A hypothetical history of Old Babylonian mathematics

places, passages, stages, development

Høyrup, Jens

Published in:
Max-Planck-Institut für Wissenschaftsgeschichte. Preprint

Publication date:
2012

Document Version
Publisher's PDF, also known as Version of record

Citation for published version (APA):
Høyrup, J. (2012). A hypothetical history of Old Babylonian mathematics: places, passages, stages, development. Max-Planck-Institut für Wissenschaftsgeschichte. Preprint, (436).

MAX-PLANCK-INSTITUT FÜR WISSENSCHAFTSGESCHICHTE
Max Planck Institute for the History of Science

2012

PREPRINT 436

Jens Høyrup

A hypothetical history of Old Babylonian mathematics: places, passages, stages, development

A hypothetical history of Old Babylonian mathematics: places, passages, stages, development

Jens Høyrup
jensh@ruc.dk
http://ruc.dk/~jensh

Contribution to the
International Conference on History
and Development of Mathematical Sciences
Maharshi Dayanand University, Rohtak
21–24 November, 2012

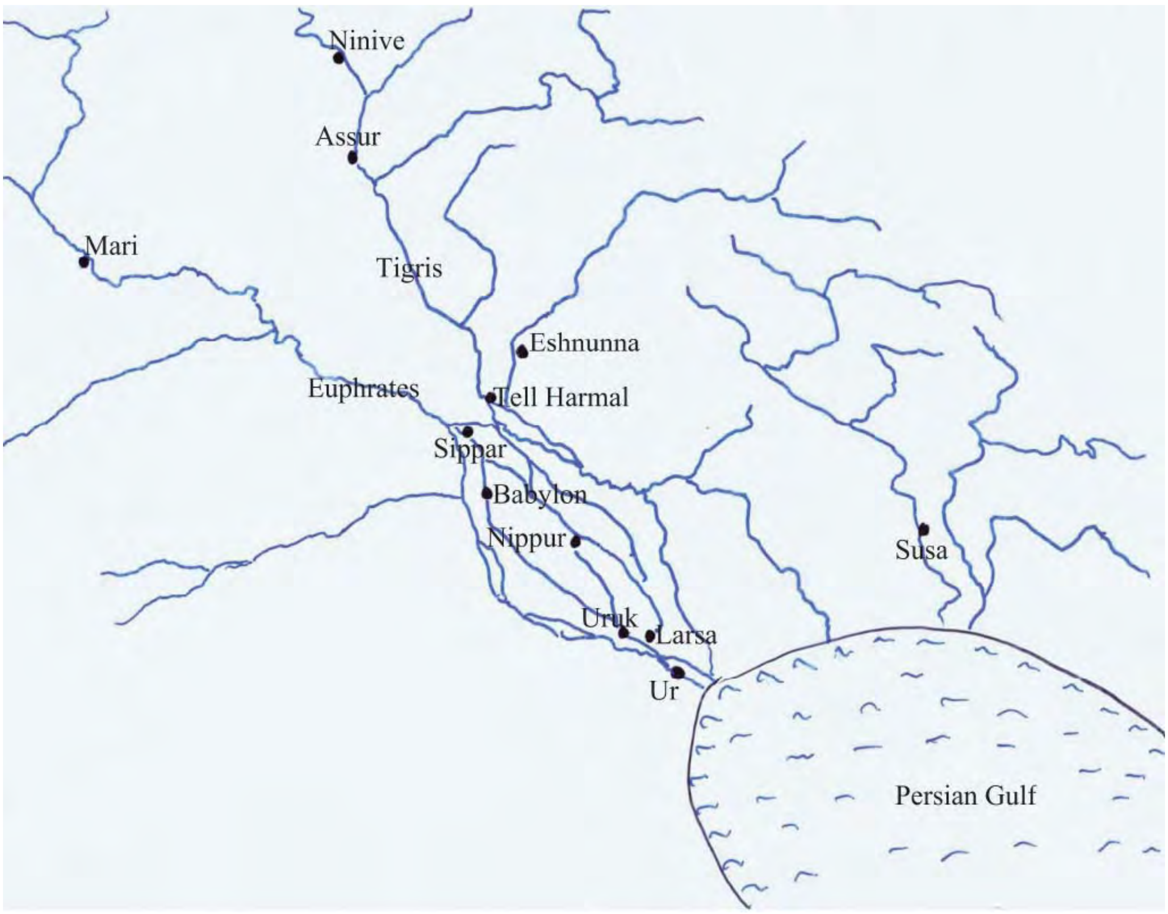
To appear in the Proceedings

Wilbur Knorr
in memoriam

Abstract

Most general standard histories of mathematics speak indiscriminately of "Babylonian" mathematics, presenting together the mathematics of the Old Babylonian and the Seleucid period (respectively 2000–1600 and 300–100 BCE) and neglecting the rest. Specialist literature has always known there was a difference, but until recently it has been difficult to determine the historical process *within* the Old Babylonian period.

It is still impossible to establish the details of this process with certainty, but a rough outline and some reasoned hypotheses about details can now be formulated. That is what I am going to present during the talk.



Non-history of Mesopotamian mathematics in general histories of mathematics

General histories of mathematics often begin with or contain a chapter on "Babylonian mathematics", or perhaps a chapter mixing up Babylonian matters with early literate mathematics in broader generality. I shall discuss only two examples, both of them much appreciated and more serious than many others: Uta Merzbach's *A History of mathematics* from [2011], a revised version of [Boyer 1968], and Victor Katz' *A History of mathematics: An Introduction*, the second edition of which appeared in [1998] (I have not seen the third edition from 2009 but do not expect it to be fundamentally different from its predecessor in this respect).

Merzbach's changes of Boyer's original text about Mesopotamia are modest – most of the chapter is taken over verbatim, including outdated interpretations, gross historical blunders, and fantasies. Some pertinent publications from recent decades are listed in the bibliography, but they seem not to have been consulted. In the preface to the new edition (p. xiv) the late Wilbur Knorr is praised for "refusing to accept the notion that ancient authors had been studied definitively by others. Setting aside the '*magister dixit,*' he showed us the wealth of knowledge that emerges from seeking out the texts"; but in spite of that Merzbach seems to have thought that regarding Mesopotamia there was nothing to add to or correct in the words of the master who wrote the prototype of the book.

