The Summit of Ancient Latin Mathematical Competence
Apuleius and Augustine
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THE SUMMIT OF ANCIENT LATIN MATHEMATICAL
COMPETENCE: APULEIUS AND AUGUSTINE

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Dedicated to
Samia Ahasniou and Saliha Mostefai

Abstract. According to all we know, Latin Antiquity was utterly unfamiliar
with the theoretical aspects of mathematics; Quintilian did not know finger
reckoning from geometry, while Cicero explains that the Romans were not
interested. Authors of handbooks in the liberal arts may know some definitions
from the Elements and perhaps some enunciations, but hardly understood what a
proof is. Symptomatic is what Latin authors have to tell about Archimedes: the
story about his death and his defense of Syracuse; the anecdote about Hieron's
crown and Archimedes's exposure of the fraud; his mechanical model of the
heavenly system; at most they know that he drew figures. There is never a hint
that such figures were connected to geometrical or mechanical proofs, theorems
or theory. But there are two exceptions to this rule, both Berbers (Africani), and
both conscious of being so: Apuleius of Madaura, and Augustine of Hippo (and
both obviously much better known for other things). Even though the Western
part of Northern Africa acquired the Latin tongue while the Eastern part spoke
Greek, some of its intellectuals were drawn to advanced Greek thought in a way
those of the remaining Latin world were not, spellbound as the latter were in the
charms of rhetoric.

Keywords: Archimedes, Apuleius, Augustine, Latin mathematical culture,
pre-Islamic Maghreb
PREAMBLE ON TWO LEGS

1. At the 3ième Colloque Maghrébin, held in Tipaza in December 1990, the two Algerian colleagues to whom this essay is dedicated asked me whether I knew anything about pre-Islamic mathematics in the Maghreb. Spontaneously I mentioned Augustine of Hippo, though at the moment I only knew his apparent familiarity with the *Elements* – admittedly a somewhat meagre foundation.

2. In 1595, Bernardino Baldi (ed. Narducci, p. 453) quoted Federico Commandino for the opinion that “that one can hardly call himself a mathematician who has not studied the works of Archimedes”.¹

3. Accordingly (and since Commandino was not alone with this opinion), familiarity with *Archimedes the mathematician* (as distinguished from the engineer, the good servant to king and country, and the fatally distracted genius) may thus be taken as an indicator of mathematical competence.

4. Having for other reasons had to work on references to Archimedes in the Latin world, I discovered supplementary reasons to ascribe mathematical competence to Augustine – but also to consider Apuleius.

ITALO-LATIN AUTHORS²

Let us begin with Cicero (106 BCE – 43 BCE) – not so much because he was, from later Antiquity and the Middle Ages onward, to embody the very idea of Latin polite style and culture but because he referred more often to Archimedes than any other ancient Latin writer.³

Cicero was evidently captivated by the Archimedes figure. In *Tusculanae disputationes* V.xxiii he tells how, being made a quaestor in Sicily,

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¹ This is my translation, as all translations not otherwise identified in what follows.
² This section draws upon (Høyrup, pp. 2–5), contracting and expanding as appropriate.
³ Obviously, this statement as well as the whole subsequent analysis refers to writings that have survived. However, the only writer who possibly knew more than Cicero about Archimedes and may even have known about his mathematics is Cicero's contemporary Marcus Terentius Varro (116 BCE – 27 BCE); but what little can be reconstructed from fragments and hearsay – see (Boissier, pp. 327–331) – speaks definitely against this assumption.
he was curious about the tomb of Archimedes, about which he knew “from some trifling verse lines inscribed upon” the monument that this monument should carry a sphere together with a cylinder (thus not from any knowledge about *De sphaera et cylindro*). After this aside, Cicero continues with a praise of the felicity of the philosopher and the mathematician as compared to that of the tyrant Dionysios.

Also in the *Tusculanae disputationes*, namely in I.xxv, Cicero expresses the opinion that without divine genius Archimedes would not have been able to imitate by his sphere – a mechanical model of the planetary system – the irregular motions of the heavenly system, themselves according to Plato's *Timaeus* a divine creation. In an intelligent-design argument against the Epicureans, *De natura deorum* II.xxxv castigates those who claim Archimedes's imitation to surpass the original (the latter being in their opinion an outcome of mere accident). *De republica* I.xiv once more describes the wondrous mechanism.

*Academica* II.xxxvi calls in a hypothetical “Archimedes” (a representative of the category of geometers with their knowledge beyond dispute) proving by drawings that the sun is much larger than the earth. Real though quite unspecified drawings (geometrical or astronomical?) occur when *De finibus* V.xix cites the story of Archimedes being so occupied with “something he was tracing in the dust” that he did not notice his native city was taken.

