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Spengler and Mathematics in a Mesopotamian Mirror

Beitrag zur internationalen Tagung

**Stadien menschlicher Entwicklung – Ansätze
zur Kulturmorphologie heute**

Wöltingerode, Harz, 28.09 – 02.10.2014

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Mathematics in Spengler and in other grand historical syntheses

Mathematics plays a major role in *Der Untergang des Abendlandes* – in outspoken contrast to two other grand and famous syntheses from the same epoch. In total, H. G. Wells' slightly more extensive *Outline of History* from [1920] thus offers no more than 12 references to the topic, all of them with any depth:

- In Confucius' China, the literary class was taught mathematics as one of the "Six Accomplishments" (p. 132);
- sound mathematical work was done in Alexandria (p. 197);
- Arabic mathematics built on that of the Greek (p. 336),
- and al-Khwārizmī was a mathematician (p. 336);
- the Mongol court received Persian and Indian astronomers and mathematicians (p. 374);
- mathematics and other sciences have been applied in war (p. 448);
- Napoleon had been an industrious student of mathematics as well as history (p. 487);
- James Watt was a mathematical instrument maker (p. 506);
- the mathematical level of English post-Reformation universities was poor (p. 525),
- but mathematics was compulsory at Oxford (p. 526);
- in post-1871 Germany, mathematics teaching might be interrupted by "long passages of royalist patriotic rant" (p. 551);
- and finally, without the word "mathematics", our "modern numerals are Arabic; our arithmetic and algebra are essentially Semitic sciences" (p. 108).

Arnold Toynbee's even more monumental *Study of History* (12 volumes) from [1934] onward is not very different on this account. He, no less than Wells, belongs to

die Idealisten und Ideologen, die Nachzügler des humanistischen Klassizismus der Goethezeit, welche technische Dinge und Wirtschaftsfragen überhaupt als außerhalb und unterhalb der Kultur stehend verachteten.¹

Toynbee's volume 12 ("Reconsiderations") contains a number of passages explaining that the study of history cannot be formulated as abstract mathematics,

¹ [Spengler 1931: 2] – "the belated stragglers of the humanistic Classicism of Goethe's age, who regarded things technical and matters economic as standing outside, or rather beneath, 'Culture'", in Charles F. Atkinson's translation ([Spengler 1932], p. 6 of the version found on

<https://archive.org/details/ManTechnics-AContributionToAPhilosophyOfLife193253>)

and a statement that the author's purely classical education and ensuing ignorance of mathematics has not been fatal to the inquiry. In volume 3 ("The Growth of Civilizations"), mathematics turns up in quotations from Spengler and Bergson on pp. 185, 381 and 388–89, and it is claimed that "Our western world inherited [...] the Greek science of mathematics [...]" without any "break of continuity" in spite of the intervening social cataclysm. Vol. 7 ("Universal churches") believes on pp. 305–07 that Sumerian counting was duodecimal, and that this Sumerian system was conserved in later metrologies until being supplanted by the less rational French metric system (except in the British division of the weight pound in 12 ounces and the shilling in 12 pence) – no source being offered for this fantasy. Finally, in connection with the analysis of civilizations and historical process, volume 9 ("Contacts between Civilizations in Time – Law and Freedom in History – The Prospects of the Western Civilization") speaks on pp. 697–704 about mathematics and its relations to the social milieu, namely in polemics with Spengler, claiming (p. 700) that

It would, indeed, be as fantastic to suggest that Geometry and the Calculus are diverse, alternative, and incompatible systems of Mathematics as it would be reasonable to say that these are different aspects of one identical object of mathematical study that can properly be called "Number-in-Itself",

admitting only that

the several provinces of this realm of Mathematical Science have been opened up at different times and places by divers members of a single mathematical fraternity whose choices of their particular fields of mathematical research have been always influenced, and sometimes virtually determined, by a mental penchant or habitus imparted to the individual mathematician by his social milieu

but maintaining (p. 701) with no argument beyond Gibbon's authority that, as the result of a

Collective Human Intellect's cumulative achievement ... The Mathematics are distinguished by a peculiar privilege that, in the course of ages, they may always advance and can never recede

without making it clear whether this means that results once obtained remain valid in something like Popper's Third World or that they can never be forgotten, and thinking that

we have now disposed of Spengler's contention that Mathematics are subject to the same law of social relativity as social human affairs.

Seen from this perspective (and not only), Spengler's emphatic declarations do seem provocative. Spengler certainly goes more into historical detail than

Toynbee, but there are still immense gaps between exemplifying details and the conclusions derived from them, and more gaps between these conclusions and the ultimate generalizations.

First of all, there is the passage which scandalizes Toynbee (p. 81):²

Eine Zahl an sich gibt es nicht und kann es nicht geben. Es gibt mehrere Zahlenwelten, weil es mehrere Kulturen gibt. Wir finden einen indischen, arabischen, antiken, abendländischen Typus des mathematischen Denkens und damit Typus einer Zahl, jeder von Grund aus etwas Eigenes und Einziges, jeder Ausdruck eines andern Weltgefühls, jeder Symbol von einer auch wissenschaftlich genau begrenzten Gültigkeit, Prinzip einer Ordnung des Gewordenen, in der sich das tiefste Wesen einer einzigen und keiner andern Seele spiegelt, derjenigen, welche Mittelpunkt gerade dieser und keiner andern Kultur ist. Es gibt demnach mehr als eine Mathematik.³