And what were then the words of this master concerning the historical development of mathematical thought and techniques in the area? It is acknowledged that the place value system was an *invention*, which is supposed to have taken place some 4000 years ago [Boyer 1968: 29; Merzbach & Boyer 2011: 24]. It is also recognized that "most of [the source material] comes from two periods widely separated in time. There is an abundance of tablets from the first few hundred years of the second millennium BCE (the Old Babylonian age), and many tablets have also been found dating from the last few centuries of the first millennium BCE (the Seleucid period)" – thus [Merzbach & Boyer 2011: 24].[1] But neither edition sees any difference between what was done in the two periods, even though some of the mathematical formulae Boyer and Merzbach extract from the texts belong to the Old Babylonian only and others exclusively to the Seleucid period – apart from one contribution "not in evidence until almost 300 BCE", the

[1] In [Boyer 1968: 29], instead of "many tablets have also been found dating" we find "there are many also". Actually, that "many" Seleucid mathematical have come down to us is only true if we count "one, two, many", or if we count astronomical texts (which are not mentioned in the book).

use of an intermediate zero in the writing of place value numbers.[2] The "Orient", as we have often been told, is eternal, be it in wisdom or in stubborn conservatism – and much more so in the periphery of Orientalism than according to those scholars whom Edward Said [1978: 18] justly praised in spite of what he could say about their ideological context.

Merzbach and Boyer at least know that Pharaonic Egypt, Mesopotamia, pre-Modern China and pre-Modern India have to be treated separately and not (as in [Katz 1998]) pêle-mêle as the mathematics of "ancient civilizations", structured only according to mathematical topic – namely "counting", "arithmetic operations", "linear equations", "elementary geometry", "astronomical calculations", "square roots", "the Pythagorean theorem", and "quadratic equations". Katz, like Boyer / Merzbach, recognizes that the great majority of Mesopotamian mathematical tablets are from the Old Babylonian period, a few being of Seleucid date (and still fewer from other periods, not always correctly identified in the book) – but even he sees no change or development beyond the introduction of the intermediate zero, similarly believed to be a Seleucid invention.

Before leaving this section I shall point out once more that I did not choose these two books because I find them particularly faulty but because I regard them as better than most on the Mesopotamian topic.

What is known to those who care about the historical development of Mesopotamian mathematics

The earliest formation of something like a state in Mesopotamia took place in the later fourth millennium BCE around the city Uruk, as a bureaucratic system run by the high priests of the temples.[3] The organizational innovation was intertwined with the creation of a script and the creative merging of earlier mathematical techniques into a coherent system of numeration, metrologies and computational procedureses (procedures which we know next to nothing about beyond the results they yielded). Writing and computation served solely in accounting, which is amply represented in surviving clay tablets; there is not trace whatever of interest in mathematics going beyond that, nor of the use of writing for religious, literary or similar purposes. The only thing we need to know (because of its importance below) is that the number system had separate notations for 1, 10, 60, 600, 3600, and 36000; we may characterize it as "sexagesimal" (that is, having base 60) or as alternatingly decimal-seximal (as the Roman system can

[2] Actually, this "zero" (which often replaces not a missing sexagesimal place but a missing level of ones or tens) is already used in a few late Old Babylonian texts from Susa.

[3] Sources for this ultrashort summary, as well as a somewhat broader exposition, can be found for instance in [Høyrup 2009].

be considered decimal or alternatingly quintal-dual). Metrologies were not sexagesimal, but their step factors were compatible with the sexagesimal system.

Around the beginning of the third millennium BCE, the Uruk state gave place to a system of city-states competing for water and other resources and mostly ruled by a military leader (a "king"). For a couple of centuries, writing and computation disappear from the archaeological horizon, but they return forcefully around 2550,[4] at a moment when a scribal *profession* begins to separate from the stratum of priestly managers. Slightly earlier already, the first short royal inscriptions turn up, and in the cities Shuruppak and Abu Ṣalābīkh we find the earliest instances of literary texts (a temple hymn, and a collection of proverbs) as well as the earliest specimens of "supra-utilitarian" mathematics – that is, mathematics which formally looks as if it could serve in scribal practice, but whose substance goes beyond what could ever be needed. One problem deals with the distribution of the barley contained in a silo of 1,152,000 "litres" (40×60 "tuns", each containing 480 "litres") in rations of 7 "litres" to each worker. The resulting number of 164571 workers exceeds the population of the city state. More telling, a "divisor" 7 would never appear in genuine practice; its merit here is exactly that it is *not* compatible with the metrologies involved, and that the solution builds on a technique which a working scribes would never need to apply.

The use of writing for the recording of "literature" is equally supra-utilitarian; the appearance of both genres can be presumed to depend on the professional (and ensuing intellectual) semi-autonomy of the scribes. Temple managers could be proud of being powerful and associates of the gods; the scribe had to be proud being a scribe, that is, of mastering the two techniques for which he was responsible, and his professional pride was best served if he mastered them with excellence, that is, beyond what was needed in trite daily practice.

But the *raison d'être* for the profession of scribes was of course this daily practice, and its expansion beyond its earlier scope. Really utilitarian mathematics did not disappear, and throughout the third millennium we see an expansion of metrologies. As Sargon of Akkad (a city located somewhere in the vicinity of present-day Baghdad) united southern and central Mesopotamia into a single territorial state around 2350 (expanding into a genuine empire under his successors), literature – in the shape of rewritten versions of the mythology fitting the new political conditions – came to function as propaganda. Supra-utilitarian mathematics could offer no similar service, but from the Sargonic school we still know a number of supra-utilitarian problems dealing with rectangles and squares (for instance, giving the area and one side of a rectangle and ask for the other side – a problem no real-life surveyor of tax collector would ever encounter).

[4] As all dates in what follows BCE, and according to the so-called middle chronology.

Concomitantly, a new "royal" metrology for use in interregional affairs was introduced (probably neither replacing earlier local metrologies nor meant to do so).

Third-millennium upward and downward extensions of metrologies were made sexagesimally (and a new weight metrology was almost fully sexagesimal from the start). Other changes were apparently made so as to facilitate administrative procedures, and were therefore not made according to the sexagesimal principle. That contradiction was overcome during the next centralization, following after the demise of the Sargonic empire around 2200 and a decentralized interlude lasting until c. 2100. The new centralized state, called "Ur III" (short for "Third Dynasty of Ur"), introduced a military reform around 2075, and immediately afterwards an economic and administrative reform.

The gist of this reform was the organization of large part of the working force of the country in labour troops supervised by overseer scribes. These were responsible for the produce of their workers according to fixed norms – so many bricks of a certain standardized type produced per day, so much dirt carried a certain distance in a day, etc. The control of all this involved an enormous amount of multiplications and divisions, as can be imagined. The way to make this work was to have tables of all the technical norms involved, to translate (using "metrological tables") all traditional measures into sexagesimal numbers counting a standard area, a standard length, etc. (corresponding to the translation of 2 yards, 2 feet and 3 inches into 99 inches). These sexagesimal numbers were of a new type, written in a floating-point place-value system (absolute orders of magnitude thus had to be kept track of separately); with these, multiplications and divisions (the latter as multiplication by the reciprocal of the divisor) could be performed be means of tables of reciprocals and multiplication tables. Finally, the resulting place-value outcome could be translated back into current metrologies (e.g., weight of silver, as a value measure) by means of a metrological table.

