**Comment on “The motion of an arbitrarily rotating spherical projectile and its application to ball games”**

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**Abstract**

In a recent paper (Robinson G and Robinson I 2013 *Phys. Scr.***88** 018101) the authors developed the differential equations which govern the motion of a spherical projectile rotating about an arbitrary axis in the presence of an arbitrary wind, assuming that both the drag force and the lift force are independent of the Reynolds number and proportional to the square of the projectile`s velocity. In this paper, by dimensional analysis, the latter assumption is shown to be incorrect for forces dependent on the angular velocity of the projectile, e.g. the lift force.

**1. Introduction**

In a recent paper (Robinson and Robinson 2013) the differential equations governing the motion of a spherical projectile rotating about an arbitrary axis in the presence of an arbitrary wind are developed. Integrating the equations the authors derive different types of motion experienced in ball games. The paper opens up a fascinating arena for students interested in ball game practice, on the one hand, and in physics and numerical solution of differential equations, on the other.

However, dimensional analysis shows that some of the basic assumptions in the paper are incorrect. Here we restrict ourselves to a discussion of these basic assumptions. Numerical integration of corrected equations of motion has to be carried out in order to evaluate the consequences of the points made here for the results derived by Robinson and Robinson (R&R).

In eq. (3) of the R&R paper the drag force from the air on the projectile is given as

FD = ½ρACDV2. (1)

In eq. (5) of the R&R paper the lift force from the air on the projectile is given as

FL = ½ρACLV2. (2)

In both expressions ρ is the density of the air, A the cross section area of the projectile and V the velocity of the projectile. The dimensionless coefficients CD and CL are named drag coefficient and lift coefficient, respectively. The basic assumptions made by R&R are that both CD and CL are independent of the Reynolds number and independent of V. For dimensional reasons this is not possible if the coefficients depends on the angular velocity, ω, of the rotation of the projectile, which certainly is the case for CL, and possibly for CD.

**2. Dimensional analysis**

We use square brackets to denote the dimensions of a quantity, and the symbols L, T, and M for the fundamental dimensions length, time, and mass, respectively. Following this notation, [ρAV2= ML-3•L2•L2T-2 = MLT-2 = [FD] = [FL] proves that CD and CL are both dimensionless. In this section we discuss CD and CL simultaneously, giving them the common name C.

In order to develop a general formula for C we ask which physical quantities C can depend on in an arbitrary medium. Physical quantities of relevance are R, V, ω, ρ, η, and θ, the radius, velocity and angular velocity of the projectile, the density and viscosity of the medium, and the angle between the directions of velocity and angular velocity. Assuming the medium incompressible, i.e. V small compared to the velocity of sound, one can hardly imagine other input variables of relevance for the size of C.

In general, if a physical formula expresses some output quantity Q1 in terms of several input quantities Q2, Q3,… then this formula must take the form

Q1 = g•(Q2)α•(Q3)β•(Q4)γ…, (3)

where the powers α, β, γ,…yield the correct dimensions for Q1 and the dimensionless factor g can be either a pure number or a function of dimensionless power-law combinations of Q2, Q3,…,(if such combinations exist, see, e.g., Jensen (2013) and its references). It may be possible to choose the powers α, β, γ,… in different ways, resulting in different physical quantities, A, B,… ,all having the dimensions for Q1,allowing Q1 to be given the form g times a linear combination of A, B, …. But then, since aA+bB+… = A(a+bB/A+…), we can regain the form given in eq. (3) by including the dimensionless parenthesis in g. Thus, eq. (3) represents all physical formula.

Following eq. (3), the formula for C must therefore take the form

C = g•Rα•Vβ•ωγ•ρδ•ηε•θζ. (4)

Noting that [C] = M0L0T0, [R] = L, [V] = LT-1, [ω] = T-1, [ρ] = ML-3, [η] = ML-1T-1, and [θ] = M0L0T0, the dimensional claim [C] = [RαVβωγρδηεθζ] implies that

M: 0 = δ + ε

L: 0 = α + β - 3δ – ε (5)

T: 0 = - β – γ – ε,

since none of the fundamental dimensions M, L, and T can be written in terms of the others, and the powers of each dimension consequently must match up separately between the two sides. From these three equations α, β, and δ can be determined in terms of γ and ε. From the first equation we get δ = - ε, from the third β = - γ – ε, which in combination with the second gives α = γ – ε. Dimensional analysis thus restricts the product function in eq. (4) to take the form