Obviously, this has nothing directly to do with mathematical *results* that may be cumulative or at least conserved once they are reached – at most but not necessarily it provides a framework for these. This is also clear on p. 79:

Gotische Dome und dorische Tempel sind steingewordne Mathematik. Gewiß hat erst Pythagoras die antike Zahl als das Prinzip einer Weltordnung greifbarer Dinge, als Maß oder Größe, wissenschaftlich erfaßt,⁴

an obvious reference to the fundamental role played by limit and proportion in Greek philosophy and ideology of mathematics and to the likely links between this conception of mathematics and the canonic proportions of sculpture (p.88). Nor does *theory-building* constitute the substance of Spengler's notion of mathematics (p. 80):

² Since almost all of my quotations from *Der Untergang ...* come from volume I (revised edition, [Spengler 1923]), these will for simplicity just be referred to by page. All translations are taken from that of Atkinson [Spengler 1927], to which the page numbers for translations refer; my corrections of obvious mistakes and omissions in Atkinson's translation stand in < >.

³ “*There is not and cannot be, number as such.* There are several number-worlds as there are several Cultures. We find an Indian, an Arabian, a Classical, a Western type of mathematical thought and, corresponding with each, a type of number – each type fundamentally peculiar and unique, an expression of a specific world-feeling, a symbol having a specific validity which is even capable of scientific definition, a principle of ordering the Become which reflects the central essence of one and only one soul, viz., the soul of that particular Culture. Consequently, there are more mathematics than one” (p. 59).

⁴ “Gothic cathedrals and Doric temples are mathematics in stone. Doubtless Pythagoras was the first in the Classical Culture to conceive number scientifically as the principle of a world-order of comprehensible things” (p. 58).

Eine hohe mathematische Begabung kann auch ohne jede Wissenschaft technisch produktiv sein und in dieser Form zum vollen Bewußtsein ihrer selbst gelangen. [...] Die Eingebornen Australiens, deren Geist durchaus der Stufe des Urmenschen angehört, besitzen einen mathematischen Instinkt oder, was dasselbe ist, ein noch nicht durch Worte und Zeichen mitteilbar gewordenes Denken in Zahlen, das in bezug auf die Interpretation reiner Räumlichkeit das griechische bei weitem übertrifft. Sie haben als Waffe den Bumerang erfunden, dessen Wirkung auf eine gefühlsmäßige Vertrautheit mit Zahlenarten schließen läßt, die wir der höheren geometrischen Analysis zuweisen würden. Sie besitzen dementsprechend [...] ein äußerst kompliziertes Zeremoniell und eine so feine sprachliche Abstufung der Verwandtschaftsgrade, wie sie nirgends, selbst in hohen Kulturen nicht wieder beobachtet worden ist.⁵

Via a double contrast to Pericles's Greece this unexplicit mathematical thought is then presented as a parallel to the mixture of explicit and implicit supposed mathematical thought of the Baroque,

das neben der Analysis des Raumes den Hof des Sonnenkönigs und ein auf dynastischen Verwandtschaften beruhendes Staatensystem entstehen sah.⁶

Evidently, this has nothing to do with Gibbon's "Collective Human Intellect's cumulative achievement". Objections to "Spengler's contention" can certainly be formulated, also beyond his very delimitation of the concept of mathematics – but Toynbee and those whom he represents miss them completely.

In any case – this is liable to provoke the interest of historians of mathematics, as well as such mathematicians who doubt the Anglican whiggism of a Toynbee – Spengler offers one of the few global historical syntheses where mathematics plays a central role.

⁵ "A high mathematical endowment may, without any mathematical science whatsoever, come to fruition and full selfknowledge in technical spheres. [...] The Australian natives, who rank intellectually as thorough primitives, possess a mathematical instinct (or, what comes to the same thing, a power of thinking in numbers which is not yet communicable by signs or words) that as regards the interpretation of pure space is far superior to that of the Greeks. Their discovery of the boomerang can only be attributed to their having a sure feeling for numbers of a class that we should refer to the higher geometry. Accordingly [...] they possess an extraordinarily complicated ceremonial and, for expressing degrees of affinity, such fine shades of language as not even the higher Cultures themselves can show" (p. 58).

⁶ "presents us with a mathematic of spatial analysis, a court of Versailles and a state system resting on dynastic relations" (p. 58). Cf. also p. 8, "Wer weiß es, daß zwischen der Differentialrechnung und dem dynastischen Staatsprinzip der Zeit Ludwigs XIV. [...] ein tiefer Zusammenhang der Form besteht?" ("Who [...] realizes that between the Differential Calculus and the dynastic principle of politics in the age of Louis XIV [...] there are deep uniformities?" (p. 7).

No wonder, therefore, that Spengler's views of mathematics finds explicit echoes as well as parallels among sociologists of mathematical knowledge⁷ as well as students of ethnomathematics and the history of mathematics – many of whom will however have been quite unaware of the parallel.