The place-value *idea* may be older, but only the integration in a system of arithmetical, metrological and technical tables made its implementation worthwhile. This implementation presupposed the whole system to be taught in scribal training. However, it appears that rank-and-file scribes were taught no mathematics beyond that – the whole tradition of mathematical *problems*, not only of supra-utilitarian problems, appears to have been interrupted; beyond training (and learning by heart) of the tables belonging to the place-value system, mathematics teaching seems to have consisted in the production of "model documents", emulating such real-life documents as the scribe would have to produce once on his own. Mathematics teaching was apparently directed exclusively at the drilling of routines – even the modicum of independent thought which is required in order to find out how to attack *a problem* was apparently unwanted.

Old Babylonian political history in rough outline

Around the time of the military reform, Susa in the Eastern periphery and the area around Eshnunna in the north-to-east had been conquered by Ur III; in 2025 they rebelled, and around 2000 – the beginning of the "Old Babylonian period" – even the core disintegrated into smaller states, dominant among which was first Isin, later Larsa.[5] In both, but most pronounced in Larsa, the politico-economical structure was gradually decentralized. In the north, the cities Sippar and Eshnunna became important centres, and in the early 18th century Eshnunna had subdued the whole surrounding region. In the north-west, Mari (which had never been subject to Ur III) was the centre of a large territorial state. Between Sippar and Eshnunna to the north and Larsa to the south, Amorite chiefs had made Babylon the centre of another state.

In 1792, Hammurabi became king of Babylon. A shrewd diplomat and warrior, he took advantage of existing conflicts between the other powers in the area to subdue Larsa and Eshnunna in c. 1761, and Mari in c. 1758.[6] Isin had already been conquered by Larsa in c. 1794, which had taken over from Isin the control of Ur already before 1900.

With Hammurabi's conquests, southern and central Mesopotamia thus became "Babylonia", and it remained so for some two thousand years (Assyria, in northern Mesopotamia, is a different entity which we do not need to take into account here).

None the less, Hammurabi's Babylonia was not politically stable. In 1740, ten years into the reign of his son, Larsa revolted, and the first emigration of scholar-priests from the south toward Sippar began. The revolt was suppressed (possibly with great brutality), but twenty years later the whole south seceded definitively, and formed "the Sealand", where scribal culture appears to have become strongly reduced.[7]

After another century of increasing internal and external difficulties, Babylon was overrun by a Hittite raid in 1595, and afterwards the Kassite tribes (already familiar in the preceding century, both as marauders and as migrating workers)

---

[5] This section synthesizes information that is more fully documented in [Liverani 1988], [van de Mieroop 2007], and various articles in [RlA].

[6] I repeat that these dates are according to the "middle chronology". According to the "high" and the "low" chronologies, they fall c. 60 years earlier respectively later. This has no importance for the present argument.

[7] A newly collection of texts [Dalley 2009] shows that literacy did not disappear completely (which would anyhow be difficult to imagine in a situation where statal administration did not vanish) – but the writing style of the same texts shows that the level of erudition was low (p. 13).

took over power. This marks the end of the Old Babylonian period, which thus roughly spans the four centuries from 2000 to 1600.

Old Babylonian mathematics, and mathematics teaching

Administrative and economic records from this as well as other epochs present us with evidence of computation and area determination, but mostly tell us little about the mathematical procedures involved (even the place value system, serving only for discarded intermediate calculations, is absent from them). Our entrance to mathematics as a field of *practised knowledge* thus goes via texts which, at least by their format, can be seen to be connected to scribal teaching.

Apart from a huge lot found *in situ* in Nippur (since the third millennium an important temple city, but in the northern part of what was to become the Sealand), a rather large lot (badly) excavated in Susa and smaller batches from Ur, from Mari and from various towns in the Eshnunna area, almost all Old Babylonian mathematical texts have been "found" on the antiquity market – that is, they come from illegal diggings in locations which dealers did not identify or did not identify reliably. However, orthographic and terminological analysis allows to assign most of the important texts to a rather small number of coherent groups and to determine their origin.[8]

The Nippur corpus gives us a detailed picture of the general scribal curriculum, literary as well as mathematical [Veldhuis 1997; Proust 2004; Proust 2008]; the texts must predate 1720, where Nippur was conquered by the Sealand and soon deserted, and they are likely to be from c. 1739 [Veldhuis 1997: 22]. As far as the mathematical aspect is concerned, it shows how the Ur III curriculum must have looked (not our topic here), but it contains only three verbal problem texts (the ones which allow us to discern historical change) – too little to allow us to say much.[9]

---

[8] A first suggestion in this direction was made by Otto Neugebauer [1932: 6*f*]. This division into a "northern" and a "southern" group was refined by Albrecht Goetze [1945], who assigned most of the then-known texts containing syllabic writing (which excludes the Nippur corpus, whose origin was anyhow well known) into six groups, six "southern" (that is, from the former Ur III core) and two "northern".

Since then, the texts from Susa, Ur, Mari and Eshnunna have been added to the corpus. The picture which now presents itself is described in [Høyrup 2002a: 319–361]. Apart from the inclusion of new text groups it largely confirms Goetze's division though with some refinements due to analysis of terminology (Goetze had considered orthography and occasionally vocabulary, but mostly without taking semantics into account).

[9] Similar texts, though never copious enough in one place to allow us to identify deviations from the Nippur curriculum (but showing that such deviations will not have affected the fundamental substance), are found in various other places – see not least [Robson 1999:

There are, moreover, problem texts almost or totally deprived of words, of the types "*a*, *b* its reciprocal" or "*s* each square-side, what its area? Its area *A*", or simply indicating the linear dimensions of a triangle and writing its area inside it [Robson 2000: 22, 25, 29]. They are student exercises, and correspond to those written in the Sargonic of Shuruppak schools. Changes in vocabulary (more precisely the appearance of the word EN.NAM[10] meaning "what", a pseudo-Sumerogram invented in the Old Babylonian period) shows them not to be a direct continuation of Ur III school habits.

While showing the characteristics of general scribal education, the Nippur corpus is not of much help if we want to build at least a tentative diachronic *history* of Old Babylonian mathematics. For this, we need genuine word problems.

The earliest Old Babylonian mathematical text group containing genuine word problems comes from Ur (if anything, the core of Ur III). The pertinent texts belong to the 19th century – since they have been used as fill, nothing more precise can be said [Friberg 2000: 149*f*].[11] Many are simply number exercises, and thus reflect training in use of the place-value system. But there are also word problems, which provide evidence for an attempt to develop a *problem format:* the question may be made explicit (depending on grammatical case by a regular Sumerian A.NA.ÀM or by the pseudo-Sumerogram EN.NAM, both meaning "what"); a few times results are "seen" (PÀD or PAD[12]). PÀD is regular Sumerian, and is already used in certain Sargonic problem texts; the phonetically caused "misspelling" PAD speaks against transmission via the Ur III school, whose Sumerian was highly developed.

272–277; Robson 2004; Proust 2005]. Exceptionally, some of the texts in question from Larsa area dated, in part to c. 1815, in part to c. 1749 [Robson 2004:13, 19].

