A Dialogue on Time

Two Logics of Time and Physics

Kofod, Julie Lundbak

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A Dialogue on Time

Two Logics of Time and Physics

Julie Lundbak Kofod
MSc in Physics and Philosophy and Science studies
Master's Thesis

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Roskilde University,
Department of Science and Environment, IMFUFA
P.O. Box 260, DK - 4000 Roskilde
E-mail: imfufa@ruc.dk

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This master thesis examines the distinction between tense logic and first order logic concerning time. In particular it studies whether this distinction exists in physics or not.

Firstly a general test is made: translating sentences concerning time from physics into the two logics. The results here are inconclusive, as it is possible for both logics to represent the sentences, so this experiment does not reveal a similar distinction in physics. However this test also reveals some properties of the logics: tense logic holds a local view on time while first order logic holds a global view, and it is possible to translate tense logical formulas into first order logic with the standard translation.

Secondly it is examined whether properties of reversibility in physics can be compared with properties of the two logics; this is done under the conjecture that there is a correspondence between first order logic and reversibility in physics and tense logic and irreversibility in physics. Through this study it is concluded that it is not possible to see this correspondence with the tested property; time symmetry, since symmetry seems to concern the underlying structures, not the logic itself. It is, on the other hand, possible to see the correspondence while looking at the property of being fundamental. Through a presentation of Onsager's reciprocal relation, reversibility is shown to be fundamental. In logic, fundamentality of first order logic is argued for through the standard translation.

Finally it is noted that the finite model property and the decidability of tense logic gives useful properties that the more fundamental first order logic has not.

It is concluded that it is unclear whether the distinction of time between the two logics exists in physics.

Master's Thesis in
Physics and Philosophy & Science Studies

A Dialogue on Time

Two Logics of Time and Physics

Written by
Julie Lundbak Kofod, 45381
Supervisors:
Physics: Jeppe C. Dyre
Philosophy: Patrick Rowan Blackburn

Roskilde University
Department of Science and Environment
&
Department of Communication and Arts
May 8, 2017

Resumé

Nærværende speciale undersøger distinktionen af tid i tidslogik og første ordenslogik. Det er undersøgt om denne distinktion eksisterer i fysik.
Gennem oversættelser af sætninger fra forskellige grene af fysikken til de to logikker, er en generel test gennemført, som det første. Da det var muligt for begge logikker at oversætte alle sætningerne, er det derfor ikke muligt at se distinktionen af tid i fysik gennem testen. Dog blev det tydeligt at tidslogik har et lokalt syn på tid, og første ordenslogik et globalt, og at det er muligt at oversætte tidslogik til første ordenslogik gennem standard-translationen.
Dernæst er der arbejdet under hypotesen om at der er en sammenhæng mellem; reversibilitet i fysik og første ordenslogik — og irreversibilitet i fysik og tidslogik. Egenskaber ved reversibilitet er sammenlignet med egenskaber ved de to logikker. Gennem studiet konkluderes at det ikke er muligt at se ovennævnte sammenhæng ved den testede egenskab: tidssymmetri, da symmetri angår de underlæggende strukturer, ikke logikken selv. Det er modsat muligt ved fundamentalitetsegenskaben. Gennem en præsentation af Onsagers reciprocitets relation er der argumenteret for, at reversibilitet er en fundamental egenskab i fysik. I logik er fundamentalitet argumenteret for gennem standard-translationen.
Endelig bemærkes det at den finitte-model-egenskab og afgørbarheden af tidslogik bidrager med brugbare egenskaber som den, mere fundamentale første ordenslogik, ikke har.
Det konkluderes at det er uklart om distinktionen af tid mellem de to logikker eksisterer i fysik.

Introduction

Preface

This thesis is a combination of physics and philosophy. That closes many doors if one looks at the two sciences separately, but it certainly also opens many other doors too when the subjects are combined. The most obvious one of these is, I guess, the one that leads into working with time. And that is what the thesis is about.
Time has been worked with and thought about since ancient times by Aristotle. Later when physics and philosophy became divided into two separate areas; by Isaac Newton (1643-1727), Albert Einstein (1879-1955) and Stephen Hawkins (1942-) just to mention a few physicists, many more have thought and worked on describing time. These physicists had different views on what the nature of time is. Newton and Einstein saw it in fundamentally different ways. Newton saw time as absolute; not derived from any other things but a self standing entity. That view dominated both philosophy and physics until the 20th century. Einstein showed that time was not absolute, but changed according to velocity in his special theory of relativity.
Philosophers who thought about time are also abundant: to mention a few there are Immanuel Kant (1724-1804), Henri Bergson (1859-1941), Martin Heidegger (1889-1976) and the most important one for this thesis; Arthur N. Prior (1914-1969). Arthur N. Prior was a logician and developed the logic of time called tense logic. In tense logic the *present* is of fundamental importance, as you will discover while reading this thesis.
A central debate while working with time in philosophy is the one between A-theorists and B-theorists. The notion of a A series and B series of time was introduced by J. M. E. McTaggart (1866-1925) (McDaniel; 2016). The A-theorists take the *present* seriously and think of time as running from a far past to near past to present to near future to far future. The B-theorists, on the other hand, think of time as events and compare events with one another e.g. event x happened before or after event y (Markosian et al.; 2016). That discussion is central to this thesis. The two views, A and B, are worked with in the formal presentation of tense logic and first order logic respectively. Furthermore the discussion between A and B theorists is here extended into the field of physics. Physics is often drawn into the philosophical discussion with the aim of trying to show one position to be 'right' rather than the other. But I am not using physics to 'judge' the positions; A versus B. Rather

the aim is to investigate whether physics *does* hold one view or the other, or if both views are represented in physics. That interest lead me to the following question which will define the scope of the thesis.

Problem Formulation

The overall question that I wish answered during this report is:

Does the distinction of time, as it is presented through tense logic and first order logic, exist in physics?

In the course of answering the above question, the following questions are posed.

> *How is time talked about in tense logic and first order logic?*
> This question is answered in Chapter 1
>
> *Are both ways of talking about time relevant to physics?*
> In Chapter 1 this question is approached from above by applying the two logics on physical examples from different fields.
>
> *Do either of them seem more relevant to the question about reversibility?*
> With the knowledge from Chapter 1, Chapter 2 attempts to answer this question.

Some comments need to be added to the problem formulation and the scope of the thesis. To answer the problem formulation one could choose to dig in to one branch of physics and ask how the distinction would make sense in the depths of this branch, which would be interesting. But this is *not* the approach in this thesis. On the contrary the thesis works with more branches and therefore contains a wider view of physics.

Foreword

This thesis is written as a dialogue between fictional characters. There are several reasons for writing this way:
One is that the thesis is a combination of two subjects; physics and philosophy. These two subjects have different research cultures particularly concerning language and method. So I found it better to join them on foreign ground, in an attempt to avoid privileging one style over the other, or on the other hand ending up with two separate reports.
From earlier projects (Jensen and Kofod; 2015), and while writing this thesis I have experienced that the flow of arguments comes naturally from the questions that are posed by the characters and the needed answers to those questions.
The dialogue form is not unusual for philosophical writings. Ancient philosophers

like Plato did much of his writing as dialogues and later philosophers like Galilei Galileo and David Hume also embraced the dialogue. 'What the Tortoise Said to Achillies' by Lewis Caroll (Carroll; 1895) is a dialogue about logic and indeed provided a central inspiration for the dialogues for the more recently published book; Gödel, Escher, Bach by Douglas R. Hofstadter (Hofsdtadter; 1995).
In physics it is less common to find writings where the dialogue is used. However Galileo Galilei can be mentioned again in this respect, and in more recent times 'Dialogues on Modern Physics' by Mendel Sachs can be mentioned (Sachs and Evans; 1998) and 'Are Quanta Real?: A Galilean Dialogue' by J.M. Jauch (Jauch; 1989). Other forms deviating from the norm within physics can be found in the play about the meeting between Niels Bohr and Werner Heisenberg during second world war; 'Copenhagen' by Michael Frayn. The last example of recent works has the aim to describe all the things around physics, like feelings and politics. That is not the aim in this thesis.
Because of the above mentioned reasons I have taken the challenge to communicate physics and logic in the form of a dialogue in this thesis.

While reading the dialogue you will meet different animals that play different roles. The three main characters are the fox, the owl and the raven. These animals have a problem they want to solve due to their different points of view on time. The three animals are generalists, so while posing the problem they need to seek help of the specialists to get specific answers. This is in accordance with the education at Roskilde University, where we are meant to work the same way as the fox, the owl and the raven.
The thesis is aimed to be read and understood of both physics and philosophy students or other people interested in the intersection of the two subjects. Therefore most things are explained in detail and calculations are fully brought out. During the text there will be footnotes; here I step out of the story and the forest. Some of the footnotes are with references, some of them with other comments on the story. The comments are supposed to work as: inspiration for further readings as well as explanatory; for the philosophically inclined there will be comments on the physics e.g. calculations, and for the physically inclined there will be philosophical explanations to ease the understanding for both parts.
Due to the combination of subjects I ask the reader to be tolerant concerning the notation, since physics and logic have different cultures in this respect, therefore it sometimes deviates from the norm of one subject, while not of the other.
The thesis is written as the last activity on the master education at Roskilde University. As mentioned, it is an interdisciplinary thesis written within the two subject; Physics and Philosophy & Theory of Science.
It was written in the last semester of my master studies and was handed in on the 8th of May 2017.
The illustrations are drawn by the author.

I would like to thank my supervisors Patrick Blackburn and Jeppe Dyre for con-

tributing with ideas, for being open to supervising a thesis combined between philosophy and physics, as well as being positive towards the dialogue form. Furthermore I thank Heine Larsen for good help out of my frustrations concerning some of the problems in physics.
I definitely owe a thank to the group of physics and mathematics at Roskilde University; IMFUFA, for making my study in general a brilliant time, and for showing interest in this very thesis. I will take a lot of good advice, memories and knowledge with me from this place.
Thanks to the other students at the office on IMFUFA for enduring my frustrations and philosophical nonsense.

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Chapter 1

The sunset

The sun was setting behind the abundant treetops, just as it had done for all known past time, and will do for all future time. On this present evening once again the clever sly fox and the wise old owl met up, not for a hunt as one might think, but for one of their nightly conversations about the abstract world in the forest[1]. *Though this night will turn out to be slightly less abstract as the two friends search for answers in the physical world. The fox has just now found the owl sitting in an oak tree straightening its feathers.*

FOX: Good evening good old Owl!
OWL: Oh, good evening my dear friend.
FOX: I have been wondering about a thing I can not find answers to.
OWL: That does not surprise me. What is it this time?
The owl said full of dignity.
FOX: It is *time*!
OWL: Yes, time is a curious concept, not so easy to wrap your head around.
FOX: So my wise friend, what do you have to say about time? You must have learned about it at your beloved owliversity. What is time? It is no object we can see, and yet it is so apparent when the sun sets and the night turns dark, when the sun rises and it gets light again, when trees grow and animals are born or when eggs are laid and when they grow old and die.
We have such a clear idea about what it is when we do not think about it, but if you just think a little bit deeper you get almost dizzy of the number of questions piling up[2].
OWL: Well as I said it is not answered so easily if it is answered at all.

[1] If you wonder why a fox and an owl would meet up for conversations, the beginning of their friendship is explained in Jensen and Kofod (2015)

[2] The fox unknowingly has just asked what Augustin of Hippo asked long time before him: *'Quid est ergo tempus? Si nemo ex me quaerat, scio; si quaerenti explicare velim, nescio.' 'What, then, is time? If no one ask of me, I know; if I wish to explain to him who asks, I know not.'*

Figure 1.0.1: The fox and the owl meeting by the old oak for a nightly discussion about time.

1.1 A and B Theory of Time

The fox has motivated the owl to tell about its view on time. You will see when the conversation develops that the owl holds a B view of time, unfolded as first order logic. The fox does not quite agree and holds the A view of time presented in tense logic.

Owl: Let me explain to you my view of how time should be perceived.
I believe that time is just like distances, and events are just items hanging on the string, or the axis, of time. You see, when I fly above the treetops I look down on what is happening down in the forest. Now you are sitting here *below* me and over there *behind* you, and behind that tree there is a hedgehog sniffing about. Why should I not just adopt these relations of behind, in front, below and above into my understanding of time? So that things are happening before or after each other.[3]
Fox: I see your understanding of time makes sense. But I think you miss one thing. It might be so, that you see it this way when you fly around up there in the thin air. But down here on the solid ground it seems different. I do not think that you can just convert your understanding of distances and relations into time. There is the big difference: you can choose to walk, or fly back to where you came from, but you can not choose to go back to the past. That makes a big difference you can not ignore: there *is* an asymmetry between the past and the future! It is even reflected in our language. We say 'See you tomorrow' or 'it was fun yesterday'. Do you not

[3]The owl is presenting the B-series view on time.

see, that these make reference to the *present*?[4]

OWL: But why should the present be privileged? As I see it, *the present* is just one point on the axis of time just like the other points, so is your perception not just based on subjective feelings?

FOX: You can call it feelings; I would call it experience. If we try to describe some concept concerning our surroundings, is it not a bad description if it does not correspond with our experiences? [5]

OWL: I see your point, but you forget that some animals have night vision while some do not. Some can smell their predators miles away, while some can not smell anything at all. You see, experience is subjective!

FOX: So you think it is better to choose an understanding of time which does not correspond to experience at all, but is purely constructed and not real?

OWL: Yes I prefer a simple and useful conception.

FOX: So we are talking about the same thing, but have two completely different views on it. How are we to communicate like this?

The fox was rather frustrated about the direction, or lack of direction the conversation was taking, and there was silence between them for a while.

FOX: Owl, we need to get a common language to communicate rationally about time. And that language needs to be a formal language! With such a language we can maybe get a deeper understanding of time and our different positions.

OWL: Yes that sounds rational.

The owl felt on safe ground with formal languages, so it embraced the idea right away.

OWL: So if I were to formalise a sentence, that could be; 'I will sleep in the future', I would do it as follows, where I have set p to be the proposition 'I sleep' and t to be the time of utterance[6]

$$\exists t'(t < t' \land p(t'))$$

This means: There is some time t' which is later than the time of utterance t, and at this later time I sleep. You see Fox, this is just first order logic as we know it.

FOX: I see it has its beauty that you use the good old logic that we know, to formalise the sentence. The problem is just that you are not catching the essence of

[4]The fox is presenting the A-series view on time.

[5]The fox is presenting its view based on Hasle and Øhrstrøm (2016). This was A.N. Prior's motivation for developing tense logic (Prior (1967) and Prior (1968)).