But let us return to some of the objections. Workers on ethnomathematics certainly agree with Spengler's inclusion of aboriginal and similar kinship structures and appurtenant marriage regulations in mathematics – cf. for instance [Ascher & Ascher 1986: 135–139]; but they will not include practices which do not allow *us* to distinguish underlying formal structures, and nothing in what Spengler says about boomerangs (whether their production or use) suggests that. For Spengler, instead, mathematical law is “Das Mittel, tote Formen zu erkennen” (p. 4 – “the means whereby to identify dead forms”, p. 4) – where no “formalization” should be read into *Formen*, and *tot*/“dead” is everything that has not to be understood as *Welt als Geschichte*/“world-as-history” (p. 6, trans. p. 5) – the two realms being thus described by *mathematical number* and *chronological number*, respectively (p. 7).⁸

If this is taken to the letter, a historian of mathematics might skip Spengler's whole endeavour wholesale, in the way Aristotle skips Plato's “ideal numbers”, to which “no mathematical theorem applies [...], unless one tries to interfere with the principles of mathematics and invent particular theories of one's own” (*Metaphysics* N, 1090^b27–35, [trans. Tredennick 1933: II, 281]). It would hardly be justified, however, to take everything to the letter in a work which according to its preface (p. vii) is

einen ersten Versuch [...], mit allen Fehlern eines solchen behaftet, unvollständig und sicherlich nicht ohne inneren Widerspruch.⁹

So, let us turn elsewhere. The image of *one* mathematics above historical circumstance, progressing toward one inescapable goal, smacks of what is commonly thought of as “Platonism” (or, in the terminology of recent

⁷ Thus [Restivo 1983], cf. [Høyrup [1984].

⁸ Those who want to may see Spengler's delimitation of mathematics as prophetic – actually, the intervening century has seen virtually the whole domain of “dead forms” being subjected to mathematization, and even much of that living world which according to Goethe, Spengler and Habermas *ought not* to be treated thus (cf. also the discussion of Habermas in [Barnes 1977: 13–19], which *mutatis mutandis* can also equally well applied to Spengler if not to Goethe's inspired utterances).

⁹ “a first attempt, loaded with all the customary faults, incomplete and <certainly> not without <internal contradictions>” (p. xiii).

historiographic polemics, “essentialism”), and after another century’s research in the history of mathematics better counterarguments can certainly be advanced today than those advanced by Spengler – touching also at results and theories. On the other hand, Spengler’s view of cultures with their inherent culmination as “civilization” also strongly suggest essentialism (this time however Romanticist). Thus (p. 42),

jede Kultur hat ihre eigne Zivilisation. [...] Die Zivilisation ist das unausweichliche Schicksal einer Kultur. Hier ist der Gipfel erreicht, von dem aus die letzten und schwersten Fragen der historischen Morphologie lösbar werden. Zivilisationen sind die äußersten und künstlichsten Zustände, deren eine höhere Art von Menschen fähig ist. Sie sind ein Abschluß,¹⁰

and p. 29,

Jede Kultur hat ihre neuen Möglichkeiten des Ausdrucks, die erscheinen, reifen, verwelken und nie wiederkehren. Es gibt viele, im tiefsten Wesen völlig voneinander verschiedene Plastiken, Malereien, Mathematiken, Physiken, jede von begrenzter Lebensdauer, jede in sich selbst geschlossen, wie jede Pflanzenart ihre eignen Blüten und Früchte, ihren eignen Typus von Wachstum und Niedergang hat. [...] Sie gehören, wie Pflanzen und Tiere, der lebendigen Natur Goethes, nicht der toten Natur Newtons an.¹¹

Whether essentialism (Romanticist or otherwise) is objectionable must depend on arguments, and that is what I give afterwards in a specific example. But even *a priori*, essentialism can be seen to bar certain questions – in Spengler’s case such questions as concern development of general characteristics other than the ones prescribed by the fate of the culture in question,¹² or those pertaining to

¹⁰ “every Culture has its own Civilization. [...] The Civilization is the inevitable destiny of the Culture, and (here the high point is reached) from which the deepest and gravest problems of historical morphology become capable of solution. Civilizations are the (extreme and most) artificial states of which a species of developed humanity is capable. They are a (termination)” (p. 31).

¹¹ “Each Culture has its own new possibilities of self-expression which arise, ripen, decay, and never return. There is not one sculpture, one painting, on mathematics, one physics, but many, each in its deepest essence different from the others, each limited in duration and self-contained, just as each species of plant has its peculiar blossom or fruit, its special type of growth and decline. [...] They belong, like the plants and the animals, to the living Nature of Goethe, and not to the dead Nature of Newton” (p. 21).

¹² Cf. the closing words of vol. II (p. 635):

Wir haben nicht die Freiheit, dies oder jenes zu erreichen, aber die, das Notwendige zu tun oder nichts. Und eine Aufgabe, welche die Notwendigkeit der Geschichte gestellt hat, wird gelöst, mit dem einzelnen oder gegen ihn.

causal explanation (cf. Chapter II.ii, “Schicksalsidee und Kausalitätsprinzip” / “The destiny-idea and the causality principle”, pp. 154ff, translation p. 115ff) – be it the Humean insipid version, be it dynamic structural-functional causation.¹³

Mesopotamia – a case study

Spengler refers quite often to Babylonian mathematics; all he could know about, however, was the mathematics of Seleucid astronomy (contemporary with Euclid or later), which he was informed about through Carl Bezold’s *Astronomie, Himmelsschau und Astrallehre bei den Babyloniern* (which, apart from knowing about no mathematics antedating Seleucid epoch deals with nothing but this very particular aspect of mathematics). Almost all pertinent sources have indeed been published after the appearance of *Der Untergang*. The emergence and development of Mesopotamian mathematics may thus serve as that application of a theory to a *new* realm which philosophers of science often see as a decisive test,

Ducunt fata volentem, nolentem trahunt.

In translation (p. II, 507),

We have not the freedom to reach to this or to that, but the freedom to do the necessary or to do nothing. And a task that historic necessity has set will be accomplished with the individual or against him.

Ducunt Fata volentem, nolentem trahunt.