[10] For simplicity, I write all logograms, whether identified with Sumerian words or not, in SMALL CAPS (when philological questions are dealt with, Sumerian is often written in s p a c e d   w r i t i n g and "sign names" as SMALL CAPS or even CAPITALS); this should be superfluous here.

Syllabic spellings of Akkadian conventionally appears in *italics*.

[11] A few mathematical texts from Ur – found at different location, namely in a house that served for scribal training in a reduced curriculum – belong to the 18th century [Friberg 2000: 147*f*]; they are of the same types as those found in Nippur and confirm the general validity of the Nippur curriculum.

[12] Accents and subscript numbers in transliterated Sumerian indicate different signs used for homophones – or at least for terms which were homophones after Ur III, when Sumerian was a dead language taught at school to Akkadian speakers. Since Sumerian was a tonal language but Akkadian not we cannot be sure whether homophony is original or the result of phonetic impoverishment called forth by the transfer to a new linguistic environment.

In the actual case, while PÀD means "to see", PAD originally stood for a small portion of nourishment, or for "to bite". We do not know whether tone, some other small difference in phonetic quality or only contexts allowed the Sumerians to distinguish.

The themes of the word problems all fall within the sphere of Ur III scribal practice, even though some of them are certainly supra-utilitarian. In this respect they do not point toward the imminent developments that have come to be known as "Old Babylonian mathematics".[13] That, on the other hand, shows that the new attitude toward mathematics, emphasizing the role of genuine and often supra-utilitarian problems, preceded the substantial expansion of mathematical interests, know-how and know-why.[14] Moreover, it suggests that the change in attitude was a driving force behind the expansion.

The new attitude is not an isolated phenomenon characterizing only the culture of mathematics teaching. It corresponds to a general change of cultural climate at least at elite level, emphasizing so to speak individual self-conscience, expressed among other things by the appearance of private letter writing (when needed being served by free-lance scribes), and of personal seals (as opposed to seals characterising an office or function).[15] Within the scribal environment, it expressed itself in an ideal of "humanism" (NAM.LÚ.ULU, "the condition of being human"), which a scribe was suppose to possess if able to exert scribal abilities beyond what was practically necessary: writing and speaking the dead Sumerian language, knowing rare and occult meaning of cuneiform signs, etc. The texts which explain this ideal (texts studied in school and thus meant to inculcate professional ideology in future scribes) also mention mathematics, but give no particulars.

None the less, this ideology illuminates not only the strengthened reappearance of mathematical problems but also the new kind of mathematics which turns up in the various texts groups from the 18th and 17th centuries.

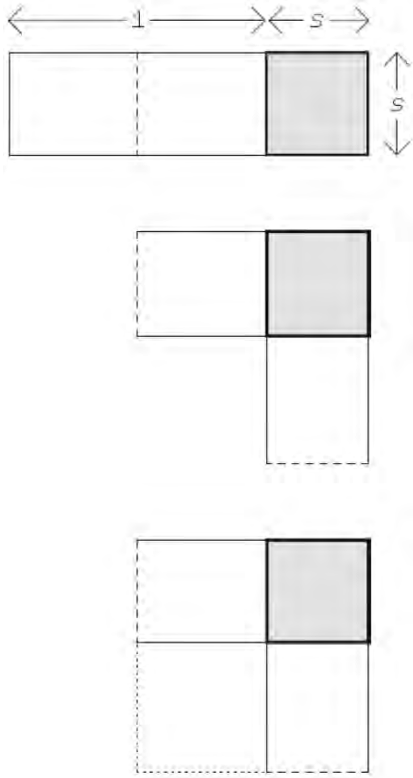
---

[13] This claim seems to be contradicted by three texts, UET 6/2 274, UET 5, and 858 UET 5, 859 [Friberg 2000: 113, 142, 143]. The first contains only numbers, but Jöran Friberg has shown that they correspond to finding the sides of two squares, if the sum of their areas and the ratio between the sides is given. This problem turns up in several later text groups in the context of metric "second-degree algebra", but is much simpler than the basic stock of this discipline. The second deals with the bisection of a trapezium by means of a parallel transversal, something which already Sargonic surveyor-calculators had been able to do correctly; but here the problem is reduced to triviality, since the ratio between the two parts of the divided side is given. The third asks for the side of a given cubic volume. Since tables of "square" and "cube roots" (better, inverse tables of squares and cubes) were already part of the Ur III system, this does not fall outside Ur III themes – and since the solution ends by transforming the resulting place value number into normal metrology (doing so wrongly!), we get extra confirmation of this.

[14] Since no development of autonomous *theory* but only of well-understood techniques and procedures takes place, these terms seem adequate.

[15] Elaboration, documentation and further references in [Høyrup 2009: 36*f*].

Before discussing these text groups, a short characterization of this mathematics will be useful. If counted by number of problem statements, with or without description of the procedure, the dominant genre is the so-called “algebra”, a technique which allows to solve problems involving the sides and areas of squares or rectangles by means of cut-and-paste and scaling operations.[16] We may look at two simple specimens, both borrowed from the text BM 13901, which contains 24 “algebraic” problems about one or more squares.[17]



The procedure used to solve BM 13901 #1

Problem #1 from the text states that the sum of a square area and the corresponding side is $^{3}/_{4}$. In the adjacent diagram,[18] the area is represented by the grey square $\square(s)$, while the side is replaced by a rectangle $\sqsubset\!\sqsupset(s,1)$. The composite rectangle $\sqsubset\!\sqsupset(s+1,s)$ thus has an area $^{3}/_{4}$.

As a first step of the procedure, the excess of length over with is bisected, and the outer half moved around so as to contain together with the half that remains in place a square $\square(^{1}/_{2})$, whose area is evidently $^{1}/_{4}$. Adding this to the gnomon into which the rectangle $\sqsubset\!\sqsupset(s+1,s)$ was transformed gives an area $^{3}/_{4}+^{1}/_{4} = 1$ for the large square, whose side must thus be 1. Removing the $^{1}/_{2}$ which was moved around

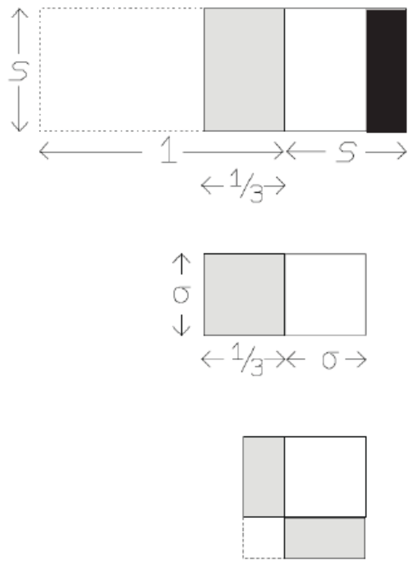
[16] A detailed analysis of this technique based on close reading of original sources is found in [Høyrup 2002a: 11–308]. A shorter (but still extensive) popular presentation in French can be found in [Høyrup 2010].

