation use by experts and novices during physics problem solving. *Physical Review Special Topics - Physics Education Research*, 4(1), 010111.

Kothiyal, A., Majumdar, R., Pande, P., AGARWAL, H., RANKA, A., & CHANDRASEKHARAN, S. (2014). How does representational competence develop? Explorations using a fully controllable interface and eye-tracking. In *Proceedings of the 22nd international conference on computers in education*(pp. 738-743). Nara, Japan: Asia-Pacific Society for Computers in Education.

Kozma, R. (2003). The material features of multiple representations and their cognitive and social affordances for science understanding. *Learning and Instruction*, 13(2), 205–226.

Kozma, R., & Russell, J. (1997). Multimedia and understanding: Expert and novice responses to different representations of chemical phenomena. *Journal of Research in Science Teaching*, 34(9), 949-968.

Kozma, R., & Russell, J. (2005). Students Becoming Chemists: Developing Representational Competence. In *Visualization in Science Education* (pp. 121–145). Springer, Dordrecht.

Kundel, H. L., Nodine, C. F., Conant, E. F., & Weinstein, S. P. (2007). Holistic component of image perception in mammogram interpretation: gaze-tracking study. *Radiology*, *242*(2), 396–402.

Landy, D., Allen, C., & Zednik, C. (2014). A perceptual account of symbolic reasoning. *Frontiers in Psychology*, 5, 275.

Landy, D., & Goldstone, R. L. (2007). How abstract is symbolic thought? *Journal of Experimental Psychology. Learning, Memory, and Cognition*, *33*(4), 720–733.

Larkin, J.H. (1979). Processing Information for Effective Problem Solving. *Engineering Education*, 70(3), 285-28.

Larkin, J., McDermott, J., Simon, D. P., & Simon, H. A. (1980). Expert and novice performance in solving physics problems. *Science*, 208(4450), 1335–1342.

Lesh, R., Post, T., & Behr, M. (1987). Representations and translations among representations in mathematics learning and problem solving. Problems of Representation in the Teaching and

Lindgren, R., & Johnson-Glenberg, M. (2013). Emboldened by Embodiment: Six Precepts for Research on Embodied Learning and Mixed Reality. *Educational Researcher*, 42(8), 445-452.

Lindner, M. A., Eitel, A., Strobel, B., & Köller, O. (2017). Identifying processes underlying the multimedia effect in testing: An eye-movement analysis. *Learning and instruction*, *47*, 91-102.

Lowe, R. (2015). Perceptual Learning in the Comprehension of Animations and Animated Diagrams. In *The Cambridge Handbook of Applied Perception Research* (pp. 692-710). Cambridge University Press.

Madden, S. P., Jones, L. L., & Rahm, J. (2011). The role of multiple representations in the understanding of ideal gas problems. *Chemistry Education Research and Practice*, *12*(3), 283–293.

Majumdar, R., Kothiyal, A., Ranka, A., Pande, P., Murthy, S., Agarwal, H., & Chandrasekharan, S. (n.d.). The Enactive equation: Exploring How Multiple External Representations are Integrated, Using a Fully Controllable Interface and Eye-Tracking. In 2014 *IEEE Sixth International Conference on Technology for Education* (pp. 233–240). Amritapuri, Clappana P. O., India: IEEE.

Malinverni, L., & Pares, N. (2014). Learning of Abstract Concepts through Full-Body Interaction: A Systematic Review. *Educational Technology & Society*, *17* (4), 100–116.

Maravita, A., & Iriki, A. (2004). Tools for the body (schema). *Trends in cognitive sciences*, *8*(2), 79-86.

Mayer, R. (1999). A Cognitive Theory of Multimedia Learning. *International Journal of Educational Research*, 31(7), 612.

Mayer, R. (2005). Cognitive Theory of Multimedia Learning. In *The Cambridge Handbook of Multimedia Learning* (pp. 31-48). Cambridge University Press.