One may think of Sartre’s *Les mouches*: Oreste returns to Argos, in a postmodern search for his *roots*. But fate is waiting for him, and eventually he accepts it as “bien à moi”. Electre has waited for revenge of her father with burning soul, but in the end she betrays – yet things happen as they are bound to (or as the myth prescribes).

¹³ Similarly, Michel Foucault’s notions of successive *épistémès* forbids questions relating, for instance, Linné and Darwin [1966: 14]:

Si l’histoire naturelle de Tournefort, de Linné et de Buffon a rapport à autre chose qu’à elle-même, ce n’est pas à la biologie, à l’anatomie comparée de Cuvier ou à l’évolutionnisme de Darwin, c’est à la grammaire générale de Bauzée, c’est à l’analyse de la monnaie et de la richesse telle qu’on la trouve chez Law, chez Véron de Fortbonnais ou chez Turgot

or, in translation [Foucault 1971: xxii–xxiii]:

If the natural history of Tournefort, Linnaeus, and Button can be related to anything at all other than itself, it is not to biology, to Cuvier’s comparative anatomy, or to Darwin’s theory of evolution, but to Bauzee’s general grammar, to the analysis of money and wealth as found in the works of Law, or Veron de Fortbonnais, or Turgot. Obviously, the guru – recently ranked as next to compulsory “theory” in the professional upbringing of US historians of science [Nappi 2013: 106] – invites the same objections as Spengler; he is likely to have read less of the material he speaks about (at least Linné and Darwin) than Spengler.

and which may allow us to discern what has to be retained, what has to be reinterpreted, and what has to be rejected in Spengler's morphology of mathematical culture.

Mesopotamian culture, as Spengler would define it, was born in Uruk in southern Iraq, at the onset of the "Uruk IV" phase – perhaps 3200 BCE, perhaps already 3400 BCE (in the absence of wood, neither carbon 14 dating nor dendrochronology allow us to know precisely, nor does the precise dating matter¹⁴). What was born was a statal social organization centred around the great temples, legitimized by a transformation of an age-old redistribution practice into a system of taxation (or tribute) coupled to distribution of land and food rations – taxation as well as distribution being precisely accounted for. The birth of the state was thus not only conditioned by the creation of writing and book-keeping – these are indeed inseparable aspects of the same process. War and slave-taking were certainly also involved, as obvious from some of the seals of high officials. However, warfare did not enter the circuit state-accounting-writing.¹⁵

For a long while, writing was the privilege and task of the priestly elite – no separate scribal profession was in existence. But writing was not used for sacred or religious purposes: it was created with the sole purpose to serve accounting, providing context for numerical and metrological notations (on their part continuing a much older accounting system based on small tokens of burnt clay). Circa 85% of all texts from the period are accounts – the remaining 15% consist of "lexical lists" used for training the script.¹⁶

¹⁴ Further on, I shall follow the "middle chronology", which does not exclude anything between 3400 and 3200.

¹⁵ References and documentation for what is said about the period of state formation and about the third millennium can be found in [Nissen, Damerow & Englund 1993] and [Høyrup 2009]. The latter publication also provides references and documentation for the later periods.

¹⁶ According to a recent interpretation [Glassner 2013], *one* historical text seems to have existed. However, this text is truly the exception that conforms the rule, being an accounting document, detailing the institution of ceremonial gifts to two (obviously high-ranking) persons and the attribution of land with appurtenant workers to an institution (presumably a temple), decided by the assembly of the city in agreement with the decision of the assembly of the gods; as it shows, no other format than that of the account was available. What shows the document to be intended and used as a historical record is, firstly, that it exists in multiple copies; secondly, that it was copied over the following millennium with additions that identify the two recipients with the culture hero Enmekar,

The lexical lists are ordered according to categories: trees and wooden objects; fish; birds; cattle; professions; etc. We may find that natural, we would probably do as much. However, the investigations of the psychologist Aleksandr Luria of the structuring of thought, undertaken in the 1930s in Soviet Central Asia, show otherwise. An illiterate peasant with no experience outside his traditional life [Luria 1976: 55f, 74f], would think in fixed situations – presented with pictures of a hammer, a saw, a log and a hatchet he refused to eliminate the log from the group because it belongs together with the tools applied to it. In his practice, these objects would go together. Young people who had gone to school and participated in the construction of the modern world of the kolkhoz or lived for a while in a larger city – that is, whose experience was not limited to fixed situations – would think in abstract categories – for example, eliminating the saucepan from a set consisting of a glass, a pair of spectacles, a bottle and the saucepan because the three first “are made of glass but the saucepan is metal”.

In this dichotomy, the lexical lists thus represent modernity. But there is something to add: taken as a whole, they represent their world as a “Cartesian product” – in one dimension, the various lists, in the other the contents of these; one list, that of professions, also has the Cartesian product as an internal condition: in one dimension, the various professions, in the other the ranks (leader, foreman, worker).

The Cartesian product is also inherent in the accounting tablets. Regularly, their obverse will carry a number of semantically parallel entries, each of which list for instance how much various persons have received of different types of beer; the reverse then shows the totals for the single types, and the grand total.

A few accounting texts can be singled out because their numbers are too nice or too large, and because they do not carry the seal or signature of a responsible official – they are *model documents*, used for teaching. Apart from these, we have no traces of mathematics teaching. Mesopotamian mathematics of the protoliterate period, Uruk IV–III, was a an fully integrated tool for accounting *and nothing but*. Since distribution of land was accounted for, area measurement was still part of it, along with metrology and arithmetical techniques.