[17] First published in [Thureau-Dangin 1936], here following the analysis in [Høyrup 2002a: 50–55]. For simplicity, I translate the sexagesimal place value numbers into modern fraction notation.

[18] These diagrams are never drawn on the clay tablets, which would indeed not be adequate for deleting and redrawing. We may imagine them to be made on a dust-board (as were the diagrams of working Greek geometers) or on sand spread on the floor of the court-yard. The diagrams sometimes drawn in the tablet serve to make clear the situation described in the statement.

leaves $1-\frac{1}{2} = \frac{1}{2}$ for the side *s*.[19]

#3 of the same text is "non-normalized", that is, the coefficient ("as much as there is of it", as expressed in a different text) is not 1. This asks for application of the "scaling" technique. The statement explains that $\frac{1}{3}$ of a square area (black in the diagram) is removed, and $\frac{1}{3}$ of the side (represented by a grey rectangle ⊏⊐($\frac{1}{3}$,*s*) in the diagram) is added. Instead of a square area □(*s*) we thus have a rectangle ⊏⊐($\frac{2}{3}$,*s*), but changing the scale in the vertical direction by a factor $\frac{2}{3}$ gives us a square □(σ), $\sigma = \frac{2}{3}s$. In the same transformation the rectangle ⊏⊐($\frac{1}{3}$,*s*) becomes ⊏⊐($\frac{1}{3}$,σ). Thereby the situation has been reduced to the one we know from #1, and σ is easily found. Finally, the inverse scaling gives us *s*.[20]



The procedure used to solve BM 13901 #3.

The procedure of #1 can also be used to solve problems where a multiple of the side is subtracted from a square area, or where the difference between the sides of a rectangle is known together with its area. The problem where a rectangular area is known together with the sum of the sides is solved by a different but analogous procedure (the diagram of *Elements* II.5 gives the gist of it). Taken together, the two techniques permit the solution of all "mixed second-degree" problems about rectangular and square areas and sides. Moreover, the method allows, and was actually used for, *representation*. If (for example) a number of workers carrying bricks according to a fixed norm per day is identified with one side of a rectangle and the number of days they work with the other,[21] then the number of bricks they bring is a known multiple of the area. From the ratio between the number of workers and that of days together with the sum of workers, days and bricks (evidently another supra-utilitarian problem, never to be encountered in scribal real life), all three entities can thus be determined. In

[19] Whoever is interested may translate the procedure into algebraic and thus discover that the steps agree with those that occur when we solve an equation $s^2+s = \frac{3}{4}$. This explains that the technique is commonly spoken of as "algebra" (even though better reasons can be given).

On the other hand, the transformations of the diagram correspond rather precisely to those used in *Elements* II.6, which explains why Euclid's technique is sometimes spoken of as "geometric algebra".

[20] Translated into symbolic algebra, this corresponds to a change of variable.

[21] AO 8862 #7, ed. [MKT I, 112*f*].

other problems we see lines representing areas or volumes (which allows the solution of bi- and bi-biquadratic problems), as well as inverse prices ("so and so much per shekel silver") or number pairs from the table of reciprocals.

The oldest text group where "algebraic" problems turn up is from the Eshnunna area (Eshnunna itself and a few neighbouring sites). Before it fell to Hammurabi, Eshnunna appears to have been the cultural centre for central and northern Mesopotamia – Mari at a certain point undertook a writing reform and adopted the orthography of Eshnunna [Durand 1997: II, 109; Michel 2008: 255], and Eshnunna produced what seems to have been the earliest Akkadian law code in the early 18th century.

The earliest problem text from Eshnunna, IM 55357 [Baqir 1950] from c. 1790, exhibits a problem format which is still rudimentary but none the less slightly more elaborate than what we know from Ur – the prescription is introduced by the phrase ZA.E AK.TA.ZU.UN.DÈ, "You, to know the proceeding". Beyond that, like one of the texts from Ur it asks the question "what" (in accusative) A.NA.ÀM, and it "sees" results – not using PÀD, however but IGI.DÙ, an unorthographic writing of IGI.DU_8, "to open the eye". All in all, it obviously shares some inspiration with the texts from Ur but does not descend from them.

This text makes heavy use of Sumerian word signs, even though the structure of phrases shows that they were meant to be read in Akkadian (corresponding to a reading of "*viz*" as "namely" and not as latin "videlicet"). The other texts from the area (written between c. 1775 and 1765) are predominantly written in syllabic Akkadian. They make an effort to development problem formats, but disagree on how this format should look. Clearly, they stand at the beginning of a tradition where conventions have not yet been settled.[22]

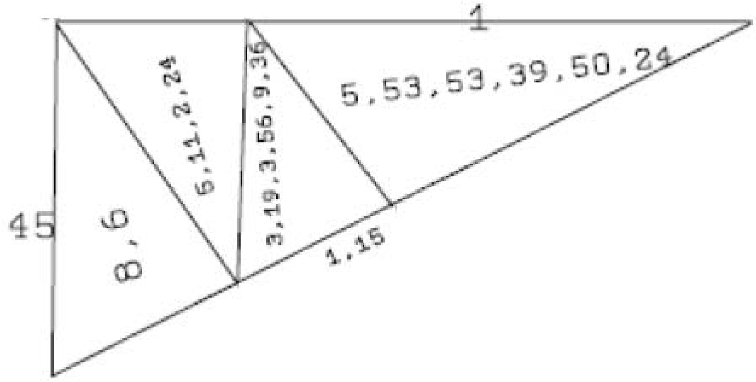


Diagram illustrating the problem of IM 55357 (as drawn on the tablet, numbers translated)

Some of the Eshnunna problems – for instance 10 problems solved on a tablet published in [al-Rawi & Roaf 1984], found in Tell Haddad (ancient Me-Turan) and most likely from c. 1775 – consider practical situations which already Ur III scribes

[22] The texts belonging to this group were published singly or in smaller batches in journal articles. A preliminary treatment of all the texts from Tell Harmal (apart from the Tell Harmal compendium) is given in [Gonçalves 2012].

would have had to deal with. But supra-utilitarian problems of various kinds dominate, many of them “algebraic”. Largely overlapping with the latter category, many deal with artificial geometric questions. IM 55357, presented above, deals with the cutting-off from a right triangle of similar subtriangles. The ratio of the sides in all triangles is 3:4:5 – that is, they are Pythagorean, but that is not used; the solution applies the scaling operation, but has no use for the cut-and-paste technique. However, the text Db_2-146 [Baqir 1962], found at Tell Dhibaʾi together with texts dated 1775, contains a partial quote of the “Pythagorean rule” in abstract terms in its proof; the general rule (and not just the 3:4:5-triangle) was thus familiar. The problem asked and solved in the latter text is to find the sides of a rectangle from the area and the diagonal.