[6]Here comes a little dictionary for the untrained reader in first order logic:

Connectives:

- $\rightarrow$ Implies, if... then
- $\land$ Conjunction, and
- $\lor$ Disjunction, or
- $\neg$ Negation, not

Quantifiers:

- $\forall$ Universal quantifier, for all
- $\exists$ Existential quantifier, there exists

the sentence; you need to take the tense seriously and not change the sentence into something tenseless as if time was any other kind of variable, like position [7].
OWL: All right, I hear what you say my friend. But there is a problem with what you are presenting. How would you ever formalise a sentence while keeping the tense?
FOX: Well...
The Fox thought for a while, and then started answering the owl's question.
FOX: I do not see how you should do it with the standard logic. That is invented by animals, why should we not just add to it?
The owl was sceptical, but it also knew that the fox was smarter than it looked. The owl was also very curious to hear about the fox's silly idea.
FOX: In propositional logic we have the well-known two-place connectives $\wedge$ (and), $\vee$ (or) and $\rightarrow$ (implies) and the one-place connective $\neg$ (not).[8] I am thinking that we could add two extra one-place connectives: F for future, meaning 'It will at some time in the future be the case that' and P for past, meaning 'it was at some time in the past the case that'. That would make the sentence 'I will sleep' Fp, where p is the proposition 'I sleep' as you named it. And Pp would be 'I was sleeping'. Look! So much more simple than all your variables.
OWL: Well well, I see that it works. But let us take another example to see if your logic holds for other sentences concerning time. I have thought about one; 'I have always been an owl'. In first order logic I would let the proposition 'I am an owl' be q and let t be the time of utterance, and then I would proceed as follows:

$$\forall t'(t' < t \rightarrow q(t'))$$

Beautiful!
FOX: Oh, but this sentence is straightforward to translate into my logic! You see that the sentence 'I have always been an owl' is the same as 'it is not the case that there was a time in the past where I was not an owl', do you agree on that?
OWL: Yes I do.
FOX: All right, that sentence that you just agreed about is just two negations like this: $\neg P \neg q$. I feel like abbreviating this to Hq, where $H = \neg P \neg$ meaning 'it *has* always been the case'. Had you said 'I will always be an owl', I would equivalently say Gq where $G = \neg F \neg$ meaning 'It is always *going* be the case'.
The fox was very excited about the very simple formalisation it had made, and to see that it worked well with the owl's sentences.
FOX: Oh, I can even talk about different past and future times with my logic, just like when we speak. Just see all the grammatical tenses I can make:

[7] This point is stated in Burgess (1984)
[8] The Fox will now present tense logic build on Fitting and Mendelson (1998) and Burgess (1984).

I am a fox	: f
I was a fox	: Pf
I will be a fox	: Ff
I have always been a fox	: Hf
I will always be a fox	: Gf
I have been a fox	: PPf
I will have been a fox	: FPf
I would be a fox	: PFf

I like this logic I will call it *tense logic*.
OWL: But Fox, this is not special for your tense logic. I can also make all the grammatical tenses you want me to with first order logic. Just see how I would do it:

I am an owl	: $\exists t'(t' = t \wedge o(t'))$
I was an owl	: $\exists t'(t' < t \wedge o(t'))$
I will be an owl	: $\exists t'(t < t' \wedge o(t'))$
I have always been an owl	: $\forall t'(t' < t \rightarrow o(t'))$
I will always be an owl	: $\forall t'(t < t' \rightarrow o(t'))$
I have been an owl	: $\exists t' \exists t''(t' < t \wedge t'' < t' \wedge o(t''))$
I will have been an owl	: $\exists t' \exists t''(t < t' \wedge t'' < t' \wedge o(t''))$
I would be an owl	: $\exists t' \exists t''(t' < t \wedge t' < t'' \wedge o(t''))$

where t is the time of utterance.
The owl and the fox looked at each other, the atmosphere between had become a bit tense. Even though the owl was a proud bird, it was starting to become a bit gloomy when it saw that the formalisation the fox presented was indeed very simple. But as it was stubborn, it kept its conviction.
OWL: So now we have formulas in the two logics, but they mean the same. Why bother inventing a new formalism?
FOX: The thing is that the two logics express two different things: your logic is tenseless, and my logic is tensed. You see that the *is* in '5 *is* a natural number' is a different *is* than 'it *is* dark'. Do you not think that logic should capture that?
OWL: I see what you mean. But I am not convinced, I do not see that my logical sentences are less correct than yours. I learned this language in Owliversity, and I am not throwing it away because a cheeky fox got an idea.
FOX: I am not telling you to throw it away, but keep it in its proper place, where it is actually useful. Which is not for sentences concerning time.
OWL: You mean I cannot use it to describe *your* view on time! My translations fit perfectly well with *my* understanding of time, as I presented it earlier.
FOX: Well yes, it seems we have not got a lot further agreeing on a formal language for time then...
The fox said crestfallen.

The owl and the fox have presented their point of view on time, and formalised their views: the owl has used first order logic and the fox has developed tense logic

to make clear how they think time should be described. The fox wished deeply that they could agree on one way of formalising and understanding time, if they were to talk about it. But both of their formalisations work for the examples they have gone through so far, so they are no closer at agreeing on one view of time.
But as we shall see the help they needed to get further with the discussion was not far away.

1.2 The Experiment

In this section you will meet the raven. The raven has a slightly different interest than the one the fox and the owl have. It wishes to compare the two views with examples from the world of physics. It will do this by presenting sentences picked from six different branches of physics, and let the fox and the owl try out their two formalisations on those sentences. This will show if one of the formalisations turns out to be better at describing the physical laws than the other, and thereby get an idea about if the distinction of time exists in physics. Through the test the three animals discover new features about their logics and the connection between them.

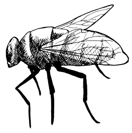
1.2.1 The Raven

The Fox and the Owl heard some rattling feathers further up in the tree where the owl was sitting. It was the raven, who had been listening a little to their conversation and was no longer able to stay out of it.

RAVEN: Excuse me, creatures. I could not help listening to your very interesting talk. You seem stuck, but I think I have an idea of how to get some more insight into your perceptions of time!
OWL: Oh, good evening good Raven, what a pleasant surprise to see you around here. Why do you find this discussion interesting? Not all animals do.
RAVEN: You are right, but *I* fly around in and outside the forest and observe how the world is working and try to describe it with your beloved formal language: mathematics. Therefore I like to call myself a physics-raven. For most processes time is a relevant variable, therefore I find your discussion interesting.
FOX: So what is your conception of time?
RAVEN: As you figured out already, it is not a question answered very easily, but I will tell you what my everyday perception is [9].
First of all, as I see it, time is something we measure, and we measure it by events that occur again and again. As you mentioned a while ago Fox, time is apparent as the night turns dark, the sun rises and it gets light again. That could be a way of measuring time; by for example counting how many days and nights there are between two full moons. We can divide the days and nights into smaller periodicities

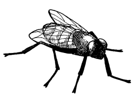
[9] The everyday perception of time that the raven has is based on Feynman (1963, 2006, 2013)

as well by counting how many times an ant walks between the anthill and the dead tree [10]. We can continue like this and measure time on a very small scale and we can continue counting periodicity on a very large scale too. So I usually just think of time like something we measure, and compare other things with, that do not happen periodically.
OWL: That sounds very much like thinking about time as an axis, like length. We can think of time like the numbers on the rulers the beavers use to measure wood when they build things[11].
RAVEN: I guess it is not too far from that conception. Except from that it sounds like that your axis actually exists, and my axis is derived from events.
Just then a fly flew by with a old piece of parchment with the following text written on it. The animals fell silent while they read the text.



> Absolute, true, and mathematical time, of itself, and from its own nature flows equably without regard to anything external, and by another name is called duration:
> Relative, apparent, and common time, is some sensible and external (whether accurate or unequable) measure of duration by the means of motion, which is commonly used instead of true time; such as an hour, a day, a month, a year.
> *Newton (1846)*

OWL: So this *Newton* already did divide time into an absolute time; mine, and a relative time; yours Raven.
RAVEN: '*This Newton*' is not just anybody! He is the the one who is considered the father of classical mechanics. But yes, it certainly seems like he was aware of the distinction we have just discussed.
And another fly came by with an even bigger piece of parchment on which the following was written.

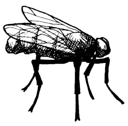


> Absolute time, in astronomy, is distinguished from relative, by the equation or correction of the vulgar time. For the natural days are truly unequal, though they are commonly considered as equal, and used for a measure of time; astronomers correct this inequality for their more accurate deducing of the celestial motions. It may be, that there is no such thing as an equable motion, whereby time may be accurately measured. All motions may be accelerated and retarded, but the true, or equable, process of absolute time is liable to no change. The duration or perseverance of the existence of things remains the same, whether the motions are swift or slow, or none at all: and therefore it ought to be distinguished from what are only sensible measures thereof; and out of which we collect it, by means of the astronomical equation.

[10]The ants in this forest are very habit regulated kind of ants, so the animals can count on their periodicity.
[11]Although the owl does not use these words, it is clear that it is now thinking in terms of standard numerical structures like the real numbers; $\mathbb{R}$ or the rational numbers $\mathbb{Q}$.

Newton (1846)

Owl: Oh, so Newton meant that time was absolute and *liable to no change.* Just like me!
Raven: Yes, it seems like that...
And yet another fly came buzzing by with a piece of paper with the following text printed on it:



> What really matters anyway is not how we define time, but how we measure it. One way of measuring time is to utilize something which happens over and over again in a regular fashion—something which is periodic.
> *Feynman (1963, 2006, 2013)*

And it seems that Mr. Feynman agrees a great deal with me then!
But that distinction is not important for our discussion, since we worry about the distinction between *your* two logics. And all these flies flying around with quotes from physicists are only representations of mine and the owls view, which have a lot in common: I can just count periods and write them down and then I have an axis like yours Owl.[12] I think we should start on comparing it with the lovely physical world!

After the fox and the owl had met the raven, were presented for its view on time, realized that the physicist Isaac Newton held the same view on time as the owl and that Feynman the same view as the raven, they moved on. The ravens idea was that they should compare the fox's and the owl's view on time with the physical world, and so they will do in the following section.

1.2.2 The Physical Examples

Raven: I was thinking that you should ask the world around you for help to get further in your discussion.
Fox: What do you mean 'asking the world around us' for help?
Raven: What you have shown so far only concerns ordinary forest sentences. I was thinking: what will happen if we take sentences that describe the physical world and translate them into your two different logics? Maybe one of your logics will prove more worthy at describing this world than the other! From my point of view, what is a logic of time worth if it does not describe the physical world as we have come to understand it? And even more interesting is: to see if your two logics and thereby your two different perceptions of time, can both make sense of the modern scientific conception of the physical world.

[12]According to van Benthem (1983) the ravens stipulation is not quite right, because the two views constitute two different structures of time; *point structures* and *period structures*, that do have different properties. What can be proved, however, is that the ravens method gives rise to structures that are mathematically isomorphic to the kinds of structures the owl has in mind, like $\mathbb{R}$ or $\mathbb{Q}$.

OWL: That sounds like a good idea Raven.
The Owl was very certain that this could prove it to be right, and first order logic had never let the owl down before.
FOX: This sounds like a challenge to me, I like that!
The fox was equally pleased with the idea. He was not so certain about the outcome, but always thrilled by a challenge.
RAVEN: Glad you like the idea. I think we should ask different realms of the physical world. So I will think of sentences from classical mechanics, thermodynamics, electrodynamics, the special theory of relativity and quantum mechanics.
FOX: I am not sure about all these 'realms', but say the sentences and we will see where it leads us.
RAVEN: I know. Do not worry. I will answer the questions you will have as well as I can as we go along.

1.2.2.1 Classical Mechanics — Revolutions

RAVEN: Let us start with classical mechanics. How would you translate the sentence:

> The revolution of the moon around the earth takes 28 nights.[13]

OWL: All right, let me see. So the moon is orbiting the earth, and at a certain time it is at one position and 28 days later it is at that same position.
RAVEN: Mhmm.
The raven said consentingly.
OWL: Hm, so I am thinking something like this:

$$\forall t \forall t' (t = t' + 28 \text{ nights} \rightarrow pos(\text{earth}, \text{moon}, t) = pos(\text{earth}, \text{moon}, t'))$$

So for all ts and for all t's, if t is 28 nights later than t', then the position *pos* is the same; where *pos* is a three place function which returns the position of its second argument (here, the moon) at time t with respect to its first argument (here, the earth). For example it might return spherical coordinates.
The Owl was satisfied with its answer and a little relieved.
OWL: So now it is your turn Fox.
FOX: Yes, let me see...
The fox was walking up and down the forest floor, thinking what to do about the sentence, feeling that the logic it had presented would not suffice.
FOX: Well I think I have to add something to my logic. Yes, I need to add a

[13] What the Raven is referring to here is Newton's gravitational law $F_G = G\frac{Mm}{r^2}$, and the centripetal force $F_C = m\frac{v^2}{r}$. Then $F_G = F_C$ and the period is described by $T = \frac{2\pi r}{v}$, then velocity of the moon can then be expressed by $v = \frac{2\pi r}{T}$ which is inserted in F_C; $F_C = m\frac{\left(\frac{2\pi r}{T}\right)^2}{r} = m\frac{4\pi^2 r}{T^2} = G\frac{Mm}{r^2}$ $\Rightarrow T = 2\pi\sqrt{\frac{r^3}{GM}} \Rightarrow T\frac{1\text{hr}\cdot 1\text{day}}{3600\text{s}\cdot 24\text{hr}} \approx 28$ days

way of talking about a certain amount of time in the future or the past[14]. I think a superscript will do, so one just decides on an interval, perhaps hours, nights or years and writes it in the superscript. In our case we are using days, so if p is the proposition, $F^n p$ will mean n days in the future, p is the case.
So now I will adopt your notation, but in my tense logic $pos(earth, moon)$ is the position at the present time. As I have presented it, there should be a proposition after a tense operator. In this case it makes sense to say that the proposition in natural language is *'the moon's relative position to the earth is r'*, written as $pos(earth, moon) = r$. And your sentence Raven is then:

$$\forall r(pos(earth, moon) = r \rightarrow F^{28} pos(earth, moon) = r)$$

It works! I like this system! I will call it the *metric* tense logic
OWL: That is all fine. But what can your n be, would you say? Can it be negative?
FOX: Oh, yes it can, we just say that $P^n = F^{-n}$. I see no problem in fact it is very simple an elegant! And F^0 would just be the present. But you are right, that should be specified.
OWL: Very well, I accept this. Let us go on then.