The protoliterate statal system collapsed some 300 years after its emergence, being replaced by a network of competing city states ruled by a military leader during the Sumerian “Early Dynastic” period. Until c. 2600 BCE we have

supposed inter alia to have been the king of Uruk and the inventor of writing, and his wife Enmekarzi.

extremely few written sources, but then writing becomes copious. Around 2550 BCE, we still find the old lexical lists in use in the city state Shuruppak, but now they serve the training of a genuine *scribal profession*. We also see writing in wider use, for instance in the stipulation of private contracts, in the writing of literature (proverbs) and in “supra-utilitarian” mathematical problems – that is, problems that according to the matter dealt with seem to concern questions a scribe might encounter in his working practice, but which would never present themselves in real life – for instance (a problem that belongs to a later epoch) to determine the sides of a rectangular field from their sum and its area, or (a problem found twice in the Shuruppak material) to find the number of workers that could receive rations of 7 litres of grain from a “storehouse” supposed to contain 2400 tuns, each consisting of 480 litres.¹⁷ (The answer probably exceeds the population of the state.)

Already slightly earlier, the first royal inscriptions turn up; their social purpose is obvious. However, what was the purpose of putting proverbs – so far belonging to oral culture – into writing, and what was the purpose of training mathematical techniques that a working scribe would never have to apply? The likely answer is scribal self-consciousness or pride. Temple managers could be proud of belonging to the leading stratum of the city, and had no need to boast of their ability to use writing and computation, mere subservient tools for their status. But scribes, no longer priests at the temple, could only glory in being scribes – and they certainly did, many of the beautiful so-called “school texts” from Shuruppak seem to be *de luxe* copies made “in memory of good old school days” for scribes already well in the career.¹⁸

In order to serve *scribal* self-esteem, mathematics had to be supra-utilitarian (or utilitarian but particularly difficult). A dentist may be personally proud of being good at chess; but *qua* dentist he can only be proud of skills which are, or at least seem to be, relevant to dentistry or odontology. Some of the empty corners of the *de luxe* school texts are filled out by figurative drawings (a deer, a flower, or the stately teacher). Others carry abstract line patterns which modern mathematicians might view as connected to graph theory; actually, however, they have the same decorative purpose as the figurative drawings, as shown by the absence of accompanying text and by their location on the tablet.

¹⁷ 7 does not divide any of the factors of the metrology, for which reason it would never be used in real distribution; but for the same reason, it could give rise to “interesting” mathematical problems.

¹⁸ I owe this observation to Aage Westenholz.

Shuruppak mathematics remained supra-utilitarian; this means that it always *asks for the correct number* – which was after all what a working scribe had to provide, whether he was engaged in accounting or in surveying (two roles which were already separate in Shuruppak if not before – the scribe who made a sales contract for a house appealed to another one, specialist in the matter, to take the measurements).

Around 2350 BCE, southern Mesopotamia was united, first under a local city king, very soon however under Sargon of Akkad – Akkad being a so far unidentified locality in central Iraq. His grandson expanded the realm into a true empire encompassing the whole of present-day Iraq and much of Syria. This had consequences for mathematics – common measures (probably to be applied in transregional administration only) were introduced, and sophisticated “brick metrologies” meant to facilitate the calculation of manpower needed for brick constructions were created. Both innovations were obviously linked to the administrative functions of mathematics. Throughout the Early Dynastic period, there had also been a constant drive toward “sexagesimalization” – that is, use of the step factor 60 (the base of the Sumerian number system just as 10 is the base of our as well as the Roman system) in extensions of existing metrologies upwards and downwards and as the overall principle of the newly created weight system. This transformation reflects the partial intellectual autonomy of the teaching situation – teachers, even teachers supposedly teaching for practice, tend to know best the practice of teaching, and if they happen to teach mathematics they will pursue mathematical regularities where such present themselves (after all, this facilitates teaching). Partial autonomy of teaching and scribal self-consciousness also shines through in the continued teaching of supra-utilitarian mathematics – now mostly connected to surveying, for instance the finding of one side of a rectangular field if the other side is known together with the area (because of the complexities of the metrology this was no mere division problem – one may think of an area expressed in acres and a side in yards, feet and inches).

The Sargonic empire lived no longer than the British world empire, counted from the battle of Trafalgar to 1945. The 22nd century saw a resurgence of city states and nomadic incursions, while the 21st century gave rise to a new centralization of southern Iraq under the “Third Dynasty of Ur”. During its first 30 years, “Ur III” was probably not very different from the Sargonic predecessor, but in c. 2075 BCE, in the wake of a military reform connected to the establishment of a genuine empire encompassing central Iraq as well as Elam in the Zagros area, an administrative reform was introduced. From now on, the larger part

of the working population at least in the core area was drafted into labour troops governed by scribal overseers, who were responsible for their produce calculated according to fixed norms with painstaking precision. As a tool for this accounting, a place-value system with base 60 was created, and all measures were expressed as such place-value multiples of “basic units”;¹⁹ it was a floating-point system (that is, in the likeness of a slide rule it was not provided with a “sexagesimal point” indicating absolute value), and it only served in intermediate calculations.