*De oratore* III.xxxiii speaks of the time in which Euclid and Archimedes cultivated geometry and where knowledge was not compartmentalized. *Actio in Verrem* II, IV.lviii, refers in passing and *hors propos* to Marcellus's admiration for Archimedes and his distress when he learned the genius had been killed. *Oratio pro Cluentio* xxxii performs the multiplication $16 \times 40000 = 640000$ correctly and then claims that Archimedes could have done no better. Two letters to Atticus (XII.4, XIII.28) refer to a tangled problem – how to make the funerary oration for an arch-enemy of Caesar in the presence of Caesar himself (Simms 1989) – as a “problem for Archimedes”. Archimedes's unspecific ingenuity was apparently proverbial at the moment, in Cicero's circle at least.

No other Italo-Latin author returns to Archimedes nearly as often. In Augustus's time, Vitruvius (* before c. 70 BCE, † c. 25 BCE or later) speaks in
De architectura I.i about technical manuals written by Ctesibios, Archimedes and others, which however one cannot understand without having learned natural philosophy; given Vitruvius’ profession as a supervisor of military engineering, this piece of information, not found elsewhere, could be trustworthy. In the same chapter Vitruvius speaks again of mechanics writings by Aristarchos, Philolaos, Apollonios, Archimedes and others. Book IX tells in its introduction the anecdote of Hieron's crown and Archimedes's exposure of the fraud.

Titus Livius (59 or 64 BCE – 17 CE, thus also Augustean) refers to Archimedes as a unique observer of the heavens and stars in Ab urbe condita XXV.xxxiv and then goes on with details about Archimedes's war machines and stratagems. XXV.xxxi tells how Archimedes was killed while “eagerly describing some shapes in the dust”. In Fasti VI.277, Ovid (43 BCE – 18 CE) speaks about the sphere “made by Syracusan art” without identifying the creator by name.

Under Tiberius, Valerius Maximus (fl. 27 CE) speaks in Facta et dicta memorabilia VIII.vii.7 generically about Archimedes's efficient war machines, about Marcellus's admiration for his genius, and about Archimedes's death. In agreement with the character of the work, all of this could be drawn from Cicero and Livius.

In the introductory description of his Historia naturalis, the Elder Pliny (23 – 79 CE) lists Archimedes as one of his many sources for the cosmology of book II; the text of that book, however, does not cite Archimedes. In VII.xxxvi.125, Pliny calls Marcellus in as witness of Archimedes's knowledge of the sciences of geometry and machines. In the later first century CE, Quintilian (c. 35 – c. 100 CE) speaks in Institutio oratoria (I.x) of the cosmological insights provided by geometry, adding that he will not go into the details of tactics nor speak about Archimedes's single-handed defense of Syracuse; the line of thought must be that the use of geometry in astronomy makes Quintilian think of Archimedes, but Archimedes he associates primarily with his military work. More or less at the same time, Florus (c. 74 CE – c. 130 CE) refers to Archimedes's ultimately failing defense of Syracuse in Epitome I.xxii.33 without saying anything more; as in general in this “epitome of Titus Livius”, Florus almost certainly draws on and abbreviates Livius for this.
Probably in the early to mid-third century CE, the compiler Gaius Julius Solinus mentions Archimedes's knowledge of stars and machines in his *De mirabilibus mundi* V.13, adding no details. Since much of the work in general is borrowed from the Elder Pliny, Pliny might also be the source here.

In *Res gestae* XXVI.i.8, Ammianus Marcellinus (c. 330 CE – c. 395 CE) lists Meton, Euctemon, Hipparchos and Archimedes as the most distinguished students of the stars. Ammianus was Greek-born and settled in Rome around 380 after a long military career; the last part of his historical work (beginning precisely with book XXVI) was published in 394 CE or later. As a historian, he draws not only on Latin but also on Greek material, which probably explains that he knows about Meton, Euctemon and Hipparchos, not appearing together with Archimedes in earlier Latin writings.

Already between 335 and 337 CE, Firmicus Maternus (*Matheseos libri* VIII VI.xxx.26, ed. Kroll & Skutsch, II p. 148) had spoken briefly about Archimedes's ingenious sphere and the efficacy of his machines, for which he could have taken inspiration from a variety of earlier Latin writings – but since he almost certainly used Greek sources for this astrological handbook, Greek lore is also possible.