Rγ-ε•V-γ-ε•ωγ•ρ-ε•ηε•θζ = (Rω/V)γ•(η/RVρ)ε•θζ , (6)

where γ, ε, and ζ are arbitrary numbers. The analysis also proves that the function g in eq. (4) can be expressed as a function of only the three dimensionless quantities Rω/V, η/RVρ, and θ, since eq. (6) proves that all dimensionless power-law combinations of the input quantities R, V, ω, ρ, η, and θ can be derived as power-law combinations of just those three. Consequently C can be described as a function of Rω/V, η/RVρ, and θ, with η/RVρ ≡ Reynolds number-1 ≡ Re-1.

Thus, assuming that C is independent of Re, any dependence of ω has to be via a dependence of Rω/V. Assuming that C is independent of Re, but dependent of ω without being dependent of V, is not a possible assumption.

**3. Conclusions**

Dimensional analysis does not immediately affect the expression for CD used by R&R. In ball games the velocities are typically large enough that the drag can be considered independent of the Reynolds number. With reference to observational data R&R assume CD to be a constant (=0.45) in the relevant interval of Re. With no dependence on ω a drag force proportional to the square of the projectile follows. However, R&R also refer to experimental results where ω is found to affect CD (see e.g. their eq. (4)). In that case, assuming CD independent of Re, CD is necessarily dependent of V, and thus FD cannot be simply proportional to V2.

Since, in contrast to CD, CL = 0 for ω = 0, CL´s dependence on ω cannot be neglected. R&R use an experimentally found relation between CL and ω, their eq. (10):

CL = 3.19•10-1[1 – exp(-2.48•10-3ω)], (7)

where ω is in rad s-1. However, they assume that CL is both independent of Re and V, which contradicts the dimensional analysis. If CL is independent of Re and dependent on ω, CL is also dependent on V, and consequently FL is not proportional to V2. According to R&R V = 115 km h-1and R = 0.68 inches in the measurements leading to eq. (7). Using these values eq. (7) should instead be substituted by the equation

CL = 3.19•10-1[1 – exp(-3.7•(Rω/V)]. (8)

At small values of Rω/V we can in general use a linear approximation to CL. Including the θ dependence and assuming that CL is independent of Re, we get

CL(Rω/V, Re, θ) = Rω/V•f(θ), (9)

where f(θ) is an unknown function of θ. Insertion into eq. (2) finally gives

FL = ½ρARωV•f(θ). (10)

The lift force is thus forced to be proportional to V and not to V2 as assumed by R&R.

This proportionality can also be derived using Bernoulli`s equation for a flow like that pictured in figure 1 in the paper by R&R. The flow in a not rotating frame of reference following the sphere is assumed to be a superposition of a flow created by a rotating sphere without translational motion of the air and a flow around a not rotating sphere of air passing it. The pressure difference between the high- pressure region and the low- pressure region in the flow around the rotating sphere is then, according to Bernoulli`s equation, proportional to ½ρ(V+ Rω)2 - ½ρ(V – Rω)2 = 2ρRωV. Thus, the lift force, being more or less the cross sectional area of the sphere times this pressure difference, FL ≈ ρARωV, is seen to be in accordance with eq. (10).

Since CL varies with ω, the dimensional analysis shows that CL is dependent on V if CL is assumed independent of Re. R&R explicitly assume CD to be independent of Re. Concerning CL, their assumption of independence of Re is indirect by their use of eq. (7) (their eq. (10)), independent of Re. It is dimensionally possible to maintain the assumption of CL being independent of V. But then, instead of being a function of Rω/V and θ, CL is forced to be a function of R2ρω/η (= Re•Rω/V) and θ. The only dimensionless power-law combinations of R, ρ, ω, and η, not including V as an input quantity, are (R2ρω/η)α, where α is an arbitrary number.

If CD and CL neither are assumed independent of Re, nor of V, dimensional analysis only implies, that CD and CL as functions of the six input quantities R, V, ω, ρ, η, and θ are restricted to be functions of *three* input quantities, e.g. Rω/V, Re, and θ.

We look forward to seeing more experimental results and further simulations of relevant differential equations to deepen our understanding of these fascinating phenomena.

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**References**

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