Mathematically seen, this was an impressive feat, and our own decimal fractions descend from the Ur III invention. At the same time, it appears that the mathematical training of future overseer scribes was based exclusively on model documents: mathematical *problems* seem to have been banished, as offering too much space for independent thought.²⁰ In certain ways, this last “renaissance” of Sumerian culture (probably already carried by rulers and scribes whose mother tongue was no longer Sumerian but Akkadian) returned to patterns from the proto-literate period (though in much larger scale). And whereas mathematical accounting in the proto-literate period probably gave a lustre of social “justice” to the corvée and tribute paid in kind to the temples by continuing systems originally developed in connection with age-old redistributive patterns, the king who introduced the oppressive administrative reform in 2075 BCE boasted of its appurtenant metrological reform as an aspect of his “justice” [trans. Finkelstein 1969: 67].

Common workers apparently did not share his ideas;²¹ if not falling ill or dying from starvation they ran away the best they could (all three categories are accounted for in the texts). This may be one reason that the top-heavy system collapsed around 2000 BCE. The next 200 years (the first half of the “Old Babylonian” period) produced a reshuffling of economic structure as well as scribal and general ideology. Land, even crown land, was leased and thus

¹⁹ We may think of expressing classical British monetary units in terms of pence, all weights in ounces, all lengths in inches, and all areas in square inches. That would reduce the Sargonic area problem of finding one side of a rectangle from the area and the other side to a pure division problem.

²⁰ See [Høyrup 2002b]. Not only are problem texts (beyond model documents) totally absent from the record, which might be an archaeological accident; as can be seen from the terminology of the subsequent period, the very vocabulary in which to express the *format* of problems disappeared and had to be reinvented.

²¹ A later epic which however reflects the social conditions of Ur III and not those of its own times relates a wild-cat strike with so much insight in the psychology of such strikes that it must build on historical experience [ed. trans. Lambert & Millard 1969: 42–55].

cultivated privately, and also in other respects the economy was individualized. At the ideological level, the individual also became more visible: the seal became a token of private identity, not only of office; and private letters (often written by “street-corner scribes”, a new category) turn up in the record. The scribe school inculcated an ideology of scribal identity (na m.lú.u lù, meaning “humanism”!): the scribe should not only be able to write the current Akkadian language phonetically (even some laymen were able to do so) but also know all ideographic values of characters – even values so secret that we do not know what is meant; he should be able to read and speak Sumerian (which only other scribes would understand); and he should know about mathematics. In the latter domain, the ideological texts offer no specification, but we may feel confident that a new, surprisingly high level of supra-utilitarian mathematics falls under the “humanist” heading.

This supra-utilitarian type of mathematics is what is mostly spoken of as “Babylonian mathematics” (during the 1760s, Hammurabi of Babylon subdued the whole of southern and central Iraq, and from then on it is customary to speak of that region as “Babylonia”). A main component is often referred to as “Babylonian algebra”; it is actually a technique dealing with square and rectangular areas and their sides,²² but other questions which we would express in terms of second-degree algebra can be *represented* by these geometric entities and thereby solved.

The starting point was apparently a deliberate attempt to (re-)establish a culture of mathematical problems in the school. For this purpose, mathematical riddles were borrowed from non-scribal mathematical practitioners – in particular, it appears, from Akkadian-speaking surveyors of central Iraq.²³ These riddles were, for instance:

- I have put together the side and the area of a square, and 110 resulted;
- I have torn out the side from the area of a square, and 90 resulted;
- I have put together the four sides and the area of a square, and 140 resulted;
- I have put together the sides of a rectangle, and s resulted, and the area is A ;
- The length of a rectangle exceeds the width by d , and the area is A ;

²² Literally, square and rectangular *fields* and their sides; but the terminology of the texts distinguish sharply between these “formal” fields and real agricultural plots and their dimensions.

²³ See, for instance, [Høyrup 2011; 2012]. Adoption of oral traditions into the new scribe school also affected other areas such as divination – see [Richardson 2010].

- The diagonal of a rectangle is D , and the area is A ;

Other riddles dealt with two squares with known sum of or difference between the sides and known sum of or difference between the areas, and with a circle for which the sum of perimeter, diameter and area is given. In total, the number of these riddles will not have exceeded 15.²⁴ In a school which (since the proto-literate training by means of lexical lists) had always emphasized systematic variation and learning by heart, however, a small number of riddles would not serve as a convincing foundation for professional pride. Very soon, therefore, the adopted riddles gave rise to the creation of a genuine discipline involving also further experiments (including experiments with problems of the third degree). We find no traces of theoretical investigation, for instance of conditions for solvability,²⁵ even though we know texts that aim very clearly at didactical explanation and concept formation. There are also no problems about geometrical constructibility of the kind that abounds in Euclid's *Elements*. Everything, as in Shuruppak, asks for the finding of a numerical solution.

Toward the end of the Old Babylonian period we encounter a new phenomenon: serialization, that is, collection of sequences of analogous problem statements first on one tablet, then (that is where the term really applies) on series of numbered tablets. Similar serializations begin in other areas such as medicine and divination. Mathematics, however, offers a possibility available only to a limited extent where the object is not freely constructible: ordering in Cartesian product. We may look at the sequence #38–53 from the tablet YBC 4668 – see [Høyrup 2002a: 201–203]. The first problem contains a linear condition that can be expressed in symbols as

$$\frac{1}{19} \cdot (L - W) + L = 46\frac{2}{3},$$

where $L = (\frac{\ell}{w}) \cdot \ell$ and $W = (\frac{w}{\ell}) \cdot w$, ℓ and w being the sides of a rectangle with area 600. Here,

- $\frac{1}{19} \cdot (L - W)$ may be replaced by $\frac{1}{7} \cdot (L + W)$.