Other supra-utilitarian geometric problems apply the “algebraic” technique to trapezia or triangles divided by of parallel transversals. Beyond that, the “Tell Harmal compendium”[23] list a large number of “algebraic” statement types about squares (there are no prescriptions, and even the statements leave the numerical parameters undetermined). So, the “basic representation” for the “algebraic technique” was well known as such, even though the full problems only show it at work in more complex problems. On the other hand, *representation* never occurs in the Eshnunna texts.

Many of the problems open as riddles, “If somebody has asked you thus: ...”. This betrays one of the sources from which the “new” Old Babylonian school drew its “humanist” mathematics, namely the mathematical riddles of non-scribal mathematical practitioners – not only surveyors (certainly the most important source, according to the statistics of surviving problems[24]) but also, it appears,

[23] Three badly broken tablets left on the ground as worthless by illegal diggers [Goetze 1951]; being found under such circumstances they are obviously undated, but certain aspects of the terminology shows them to be early [Høyrup 2002a: 324] – probably from the 1770s.

[24] Comparison with later sources allows us to identify a small set of surveyors’ riddles that survived until the late first millennium CE or later, and left an impact on Greek ancient mathematics (theoretical as well as supposedly practical), in medieval Arabic surveying texts, and even in Jaina mathematics – see [Høyrup 2001; 2004].

The earliest members of the set appear to have dealt with rectangles, whose area was given together with (1) the length, (2) the width, (3) the sum of length and width, or (4) the difference between these. (1) and (2) were already adopted into the Sargonic school, as we remember. The trick to solve (3) and (4), the geometric quadratic completion, appears to have been discovered somewhere between 2200 and 1900, after which even they, and a few other simple problems that could be solved by means of the same trick (e.g., known sum of or difference between a square and the corresponding side, cf. above), were taken up by the Old Babylonian school, where they provided the starting point for far-reaching further developments.

travelling traders (the grain filling Eshnunna problem IM 53957[25] is obviously related to problem 37 from the Rhind Mathematical Papyrus – see [Høyrup 2002a: 321]).[26]

The finding of texts in elaborate problem format and often with intricate supra-utilitarian contents in several sites suggests that the Eshnunna region as a whole was the hotbed where the new type of mathematics developed. That this new type arose in continuity with and as a graft upon the heritage from Ur III is obvious: everything makes use of the place value system, and the Tell Harmal compendium contains long sections with technical constants.[27]

We may compare Eshnunna with Mari to its west (close enough to prompt repeated military conflict). Mari had never been part of the Ur III empire, though certainly kept for a while under political control. None the less, a batch of mathematical texts from between 1800 and 1758 [Soubeyran 1984] shows that many of the place-value techniques were adopted. We also find a reflection of the general interest in supra-utilitarian mathematical skills. One text, indeed (not written in problem format), calculates 30 consecutive doublings of a grain of barley. There can be no doubt that this is the first known version of the "chess-board problem" about continued doublings of a grain of barley, widely circulating in subsequent millennia.[28] The diffusion in later times coincides with the Eurasian caravan trading network [Høyrup 1990: 74]; this, as well as the theme, indicates that the Mari scribes have borrowed it from a merchants', not a surveyors' environment.

Eshnunna was conquered and destroyed by Hammurabi (in contrast to texts on papyrus, vellum, palm leaves and paper, those written on clay are best conserved when the library burns). We know that the conqueror emulated the idea of the law code; whether he brought captive scholars to Babylon we do not know – the Bronze Age strata of the city are deeply borrowed under the remains

---

[25] [Baqir 1951: 37], revision and interpretation [von Soden 1952: 52].

[26] The kinship only concerns *the question* – the solution in the Rhind Papyrus builds on orthodox and highly sophisticated use of the Egyptian unit fraction system and its algorithms, while the Eshnunna solution is no mathematical solution at all but a mock reckoning which takes advantage of the known result – a type which is not rare in collections of practitioners' riddles. Since these are meant to dumbfound the non-initiates and not supposed to represent a category of "mathematics", it is not strange that they may distort usual procedures as much as the riddle of the sphinx (even more clearly a "neck riddle") distorts the normal sense of words.

[27] Certain linguistic particularities may also suggest direct continuity with pre-Ur-III mathematics, but local writing habits (in particular the frequent use of unorthographic Sumerograms) might also have induced a local transformation of the Ur-III heritage.

[28] Until the diffusion of chess, other variants also have 30 doublings. After that, 30 and 64 compete.

of the first-millennium world city. What we do know is that another text group ("group 1" according to [Goetze 1945]) carrying all the characteristics of a tradition *in statu nascendi*[29] can be located in Larsa, and that one of the most important texts belonging to it (AO 8862) is very similar to another one containing tables of squares, inverse squares and inverse cubes and dated to 1749, both being written not on flats tablets but on clay prisms [Proust 2005, cf. Robson 2002].[30] Literary school texts from Larsa inculcating the ideal of scribal humanism written on similar prisms are dated to 1739.

The Larsa text group, though still in search for a definite style or canon, differs from the Eshnunna group in one important respect: it uses the "algebraic" area technique for *representation* – the above-mentioned problem about workers, working days and bricks comes from the prism AO 8862.

We do not know whether the Larsa tradition ever reached maturation; the texts we have were found by looters and acquired by museums on the antiquity market, and what looters find is of course rather accidental. In any case, the time for development will have been restricted – in 1720, as told above, the whole southern region seceded as the Sealand, and scholarly culture withered away.

Two text groups from Uruk (Goetze's "group 3" and "group 4") did reach maturity before the collapse. They betray no groping similar to what we find in the Larsa group, and are therefore likely to be somewhat later in time. Striking is, however, that while each of the groups is very homogeneous in its choice of canonical format and terminology, the two canons differ so strongly from each other that intentional mutual demarcation seems the only explanation [Høyrup 2002a: 333–337] – we may imagine two teachers or schools in competition, but there could be other reasons. None of them can descend from the other.

Goetze's "group 2" [Høyrup 2002a: 345–349] can only be located unspecifically in "the south". It consists of texts containing either a long sequence of complete problems (statement+procedure) dealing with related topics (mostly right parallelepipedal "excavations" (KI.LÁ) or "small canals" (PA$_5$.SIG)[31]), or similar

[29] Vacillating format, vacillating terminology, an experiment with an abstract formulation of a rule, etc. See [Høyrup 2002a: 337–345].

[30] As mentioned in note 9, other mathematical texts – but exclusively arithmetical tables in Ur III tradition – from Larsa can be dated to c. 1815. This does not contradict the impression coming from the problem texts from c. 1749 that they represent the very beginning of a tradition.

[31] The text YBC 4612 [MCT, 103*f*] dealing with simple rectangle problems is written more coarsely than the indubitable members of the group, but it is likely to belong to the same family.