1.2.2.2 Thermodynamics — Entropy

RAVEN: Now let us look at a different branch on the physics tree, one which is not too far away from our everyday understanding of the world either, namely thermodynamics. I am thinking of the fundamental sentence:

Entropy increases with time

FOX: 'Entropy'. That is a strange word. What does it mean?
RAVEN: I am happy that you ask. An explanation is not strictly needed for you to translate the sentence, but it is a central element of the understanding of time in physics so I will give you a short explanation of entropy [15].
Entropy is microscopically a measure of the disorder of a system. Entropy is therefore zero at 0 Kelvin, since then everything is well ordered, but as the system increases its energy, heats up for example, the elements of the system will start to move and the disorder begins.
Entropy is defined as

$$S = k_b \ln(\Omega)$$

Where Ω is the multiplicity, that is the number of microstates a system can be in.
FOX: What is a microstate?
RAVEN: Well, let me give you an example. This tree, where the owl and I are sitting, has 10 good sitting branches, and we only sit at two of them. If the branches are equally good sitting branches there is an equally big probability that we will

[14] The fox will now present metric tense logic as it is given in McArthur (1976), it is originally introduced by Arthur N. Prior in Prior (2003) and Prior (1967).
[15] The presentation of entropy is based on Schroeder (2014).

sit at any particular branch if we have the same energy. A microstate is then all the different ways we can place ourself at the branches. We can both sit in the two top branches or we can sit one at the top and one at the bottom, or any other combination you can think of in the system.
FOX: I see!
RAVEN: So for a thermodynamical system we therefore say that at zero kelvin there is only one microstate the system can be in, and since $\ln(1) = 0$, $S(\Omega = 1) = 0$. Ω changes when the energy of the system changes. That is, when the temperature T, the pressure p or the volume V changes. When these parameters change the number of microstates the system can be in increases, as does the entropy. If you again imagine Owl and me in here in the oak, a change in energy would then be a change in the number of good sitting branches. At the lowest energy there would only be two branches for us to sit in and higher energies would give us a larger number of branches to sit in.
We do not see all the different microstates here in the forest, but we can measure difference in temperature, pressure and volume, so a change in entropy is experienced when you lay in the snow in the winter and it starts melting around you, it is because you deliver energy in the form of heat to the environment and heat up the snow, since heat tends to travel from hot to cold, and the entropy of the snow increases, and in fact the entropy of the universe increases. You see why this is relevant for time?
FOX: Yes! If entropy always increases and *never* decreases, then that is a way of understanding the *present*! The present moves together with the entropy in one direction; that creates the arrow of time!
RAVEN: Yes, though I would phrase it differently. It is what defines irreversibility. Systems where entropy goes up are irreversible and systems where there are no change in entropy is reversible.
FOX: Wow, I like that concept!
RAVEN: And there is plenty more to say about entropy, but I think we should not continue any further out on this branch at the moment, but jump back to the translations, since that is our scope now. Later it could be interesting to continue to look more at entropy[16].
OWL: Yes. I have a clear idea about how to turn your sentence into first order logic. Again I use the same notation, $<$, to express that one time t' is later than another time t. I use another variable x, which ranges over thermodynamical systems. The symbol S should be thought of as the entropy function, that maps a thermodynamical system and a time to the real numbers. So $S(x,t)$ is the entropy that the system x has at time t[17].

$$\forall t \forall t' \forall x (t < t' \rightarrow (S(x,t) \prec S(x,t')))$$

FOX: All right. This is simpler than the previous one: I do not need the metric

[16]See chapter 2.
[17]In the report we distinguish between '$<$' and '$\prec$'. '$<$' is only used when one time is related to another, whereas '$\prec$' is used to relate one number to another. Both signs should be read as 'less than'.

tense logic. The proposition at the present is *'the entropy is r'*, and I will write this as $S(x) = r$, meaning the system x has entropy of value r. So the tense logical sentence we need is:

$$\forall x \forall r (S(x) = r \to G(r \prec S(x)))$$

You see, if the system x has a value of entropy r at the *present* then it is always going to be the case that the system x has a greater value than r in the future.
This is going great! What is the next sentence?

1.2.2.3 General — Conservation of energy

RAVEN: Now we turn to a universal statement

> The energy of a system is conserved

OWL: Oh, this is not too different from earlier sentences. I will keep my notation from before. Now E denotes the energy of a system at some time, so it is a function mapping a system and a time to the real numbers.

$$\forall t \forall t' \forall x (E(x,t) = E(x,t'))$$

My notation is working very well I must say. What do you say Fox?
FOX: This is very similar to the previous statement. I will use your notation too Owl. So if $E(x)$ is the energy of the system x at the present time, then just like before:

$$\forall x \forall r (E(x) = r \to (G(E(x) = r) \wedge H(E(x) = r)))$$

Wait a minute! I can make this even clearer! I could make a new operator 'A' meaning 'at *all* times', so $Ap = Gp \wedge p \wedge Hp$, and the sentence would be

$$\forall x \forall r (E(x) = r \to A(E(x) = r))$$

It is getting neater and neater!
OWL: Could you just put the 'always' operator outside, together with the quantifiers?
FOX: Oh yes I could, and that would make the formula even simpler. But we need to be a little careful. When I put the operator outside I can throw away the $E(x) = r$ before the '$\to$' and get this:

$$\forall x \exists r A(E(x) = r)$$

This means that all systems have an energy of some value r which is the same at all times, whereas if the operator was put in front of it all like this

$$A \forall x \exists r (E(x) = r)$$

it means something very different and rather stupid; namely it is always so that we can find some energy of any system.
So now I think I have found the simplest way of putting your sentence into tense logic Raven.

1.2.2.4 Electrodynamics — A capacitor

Raven: Very well Fox, but now we are moving to a branch of physics which you probably will find rather unfamiliar, since we do not use a lot of electricity in this forest. This branch is electrodynamics.
As before I will just say the statement, since the translation is the important part here, and then we can turn to a deeper explanation if it is needed.
I fly outside this forest sometimes, and once I found a shining object — I have an eye for that stuff. This specific object consists of two metal plates parallel to one another and it is called a capacitor. It has this property when connected to an electrical potential difference:

> A charging capacitor draws a large current at the start and a lower current later.[18]

Owl: Hm, this indeed is unfamiliar but I will try to translate this strange sentence anyway.
So $C(x,t)$ means we start to charge the capacitor x at time t, and $I(x,t)$ is the current drawn by capacitor x at time t. So I can express the sentence by:

$$\forall t' \forall x (C(x,t) \wedge t < t' \rightarrow I(x,t') \prec I(x,t))$$

Fox: Well, if we say that I is the current pulled by an object x, and C is a capacitor, then we can also do it like this:

$$\forall x \forall r (C(x) \wedge I(x) = r \rightarrow G(I(x) \prec r))$$

Owl: So far the two logics are pretty alike I must say.
Raven: Wait and see how you find the next one.

1.2.2.5 Special Theory of Relativity — Time Dilation

Raven: The sentence I think of now will appear even more peculiar to you than the other sentences have done. This is because it is about the nature of time itself, and it is not according to your intuition.
Fox: It has all ready been peculiar enough for me... But let us hear about the 'nature of time itself'.
Raven: Here is the sentence I want you to translate:

> Time goes slower for an object moving with a velocity close to the speed of light, compared to time measured here in the forest.[19]

[18] What the Raven is presenting is the described by $I(t) \propto e^{-t}$, and is true for a circuit with a resistor R and a capacitor C: $I(t) = \frac{\varepsilon}{R} e^{-\frac{t}{RC}}$

[19] The Raven is referring to time dilation: $\Delta t' = \frac{\Delta t}{\sqrt{1-\frac{v^2}{c^2}}}$, which will be derived later.

OWL: I do accept that time is something you can measure, but how in the forest is it possible that time can be different for different beings, with different velocities?
FOX: Yes, how is that possible? And what do you mean when you say that time goes slower for one object than for another; how is that measured?
RAVEN: I am happy that you ask! There is a very nice explanation of this. The important point here is that the speed of light is the same no matter if you are here in the forest or if you are moving with a moving object.
FOX: But normally I would think that the speed of anything would appear slower for me if I would run towards it, than if I were standing still; how would I ever get to catch any prey if that was not the case? But is that not the case for light?
RAVEN: You are right, that is our experience from every day life. But let me show you that it is not like that with light.
FOX: Let us hear then!
RAVEN: We measure the time exactly by using the fundamental speed of light. Then here in the forest, we reflect the sunlight between you two.
The raven was pointing its feathers at the owl and the fox who were both looking perplexed and interested in what insights the raven could bring them.
RAVEN: So Fox, imagine that you hold a light reflecting plate and so do you Owl, then you do a tick sound when the light beam reaches your plate.
The time it takes for the light to travel from Fox to Owl and back again when you are standing in the forest is $\Delta t = \frac{2L}{c}$, where L is the distance between you, and c is the speed of light. Do you follow this?
FOX: Yes this is clear!
It was clear to the fox since it was picturing the scenario shown in figure 1.2.1 inside its furry head.

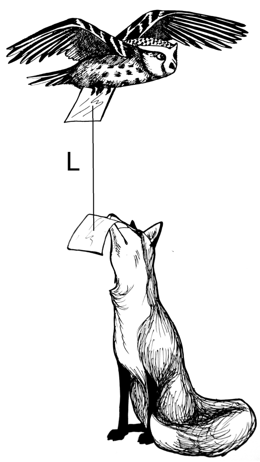


Figure 1.2.1: Light travelling the length $2L$; to and fro the fox. The fox and the owl are standing still in the forest frame.

RAVEN: All right then. Now we move both of you to a moving object, that is

travelling with the velocity v. You are in the same relative position as before, still having the reflective plates. When the light is reflected from Fox's plate and the light is travelling between you, the plates are moving, seen from my point of view, if I am still standing in the forest. Therefore the light has to travel further than in the forest. Can you imagine that?
The fox and the owl tried to imagine what the raven told them, and inside their heads was something similar to figure 1.2.2.

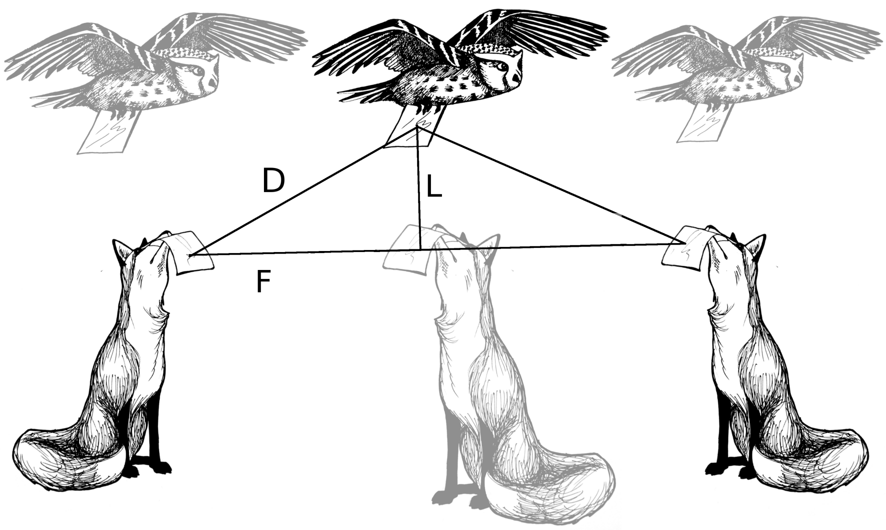


Figure 1.2.2: The geometry of time dilation, where the light is travelling the distance $2D$, from the fox to the owl and back to the fox. The fox and the owl are moving with same speed according to the forest frame.

Here the light has to travel the distance $2D$ instead of $2L$, and the time that takes is then $\Delta t' = \frac{2D}{c}$. We can calculate D like this: $D = \sqrt{L^2 + F^2}$, and $F = \frac{1}{2}\Delta t' v$, where $\Delta t'$ is the time it takes from when the Fox reflects the light until it receives it back. That makes $D = \sqrt{L^2 + (\frac{1}{2}\Delta t' v)^2}$. Then $\Delta t' c \frac{1}{2} = \sqrt{L^2 + (\frac{1}{2}\Delta t' v)^2}$, and isolating $\Delta t'$ gives $\Delta t' = \frac{\frac{2L}{c}}{\sqrt{1-\frac{v^2}{c^2}}} \Rightarrow \Delta t' = \frac{\Delta t}{\sqrt{1-\frac{v^2}{c^2}}}$.
Owl: That does indeed make sense!
Fox: Ah, I see that when I hunt my speed is very, very much smaller than the speed of light, and $\frac{v}{c}$ becomes very, very small and $\Delta t'$ goes towards Δt.
Raven: Yes Fox, that is the reason!
Owl: So your sentence that we started with is then: the size of $\Delta t'$ is smaller than Δt, when $\Delta t'$ is measured on a moving object with respect to the forest and Δt is measured in the forest.
Raven: Yes, you could say that.
Owl: Hm, how can I handle two different frames of reference in first order logic?
The Owl was thinking for a while, not having learned about handling different refer-

ence frames in the logic classes.
OWL: So what do you think about this solution?
I say that m and f are frames of reference, and then I will tie a predicate to a frame and some event e. As you explained it Raven, the event is the light travelling from Fox to me and back to Fox. The predicate is telling whether the event is measurable or not. The ticks are how the event is measured, so ticks are measurable in the *forest* frame f and the *moving* frame m. I will call the predicate 'measurable', abbreviated 'meas'. Another part is that one frame is moving with respect to another. I will describe this as a function $v(f)$ and $v(m)$ mapping to the real numbers which simply tells us the speed of that reference frame, so $v(f) = 0$ when f is the initial frame of reference — the forest. This gives us the antecedent of the implication — the conditions that have to be true for it to apply. Δt is a function taking the difference between the *end* of the event and the *start* of the event in one of the frames. It all looks like this when written in first order logic:

$$\begin{aligned}&\forall e \forall f \forall m\\&(\text{meas}(e, f) \wedge \text{meas}(e, m) \wedge v(f) = 0 \wedge 0 \prec |v(m)| \rightarrow\\&\Delta t(\text{start}(e, m), \text{end}(e, m)) \prec \Delta t(\text{start}(e, f), \text{end}(e, f)))\end{aligned}$$

I have put the lines $|v(m)|$ because it should not matter whether the moving frame is moving with a positive or negative direction from my point of view, it should be a positive number.
I think this should catch it all.
The Owl said relieved, as again first order logic had shown itself powerful.
FOX: This is indeed more difficult and it is a very different kind of nature than the other examples. There are parts of this which are very appealing and parts which are very appalling. All this focus on the observer seems very compatible with tense logic; am I in the forest or am I on some moving object. On the other hand it seems incompatible that the future is different in the forest than on the moving object and nonetheless to compare those two times.
But there is a big difference between this formula good Owl and all of the previous ones. Here you do not quantify over time like you have done in the other ones. It would feel kind of circular to use time to describe time.
OWL: Yes, you are right my friend. Time only comes in as a function which takes in the beginning and end of an event.
RAVEN: And that is essential, since in the other examples time was thought of as something fixed, but here it is the speed of light that is fixed, as the light beam travels to and from Fox, so it is meaningful to quantify over the event.
FOX: Exactly! And if you look back on the previous translations, then all the quantisations over time that the dear owl has made I have eaten with my tense operators, and if there are no time quantification to eat, how should those operators come into play?
RAVEN: Yes, I see.
OWL: So you think you should not translate this one, and that your logic can not describe special theory of relativity?