²⁴ The riddles turn up in agrimensor writings from classical Antiquity and the Indian, Islamic and Latin Middle Ages in ways that exclude descent from the Old Babylonian school – see [Høyrup 2001]; these later sources allow us to identify them.

²⁵ Since problems were constructed backwards from known solutions, they could not fail to have one. That, however, is no guarantee that the solution could be found by legitimate methods – how would one know, for instance, that the dimensions of a rectangle can be found from its area and the area of another rectangle whose length is the cube on the original length and whose width is the original diagonal? In order to realize that this problem is solvable as a cascade of second-degree problem one needs some kind of theoretical insight – but such insights were apparently never written down.

- The second member L may be replaced by W .
- The first member may be subtracted instead of added.
- The first member may be taken twice instead of once.

Since the solution is always $\ell = 30$, $w = 20$, the number to the right changes accordingly. In total, this gives 2^4 different problems.

The Cartesian product, of course, did not pop up from nowhere after having been forgotten for a millennium. The implicit Cartesian product was known from the training of the place-value system in the scribe school: here, strictly parallel multiplication tables for different multiplicands were copied so often that they had been learned by heart. Only the mathematical series texts, however, allowed the principle to unfold to the full and in more than two dimensions.

After a protracted economical, political and social crisis, the Old Babylonian state was destroyed by a Hittite raid in 1595. The raid resulted in general chaos and eventual takeover of power by Kassite tribes, which had already been present in the area as hired workers, soldiers and marauders for quite some time. This led to a general decline of urban life and scribal culture (it has been estimated that the ratio between town- and countryside dwellers fell to fifth-millennium levels!). Scholar-scribes were henceforth taught within their family, not in a school. We know about these families from testimonials coming from the scribal families of the outgoing second and the earlier first millennium BCE; these testimonials make it clear that there was some continuation of the tradition but do not inform about how few people were involved (in any case they will have been few, and they may have lived from the land owned by the family and not from scribal services). They kept alive part of what the scribes of the late Old Babylonian scribes had produced – literature (like the Gilgamesh epic), divination, and medical texts. From mathematics, however, they only remembered the metrology shaped in Sargonic and Ur III times and the essentials of the place-value system. Genuine practical mathematics as needed in trade, taxation and surveying was probably taken care of by people who had been taught only basic writing, and who produced new metrologies more intimately linked to agricultural-managerial practice (like areas measured by the seed needed for ploughing and sowing them); that at least was the situation in the first millennium BCE.

Assurbanipal (668–631 BCE), the last significant ruler of the Assyrian empire and in his youth an eager pupil of scholar-scribes (originally he had been meant to become a high priest, not a ruler), boasts that he is able to find reciprocals²⁶

²⁶ Since the Ur-III invention of the place-value system, division by a number n was

and to perform difficult multiplications; in the same text he claims he can read tablets from “before the Flood” (that is, Early Dynastic texts); his scholar-scribes at least knew to do it, and even to emulate them. We may conclude that even the scholar-scribes knew no mathematics beyond multiplication and the division by means of reciprocals.

In two unconnected episodes, sophisticated supra-utilitarian mathematics produced by scholar-scribes turns up, once in the fifth century and once in the third or second century BCE. As can be seen from the terminology, both episodes draw on material handed down within environments not trained in Sumerian; it appears that these Late Babylonian scholar-scribes were aware of what had once, more than a millennium ago, belonged to scribal learning, and tried to resurrect what had been lost. Once more they drew on the surveyors’ riddles – but they never developed a discipline from them, nor anything that can be characterized as an “algebra”. The main text from the latest group also contains a problem (about a cup produced from two different metals)²⁷ that points forward to what was to become the grand medieval tradition of practical arithmetic reaching from India to the Mediterranean.

Summing up

How does this agree with Spengler’s views of mathematics? And with Spengler’s views of Mesopotamia?

Firstly, it verifies (against Spengler himself) what is said on p. 23:

Wir wissen, daß nur scheinbar eine Wolke um so langsamer wandert, je höher sie steht und ein Zug durch eine ferne Landschaft nur scheinbar schleicht, aber wir glauben, daß das Tempo der frühen indischen, babylonischen, ägyptischen Geschichte wirklich langsamer war als das unsrer jüngsten Vergangenheit. Und wir finden ihre Substanz dünner, ihre Formen gedämpfter und gestreckter, weil wir nicht gelernt haben, die – innere und äußere – Entfernung in Rechnung zu stellen.²⁸

performed as a multiplication by the reciprocal $\frac{1}{n}$. Assurbanipal thus find it worthwhile to boast that he is able to use a table of such reciprocals, since that is where they are found.

²⁷ BM 34568 #16 [Neugebauer 1935: III, 16, 19].

²⁸ “We know quite well that the slowness with which a high cloud or a railway train in the distance seems to move is only apparent, yet we believe that the tempo of all early Indian, Babylonian or Egyptian history was really slower than that of our own recent past. And we think of them as less substantial, more damped-down, more diluted, because we have not learned to make the allowance for (inward and outward) distances” (p. 17).