Somewhere around 400 CE, Claudianus (*Shorter Poems*, LI) disparages Archimedes sphere as a poor imitation of the divine creation – probably an echo of Cicero's *De natura deorum*, as befits an Alexandrian-born writer eager to work himself into the Roman elite.

Macrobius's Neoplatonic *Commentarii in Somnium Scipionis* I.xix from the earlier fifth century CE enrolls Archimedes and the Chaldaeans as supporting Cicero's opinion about the order of the spheres of the planets. Martianus Capella's similarly Neoplatonic and similarly badly dated but in any case roughly contemporary *De nuptiis* speaks in II.213 about Plato and Archimedes who rotate golden spheres. This could but need not go back to distorted memories of various Ciceronian passages. Pseudo-Priscian, *Carmen de ponderibus* (ed. Hultsch II, pp. 95–97), probably a product of the fifth century CE, tells the story of the crown, which could be inspired by Vitruvius.
Leaving out Augustine for the moment, the Patristic (Christian) Latin material, not exclusively Italo-Latin, is even more meagre. Tertullian (c. 160 CE – 220 CE or later) ascribes in De anima (PL 2, col. 669) a wonderful hydraulic organ to Archimedes. In the earlier fourth century CE, Lactantius seems to borrow in Divinarum institutionum II (PL 6, col. 297) from what Cicero writes in Tusculanae disputationes I.xxv about the sphere – as a prestigious teacher of rhetoric he will have known his Cicero well. In Historia IV.xvii (PL 31, col. 896), Orosius speaks in the earlier fifth century CE about Archimedes's machines and their efficiency in the defense of Syracuse – according to the wording a borrowing from Valerius Maximus; however, Valerius's account of Archimedes's death is omitted.

Claudianus Mamertus († c. 474 CE) refers to Archimedes's use of the radius (PL 52, col. 781), in parallel to Orpheus's use of the plectrum (etc.); since the radius goes together with drawing in the dust (which was the standard medium for geometric and similar drawings), this could come from Cicero's De finibus, but also from other sources. 4

Cassiodorus's (* c. 480 CE, † perhaps c. 550 CE (Neugebauer 1982)) Institutiones II.vi.3 mentions Archimedes along with Euclid and Apollonios “and other writers” as Greeks who have written about geometry (ed. Mynors, p. 152). His Epistola XLV (ed. Mommsen, p. 40) states that Boethius translated “the mechanician Archimedes”. This is almost certainly false, and if so, Cassiodorus will have known it to be; the letter to Theodoric may have had a political purpose – which could indicate that the Byzantine-bred Theodoric was supposed to respect Archimedes’s name.

In summary we see that few of the authors after Cicero and Livius go beyond what they could read in these two about Archimedes. The exceptions are,

– Vitruvius, who through his profession may actually have known or known about mechanical technical writings that have gone lost; 5

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4 The passage is also found in a letter to Claudianus from Sidonius Apollinaris (* c. 430 CE; Epistulae IV.iii.5), which is thus likely to be Claudianus's direct source.
5 We should always remember that the surviving Greek corpus has been filtered through what the Byzantines found worth-while copying at least until it was saved in Arabic during the Abbasid translation wave, and that the survival of Latin material is
Ammianus Marcellinus, whose Greek background may have allowed him to know the names of some early Greek astronomers;

Tertullian the Carthaginian, who may have drawn on sources unknown to us for his Archimedean *organum hydraulicum* – not totally to be discarded according to (Farmer, p. 13).

In any case, none of them, nor Cicero nor Livius, know about Archimedes as a producer of mathematical *theorems* or *theory*.

We may even ask ourselves whether they had any idea about what mathematical theory should be. An oft-quoted passage from Cicero (*Tusculanae disputationes* I.ii.5, trans. King, p. 7) states that

> With the Greeks, geometry was regarded with the utmost respect, and consequently none were held in greater honour than mathematicians, but we Romans have restricted this art to the practical purposes of measuring and reckoning.

The consequences of this attitude are encountered in Quintilian's *Institutiones* I.x.34–37. According to Quintilian, geometry deals with finger reckoning as well as figures – probably because the teacher of both in elementary teaching was called a “geometer”. Figures should be taught because law-suits often regard boundaries and measurement of landed property.