FOX: Oh yes I think I should translate it, and I can! But your are right, the tense logic is not strictly useful here. I have got another idea which has the same kind of thought as in tense logic, just more general.
I will start focusing on the initial frame of reference, that you Owl denoted f and take that as the place I evaluate from. Before I focused on where in time I evaluated from. I start in the forest frame of reference f. Here the event e happens where the time difference Δt is measured by the ticks. Since I am standing in f everything that I see moving will be a different reference frame, so I put all these other frames in a box, $\Box$. This box lets me talk about all the frames reachable from my initial frame. So for all other frames that have a velocity greater than zero from f, where this very same event is being measured in exactly the same way, will get a smaller value for Δt than in what was measured in f, so I write it like this[20]:

for f :
$$\forall e \forall r(\text{meas}(e) \wedge \Delta t(\text{start}(e), \text{end}(e)) = r \rightarrow \Box(0 \prec v \wedge \Delta t(\text{start}(e), \text{end}(e)) \prec r))$$

RAVEN: Very cleverly done Fox!
OWL: Yes indeed! And actually is seems like there is a pattern between this and first order logic. You almost said it your self dear fox; you put *all* the frames in a box. So your box $\Box$ behaves like my for all $\forall$. Then we can just translate between the two languages.
FOX: Oh that would be great! Then we talk the same language about time after all, just what I wanted! So how do you think the translation could work?
OWL: [21] You see, every time I use a function or a predicate P assigning a variable to itself, you just say the function or the predicate without a variable because your variables are included in your operators. In your last sentence it was the frames of reference that were included in the operator $\Box$ and in the earlier example it was the times themselves that were included in the tense operators, as you said just before. That means that we can translate p, simply to $P(t)$, where t is the time variable, in first order logic.
When I say $\forall$ you say $\Box$ in the previous translation, so you should have a operator for $\exists$ as well.
FOX: Oh yes, what about $\Diamond$? Yes — a diamond is good.
OWL: You see, then we have a translation between your 'box and diamond' logic and first order logic. Let us call this the standard translation, and write ST in front

[20] The fox is now using modal logic as presented in Fitting and Mendelson (1998).
[21] The owl and the fox will now introduce the standard translation (ST) from modal logic to first order logic and from tense logic to first order logic, based on van Benthem (1983) and Blackburn et al. (2006).

like this:

$$ST_x(p) = P(x) \tag{1.1}$$
$$ST_x(\neg p) = \neg ST(p) \tag{1.2}$$
$$ST_x(p \to q) = ST(p) \to ST(q) \tag{1.3}$$
$$ST_x(\Box p) = \forall y(xRy \to ST_y(p)) \tag{1.4}$$
$$ST_x(\Diamond p) = \exists y(xRy \land ST_y(p)) \tag{1.5}$$

Where R is some binary relation, and the y in ST_y is a new variable that has not yet been used in the translation.
FOX: So, I could just have stated my sentences and then you could have translated that into first order logic, just as a Turing tortoise could have done[22]. Let me try to do it with the previous sentence. So I just start from the beginning and put the ST machinery in front. ST will translate over the variable f:

$$ST_f(\forall e\forall r(\text{meas}(e) \land \Delta t(\text{start}(e), \text{end}(e)) = r \to \\ \Box(0 \prec v \land \Delta t(\text{start}(e), \text{end}(e)) \prec r))$$

ST does not do anything with the quantifier $\forall e$, neither for $\forall r$, so that yields:

$$\forall e\forall r ST_f(\text{meas}(e) \land \Delta t(\text{start}(e), \text{end}(e)) = r \to \\ \Box(0 \prec v \land \Delta t(\text{start}(e), \text{end}(e)) \prec r))$$

As it says in the translation rule 1.3 for an implication we now get:

$$\forall e\forall r(ST_f(\text{meas}(e) \land \Delta t(\text{start}(e), \text{end}(e)) = r) \to \\ ST_f(\Box(0 \prec v \land \Delta t(\text{start}(e), \text{end}(e)) \prec r)))$$

And now comes the interesting part, I use translation rule 1.4:

$$\forall e\forall r(ST_f(\text{meas}(e) \land \Delta t(\text{start}(e), \text{end}(e)) = r) \to \\ ST_m(\forall m(mRf \to ST_m(0 \prec v \land \Delta t(\text{start}(e), \text{end}(e)) \prec r))))$$

Now I will remove all the STs according to rule 1.1:

$$\forall e\forall r(\text{meas}(e, f) \land \Delta t(\text{start}(e, f), \text{end}(e, f)) = r \to \\ (\forall m(mRf \to (0 \prec v(m) \land \Delta t(\text{start}(e, m), \text{end}(e, m)) \prec r)))$$

I can move the $\forall m$ in front:

$$\forall e\forall r\forall m(\text{meas}(e, f) \land \Delta t(\text{start}(e, f), \text{end}(e, f)) = r \to \\ (mRf \to (0 \prec v(m) \land \Delta t(\text{start}(e, m), \text{end}(e, m) \prec r))$$

[22]The fox and the owl went thoroughly through computability when they met each other one other night. Here the Turing tortoise is what the animals call a Turing machine (Jensen and Kofod; 2015).

This is not exactly what you proposed. But it means something very similar.
OWL: If the translation should work properly should it not be exactly the same?
FOX: Yes it should, and it does, it just has inherited my point of view from the modal language.
OWL: Yes of course. I see that your translated sentence turns into my old sentence if we add a quantifier over the reference frame f, which is not included in the modal language. Since the modal language has a local point of view and the first order language has a global point of view.
Furthermore it has inherited the value r that is assigned to all your sentences. The r is not needed in first order logic, so I erase that and insert $\Delta t(\text{start}(e, f), \text{end}(e, f))$ at the last standing r.
Now I have to do something about the binary relation. It is important that the same event e is measured in both frames, and it is important to point out which frame of reference is the initial. Therefore it is meaningful to say that $mRf = (\text{meas}(e, m) \wedge v(f) = 0)$, that yields an expression:

$$\begin{aligned}
&\forall e \forall f \forall m \\
&(\text{meas}(e, f) \wedge \Delta t(\text{start}(e, f), \text{end}(e, f)) \rightarrow \\
&(\text{meas}(e, m) \wedge v(f) = 0) \rightarrow \\
&0 \prec v(m) \wedge \Delta t(\text{start}(e, m), \text{end}(e, m)) \prec \Delta t(\text{start}(e, f), \text{end}(e, f)))
\end{aligned}$$

since $a \rightarrow b \rightarrow c = a \rightarrow b \wedge a \rightarrow c = a \wedge b \rightarrow c$

$$\begin{aligned}
&\forall e \forall f \forall m \\
&(\text{meas}(e, f) \wedge \Delta t(\text{start}(e, f), \text{end}(e, f)) \wedge (\text{meas}(e, m) \wedge v(f) = 0 \wedge 0 \prec v(m) \rightarrow \\
&\Delta t(\text{start}(e, m), \text{end}(e, m)) \prec \Delta t(\text{start}(e, f), \text{end}(e, f)))
\end{aligned}$$

And it is the same thing.
The standard translation does indeed work. Very mechanical and beautiful!
FOX: Yes, And I think it will work very simply and more intuitively with tense logic. For tense logic this binary relation R is just greater or smaller than, let us fix it at smaller than, and then t is always the variable. Furthermore $\Diamond$ is existential like F and P and $\Box$ is universal like G and H. So the translation between tense logic and first order logic is as follows.

$$ST_{t_0}(p) = P(t_0) \tag{1.6}$$
$$ST_{t_0}(\neg p) = \neg ST_{t_0}(p) \tag{1.7}$$
$$ST_{t_0}(p \rightarrow q) = ST_{t_0}(p) \rightarrow ST_{t_0}(q) \tag{1.8}$$
$$ST(Fp) = \exists t(t_0 < t \wedge ST_t(p)) \tag{1.9}$$
$$ST(Pp) = \exists t(t < t_0 \wedge ST_t(p)) \tag{1.10}$$
$$ST(Gp) = \forall t(t_0 < t \rightarrow ST_t(p)) \tag{1.11}$$
$$ST(Hp) = \forall t(t < t_0 \rightarrow ST_t(p)) \tag{1.12}$$

OWL: It really is the same thing!
FOX: So we did not speak different languages after all.
Very happy the two friends contemplated over their new discovery[23].

1.2.2.6 Quantum Mechanics — The Uncertainty Principle

RAVEN: So with your new insight in your common but different logical languages, would you like to try another sentence?
FOX: Yes, now we are like one brain just thinking differently, that can only be better!
RAVEN: Fine, I have thought of one last sentence for you, this is possibly even more against your intuition. And it is also beyond the capabilities of your senses, for it concerns those very tiny objects that every thing consist of, elementary particles and waves. When observing this tiny world, the physical laws are not governed by the same laws as we know from classical mechanics. The sentence I will challenge you with is an instance of this.

> It is impossible simultaneously to know the momentum and the position of a system exactly[24].

FOX: So the time dimension here is the simultaneity?
RAVEN: Yes indeed.
FOX: But in the forest as I know it I find no problem defining Owl's position and velocity at the same time, but this does not count in 'tiny world'?
RAVEN: Nope! This is very essential for quantum mechanics.
FOX: That gives me an idea; so it is possible for me to know the owl's position and velocity and therefore momentum simultaneously, but not possible in the quantum world.
Since I can easier grasp the classical world I will start there. Look, there is a system s for which we always want to know the position x and the momentum p. Since we are saying 'always' I can use my previous invented operator A. For the classical system it makes sense to say:

$$\forall s \forall r \forall r' A(p(s) = r \wedge x(s) = r')$$

Where r and r' are both some real number.
RAVEN: It makes sense yes.
FOX: Fine, then in the quantum mechanical system Ψ, it is not the case that one can

[23]It is very easy to check that this translation (and the earlier one given for modal logic too) is correct in the following sense:
A tense logical formula ϕ is true in some model at a time t if and only if its standard translation $ST_{t_0}(\phi)$ is satisfied in the same model where the unique free variable t_0 that it contains is mapped to t. You prove this by induction on the structure of ϕ, using the clauses of the translation see van Benthem (1983) or Blackburn et al. (2006).
[24]The raven is referring to Heisenberg's position-momentum uncertainty principle; $\sigma_x \sigma_p \geq \frac{\hbar}{2}$

know the position and momentum exact, that is if $x(\Psi) = r$ then $p(\Psi)$ is infinitely inexact, I will write it like this:

$$\forall \Psi \forall r A(x(\Psi) = r \to p(\Psi) = R)$$

Where R is a set with infinitely many members.
Raven: That was a clever way around it sly fox, I like the solution with the infinite set!
Owl: This is so neat, I can just make use of the standard translation now.

$$ST_t(\forall \Psi \forall r A(x(\Psi) = r \to p(\Psi) = R))$$
$$\forall \Psi \forall r ST_t(A(x(\Psi) = r \to p(\Psi) = R))$$

So I remember that your $Aq = Gq \wedge q \wedge Hq$

$$\begin{aligned}&\forall \Psi \forall r\\ &ST_t((G(x(\Psi) = r \to p(\Psi) = R) \wedge\\ &(x(\Psi) = r \to p(\Psi) = R) \wedge\\ &H(x(\Psi) = r \to p(\Psi) = R))\end{aligned}$$

According to the tense logic translation (rule 1.11, 1.12 and 1.6)

$$\begin{aligned}&\forall \Psi \forall r\\ &(\forall t'(t < t' \to ST_{t'}(x(\Psi) = r \to p(\Psi) = R)) \wedge\\ &ST_{t'}(x(\Psi) = r \to p(\Psi) = R) \wedge\\ &\forall t'(t' < t \to ST_{t'}(x(\Psi) = r \to p(\Psi) = R)\end{aligned}$$

$$\begin{aligned}&\forall \Psi \forall r\\ &(\forall t'(t < t' \to (x(\Psi, t') = r \to p(\Psi, t') = R)) \wedge\\ &(x(\Psi, t') = r \to p(\Psi, t') = R) \wedge\\ &\forall t'(t' < t \to x(\Psi, t') = r \to p(\Psi, t') = R)\end{aligned}$$

$$\begin{aligned}&\forall \Psi \forall r \forall t'\\ &(t < t' \to (x(\Psi, t') = r \to p(\Psi, t') = R) \wedge\\ &(x(\Psi, t') = r \to p(\Psi, t') = R) \wedge\\ &t' < t \to (x(\Psi, t') = r \to p(\Psi, t') = R))\end{aligned}$$

Since there is quantified over *all* times the top and the bottom lines could be excluded, and I am left with

$$\forall \Psi \forall r \forall t'(x(\Psi, t') = r \to p(\Psi, t') = R)$$

It is a little tedious and ugly and you need to work with simplifying to get a beautiful expression, but it works.
Raven: That sounds like a happy end to the problem you posed Fox, that you wanted a common language for talking about time. Though you have only shown that the translation works from tense logic or box-diamond logic to first order logic, and not the other way around, does it?
Fox: Oh, I did not think about that...
Owl: You are right Raven we need to think about that. I did not develop the modal logic, but as I see it, it does not work both ways.
It is rather simple to explain, if I can think of a first order formula that is not translatable into modal logic or tense logic. I will show it with modal logic since this is a 'bigger' logic, so if it works here it will also work for tense logic.
Modal formulas are invariant under bisimulations, but first order formulas are not always, therefore any first order formula that is not invariant under bisimulation is not translatable, and therefore the translation does not work the other way around.
Fox: What does it mean to be 'invariant under bisimulations'?
Owl: I will be happy to explain it.
If you imagine that these three sticks are forming two models.
The owl pointed at some neighbouring branches that looked like figure 1.2.3.

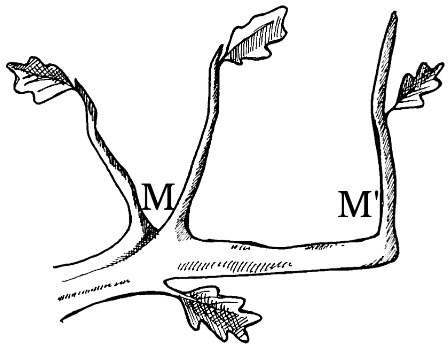


Figure 1.2.3: Two models M and M'.

In first order logic we would look at the models like *we* see them now. Do they look different to you?
Fox: Yes definitely! That is a silly question.
Owl: You might think so. You yourself have talked about that modal logic has a local point of view. When you made the time dilation modal logic expression you emphasised the power of it. And that is exactly what you would do as a modal logician; you would jump in the models and look around you. It does not matter where you would place yourself in the two models, you could not say if you were in M or in M'.
Fox: Oh, you are right, that is indeed the behaviour of modal logic.
It is just like when I hunt in the glade where the grass has grown tall. I cannot see the rabbit or the rabbits that I hunt. So what I do is to smell them, that is just

like being in the model: I know that there is some undefined amount of rabbit in the glade, but I have no clue if there is one or 10 of them, since I cannot distinguish them.
Owl: And when *I* hunt, I fly above everybody and have very good night vision, so I can see all the mice I'm hunting.
Fox: Yes I understand!
Owl: Well, then I have found a model that in first order logic is not invariant under bisimulation. Therefore I have shown that the standard translation cannot translate every first order logical sentence into modal logic. In fact modal logic is a subset of first order logic[25].
Raven: Is that a satisfying end then? It seems like first order logic is privileged?
Owl: And it is since it is stronger due to the explanation!
The owl said proudly, being very secure with itself and its viewpoint.
The fox, on the other paw, was not so pleased about the outcome of the explanation. The locality of modal logic and tense logic, was exactly what it had wished the logic to contain, and now this very property was the reason that showed it a smaller logic. The fox found it hard to admit its defeat. At the same time the fox was happy about that they had reached a common language for talking about time, but found it hard to accept that it was the owl's formalisation and therefore that view that won. It was not ready to give up its own formalisation just yet, after all it did prove worthy describing all the raven's examples.
Meanwhile the raven was thinking about what to do with the result of the experiment.
Raven: All right let us sum up what we have figured out during this experiment:
First of all you showed that it is possible to translate the sentences about the physical world into first order logic and tense logic or modal logic. That could tempt one to draw the conclusion that the distinction of time that you two hold does not make sense in physics. But I think that would be a too hasty conclusion, I see this little experiment as a first order approximation, and now we need to look at some finer details, since this experiment looks like it has been too crude. So now I will present what outcomes of the experiment I find interesting and worthwhile making a finer investigation of:
There is an asymmetry in time that makes the present special, due to the explanation I gave about entropy, so it would be interesting to make a closer investigation of the asymmetry of time in physics [26].
I also liked the discussion about the time dilation in special relativity. Where it got clear that the first order logic holds a global view on time and pointed out two frames. The modal logic holds on that there is *a point of view* and has therefore a local view. From that you found the standard translation, which might come in handy.