Menary, R. (2010). Introduction to the special issue on 4E cognition. *Phenomenology and the Cognitive Sciences*, *9*(4), 459-463.

Newell, A., & Simon, H. A. (1972). *Human problem solving* (Vol. 104, No. 9). Englewood Cliffs, NJ: Prentice-Hall.

Gilmartin, K. J., Newell, A., & Simon, H. A. (1976). A program modeling short-term memory under strategy control. *The structure of human memory*, 15-30.

Nitz, S., Nerdel, C., & Prechtl, H. (2012). Modeling the Relationship between Representational Competence and Domain Knowledge. In E. de Vries & K. Scheiter (Eds.), *of Representational Technologies in Education and Training?* (pp. 160–162).

Nitz, S., & Tippett, C. D. (2012). Measuring Representational Competence in Science. In E. de Vries & K. Scheiter (Eds.), *of Representational Technologies in Education and Training?* (pp. 163–165).

Ottmar, E.R., Landy, D., Goldstone, R., & Weitnauer, E. (2015). Getting from here to there: Testing the effectiveness of an interactive mathematics intervention embedding perceptual learning. *Proceedings of the 37th Annual Conference of the Cognitive Science Society*. Pasadena, CA: Cognitive Science Society.

Ozogul, G., Johnson, A. M., Moreno, R., & Reisslein, M. (2012). Technological Literacy Learning With Cumulative and Stepwise Integration of Equations Into Electrical Circuit Diagrams. *IEEE Transactions on Education*, *55*(4), 480–487.

Paivio, A. (2006). Dual Coding Theory and Education. *The Conference on Pathways to Literacy Achievement for High Poverty Children*. 1-20.

Pande, P., & Chandrasekharan, S. (2017). Representational competence: towards a distributed and embodied cognition account. *Studies in Science Education*, 53(1), 1– 43.

Pape, S. J., & Tchoshanov, M. A. (2001). The Role of Representation(s) in Developing Mathematical Understanding. *Theory into Practice*, 40(2), 118–127.

Papert, S., & Harel, I. (1991). Situating constructionism. *Constructionism, 36*(2), 1-11.

Pretz, J.E., Naples, A.J., & Sternberg, R.J. (2003). Recognizing, Defining and Representing Problems. In J.E. Davison & R.J. Sternberg (Eds.) *The Psychology of Problem Solving* (pp 3-30). Cambridge, UK: Cambridge University Press.

Rahaman, J., Agrawal, H., Srivastava, N., & Chandrasekharan, S. (2017). Recombinant Enaction: Manipulatives Generate New Procedures in the Imagination, by Extending and Recombining Action Spaces. *Cognitive Science*. https://doi.org/10.1111/cogs.12518

Reid, N. (2008). A scientific approach to the teaching of chemistry. What do we know about how students learn in the sciences, and how can we make our teaching match this to maximise performance? *Chemistry Education Research and Practice*, 9(1), 51-59.

Rivera, J., & Garrigan, P. (2016). Persistent perceptual grouping effects in the evaluation of simple arithmetic expressions. *Memory & Cognition*, *44*(5), 750–761.

Rutten, N., van Joolingen, W. R., & van der Veen, J. T. (2012). The learning effects of computer simulations in science education. *Computers & Education*, 58(1), 136–153.

Sanger, M. J. (2005). Evaluating students' conceptual understanding of balanced equations and stoichiometric ratios using a particulate drawing. *Journal of Chemical Education*, *82*(1), 131–134.

Sharlin, E., Watson, B., Kitamura, Y., Kishino, F., & Itoh, Y. (2004). On tangible user interfaces, humans and spatiality. *Personal and Ubiquitous Computing*, *8*(5), 338–346. http://doi.org/10.1007/s00779-004-0296-5

Schubotz, R. I. (2007). Prediction of external events with our motor system: towards a new framework. *Trends in Cognitive Sciences*, *11*(5), 211–218.

Schwartz, D., & Holton, D. (2000). Tool Use and the Effect of Action on the Imagination. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, 26(6), 1655-1665.