On the basis of what could be read in Eduard Meyer's *Geschichte des Altertums*, Spengler's main source for what he writes in general about Mesopotamia, it might perhaps seem reasonable to see this area as carrying *one* culture culminating and ending in a phase of civilization. However, the discoveries made during the intervening century shows this to be an illusion produced by distance. *If* history can be fitted into Spengler's scheme, Ur III may probably be seen as a phase of civilization, and even as one of *Imperialismus*. But to include post-Ur-III Mesopotamia together with (p. 50)

Reiche wie das ägyptische, chinesische, römische, die indische Welt, die Welt des Islam[, die] noch Jahrhunderte und Jahrtausende stehen bleiben und aus einer Erobererfaust in die andere gehen können – tote Körper, amorphe, entseelte Menschenmassen, verbrauchter Stoff einer großen Geschichte²⁹

is misleading. Already Old Babylonian culture, for whose emergence Amorrite tribal structures were important, is no mere imposition of the conqueror's fist on a petrified social body, and the culture of the Assyrian empire is certainly as much a *new* culture as was that of the Latin Middle Ages with respect to Greek Antiquity. Probably as much could be said about China and India, but that is outside my topic (yet see David Engels' contribution to the present volume) – and on the whole, this consideration belongs with a general evaluation of Spengler's morphology.

So, let us concentrate on mathematics. Do we find a particular kind of mathematics, or more modestly a characteristic Mesopotamian mathematical mind-set?

To some extent we do – or at least we are easily led to believe so from *our* particular stance. We find no formulation of theorems and no explicit demonstrations. But perhaps it is the Euclidean type that is an exception. The Egyptian Rhind Mathematical Papyrus [ed. trans. Peet 1923] also teaches to find the correct number; so do the Chinese *Nine Chapters on Arithmetic* [ed. trans. Chemla & Guo 2003]; and so did my own middle school arithmetic in the 1950s (etc.). This interest in finding the correct number follows from the purpose of the teaching – namely to train for work where finding the correct number is essential. In all three cases we find supra-utilitarian problems that also ask for a numerical solution; that is a consequence of the dynamics of the school situation.

²⁹ “the Egyptian empire, the Roman, the Chinese, the Indian[, which] may continue to exist for hundreds or thousands of years (and be taken over from one conqueror's fist by another one) – dead bodies, amorphous and dispirited masses of men, scrap-material from a great history” (p. 36).

If we scrutinize the Old Babylonian “algebraic” technique in depth we shall also find an organization of mathematical thought so different from ours that for long it was only interpreted in term of modern equation algebra, which could show why results were correct and make sense of the numbers occurring in the texts but could not account for their words.³⁰ But this was not characteristic of the long run of Mesopotamian mathematical culture but only in existence for a couple of centuries. At a pinch we could link it to the field plans we know from Ur III, which would give us half a millennium at least – but then we end up seeing it as a supra-utilitarian outgrowth and expression of pre-modern agrimensorial mathematical thought in general, always based on partition into rectangles and right-angled triangles.

The repeated appearance of the Cartesian product is a more significant characteristic, long-lasting and specifically Mesopotamian (even though it has affected later cultures through their direct or indirect familiarity with Seleucid astronomical tables). Of course this does not in itself suggests a unique *Zahlenwelt*/“number-world”, and it hardly expresses a particular *Weltgefühl*/“world-feeling”; but at least it connects the mathematical thinking of scribes to other aspects of scribal training in a rather specific way and to the roots of Mesopotamian mathematics in bureaucratic accounting.

This leads to what is probably the most serious objection to/revision of Spengler’s postulated separate mathematical universes: mathematical thought is not carried by a general “culture” as expressed by its “Bauerntum (und dessen höchste Form, der Landadel)” (“the countryman and especially that highest form of countryman, the country gentleman” – p. 44, translation p. 32); it was always a matter for specialists (Wells and Toynbee were neither the first nor the last to leave mathematics to these). Mathematical practitioners, moreover, participate in cultures of their own that often intersect with several “cultures” defined by mythology and priesthood instead of being contained within one of them – not to speak about coinciding. They were, for instance, travelling merchants – military engineers and tax officials following the conquerors or selling their services to them (sometimes conquered as booty themselves) – and master builders hired by whoever needed them and could pay. That is not only a difficulty if we try to apply Spengler’s ideas to Mesopotamian mathematics – it is no accident that what he has to say about Greek mathematics and its *Weltgefühl* fits sculpture and the opinions of Platonizing and Neopythagorean philosophers like Plutarch

³⁰ See [Høyrup 2013], Introduction and Chapter 5.

and Iamblichos³¹ much better than Aristotle, not to speak of Euclid, Archimedes and Apollonios, and that Hypsicles and other mathematicians based in Alexandria have to be written off as “zweifelloos sämtlich Aramäer”/“all without doubts Aramaeans”, carriers of Syrian thought and “Widerschein früharabischer Innerlichkeit”/“early Arabic Inwardness” (p. 86 and II, p. 240f, quotations pp. 86 and II, p. 241, translated pp. 63 and II, 200).

All in all, Spengler’s Romanticist essentialism with its belief in over-arching “cultures” becomes a deforming straitjacket when applied to the history of mathematics; but Spengler’s insistence that mathematics are plural, and not only in the etymological sense that mathematics encompasses a plurality of disciplines, remains a fundamental insight and corrective, not least to still prevailing, equally essentialist “mathematicians’ historiography of mathematics”.

At least when it comes to mathematics, the teaching of *Der Untergang* is, like positivist scepticism, a medicine – the latter against theoretical drunkenness, the former against unidimensional teleological simplification of its history.

Medicine is not food, and nobody can live from medicine alone. But medicine may still be needed.

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³¹ Even these, of course, only express the culture of imaginary peasants and country gentlemen. Egyptian fractions and the canonical system for the proportions of sculpture and architecture were borrowed not by peasants staying at home but by merchants, artists and master builders travelling one way or the other.

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