The raven has presented six sentences taken from different branches of physics and

[25]What the creatures did not know about the range of this property was that any formula in first order logic that is bisimilation invariant is equivalent to a formula in modal logic, this is The van Benthem-Rosen characterisation theorem (Blackburn et al.; 2006).

[26]This is investigated further in Section 2.1.

Figure 1.2.4: The raven catching up on the three animals' experiment.

let the fox and the owl translate those sentences into first order logic and tense logic to see if one of the two logics could describe the sentences better than the other. The animals have shown that both of the two logics are able to describe the sentences, but the fox had to invent modal logic to translate the sentence about time dilation. Furthermore they have figured out that it is possible to translate from tense logic and modal logic to first order logic, and the owl has shown that it is not possible to translate from first order logic to tense and modal logic.
They have seen that first order logic is a bigger logic than tense and modal logic, so the two logics are not the same thing, but one is embedded in the other. This is due to that modal and tense logic holds a local point of view where first order logic holds a global point of view. This is the distinction between the two logics.
It is still not clear if the distinction the owl and the fox keeps emphasising is meaningful in physics. Therefore the raven has pointed out some areas that would be interesting to investigate further.
In the next chapter, the three animals will try to dig deeper into some of these areas. They will dig into reversibility in physics, and figure out if there are properties of the reversible processes and irreversible processes that are similar to the properties of the two logics.

Chapter 2

Reversibility?

In this chapter the animals will try to answer the question of whether there is a correspondence between reversibility in physics and first order logic, and irreversibility in physics and tense logic, to see if the distinction between the two logics exists in this respect. They will do this by looking at different properties of reversibility and irreversibility. By diving into some laws of physics and time reversing them, the animals realize in which regions of physics time is and is not reversible, and therefore which physical laws are symmetric and asymmetric. The animals will discuss whether the two views on time that are represented in the two kinds of logic in Chapter 1 have a similar property.
The animals also look at Onsager's reciprocal relation and learn that reversibility is more fundamental than irreversibility. They will discuss how this property is connected to the two logics. They argue that first order logic can be seen as more fundamental than tense logic due to the standard translation.

RAVEN: Our experiment did not enlighten us much more about whether the distinction of time your two logics represent exists in physics. It seems as if the experiment was too crude, so I do not want to give up just yet, but instead I think we should try to make a more fine-grained investigation.
I think we need to consult a peculiar little frog I know who lives by the lake[1]. It is a very clever frog, and maybe it can help us get closer to an answer about the time distinction in physics. But you need to be polite towards the frog, else it might not want to help us.
FOX: Then let us go and find this frog. I am very excited to see the outcome of this!
The three animals headed towards the lake where the frog lived. While they walked, or flew, they talked about their standpoint and what they were going to ask the frog about.

[1] This frog is not to be confused with its relative calling itself Mr. Frog, who you can meet in the stories by Egesborg et al. (2016a) and Egesborg et al. (2016b). Sir Frog was by some animals considered as the most clever animal in the forest, not the least by itself.

RAVEN: Fox, in your logic the present is a privileged time, right?
FOX: Yes! Since tense logic has a local view with respect to time it makes the present privileged; the present is where we evaluate from; here the forest.
RAVEN: Good, and from your perspective Owl, the present is not privileged at all.
OWL: That is correct. The present, in my point of view, is a point on the axis of time like any other.
RAVEN: All right, so the role of the present is definitely an important distinction in your two logics.
The owl and the fox nodded confirming.
RAVEN: To figure out whether your distinction regarding time has any parallel in physics, we need to figure out if there is such a thing as the *present* in physics then.
FOX: Yes! by the oak we figured out that entropy defines the present!
The fox had come to like this concept of entropy, since it felt that it was here tense logic and physics would agree.
RAVEN: That is right. I said then that entropy increases for irreversible processes and stays constant for reversible processes.
OWL: So what we need to ask the frog is whether there is such a thing as a reversible or an irreversible process?

RAVEN: Yes, that would be a good question for the frog.
FOX: All right, so if the frog's answer is that there is no such thing as a reversible process, and everything is irreversible, what then?
RAVEN: Then *all* times could be mapped to a value of entropy and the largest value would constitute the *present* since the entropy increases. That would be a hint about that it does make sense to talk about a *present*. And tense logic would then be a good way to describe how time is viewed in physics — at least when it comes to reversibility.
OWL: And oppositely, if the frog's answer is that there is no such thing as irreversibility?
RAVEN: Then it's a hint about that only the view of time represented in first order logic is relevant. Since a mapping of the value of entropy to time would not be meaningful if entropy never changes, and a largest value would not define the present! But this solution is not relevant, since entropy *does* change!
FOX: And if the frog's answer is that there are both reversible processes and irreversible processes?
RAVEN: Then the present makes sense for the irreversible processes and the *present* is meaningless for reversible processes, hence the distinction of time in tense logic and first order logic would make sense respectively for irreversible processes and for reversible processes. This is our hypothesis, we will see if this is meaningful or not when we have talked with the frog.
OWL: Maybe it is not that simple to say if there are reversible or irreversible processes in physics and describe how the nature of those processes are.
RAVEN: No, you are right Owl. But then we will learn about the properties of reversible and irreversible processes, and we have the properties of your two logics

in fresh memory. Then we can compare those properties with one another, and a picture might form.

The animals have presented and argued for their hypothesis about how the distinction of time might exists in physics. Now they will meet Sir Frog, learn more about reversibility and see whether the hypothesis about the correspondence between reversibility and first order logic and irreversibility and tense logic is meaningful.

2.1 Symmetry

In this section you will meet Sir Frog, he will show how physical laws are time symmetric within classical mechanics, electrodynamics and quantum mechanics. The three other animals will see the connection between what the frog presents and the sentences the raven presented in chapter 1. He will also show that the second law of thermodynamics and the Diffusion equation are asymmetric.
The animals compare the symmetry property of the physical laws with the same properties of first order logic and tense logic. They argue that it is not meaningful to talk about a symmetric logic, but about a symmetric time structure. The structure of time is not different in the two logics, the correspondence between first order logic and reversibility and tense logic and irreversibility is therefore not apparent with respect to symmetry.

Figure 2.1.1: Sir Frog enthroned on his rock.

The three animals reached the frog enthroned on a rock, well aware of its intelligence.
RAVEN: Good evening Frog.
FROG: Correction; *Sir* Frog!
RAVEN: Oh, I'm sorry; Good evening *Sir* Frog.
FROG: What do you want from me?

RAVEN: We have some questions for you, clever Sir Frog!
FROG: So, What is your question? I will enlighten you on whatever topic it might be, because I am the wisest in this forest!
FOX: What is the *now*?
The fox hurried to pose the question before the two others could say anything.
OWL: Or rather *is* there any meaning in a *present* in physics? Or as I said earlier, is there such a thing as reversibility or irreversibility in physics?
the owl specified.
FROG: As I said I will enlighten you.
RAVEN: Go ahead Sir Frog.
FROG: The short answer is yes, there is such a thing as reversibility *and* there is such a thing as irreversibility. But I sense you do not want the short answer.
RAVEN: No, we want to understand how these terms are described, so we can compare them with these two creature's understanding of time.
The raven pointed at the owl and the fox.
FROG: Very well, I will start at the most beautiful place, talking about symmetry with respect to time in the physical laws.
OWL: This sounds like a very good place to start, symmetry is indeed a very beautiful thing.
FOX: Maybe so, but what do you mean when you say symmetry in the physical laws? I think of symmetry as something visual and geometrical.
FROG: Yes, most creatures do, but we can talk about symmetry of laws by looking at which conditions can be changed and yet the law considered will still work the same way. So a physical law can be symmetric according to position; if I have some system...
The fox interrupted.
FOX: For example the moon rotating around the earth?! Like the first sentence we had by the oak[2].
The frog looked sharply at the fox, not pleased with the interruption.
FROG: Will you not interrupt me while I enlighten you?!
FOX: Oh, I'm sorry *Sir* Frog!
The frog cleared its throat and continued.
FROG: That could be an earth-moon system. If we would move that to some other place in space it would still work the same, as long as all the relevant parts of the solar system moved with it.
A physical law can also be symmetric with respect to rotation; if we consider the same earth-moon system, the moon would rotate around the earth in exactly the same way if we would turn it 90 degrees.
Finally a law can be symmetric according to time — and that is the part that is important here. That means that we can let the moon rotate around the earth at one time, then we can imagine that we stop the rotation, and wait a while and start it again, and nothing will have changed it will rotate around the earth in the same way as earlier. You could also, if it was possible, reverse the time and the system

[2]See section 1.2.2.1

would work the same and not break any physical laws[3].
Time symmetry is obviously the one which is interesting for you.
RAVEN: Yes that is indeed the interesting one.
FROG: Then let us take a closer look at that.
First I will show you that the laws of classical mechanics are time symmetrical, or you could say that they are invariant under time reversal.
The excited fox could not resist interrupting once more.
FOX: Let me guess; invariant under time reversal means that if you look at the process, you cannot tell whether time is going one or the other way, just as I could not distinguish the two models from each other while showing the invariance under bisimilation[4].
RAVEN: Exactly Fox!
The raven said encouragingly, and looked anxious at how the frog turned a slightly more red of annoyance under its otherwise very green skin. Nevertheless the frog continued.
FROG: I would say it more formally: If the solution for t and for $-t$ are both solutions to Newton's second law, and the equation itself is unchanged, we say that the equation is time symmetric.
OWL: Please, could you show how that is done formally Sir Frog?
FROG: Naturally I can! As I said Newton's second law is time invariant. Which means that if the position function $\mathbf{r}(t)$[5] is a solution $\mathbf{r}(-t)$ is also a solution. I will show it by testing it. So this is Newtons second law without time reversal:

$$\mathbf{F}(\mathbf{r}(t)) = m\frac{d^2}{dt^2}\mathbf{r}(t) \tag{2.1}$$

If I time reverse that, it looks like this:

$$\mathbf{F}(\mathbf{r}(-t)) = m\frac{d^2}{d(-t)^2}\mathbf{r}(-t) = m\frac{d^2}{dt^2}\mathbf{r}(-t) \tag{2.2}$$

Look, nothing has changed other than the t in the position function; the minus disappears in the denominator in the derivative because t is squared in equation 2.1. So it behaves as we wanted it to behave to be symmetric. Therefore Newton's second law is invariant under time reversal.
RAVEN: That means that the first sentence we worked with earlier[6] is time symmetric; I think that is clear by now.
OWL: I like this formal way of showing symmetry in the physical laws! Is it possible to argue like this for other things besides classical mechanics?
FROG: Oh yes it is, you can do the same in electrodynamics and quantum mechanics[7].
RAVEN: How can we show it in electrodynamics then?

[3]The presentation of symmetries that the frog just gave is based on Feynman (1963, 2006, 2013).
[4]See section 1.2.2.6.
[5]The bold fond denotes vectors
[6]See section 1.2.2.1.
[7]This is done in Snieder (2002).

FROG: The central laws in electrodynamics are *Maxwell's equations.* They look like this:

$$\nabla \cdot \mathbf{E}(r,t) = \frac{\rho(r,t)}{\varepsilon_0} \tag{2.3}$$

$$\nabla \cdot \mathbf{B}(r,t) = 0 \tag{2.4}$$

$$\nabla \times \mathbf{E}(r,t) = -\frac{\partial \mathbf{B}(r,t)}{\partial t} \tag{2.5}$$

$$\nabla \times \mathbf{B}(r,t) = \mu_0\varepsilon_0\frac{\partial \mathbf{E}(r,t)}{\partial t} + \mu_0\mathbf{J}(r,t) \tag{2.6}$$

Where $\mathbf{E}$ is an electric field, ρ is charge density, $\mathbf{B}$ is a magnetic field, ε_0 and μ_0 are the electric permittivity and magnetic permeability respectively and $\mathbf{J}$ is the current density.
I will show that the equations are invariant under time reversal in exactly the same way as I did with Newton's second law.

$$\nabla \cdot \mathbf{E}(r,-t) = \frac{\rho(r,-t)}{\varepsilon_0} \tag{2.7}$$

$$\nabla \cdot \mathbf{B}(r,-t) = 0 \tag{2.8}$$

$$\nabla \times \mathbf{E}(r,-t) = -\frac{\partial \mathbf{B}(r,-t)}{\partial(-t)} = \frac{\partial \mathbf{B}(r,-t)}{\partial t} \tag{2.9}$$

$$\begin{aligned}\nabla \times \mathbf{B}(r,-t) = \mu_0\varepsilon_0\frac{\partial \mathbf{E}(r,-t)}{\partial(-t)} + \mu_0\mathbf{J}(r,-t) = \\ -\mu_0\varepsilon_0\frac{\partial \mathbf{E}(r,-t)}{\partial t} + \mu_0\mathbf{J}(r,-t)\end{aligned} \tag{2.10}$$

OWL: But equation 2.5 has a minus in front and equation 2.9 has not, so they are not the same.
FOX: That is true; equation 2.6 and 2.10 are not the same either, since the first term changes sign!
FROG: Of course it is the same! Just listen! They are the same if $\mathbf{B}$ and $\mathbf{J}$ change sign under time reversal; I will note time reversal by '$\xrightarrow{t}$' then

$$\mathbf{B}(r,t) \xrightarrow{t} -\mathbf{B}(r,-t) \tag{2.11}$$

and

$$\mathbf{J}(r,t) \xrightarrow{t} -\mathbf{J}(r,-t) \tag{2.12}$$

that makes Maxwell's equations look like this under time reversal:

$$\nabla \cdot \mathbf{E}(r,-t) = \frac{\rho(r,-t)}{\varepsilon_0} \tag{2.13}$$

$$\nabla \cdot -\mathbf{B}(r,-t) = 0 \tag{2.14}$$

$$\nabla \times \mathbf{E}(r, -t) = -\frac{\partial \mathbf{B}(r, -t)}{\partial t} \tag{2.15}$$

$$\begin{aligned}
\nabla \times -\mathbf{B}(r, -t) &= -\mu_0 \varepsilon_0 \frac{\partial \mathbf{E}(r, -t)}{\partial t} - \mu_0 \mathbf{J}(r, -t) \\
&= -\left(\mu_0 \varepsilon_0 \frac{\partial \mathbf{E}(r, -t)}{\partial t} + \mu_0 \mathbf{J}(r, -t) \right) \\
\Rightarrow \nabla \times \mathbf{B}(r, -t) &= \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}(r, -t)}{\partial t} + \mu_0 \mathbf{J}(r, -t)
\end{aligned} \tag{2.16}$$

FROG: Now it is obvious that the equations are invariant under time reversal.
RAVEN: So that also means that the sentence from section 1.2.2.4 is time reversible.
FROG: What is that for a sentence?
RAVEN: By the oak we translated the sentence:

> A charging capacitor draws a large current at the start and a lower current later.