The text BM 13901, which supplied the two problems used above to demonstrate the "algebraic" techniques, was assigned to the group by Goetze for reasons which he himself sequences of problem statements only. They show the impact of another characteristic of later Old Babylonian scribal scholarship: a quest for systematization.

All southern texts share one striking characteristic: they do not announce results as something "seen", even though an oblique reference in one text[32] and a few slips in another one[33] shows the idiom to have been familiar. An ellipsis in a kind of folkloristic quotation (BM 13901 #23) also shows that the "riddle introduction" "If somebody ..." was known, but apart from this single instance it never appears in texts from the south. Instead, these state the situation directly in the first person singular, "I have done so and so", and the speaking voice is supposed to be the teacher (while the voice explaining what "you" should do is the "elder brother", an instructor well known from ideology-inculcating texts about the school).

Hammurabi is likely to have carried Eshnunna scribal culture to Babylon at the conquest, and chronology suggests that the new mathematical style of the south was sparked off by this migration of knowledge or scholars. However, the deliberate avoidance of two marks of northern and lay ways suggests that the schoolmasters of the south tried to demarcate themselves from the conquering Barbarians.

Others, as we remember, were more efficient in demarcating the south from the Babylonian centre – so efficient that scholarly high culture appears to have disappeared from the south in 1720. A number of temple scholars are also known to have gone north already from 1740 onward. From 1720 onward, advanced mathematical activity is thus restricted to the northern part of the area.

Goetze assigns two text groups to the north. His "group 5" is too small to allow any conclusions – it consists of one complete and fairly well-preserved text, a fragment and a heavily damaged text. Group 6 is larger and much more informative. Already Goetze thought it might come from Sippar, a hypothesis which can now be considered well-established [Høyrup 2002a: 332, reporting Friberg; Robson 2008: 94]; the very homogeneous core of the group can be dated to c. 1630. Goetze [1845: 151], at a moment when the Eshnunna texts were not yet known, supposed that the "6th group comprises northern modernizations of southern (Larsa) originals". This hypothesis can now be discarded. The texts of

---

characterized as insufficient and "circular" [Goetze 1945: 148 n. 354]. While remaining southern it must now be excluded from the group (and from the other established groups as well).

[32] YBC 4608, "group 3", asks what to do "in order to see" a certain result.

[33] YBC 4662, from "group 2", [MCT, 72]; three times, intermediate results are "seen" here, not "given" as in the rest of text (and the group as a whole).

the group consistently "see" results, and other characteristics too shows it to descend from a "northern" mathematical culture of which the Eshnunna texts are the first known, and probably first, representatives.

But another text group may have been affected by the emigration of southern scholars, the so-called "series texts", written almost exclusively with logograms and therefore not considered in Goetze's orthographic analysis.[34] The texts consist of lists of statements only, or statements and solution. Often, long sequences of problems can be obtained from a single one be systematic variation on up to four points in "Cartesian product" (e.g., subtraction instead of addition, a denominator 19 instead of 7, length instead of width, a member being taken twice instead of once, see [Høyrup 2002a: 203]); the writing is utterly compact, often only the variation is mentioned, and in consequence the meaning of the single statement can only be grasped when those that precede are taken into account.

The style must be the endpoint of a long development, and the texts in question are thus likely to belong to the late phase, and already for that reason to be northern. Christine Proust [2010: 3, cf. 2009: 195] gives evidence that "the structure of the colophons might speak in favour of a connection between the mathematical series texts and a tradition which developed in Sippar at the end of the dynasty of Hammurabi" – more or less as the same time as "group 6" was produced in the same city. Much in the terminology excludes, however, that the series texts came out of the same school; Friberg [2000: 172], moreover, has shown that the use of logograms is related to what can be found in groups "3" and "4". All in all, it seems plausible that the texts were produced within a tradition going back to scholars emigrated from the south.

The last group to consider consists of texts that were excavated in Susa (published in [TMS]). Since the expedition leader did not care much about stratigraphy (cf. [Robson 1999: 19] and [MCT, 6. n.28]), we only know (and mainly from the writing style) that they are late Old Babylonian [TMS, 1]. Like "group 6", they clearly belong to the northern tradition first known from Eshnunna. Firstly, their results are always "seen". Secondly, the two texts TMS V and TMS VI, lists of problem statements about squares, are clearly related in style both to the Tell Harmal compendium and to the two texts CBS 43 and CBS 154+921 [Robson 2000: 39*f*], possibly from Sippar (Eleanor Robson, personal communication).[35]

[34] The term was introduced by Otto Neugebauer [1934: 192 and *passim*; MKT I, 383*f*]. In [MCT], he and Abraham Sachs discarded the term because the Old Babylonian mathematical series do not correspond to the ideal picture one had at the moment of canonical series from later times (Neugebauer had not compared the two types when he introduced the term, just observed that the single tablets were numbered as members of a series). Since other Old Babylonian series are no more canonized, there are good reasons to retain the term. Cf. [Proust 2009: 167–169, 195].

[35] This assumption is supported by the way square sides are asked for, "how much, each,

The latter texts, like the Susa catalogues and in contradistinction to the Tell Harmal compendium, specify numerical parameters, but all refer to the side of a square as its “length” (UŠ).

The Susa group contains some very intricate problems. For instance, TMS XIX #2 determines the sides of a rectangle from its diagonal and the area of another rectangle, whose sides are, respectively, the (geometrical) cube on the length of the original rectangle and its diagonal. This is a bi-biquadratic problem that is solved correctly.[36] Even more remarkable are perhaps some texts that contain detailed didactical explanations (the above-mentioned term for a coefficient, “as much as there is of it”, comes from these).[37] They make explicit what was elsewhere only explicated orally (sometimes with set-offs in the prescriptions which only comparison with the Susa texts allow us to decode). We may assume that the peripheral situation of Susa invited to make explicit what was elsewhere taken for granted.

The end

In my part of the world, the most beautiful moments of summer may occur in September. However, they are invariably followed by real autumn, and then by winter.

Not so for the Indian summer of Old Babylonian mathematics. Very shortly after the flourishing represented by the series and Susa texts, winter set in directly. After the Hittite raid in 1595 and during the ensuing collapse of the Old Babylonian political system, even “humanist” scribal culture disappeared – or at least, what survived as its textual vestiges within the scholarly “scribal families” encompassed literature, myth, ritual, omen science, but not mathematics.

Shortly after the Kassite take-over in Babylonia, there was also a dynastic change in Elam, to which Susa belonged. The details and even the precise moment are not clear [Potts 1999: 188*f*], but in any case the Old Babylonian cultural influence (and *a fortiori* the influence of the Old Babylonian cultural complex,

---

stands against itself?”, which is routinely used in “group VI”. However, the folkloristic quotation in BM 13901 #23 also refers to a square side as what is “standing against itself”, so the phrase may belonging to the parlance of Akkadian lay surveyors (which would still suggest the texts to be northern but nor necessarily from Sippar)

[36] See [Høyrup 2002a: 197–199]. The solution is only correct *in principle*. There are indeed some numerical errors due to the misplacement of counters on the reckoning board [Høyrup 2002b: 196*f*], but since square root extractions are made from the known end result, these mistakes are automatically eliminated.