Finally, Aulus Gellius's (c. 125 CE – 180 CE or later) *Noctes atticae*, collecting all a well-bred Roman ought to know about Roman as well as Greek culture, reveals what would already be considered at the limits of Roman understanding in mathematics. The preface asks the readers not to skip the passages where geometry is spoken of, and promises that they will not be difficult. Indeed they are not. The only substance is found in I.xx, where the meanings of “plane” and “solid figure” are explained together with those of “cube”, “line” and “square number”; and II.xviii, in which optics is told to explain the working of mirrors, and harmonics to deal with rhythm and melody, which gives rise to a citation of Varro about the utility of metrics (trans. Rusca equally conditioned by what some monk found it worthwhile to copy during the precarious pre-Carolingian Middle Ages, or what had at least been in the possession of some private landowner and gone with a younger son to a monastery.
1968: I, 37; I, 71f; II, 513f). That was thus what could already be supposed to scare a Roman gentleman.

Obviously, ancient Latin culture had no space for Archimedes the theoretician.

**TWO BERBERS**

Latin culture of Western North Africa may be the exception. The witnesses are two outstanding figures: Apuleius of Madaura (c. 125 CE – 170 CE) and Augustine of Hippo (354 CE – 430 CE). None of them is known as a practising mathematician, but both give offhand references to Archimedean theory suggesting that they expected that at least knowledge of its existence could be taken for granted. Moreover, as we shall see, they were not mathematical ignoramuses.

First the offhand references. In 158 CE, Apuleius was accused by the parents of his wealthy and much older wife to have attracted her by magic (he was absolved). In one point of the *Apology* he prepared at that occasion he ascribes to Archimedes a huge volume explaining rainbows and other optical phenomena, adding (ed. Krüger, p. 24) that this figure – “in everything geometric of more admirable subtlety than anybody else” – may still perhaps deserve even greater fame because of his investigations of (convex and concave) mirrors.⁶

Augustine wrote *De utilitate credendi* in 391/92 CE (O'Donnell, I p. lxix), a few years only after his conversion in 387 CE, and hence when his pre-conversion teaching and philosophical studies were in fresh memory. In Chapter 6 of these early confessions he asks rhetorically (against the habit of his youth to take the enemies of Faith as guides to the Scripture) (ed. Perl, p. 30):

Who would think of having the concealed and obscure books of Aristotle explained to him by one of Aristotle's enemies? [...] And who would read or learn with Epicuros as his master the geometrical writings of

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⁶ Such a work has not survived, but Theon of Alexandria and others also refer to it – cf. (Heiberg, II p. 550); even though lost, such a treatise is thus likely to have existed.
Archimedes, against which Epicuros spoke stubbornly, in my opinion without having understood them?

Epicuros being earlier than Archimedes, Augustine must refer to Epicurean objections to the foundations of theoretical geometry which Archimedes shared, perhaps more specifically to Eudoxean theory—cf. (Sedley) and (Cambiano, pp. 587–590).

Augustine thus knew that Archimedes was engaged in a theoretical field with foundations and was not merely making drawings.

What else do we know that links Apuleius and Augustine to mathematics? As regards Apuleius, from his extant writings not too much. In Florida II.xv (ed. Hildebrand 1842: II, 60), Pythagoras is stated to have learned about numbers and geometry from the Egyptians; in Florida IV.xviii that Thales explored geometry as well as the stars (etc.) (ed. Hildebrand, II p. 87); in Florida IV.xx, that Apuleius himself drunk from the “Athenian cups” the fictions of poetry, limpid geometry, sweet music, harsh dialectic (ed. Hildebrand, II p. 96); and in De dogmate Platonis I.iii, that after Socrates had passed away, Plato studied geometry with Theodoros; astrology (probably meant in the double sense) with the Egyptians; etc. Most are doxographic commonplaces, but the “Athenian cups” show that Apuleius himself studied these disciplines, finding geometry limpid but dialectic harsh. Moreover, Cassiodorus's Institutiones II.iv.7 (ed. Mynors, p. 140) ascribes to Apuleius a translation of Nicomachos's Introduction to Arithmetic, which would fit his kind of Platonizing philosophy—say, Neoplatonism ante litteram—and can therefore be considered reliable (normally, indeed, it is considered so). Nicomachos is obviously a far cry from Archimedean mathematics, but at least evidence of mathematical interest and competence well beyond what can be found in any surviving Italo-Latin writer before Boethius.

From Augustine's Confessions IV.xvi we know that he found no competent teacher beyond rhetoric (and even for that nobody who understood

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7 The passage in Cicero, Academica II.xxx, about Polyaenos the geometer converted to Epicureanism and then claiming the whole of geometry to be false, cannot be Augustine's source.
8 From hearsay, Cassiodorus also ascribes to Apuleius a treatise on music (Institutiones II.iv.10, ed. Mynors, p. 149).
Aristotle's *Categories* without difficulty); however, whatever “was written, either of the art of rhetoric, of logic, whatever of geometry, music, and arithmetic, I attained the understanding of by myself” (trans. Rouse & Watts, I p. 199). The outcome of this reading about mathematical subjects is shown, on one hand by a casual remark in *De civitate Dei*, on the other by his treatise on music.