Sella, F., & Cohen Kadosh, R. (2018). What Expertise Can Tell About Mathematical Learning and Cognition. *Mind, Brain, and Education (Special Issue)*, 1-7.

Sfard, A. (1991). On the dual nature of mathematical conceptions: Reflections on processes and objects as different sides of the same coin. *Educational studies in mathematics*, *22*(1), 1-36.

Sfard, A. (2000). Steering (dis) course between metaphors and rigor: Using focal analysis to investigate an emergence of mathematical objects. *Journal for Research in Mathematics Education*, 296-327.

Sinclair, N., & de Freitas, E. (2014). The haptic nature of gesture: Rethinking gesture with new multitouch digital technologies. *Gesture*, *14*(3), 351-374.

Skulmowski, A., & Rey, G. (2018). Embodied learning: Introducing a taxonomy based on bodily engagement and task integration. *Cognitive Research: Principles and Implications*, 3(1), 1-10.

Stieff, M. (2011). Improving representational competence using molecular simulations embedded in inquiry activities. *Journal of Research in Science Teaching*, 48(10), 1137-1158.

Stigler, J. W. (1984). “Mental abacus”: The effect of abacus training on Chinese children's mental calculation. *Cognitive Psychology*, *16*(2), 145-176.

Strobel, B., Lindner, M. A., Saß, S., & Köller, O. (2018). Task-irrelevant data impair processing of graph reading tasks: An eye tracking study. *Learning and Instruction*, *55*, 139-147.

Sweller, J. (1998). Can we measure working memory without contamination from knowledge held in long-term memory? *Behavioral and Brain Sciences*, 21(6), 845-846.

Sweller, J., Van Merrienboer, J., & Paas, J. (1998). Cognitive Architecture and Instructional Design. *Educational Psychology Review*, 10(3), 251-296.

Tsui, C.-Y., & Treagust, D. F. (2013). Introduction to Multiple Representations: Their Importance in Biology and Biological Education. In *Multiple Representations in Biological Education* (pp. 3–18). Springer, Dordrecht.

Van Dijk, J., Van Der Lugt, R., & Hummels, C. (2014, February). Beyond distributed representation: embodied cognition design supporting socio-sensorimotor couplings. In *Proceedings of the 8th International Conference on Tangible, Embedded and Embodied Interaction* (pp. 181-188). ACM.

Weisberg, S. M., & Newcombe, N. S. (2017). Embodied cognition and STEM learning: overview of a topical collection. *Cognitive Research: Principles and Implications, 2*, 38. https://doi.org/10.1186/s41235-017-0071-6.

White, T., & Pea, R. (2011). Distributed by Design: On the Promises and Pitfalls of Collaborative Learning with Multiple Representations. *Journal of the Learning Sciences*, 20(3), 489–547.

Wilensky, U., & Reisman, K. (2006). Thinking like a wolf, a sheep, or a firefly: Learning biology through constructing and testing computational theories—an embodied modeling approach. *Cognition and instruction*, *24*(2), 171-209.

Wu, H.-K., Krajcik, J. S., & Soloway, E. (2001). Promoting understanding of chemical representations: Students’ use of a visualization tool in the classroom. *Journal of Research in Science Teaching*, *38*(7), 821–842.

Wu, H. K., & Shah, P. (2004). Exploring visuospatial thinking in chemistry learning. *Science Education*, *88*(3), 465–492.