FROG: For forest sake! That is not a reversible system! This is true if it is a resistor, capacitor circuit. And if there is a resistor it dissipates energy, and is therefore not reversible.
RAVEN: Oh yes, that is true, I did not think about that.
FOX: Sir Frog, not to be impolite, but just saying that **B** and **J** get negative under time reversal, to me it just seems like a funky trick. *Is* it really like this here in the real forest world?
FROG: Yes! It is really like this. The current density obviously needs to change sign, since when time is reversed the direction the charges move also changes sign. And therefore the magnetic field also changes sign because it is produced by the current[8].
FOX: All right then. Why does the electric field **E** not change sign then?
FROG: My goodness! The electric field is not a result of moving charges, but simply just charges moving or not. The direction of the electric field is only dependent on the sign of the charge[9].
RAVEN: You said you could make the same time reversing operations for quantum mechanics as well.
FROG: Of course I can!
The central equation in quantum mechanics is the Schrödinger equation, it looks like this:

$$-\frac{\hbar^2}{2m}\nabla^2\Psi(r,t) + V(r)\Psi(r,t) = i\hbar\frac{\partial}{\partial t}\Psi(r,t) \tag{2.17}$$

Where Ψ is the wave equation, V is potential, $\hbar$ is the Planck constant and m is the mass of the particle.
When I perform time reversal on the Schrödinger equation it changes like this:

$$-\frac{\hbar^2}{2m}\nabla^2\Psi(r,-t) + V(r)\Psi(r,-t) = i\hbar\frac{\partial}{\partial(-t)}\Psi(r,-t) = -i\hbar\frac{\partial}{\partial t}\Psi(r,-t) \tag{2.18}$$

[8](Rachidi and Rubinsein; 2013)
[9](Griffiths; 1999)

Fox: Again, equation 2.17 and 2.18 are not the same! The term on the right of the equality sign has gotten a minus in front that it did not have before.
The frog sighed
Frog: Yes they are the same Fox! If I complex conjugate[10] equation 2.18 like this

$$-\frac{\hbar^2}{2m}\nabla^2\Psi^*(r,-t) + V(r)\Psi^*(r,-t) = i\hbar\frac{\partial}{\partial t}\Psi^*(r,-t) \tag{2.19}$$

they are the same, if the wave equation behaves like this when it is time reversed:

$$\Psi(r,t) \xrightarrow{t} \Psi^*(r,-t)$$

Fox: This again just seems like a trick to me...
Frog: But it is not! And it is due to the way you determine an observable in quantum mechanics. An observable is something you can measure; that could be position or momentum. The expectation value[11] of an observable $\mathbf{Q}$, is calculated by the integral of the observable squeezed in between the complex conjugated wavefunction and the unchanged wavefunction:

$$\int \Psi^* \ \mathbf{Q}\Psi = \langle Q \rangle$$

So it does not matter that I have complex conjugated it[12]. But you probably do not understand that anyway[13].
Raven: We are satisfied with your explanation Sir Frog.
Sir Frog mumbled some undefinable, but certainly grumpy things inside its big frog mouth.
Owl: So what you have shown so far has all been invariant under time reversal, it seems that all physical laws are time reversible.
Frog: No, that is not the case! How can you think that? Have you ever grown younger?
The frog was almost falling down it's stone with impatience.
Owl: No I have not, but why is that?
Frog: There are physical laws that are not time reversible. Entropy for example[14]:

$$\frac{dS(t)}{dt} \succeq 0 \tag{2.20}$$

[10] Complex conjugated means to transpose the matrix and change sign on the complex members (the parts containing an i) of the matrix. For example:

$$\mathbf{A}^* = \begin{pmatrix} 0 \\ i \end{pmatrix}^* = \begin{pmatrix} 0 & -i \end{pmatrix}$$

(Griffiths; 1995).

[11] The expectation value is to be understood as the average of the measurements of particles in the same state.

[12] (Snieder; 2002)

[13] What Sir Frog really means is that it is not the purpose of this report to go deeply into this, but we are satisfied with the current explanation.

[14] I remind the reader that the sign '$\preceq$' is used to denote '$\leq$' that does not concern times, but other outcomes of functions.

If I time reverse this it is not the same:

$$\frac{dS(-t)}{d(-t)} = -\frac{dS(-t)}{dt} \succeq 0 \Rightarrow \frac{dS}{dt} \preceq 0 \tag{2.21}$$

[15]This clearly violates the second law of thermodynamics! In fact it says exactly the opposite of the law.

FOX: And you can not just say that the entropy changes sign under time reversal, like you did before with the magnetic field and the current density in Maxwell's equations?

The frog made a deep and arrogant sigh.

FROG: No! You silly fox! How in the forest would you explain that change in entropy would change sign at time reversal?

FOX: Well, I don't know...

FROG: No! Because it is illegal to violate the second law of thermodynamics!

The frog looked sharply at the fox, that was embarrassingly scratching its paw in the forest floor.

OWL: Is there other laws that are time asymmetric?

FROG: Yes there is! Diffusion for instance is time asymmetric.

OWL: Will you show it?

FROG: You do it in exactly the same way. The law is the diffusion equation[16]:

$$\frac{\partial f(x,t)}{\partial t} = D\frac{\partial^2 f(x,t)}{\partial x^2} \tag{2.22}$$

Where $f(x,t)$ is a function describing the concentration and is dependent on position x and time t; D is the diffusion constant.

If you have listened properly it should not be difficult for you to do it yourself.

RAVEN: Yes, I can do it like this

$$\frac{\partial f(x,-t)}{\partial(-t)} = -\frac{\partial f(x,-t)}{\partial t} = D\frac{\partial^2 f(x,-t)}{\partial x^2} \tag{2.23}$$

That is not the same law since there is a minus on the left side of the equality sign that was not there before.

FROG: Correct.

Now the owl and the raven were thinking about the outcome of what the frog had said about symmetries in the laws of physics. The fox had become rather silent...

RAVEN: Well then; the fundamental laws of classical mechanics, electrodynamics and quantum mechanics are invariant under time reversal hence symmetric. But the change in entropy; the second law of thermodynamics and the diffusion equation is not.

Owl and Fox, would you say that your logics are symmetric?

[15]The reversion of the change in entropy and the following diffusion equation is done by the author.

[16]The diffusion equation is shown only in one dimension for simplicity, the result is the same in three dimensions: $\frac{\partial f(x,t)}{\partial t} = D\nabla^2 f(x,t) \xrightarrow{t} -\frac{\partial f(x,-t)}{\partial t} = D\nabla^2 f(x,-t)$.

The fox and the owl looked at each other, neither of them really knew what to say to that.

OWL: Well, I am not sure if it makes sense to say that a logic is symmetric.

A logic is a way of describing a structure. The only sensible way that I can think of saying that a logic is symmetric, is saying; if we flip around the *structure* it is talking about then the same sentence stay true.

FOX: What do you mean about flipping time?

OWL: For example consider the structure $(\mathbb{N}, <)$. Maybe time looks like this, with 0 being the first point in time.

FROG: Yes, that is the big bang.

OWL: The structure of time is then $\exists t \forall t' (t \neq t' \rightarrow t < t')$.

FOX: Then t has to be equal to zero!

OWL: That is true, but then if we flip around time, the first time becomes the last and there will be no first time, therefore $\exists t \forall t' (t \neq t' \rightarrow t < t')$ is false, since it is no longer true that it always is possible to find a bigger t'! And $\exists t \forall t' (t \neq t' \rightarrow t > t')$ is correct.

So first order logic is *not* symmetric if the structure of time is the natural numbers. But sometimes flipping structures around in the same way will keep the same things true.

For example, if the time structure is the real numbers, it does not matter if we use the structure $(\mathbb{R}, <)$ or $(\mathbb{R}, >)$ there is no *first* of *last* point in the real numbers, they continue out in infinity on both sides!

First order logic does not see the difference, between those two structures, and is therefore symmetric if the structure is.

So logical symmetry seems to depend on structural symmetry.

RAVEN: I see, in your first example you broke the structural symmetry. First order logic saw this! It saw the big bang, and noticed that it was gone when we flipped it around.

And in your second example with the real numbers, it saw no difference — flipping the order makes no difference.

FOX: That is first order logic, let me think a little about my tense logic then...

I think it would make no difference because of the standard translation. I can just translate my tense logical sentence into first order logic and then all the same arguments will obviously apply.

OWL: Yes, I would say the same thing. Suppose first order logic can not see the difference between a structure and the same structure flipped over, just like the real numbers. Then your logic can not either. Because if you had a magic formula that *could* see the difference, we could translate it using the standard translation. But that would yield a first order formula which would see the difference as well — we have a contradiction!

FOX: Well, so my logic can not see flipping differences that yours can not. Or to put it the other way around, if my tense logic sees a difference so does your logic.

Logics are languages designed to be used with any conceivable mathematical structure. You can talk about anything with the logics.

Raven: But physicists do not think about such tasks. They want to describe the world around us.
Fox: Yes that seem to be very different; the logics and the physics. Two logical formulas can look very similar, and therefore talk about the same structure, but express two different very different things in physics.

In this Section the frog has shown that the central equations in classical mechanics (Newtons second law), electrodynamics (Maxwell's equations) and quantum mechanics (The Schrödinger equation) are symmetric according to time. It is shown that the second law of thermodynamics and the diffusion equation is not symmetric according to time. The symmetry of first order logic and tense logic has been discussed through the structures of time. Since first order logic and tense logic can describe the same structures, there is no difference between them in this sense and the symmetry of the logic is therefore the symmetry of the structure.
Through this comparison it is therefore not possible to find a correspondence between first order logic and reversibility and tense logic and irreversibility in respect to symmetry.

2.2 Fundamentality

In this section the animals will look at Onsager's reciprocal relation. Through this they will come to realise that reversibility seen on a microscale is a more fundamental property than irreversibility seen on a macroscale,. It is argued for that first order logic is more fundamental than tense logic due to the standard translation.

Raven: I got to think about that since tense logic is a subset of first order logic we kind of scale up the logic when we translate from tense logic to first order logic, that is the reason the translation only works one way, but still the two logics seems different. Does there happen anything diffent in physics when you scale from small scale physics to bigger scale physics?
Frog: For Frog's sake; 'big scale and small scale physics' so imprecise! You are lucky that you have found a frog that is clever enough to dissect your questions and get some sense out of them!
Raven: Oh yes we are very lucky! So what sense do you make out of my imbecile question, Sir Frog?
Frog: I assume that you mean length scales when you talk about large and small scale physics. If so, the answer is yes; reversibility on a microscopic level yields irreversibility on a macroscopic level.
Raven: Oh, how is that?
Fox: That indeed sounds very weird!
Frog: I can explain that reversibility yields irreversibility by looking at a system which is not in equilibrium, and where two kinds of fluxes exists. Through that

system I can show that there is irreversibility on a big scale and reversibility on a small scale.
Fox: So the distinction of reversible and irreversible is the distinction between different sizes...?
The fox said thoughtfully. It will catch up on this observation in Section 3.1. Raven: It certainly sounds like it, but let us figure out where this distinction comes from.
The raven said while facing the fox and the owl. Then turning to back the frog again.
Frog: Very well. Listen[17]. When it becomes fall we see many birds flying over this forest. I will say that there is a flux of birds towards the south, since they are attracted to places with warmer temperature. In the fall there is a lot of wind here in the forest as well, or more precisely; through the same cross sectional area, wind is blowing because of different pressure in the north and the south[18]. Then let us assume that the force that control the flux of birds and the force that control the flux of air particles are linearly dependent on the relevant fluxes. Then F_b is the former force with the relevant flux of birds φ_b and F_a is the latter force with the relevant flux φ_a of air. If the two fluxes are independent on each other their relation are as follows:

$$F_b = R_b \varphi_b \tag{2.24}$$

$$F_a = R_a \varphi_a \tag{2.25}$$

Where R_b and R_a are the proportionality factor.
Raven: This looks like an analogue to Ohm's law, where the R's represent some kind of resistance, and F is analogue to voltage and the fluxes analogue to current[19].
Frog: Yes, and those analogues are meaningful, but *not* for discussion now!
Raven: No, of course not.
Frog: The thing is that the two fluxes are not independent, even though they might seem so. If they are not independent equation 2.24 and 2.25 are incorrect, and need an extra term showing the mutuality through two new proportionality factors, R_{ba} and R_{ab}

$$F_b = R_b \varphi_b + R_{ba} \varphi_a \tag{2.26}$$

$$F_a = R_a \varphi_a + R_{ab} \varphi_b \tag{2.27}$$

If I then look at a very little slice of the stream of air and birds I assume that here there is locally equilibrium, I can therefore turn the time as we did before without fundamental effects on the considered system. If I assume equilibrium, then an important thing occurs, namely that the two mutuality constants are the same

[17]What the frog is going to present is Onsager's reciprocal relation, the presentation is based on Onsager's paper Onsager (1931), the derivation done in Mamedov (2003) and with inspiration drawn from the course Physical Modelling taught at Physics at Roskilde University.
[18]In Onsager's paper (Onsager; 1931) the two fluxes are heat and current.
[19]If this little notion on analogy between fields in science catches your interest you can read more in Voetman (2005).