*De civitate Dei* XI.30 explains how the six days of the Creation symbolize the perfection of this very creation, six being a perfect number. The same point had already been made by Philo of Alexandria and was hardly original. Augustine's explanation of what a perfect number is seems to reflect his pedagogical skills, but might still be borrowed from some handbook. Most interesting is his distinction between two meanings of the word “part” – one corresponding to what we find in the common notions of *Elements* I, the other one defined in *Elements* VII.

This, however, is only a trace of thoughtful reading (of Euclid or of some epitome). Stronger evidence of genuine mathematical competence might have been offered by the dialogue *De musica* (PL 32, col. 1081–1194), written in 388/90 CE (O'Donnell, I p. lxviii), if only this work had been finished. The six books that were written deal with other things than mathematical harmonics – the nature and metaphysics of music, metrics, rhythm. Nonetheless, even the treatment of rhythm betrays an underlying aim of mathematical treatment, as also summed up in book VI (meant to lead from the inferior numbers regarding mutable things to immutable divine numbers).

Apuleius and Augustine were not typical representatives of their intellectual environment. Apuleius had travelled extensively to the East, including Athens, and Augustine during the typical study of rhetoric discovered philosophy on his own. But they show that such initiatives and discoveries were at least possible in their world, as they were apparently not in the rest of the ancient Latin world; certainly, its elite intellectuals were supposed to learn Greek, but this did not influence what they wrote in Latin in the area we have considered. Differently, according to Apuleius (*Florida* IV.xx, ed. Hildebrand II p. 96), in Carthago “our venerable teacher, the celestial muse of Africa”, “the kids learn all disciplines, the youngsters display them, the old teach them”.

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HERITAGE?

In the earlier ninth century CE, al-Jaḥīẓ wrote as follows (trans. Gutas, pp. 86f):

The difference between the Christians and the Jews is that the latter consider that the study of philosophy is a cause of unbelief, that the application of dialectic to the study of religion is a heresy and the very fountainhead of doubt, that the only true learning is that contained in the Pentateuch and the writings of the Prophets, and that the belief in the efficacy of medicine and faith in astrologers' predictions are likewise causes of heresy, leading towards heterodoxy and away from the path trodden by their forefathers and models. They go to such extremes in the matter that they suffer the blood of those who do those things to be spilt with impunity, and silence any who are tempted to follow their example.

Had the common people but known that the Christians and the Byzantines have neither wisdom nor clarity [of mind] nor depth of thought but are simply clever with their hands in wood-turning, carpentry, plastic arts, and weaving of silk brocade, they would have removed them from the ranks of the literati and dropped them from the roster of philosophers and sages because works like the *Organon*, *On Coming to Be and Passing Away*, and *Meteorology* were written by Aristotle, and he is neither Byzantine nor Christian; the *Almagest* was written by Ptolemy, and he is neither Byzantine nor Christian; the *Elements* was written by Euclid, and he is neither Byzantine nor Christian; medical books were written by Galien, who was neither Byzantine nor Christian; and similarly with the books by Democritus, Hippocrates, Plato, and on and on. All these are individuals of one nation; they have perished but the traces of their minds live on: they are the Greeks. Their religion was different from the religion of the Byzantines, and their culture was different from the culture of the Byzantines. They were scientists, while these people [the Byzantines] are artisans who appropriated the books of the Greeks on account of the geographical proximity.
The first part of the diatribe may be explained away if one wants to as an oblique attack on al-Jaḥiẓ's Ḥanbalite enemies. The second part, however, cannot be reinterpreted in this way; it also corresponds quite well to the 600 years' suppression of everything philosophical not controlled by Christian orthodoxy in the Byzantine theocracy. In spite of this, our present-day modern Greek colleagues understand ancient Greek philosophy and mathematics as their heritage – and not only because it facilitates funding.

Can the Berbers Apuleius and Augustine – or the environment they represent – similarly be counted as part of Maghreb heritage? That question I leave to those who are concerned.
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Greek and Latin classics, where the text makes use of standard reference systems, are not included unless I quote a translation. Any scholarly edition will do.

Dates and basic biographic information can be found in any major handbook about classical Antiquity. I have mostly relied on *Der kleine Pauly*.


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