Figures

--Figure 1--

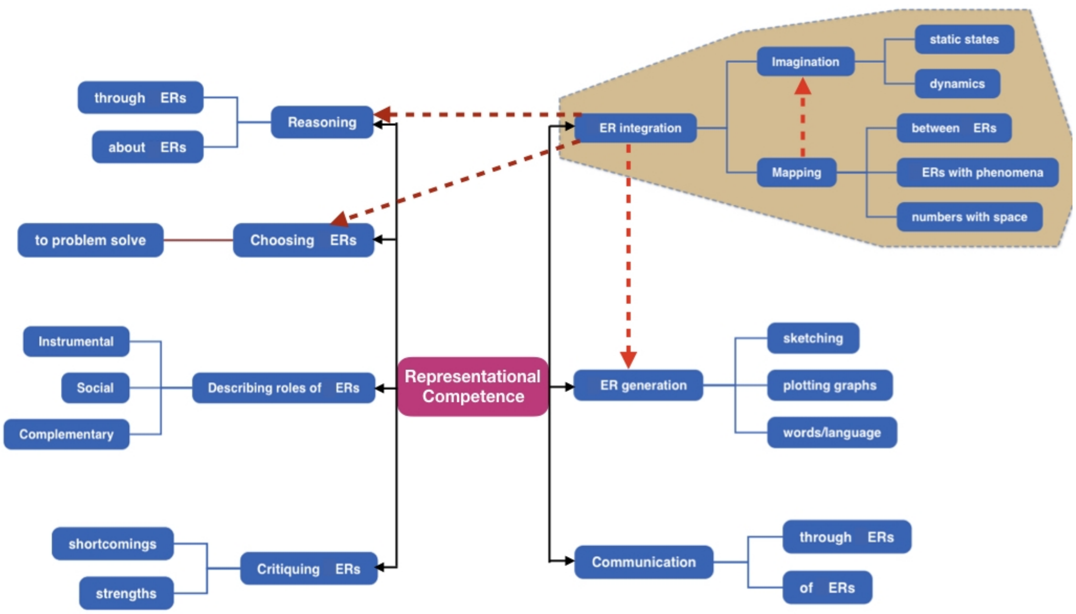

Figure 1. The different facets of RC abilities as identified in the literature. The scope of this paper is limited to ER integration (shaded area). Due to the interconnections between the different facets of RC (not indicated in the figure to avoid complexity), the thesis presented in this paper implicitly extends to such concepts as reasoning around ERs, choice of ERs or the relationships between them, and ER generation.

--Figure 2--

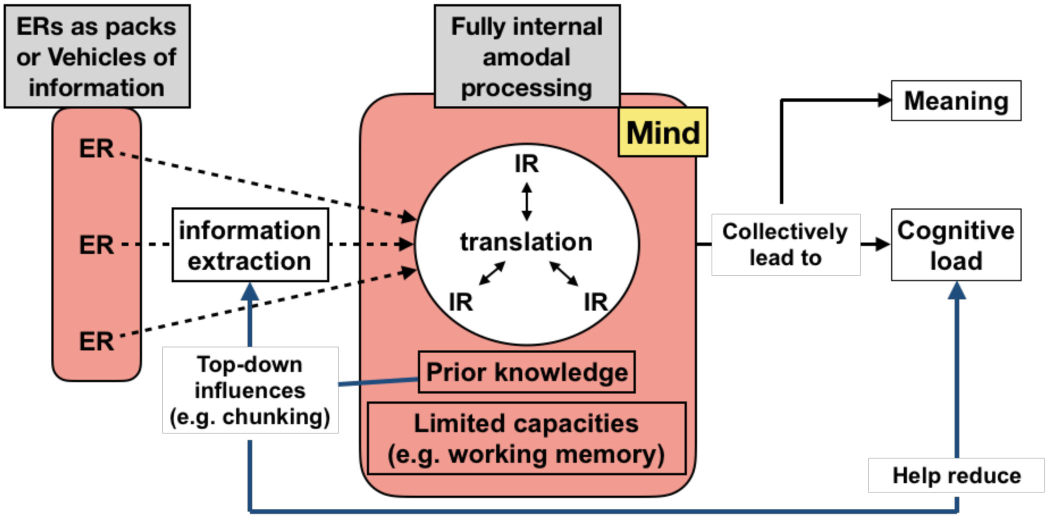

Figure 2. A classical information processing model of ER integration. In this model, meaning is 'extracted' through amodal and syntactic processing (e.g. Newell & Simon, 1972) of information contained in ERs.