[20].

$$R_{ba} = R_{ab}{}^{21} \tag{2.28}$$

This is the reciprocity relation.
You see these are entities in equation 2.26 and 2.27. With the assumption that the system is in microscopic equilibrium and therefore is reversible on a microscale it is possible to derive equation 2.28.
We can now go on to show, using the equations 2.26 and 2.27, that entropy changes, and if the entropy changes the system is irreversible on a macroscale.
So with an assumption about microscopic reversibility we end with an expression for macroscopic irreversibility.
FOX: Wow! What in the forest?! I am blown away! You really have to explain that! How can the system be reversible on a microscale but irreversible on a macroscale?
The otherwise so controlled owl also could not resist a little gasp and needed to concentrate not to fall down its branch. The frog, normally very arrogant, softened up a little bit by the sight of their excitement.
FROG: I will explain: when something is in equilibrium there is an equal amount of fluctuations out of the system as into the system, that can be fluctuations of for example energy or matter. If we look at a small volume of the bird-air system, then there will be movements in and out of the that volume, but they will average to zero.
OWL: So that is because the birds fly out of the volume in one side, but the equal amount flies in in the other side?
FROG: It does not matter where they fly in or out.
OWL: Does it matter if it is the same bird that flies in or out?
FROG: No it does not. I will show you how we can see change in entropy from the previous equations. And therefore it will be clear to you creatures that the system is irreversible.
As you said Raven, the previous equations are analogous to Ohm's law, and you also know that voltage times current gives the power.
RAVEN: Yes I do know that!
FROG: Then you also see that if I multiply equation 2.26 with φ_b and equation 2.27 with φ_a I get an analogue power P_b and P_a respectively. Summing P_b and P_a gives me the total power P_t, like this;

$$P_t = F_b\varphi_b + F_a\varphi_a = R_b\varphi_b^2 + R_a\varphi_a^2 + (R_{ab} + R_{ba})\varphi_a\varphi_b \tag{2.29}$$

RAVEN: I see where you are going!
FROG: So, where am I going?
RAVEN: The units of power is energy per time; J/s, and the units of change in entropy is energy per time per temperature; $J/(t\ K)$. You can just divide equation

[20]Due to the target group I do not derive this here; I believe it would be a too lengthy derivation for the purpose, so here we have to trust the Frog. It is derived in plenty of articles and books: Patitsas (2014) and Reichl (2016)
[21]This is Onsager's reciprocal relation.

2.29 by temperature, then you have the change in entropy!
FROG: Yes, that is correct Raven, like this:

$$\frac{dS}{dt} = \frac{1}{T}P_t = \frac{1}{T}(F_b\varphi_b + F_a\varphi_a) = \frac{1}{T}(R_b\varphi_b^2 + R_a\varphi_a^2 + (R_{ab} + R_{ba})\varphi_a\varphi_b) \tag{2.30}$$

FOX: By the oak the raven taught us that entropy increases with time; $\frac{dS}{dt} \succeq 0$ [22].
OWL: Oh, that means that equation 2.30 has the same inequality[23]:

$$R_b\varphi_b^2 + R_a\varphi_a^2 + (R_{ab} + R_{ba})\varphi_a\varphi_b \succeq 0 \tag{2.31}$$

FROG: Indeed. Then I need to state under what conditions for φ and R equation 2.31 is satisfied. That is obviously when $\varphi_a = \varphi_b = 0$. When $\varphi_a \neq 0$ and $\varphi_b \neq 0$ I can say something about how the Rs behaves. Then $R_b \succeq 0$, $R_a \succeq 0$ and $(R_{ab}+R_{ba})^2 \preceq 4R_bR_a$. The two first statements hold because the φs are squared, so they are always positive, therefore R_a and R_b also have to be positive. I can show that the last one holds by setting $\varphi_a = 1$ and let φ_b vary, then equation 2.31 is

$$R_b\varphi_b^2 + (R_{ab} + R_{ba})\varphi_b + R_a \succeq 0 \tag{2.32}$$

By differentiating it according to φ_b and putting it equal to zero I find φ_b with which I can find the minimum value for the term $(R_{ab} + R_{ba})$:

$$\frac{d}{d\varphi_b}R_b\varphi_b^2 + (R_{ab} + R_{ba})\varphi_b + R_a = 2R_b\varphi_b + (R_{ab} + R_{ba}) = 0 \tag{2.33}$$

Then $\varphi_b = -\frac{(R_{ab}+R_{ba})}{2R_b}$, this I plug in equation 2.32:

$$R_b\left(-\frac{(R_{ab} + R_{ba})}{2R_b}\right)^2 + (R_{ab} + R_{ba})\left(-\frac{(R_{ab} + R_{ba})}{2R_b}\right) + R_a \succeq 0$$

$$-\frac{(R_{ab} + R_{ba})^2}{4R_b} + R_a \succeq 0$$

$$4R_aR_b \succeq (R_{ab} + R_{ba})^2$$

OWL: So now you have shown what conditions all the constants and variables need to obey to, for equation 2.30 to be satisfied.
FROG: Indeed!
RAVEN: Sir Frog started out by assuming microscopic reversibility for an irreversible process and has shown that it satisfies the second law of thermodynamics. Reversibility implies irreversibility!
OWL: That means that reversibility is a more fundamental phenomenon than irreversibility!
FROG: Yes!

[22] In section 1.2.2.2
[23] T has been multiplied on both sides of the inequality sign, and $\frac{1}{T}$ vanish.

RAVEN: Good now we have learned another property about reversibility, can you see any similar properties in your logics?
OWL: According to the standard translation it seems that first order logic is a more fundamental logic than tense logic since everything one can say in tense logic can also be said in first order logic, but everything one can say in first order logic cannot be said in tense logic.
FOX: But that just has something to do with size, I do not see that it has anything to do with being fundamental.
OWL: It does have something to do with being fundamental, since first order logic contains all the building blocks for creating your tense logic, hence it is fundamental for tense logic.
FOX: I see your point...
The fox accepted the owl's argument, but did think further about the change in size they had seen in this Section. The fox will reveal its thoughts in Section 3.1.

In this section it is stated, through Onsager's reciprocal relation, that reversibility is more fundamental than irreversibility. It is discussed if this property is connected to first order logic or tense logic. Since tense logic is embedded in first order logic, first order logic can be said to be more fundamental than tense logic. Therefore in respect to fundamentality it seems to be possible to relate first order logic to reversibility and tense logic to irreversibility in physics. The properties of the logics due to change in size is looked further into in the following Chapter 3.

Chapter 3

Logic on the Difference Between Micro- and Macroscale

So far the animals have learned about some physical laws that are reversible and some that are not, they have also learned about that reversibility on microscale yields irreversibility on macroscale. In this chapter the three animals; the fox, the owl and the raven will discuss different logical implications of the change in size that is the case in Onsager's reciprocal relation: they will discuss whether the predicate 'reversible' is context dependent or not in section 3.1 and they will discuss it in relation to decidability in section 3.2.

3.1 Context dependency

The three animals — The fox, the owl and the raven, said goodbye and thanked Sir Frog politely for his help. Now they walked undetermined into the forest talking about the outcome of the talk with Sir Frog and what it had started in their heads. Neither of them felt that they had got a lot further in figuring out if their distinction of time really also exists for physicists in the forest.
The three animals will in the following section look at the predicate 'reversible' in logic inspired by and compared with Section 2.2. Here the property of being reversible changes when the reference group is turned from small to big — from microscopic to macroscopic; it seems. Even though by first sight it seems to be dependent on the reference group, the three animals argue that it is not, and it is not possible to find a correspondence between reversibility and first order logic and irreversibility and tense logic in this case, since the analysis yields the same result for reversibility and irreversibility. [1]

[1] I want to emphasize that what is done in this Section is an experiment, and the way of treating cross-contextuality with sets instead of individual members is not the approach van Benthem (1983) has.

FOX: I got to think about something after Sir Frog's last presentation[2]. That presentation was all about that the properties of the system changed from reversible at microscale to irreversible at macroscale. We concluded then that reversibility is more *fundamental* than irreversibility. I would like to think of it slightly different; not to think about the property of being fundamental. I would instead like to focus on the fact that the property of being reversible changes when we changes the size of the system. Because if a system is reversible at a small scale and irreversible on a bigger scale then the property of being reversible is a matter of the size of the picture we look at — a matter of the reference group[3].
RAVEN: Well yes it is, that was the whole point of the frogs second presentation. So where are you going with this?
FOX: I like to think that way, that things are dependent on where you evaluate from. I felt that about time, then I moved to modal logic, where the importance of the place of evaluation was clear as well. Now I feel that there is something similar going on in this changing of size. Let me just contemplate a bit about this and see where it takes me.
OWL: We will listen to your contemplation.
FOX: Thanks. I will focus on the relation between only two members of a set, and then change the set around them.
Look here on the forest floor, where plenty of sticks in different sizes lay.
You see that one — I will call that stick x. Then x is bigger than that one, lets call it stick y.
The fox was pointing at the sticks on the ground, you can see what the animals are looking at on figure 3.1.1
Or I could say that x is big while y is not. But you see that if I say that x is big

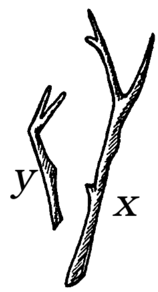


Figure 3.1.1: The forest floor with the sticks.

while y is not it is completely dependent on what I compare with. If I only look at these two sticks then x is big while y is not. If I look at a collection of all *these*

[2]See section 2.2
[3]The fox's way of thinking on micro- and macroscales in logic is based on the presentation of *comparatives* by van Benthem (1983) Chapter I.1.

sticks with them.
The fox drew a big circle around a bunch of sticks including stick x and stick y like on figure 3.1.2.
In this collection of sticks x is not big at all and y certainly not, they both are

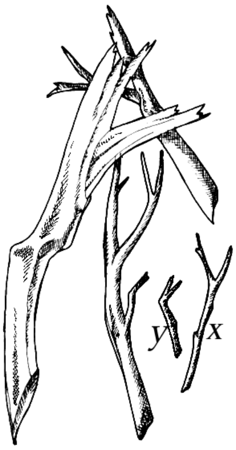


Figure 3.1.2: The forest floor with the sticks.

small. But if I only look at this collection:
Again the fox drew a circle around stick x and stick y now in a different region of

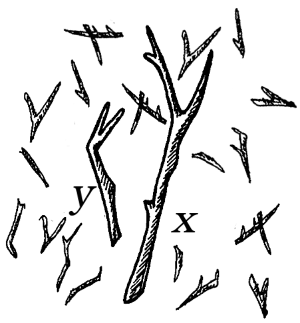


Figure 3.1.3: The forest floor with the sticks.

the forest floor where there were only tiny twigs laying, like on figure 3.1.3.
Here both stick x and y seem to be big since they are much bigger than the rest of the sticks.
OWL: Yes that is true when you use the word *big*. If you used the word *bigger* it would not be dependent on what context you refer to.
FOX: That is right. But if the predicate *reversible* changes according to the size, then it seems that that predicate *is* context dependent, just like big is.
OWL: So there must be some conditions a predicate needs to obey to be context-independent. If it is context-independent then I can just as well replace the predicate big with the relation bigger.

FOX: Yes there must be! Let me think...
The owl and the raven let the fox think.
FOX: So first of all we want the relation to stay the same between two members in all contexts.
OWL: Yes that is what we want. Saying on thing is bigger than another seems to mean that there is at least one context where one is big and the other is not.
FOX: Then there needs to be a principle that states that the relation does not get inverted in any context. I will call it the *no-reversal principle*, and keep the 'big' predicate for simplicity, then it says:

> NO REVERSAL: If x is 'big' in context c, while y is not, then in no context c' will y be 'big' while x is not.

OWL: Yes, it is obvious.
FOX: To avoid that both x and y becomes big or small I need two principles more: I will call them *Upward Difference* and *Downward Difference* respectively:

> UPWARD DIFFERENCE: If x is 'big' in context c, while y is not, then in each context c' containing c there will be some x' that is 'big' while y' is not.

So in a bigger context there will always be the relation that something is big while something is not. So it is not necessary that $x' = x$ or $y' = y$.

> DOWNWARD DIFFERENCE: If x is 'big' in context c, while y is not, then in each context c' contained in c there will be some x' that is 'big' while y' is not. As long as x and y are members of c'.

It is really important that we keep x and y in the context for downward difference, a context where they are thrown away is obviously not relevant.
So if these principles are obeyed a system is *cross-contextual.*
RAVEN: But then reversibility *is* cross-contextual.
FOX: How can that be when it changes from reversible to irreversible?
RAVEN: I understand it this way: If I say that x and y are two systems, x is reversible while y is not — That would mean that x is a microscopic system and y is a macroscopic system. Then I can think of no other size-context where x would be irreversible and y would be reversible, because it depends on the dominating laws in that set, and that does not change if we change the size of the context I look at. About the principle of *upward difference*; if I think of a much bigger set the governing laws will again be reversible since Newton's laws are reversible, hence Sir Frog's first presentation[4]. But that only shows that it would be possible to find a system that is reversible and one that is not since y would still be irreversible, but there might be some bigger system that is reversible that is not the same as x. That is not a problem for the principle of upward difference.
FOX: That is right, what about downward difference then?
RAVEN: That does not change it either, as long as x and y are members, x is re-

[4]See section 2.1.

versible where y is not.
FOX: I see. So the reversible predicate does not depend on the context.
RAVEN: Not the context of size at least, there might be a different contexts where it breaks down[5].
FOX: All right so the reversible predicate does not depend on a size-context change, even though it seemed like that in the beginning.
RAVEN: It certainly seems like that.
OWL: So what does that mean for our problem about the two logics and physics?
RAVEN: As you said, it is in accordance with your view that things are dependent on the place you evaluate from Fox, and I assume that Owl's view is, that it is independent of the place of evaluation.
OWL: Yes it is! You remember; first order logic has a global view and tense logic has a local view. So I would maybe say that in first order logic I always evaluate from the *same* place which is where it is possible to have the global view.
RAVEN: From the analysis it seems that reversibility is independent of where we evaluate from. And therefore mostly represents Owl's view.
FOX: But then irreversibility is also mostly Owl's view, because, as we have treated it here, irreversible is just the negation of reversible.
RAVEN: You are right! And our conjecture was that we could pair tense logic and irreversibility and first order logic with reversibility — that does certainly not seem to be the case at all here.
OWL: What would it mean for that conclusion if reversibility was not cross-contextual in respect to other contexts than size?
RAVEN: We have only looked at the change in size, since it is here the change from reversible to irreversible happens according to Sir Frog. If we would change context with respect to something else, like from a forest frame to an accelerated frame it is probably different and we do not know how the property of being reversible changes then.

The three animals have in this section presented the principles needed for a predicate to be cross-contextual. They have discussed whether the predicate 'reversible' is cross-contextual with respect to size, and therefore independent on the reference group. Since the predicate 'irreversible' is equal to 'non-reversible', the same holds for the predicate irreversible. Therefore it has not been possible to see the correspondence between first order logic and reversibility and tense logic and irreversibility for the three animals through this discussion.

[5] In fact there is, according to Onsager (1931) the microscopic assumption breaks down if external magnetic fields or Coriolis forces are present. Therefore a change of context from a place without a magnetic field to a context with a magnetic field, or if it changed from an initial frame of reference to one where the Coriolis force would be present the cross-contextuality would break down.

3.2 Computability?

In this Section the animals will discuss the structure of time and within that whether time is infinite or finite. They will do that with the nature of first order logic and tense logic in mind.
They come to realize that first order logic sees infinity and tense logic is finite. With this insight they are reminded about computability and find that tense logic is decidable and first order logic is undecidable.