--Figure 3--

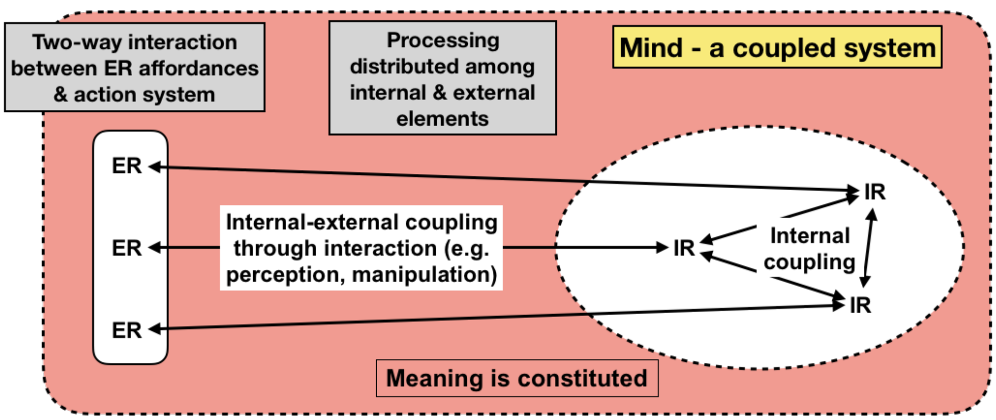

Figure 3. A general field theory model of ER integration.

--Figure 4--

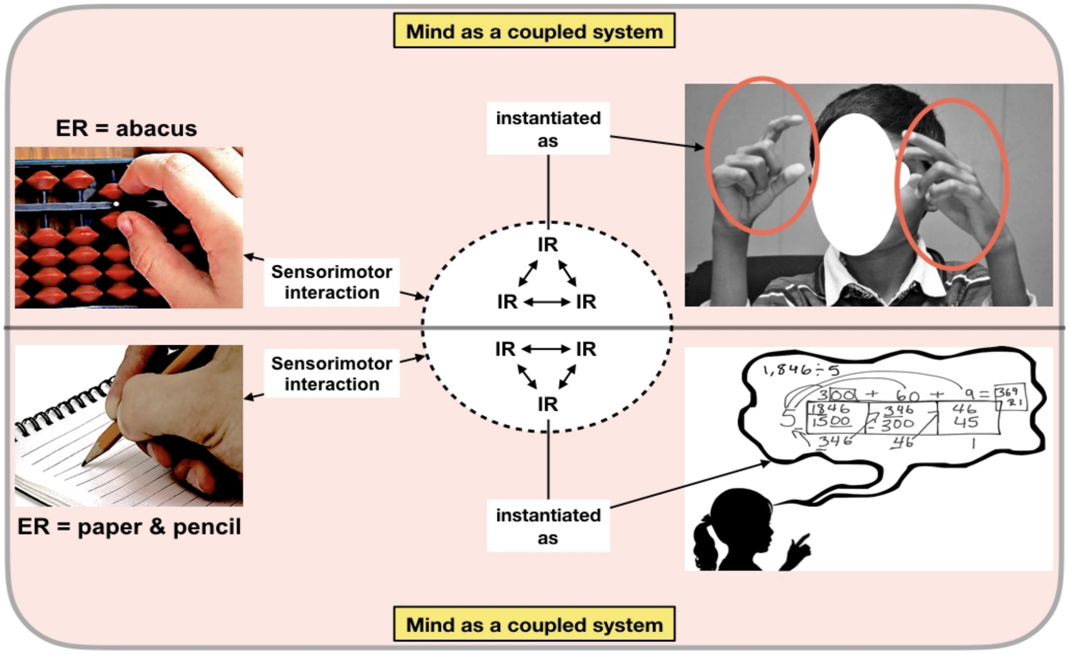

Figure 4. Development of different internal representations based on sensorimotor interaction with different ERs.