FOX: I think there is still more we have not really caught about the two logics yet. I have gotten used to the thought that your first order logic Owl, is a bigger logic and in that way a more fundamental logic than tense logic[6]
OWL: Yes, and by the oak we figured out that tense logic is invariant under bisimulation where first order logic is not. With a first order logical formula with a global view I can therefore see many more details than you can with your tense logic Fox!
The owl said rather proud of its powerful logic.
FOX: But Owl, is a bigger necessarily a better logic?
OWL: Of course! Is it not better to be able to describe more things! Do we not consider those animals the wisest who know the most words?
The owl was being a bit vain; it knew that it was considered wise because it spoke so well. Animals that spoke well were not easily found in the forest.
FOX: So the best logic is the logic where you can say the most? I think there is more to be said about that!
Do you remember back by the oak[7] you told me that time was just like a ruler measuring length? OWL: Yes, that means that there is a property called density! Given any two point you can always find one in between them; you need to be able to subdivide length. And that is a first order property, look here how its put in my language[8]:

$$\forall q \forall q'(q < q' \to \exists q''(q < q'' \land q'' < q))$$

I like this property, and maybe time *is* just like length and has the same density property — then my logic can describe it.
The owl looked very pleased with itself.
OWL: There is also another property — continuity! When a measuring ruler is divided into a 'bigger than' and a 'smaller than' parts, there is exactly one point which produced this division. And there is a very complicated formula. I seem to remember it looks like this[9]:

$$\forall A((\forall xy((Ax \land \neg Ay) \to x < y) \land \exists x Ax \land \exists < \neg Ax) \to \\ \exists z(\forall u(z < u \to \neg Au) \land \forall u(u < z \to Au)))$$

[6]See section 1.2.2.5 about standard translation.
[7]see Section 1.2.
[8](van Benthem; 1983) page 17
[9]van Benthem (1983) page 29

The three animals looked puzzled at the big formula. It was by far the most complicated formula they had seen that night, and they found it rather hard to understand.
RAVEN: Is that a first order formula? What are those As and why do they have to look different than the other variables?
OWL: I understand your confusion. I have also only learned very little about this at owliversity, and we have not talked about it at all to night. The As are sets of numbers instead of individual points which we usually quantify over.
FOX: Are you allowed to quantify over sets?
OWL: Yes I am, but not in first order logic, I need to go to second order logic to do so.
FOX: And you can say more in second order logic than in first order logic?
OWL: Yes, you can say more in second order logic. For example is it not possible to describe continuity in first order logic, but as you see it is possible in second order quantifiers.
FOX: So do you find second order logic a *better* logic than first order logic?
The owl felt where this was going, and the feeling was not pleasant, you will discover why.
OWL: Well, no I do not. You see, second order logic can describe so much, that it is very hard to do proofs with it. Actually, I was told that it is incomplete; there is no way to write a proof system for it that will prove all the logical truths it can express. The logic is so strong, that it breaks down[10].
FOX: So expressive is not always best?
OWL: No it is not, I must swallow my words again.
FOX: Good, then that is settled.
The fox looked triumphant at the owl and the raven. It was going to use the fact that little expressibility can in fact be useful.
OWL: But first order logic *does* have a prove system and *can* express a lot, so I think that seems like the Golden Mean. For instance it can work with infinities!
RAVEN: What do you mean?
OWL: First of all I think we can all agree that if we have any three moments in time t_1, t_2 and t_3 then if t_1 is earlier than t_2 and if t_2 is earlier than t_3 then t_1 is also earlier than t_3.
RAVEN: Yes that is obvious!
FOX: I can not refute that no.
OWL: Good. Then we just all agreed that time is transitive. Formally written like this:

$$\forall t_1 \forall t_2 \forall t_3 ((t_1 < t_2 \wedge t_2 < t_3) \rightarrow t_1 < t_3)$$

OWL: Very well! Then maybe we can also agree that no time t is earlier than itself?
FOX: Of course not! That would be nonsense! If that was true my operators would not make sense at all, because then one time could be both in the past and in the future at the same time — that does *not* make sense!

[10](Enderton; 2001).

OWL: Good, then this is true:

$$\forall t_1 \forall t_2 \forall t_3 (t_1 < t_2 \rightarrow t_1 \neq t_2)$$

OWL: Can we then also agree that every time t has a successor?
RAVEN: I do not see why not!?
OWL: Good! then...
The fox interrupted.
FOX: Wait! I am not so sure about that.
OWL: Why not? My axis of time definitely has this property.
FOX: Why does it definitely has that property?
OWL: Because, as I said, time is just like an axis, lets say the natural numbers. You must agree that any natural number always has a successor; you just add 1 to your number.
FOX: Yes, so *your* axis of time may be like that.
RAVEN: If you take your three axioms together; transitivity, irreflexivity and succession you get an *infinite* axis of time.
You could imagine that time was constructed out of finite periods, but then it *is* allowed for a time to be reflexive[11].
OWL: I see, so my first order logic of time sees infinity[12].
But what about your view on time then Fox?
FOX: Well, I am not sure...
The fox was thinking about its logic as it presented it in section 1.
FOX: All the formulas we talked about by the oak seems to look for some specific point and not to some random place out in infinity; that is a feature of the operators I invented.
I think I have to think it through with an example:

$$F(p \wedge Fq) \wedge G(p \rightarrow r)$$

If I evaluate the example from t_1; here both $F(p \wedge Fp)$ and $G(p \rightarrow r)$ are true. Then, because of the future operator F, I look to some other point t_2 where $(p \wedge Fp)$ is true.
RAVEN: Then $p \rightarrow r$ also has to be true. Because of the G operator $p \rightarrow r$ is true in all future from t_1.
OWL: If p is true at t_2 then r is also true at t_2!
FOX: Yes! There is another F operator. That again looks to yet another point in time t_3 where p is true, therefore r must also be true at t_3.
The fox drew a little sketch in the forest floor with its paw of the process they derived together. You can see what was drawn on figure 3.2.1[13].

OWL: What is your point with this model then?
FOX: Do you not you see? The model I put up only asks for *three* points, not an

[11]Remember that the ravens view on time from section 1.2.1 is periodic
[12]The definition of infinity is from van Benthem (1983)
[13]The notation $A \Vdash B$ is read as *B is true in A*

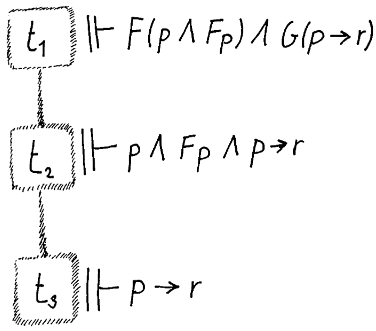


Figure 3.2.1: the fox's sktech of an example of a finite model.

infinity of points like yours[14].
OWL: I see! I also see that my logic once again shows to be bigger and more powerful.
FOX: Yes, but your logic also sees *a lot*! Maybe it sees too much.
OWL: Oh...
Something started to form inside the head of the owl; a thought that also occurred when they were translating the physics sentences in section 1.2. The owl had already then seen the simplicity of tense logic, but now the simplicity would really show its worth.
FOX: It is as if you see all the microscopic parts even when you do not need them.
RAVEN: I see what you mean Fox, in physics it is really difficult, if not impossible, to see any structures in a system if you are not allowed to idealise and cut away unimportant things, like microscopic interactions, when what you want to describe really is macroscopic.
FOX: indeed! My logic, is more macroscopic and therefore do not see all the microscopic properties of time.
RAVEN: Fox, I liked the way that you built that little three point model for your example. Do you think you always can do that? I mean, if I gave you any formula in your language, do you think you could make it true in just the way you did?
The fox and the owl looked at each other, both knowing that what the raven was asking about, was whether it is possible to build an algorithm that can determine if a modal formula is true or not [15].
FOX: You are asking me to make an algorithm!
The fox said delighted.
RAVEN: Maybe?!
The raven was confused, it had never heard the word 'algorithm' before, autodidactic as it was.
FOX: Yes you are! You are asking me if I can build a machine that can determine

[14] This is not something new that the fox discovered, it is well known that modal models are finite (van Benthem; 1983).
[15] They know this due to the time they met and talked about computability in Jensen and Kofod (2015).

if any modal formula is true!
RAVEN: So, can you?
FOX: I am not sure. It seems like it, since there are only finite steps build in to the formulas[16].
RAVEN: Owl, can you do the same with your first order logic?
OWL: No I am afraid I cannot, since first order logic is undecidable because of infinity.
RAVEN: That is a big difference between your two logics!
First order logic is undecidable due to its infinite microscopic properties, and modal logic is decidable because of its finite and macroscopic properties.
FOX: That is indeed a big difference! And Owl, here *my* logic is more powerful!
OWL: Yes, I must admit that!
RAVEN: I am sure that physics has something to say about this. And it *would* be very interesting to look further into! But I have the feeling that this would be at least a whole nights discussion, that would drag us further into reversibility, quantum mechanics and probably a lot more[17].
FOX: Look there in the horizon, the sun now shines in between the trees. I think that discussion has to wait for another nightly talk.
The fox yawned.
As you might know foxes and owls are nocturnal animals, and do not enjoy the daylight. Even though it felt like they were not done with their investigation of time in logic and physics they were off to a good days sleep. It seemed like the discussion could continue many, many nights, and maybe it does, but our travel with the three animals ends here.

Inspired by Onsager's reciprocal relation there is, in this Chapter, looked into how the difference in a macro and a micro view effects the two logics. This is done by testing the predicate 'reversible' and changing its reference group with respect to size. It is concluded that 'reversible' and also 'irreversible' is cross-contextual and therefore independent on the reference group. Furthermore, another important difference about the two logics is presented; first order logic is undecidable while tense logic is decidable. It would be interesting to look further into this property in physics.

[16] There is in fact a standard method for this called the tableau method (Fitting and Mendelson; 1998).
[17] What the raven means is, that in this topic lies a whole thesis in it self, which would indeed be interesting. Some reading about computation in physics is (Feynman; 1996)

Chapter 4

Concluding Remarks

4.1 Closure

During the dialogue we have followed the three animals; the fox, the owl and the raven. The fox and the owl held fundamentally different views on time — it seemed. After meeting the raven they were motivated to investigate how or if their distinction of time presented through their two logics existed in the real physical world. While investigating this through sentences taken from physical dependencies they also discovered more about their logics. The most important point here is that it got clear for the three creatures that first order logic has a global view on time and tense logic has a local view on time. This very property also showed that the two views on time, and therefore the two logics, were maybe not as different as they first thought, they discovered that tense logic can be translated into first order logic in a mechanical manner, but that not all first order sentences can be translated into tense logic. Tense logic is therefore a subset of first order logic.
Both logics managed to translate all of the six physical sentences, they therefore did not get closer to figuring out whether their distinction of time exists in physics.
While having discovered new properties of the two logics they looked further into physics. They did that by finding a place in physics where the *present* seemed relevant. The present seemed relevant since this concept captures the difference in locality and globality that is different in the two logics. The choice felt on reversibility. For discussing this the three animals visited the arrogant Sir Frog. He illuminated them about that some physical laws are symmetric and some are not, that is, it is possible in some equations to reverse the time without changing the equation itself, while it is not in others.
The owl and the fox thought about symmetry in the two logics and figured out that it did not make sense to talk about a reversible logic, but rather a symmetric structure. Because of the standard translation this structure can be described both by first order logic and tense logic, the two logics are therefore not different in this respect, where physics is different. Furthermore Sir Frog told them that

non-equilibrium physical systems really are in equilibrium at microscopic scale and therefore the property of being reversible is said to be more fundamental than being irreversible. This corresponds well with first order logic being more fundamental than tense logic, since all things said in tense logic can also be said in first order logic, but not the other way around. So it seems that in this case first order logic and reversibility and tense logic and irreversibility share the same fundamentality property.

The presentation of Onsager's reciprocal relation started more thoughts about the two logics.

In Onsager's reciprocal relation one goes from a microscopic view to a macroscopic view — and the reversibility changes under this context change. That yields a philosophical digression on the predicate 'reversible' and it is argued it is cross-contextual, as well as the predicate 'irreversible'. The test shows therefore that the predicates 'reversible' and 'irreversible' are equal, and can not be divided into, one being cross-contextual and the other one not, therefore one cannot be tied to first order logic and another to tense logic.

Another thought that occurred by the microscopic versus macroscopic view was the fact that first order logic sees infinity and microscopic structures in everything, where tense logic has a finite, macroscopic and discrete behaviour. That means that first order logic is undecidable where tense logic is decidable. I would be interesting to study this property of physics as well.

The above mentioned comparisons and their connection to the problem formulation (see Section) are resumed in table 4.1.1.

Table 4.1.1: Overview of the conclusions from the previous Chapters.

Test	Answer to the problem formulation
Sentence examples	No
Symmetry	No meaningful answer
Fundamentality	Yes
Cross-contextual	No meaningful answer
Computability	Not answered here

According to tabular 4.1.1 it is clear that one can not as a general statement claim that the distinction of time presented in the two logics; first order logic and tense logic, exists in physics. Though neither can one say that physics holds one view over the other, since it was possible to translate all the sentences in Chapter 1 into both logics. In Chapter 2 there were different answers to the hypothesis about whether the distinction between reversibility and irreversibility correspond to the distinction of first order logic and tense logic respectively. Therefore it is not possible to conclude on the hypothesis, one would have to look at a much wider range of properties of reversibility in physics.

The conclusion is that it is not possible to see the distinction of time, as it is presented in first order logic and tense logic, in physics with the comparisons carried out here. Even though this conclusion might seem disappointing, it still has its right; as mentioned in the preface of this thesis, physics often comes in a discussion between A theorists and B theorists for showing one position right, rather than the other. But since it has not been possible to show that physics holds one view or the other, it might also not be meaningful to use physics in the philosophical discussion the way it is done. Since it seems that physics can be used to argue for both views on time; represented in first order logic and tense logic, and therefore to some extend for A and B theorists point of view.

4.2 Conclusion

Tense logic has been presented and compared with first order logic as a logic of time. The comparison was first done through six sentence examples picked from different branches of physics; general laws, classical mechanics, thermodynamics, special theory of relativity, electrodynamics and quantum mechanics. From these examples there was no clear evidence that the distinction of time presented in tense logic and in first order logic exists in physics. Furthermore it is here concluded that the distinction of time in tense logic and first order logic is not as clear as first seemed, since tense logic is embedded in first order logic, as the standard translation shows. Nonetheless the view of the *present* in the two logics is different. In tense logic the present is privileged; hence tense logic is local. In first order logic, on the other hand, the *present* is no different from all other times; first order logic holds a global view on time.

Because the present plays different roles in the two logics I have taken a closer look at reversibility in physics, and attempted to compare properties of reversibility and irreversibility in physics with properties of the two logics. The properties discussed through reversibility in physics are time symmetry and fundamentality. Fundamentality is argued for through Onsager's reciprocal relation.

It was not possible to find a correspondence between the two logics and reversibility concerning symmetry. It is argued that the logical symmetry seems to depend on the underlying structures, which is similar for the two logics, given by the standard translation. Whereas for the property of being fundamental, it was possible to find a correspondence between first order logic and reversibility, again due to the standard translation.

Moreover there is argued for, that the predicate 'reversible' is cross-contextual, and is therefore not connected to either of the two logics.

I also noted that tense logic has the finite model property and is decidable, whereas first order logic sometimes requires infinite models and is not not decidable.

Based on the two approaches used here — sentence examples and properties of reversibility — it is not possible to conclude that the distinction of time presented in the two logics exists in physics. Other properties would have to be considered for that to be answered positively or negatively.

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