[37] See [Høyrup 2002a: 85–95].

already extinct in its homeland) was strongly reduced. Even here, the text group just discusses marks a high point as well as the end.

References

al-Rawi, Farouk N. H., & Michael Roaf, 1984. “Ten Old Babylonian Mathematical Problem Texts from Tell Haddad, Himrin”. *Sumer* **43**, 195–218

Baqir, Taha, 1950. “An Important Mathematical Problem Text from Tell Harmal”. *Sumer* **6**, 39–54.

Baqir, Taha, 1951. “Some More Mathematical Texts from Tell Harmal”. *Sumer* **7**, 28–45. Kopi: Bab, mat-hist, texter (Tell Harmal).

Baqir, Taha, 1962. “Tell Dhiba’i: New Mathematical Texts”. *Sumer* **18**, 11–14, pl. 1–3.

Boyer, Carl B., 1968. *A History of Mathematics*. New York: Wiley.

Dalley, Stephanie, 2009. *Babylonian Tablets from the First Sealand Dynasty in the Schøyen Collection*. Bethesda, Maryland: CDL Press.

Durand, Jean-Marie, 1997. *Les documents épistolaires du palais de Mari*. 3 vols. Paris: Éditions du Cerf, 1997–2000.

Friberg, Jöran, 2000. “Mathematics at Ur in the Old Babylonian Period”. *Revue d’Assyriologie et d’Archéologie Orientale* **94**, 97–188.

Goetze, Albrecht, 1945. “The Akkadian Dialects of the Old Babylonian Mathematical Texts”, pp. 146–151 *in* O. Neugebauer & A. Sachs, *Mathematical Cuneiform Texts*. New Haven, Connecticut: American Oriental Society.

Goetze, Albrecht, 1951. “A Mathematical Compendium from Tell Harmal”. *Sumer* **7**, 126–155.

Gonçalves, Carlos H. B., 2012. “A Dozen Mathematical Tablets from Tell Harmal. Uma dúzia de tabletes matemáticos de Tell Harmal”. *Preprint*, Escola de Artes, Ciências e Humanidades, Universidade de São Paulo.

Høyrup, Jens, 1990. “Sub-Scientific Mathematics. Observations on a Pre-Modern Phenomenon”. *History of Science* **28**, 63–86.

Høyrup, Jens, 2001. “On a Collection of Geometrical Riddles and Their Role in the Shaping of Four to Six ‘Algebras’”. *Science in Context* **14**, 85–131.

Høyrup, Jens, 2002a. *Lengths, Widths, Surfaces: A Portrait of Old Babylonian Algebra and Its Kin*. New York: Springer, 2002.

Høyrup, Jens, 2002. “A Note on Old Babylonian Computational Techniques”. *Historia Mathematica* **29**, 193–198.

Høyrup, Jens, 2004. “Mahāvīra’s Geometrical Problems: Traces of Unknown Links between Jaina and Mediterranean Mathematics in the Classical Ages”, pp. 83–95 *in* Ivor Grattan-Guinness & B. S. Yadav (eds), *History of the Mathematical Sciences*. New Delhi: Hindustan Book Agency.

Høyrup, Jens, 2009. “State, ‘Justice´, Scribal Culture and Mathematics in Ancient Mesopotamia.” *Sartoniana* **22**, 13–45.

Høyrup, Jens, 2010. *L’algèbre au temps de Babylone : Quand les mathématiques s’écrivaient sur de l’argile*. Paris: Vuibert & Adapt-SNES.

Katz, Victor, 1998. *A History of Mathematics. An Introduction*. Second Edition Readin, Mass.: Addison-Wesley.

Liverani, Mario, 1988. *Antico Oriente. Storia, società, economia*. Roma & Bari: Laterza, 1988.

MCT: O. Neugebauer & A. Sachs, *Mathematical Cuneiform Texts*. New Haven, Connecticut: American Oriental Society.

Merzbach, Uta C., & Carl B. Boyer, 2011. *A History of Mathematics*. Hoboken, New Jersey.

Michel, Cécile, 2008. "Écrire et compter chez les marchands assyriens du début du II^e^ millénaire av. J.-C.", pp. 345–364 *in* Taner Tarhan et al (eds), *Muhibbe Darga Armağanı*. İstanbul: Şubat.

MKT: O. Neugebauer, *Mathematische Keilschrift-Texte*. 3 vols. Berlin: Julius Springer, 1935, 1935, 1937.

Neugebauer, Otto, 1932. "Studien zur Geschichte der antiken Algebra I". *Quellen und Studien zur Geschichte der Mathematik, Astronomie und Physik.* Abteilung B: *Studien* **2**, 1–27.

Neugebauer, Otto, 1934. *Vorlesungen über Geschichte der antiken mathematischen Wissenschaften.* I: *Vorgriechische Mathematik*. Berlin: Julius Springer.

Potts, Daniel T., 1999. *The Archaeology of Elam: Formation and Transformation of an Ancient Iranian State*. Cambridge: Cambridge University Press.

Proust, Christine, 2004. "Tablettes mathématiques de Nippur. I. Reconstitution du cursus scolaire. II. Édition des tablettes d'Istanbul". *Thèse* pour l'obtention du diplôme de Docteur de l'Université Paris 7. Paris: Université Paris 7 – Denis Diderot.

Proust, Christine, 2005. "À propos d'un prisme du Louvre: Aspects de l'enseignement des mathématiques en Mésopotamie". *SCIAMUS* **6**, 3–32.

Proust, Christine, 2008. "Quantifier et calculer: usages des nombres à Nippur". *Revue d'Histoire des Mathématiques* **14**, 143–209.

Proust, Christine, 2009. "Deux nouvelles tablettes mathématiques du Louvre: AO 9071 et AO 9072". *Zeitschrift für Assyriologie und Vorderasiatische Archäologie* **99**, 167–232.

Proust, Christine, 2010. "A Tree-Structures List in a Mathematical Series Text from Mesopotamia". *Preprint*, forthcoming in K. Chemla & J. Virbel (eds), *Introduction to Textology via Scientific Texts*.

RlA: *Reallexikon der Assyriologie und Vorderderasiatischen Archäologie*. I– . Berlin etc: De Gruyter, 1928–.

Robson, Eleanor, 1999. *Mesopotamian Mathematics 2100–1600 BC. Technical Constants in Bureaucracy and Education*. Oxford: Clarendon Press.

Robson, Eleanor, 2000. "Mathematical Cuneiform Tablets in Philadelphia. Part 1: Problems and Calculations". *SCIAMUS* **1**, 11–48.

Robson, Eleanor, 2002. [Review of Høyrup, *Lengths, Widths, Surfaces*]. *MathDL. The MAA Mathematical Sciences Digital Library*. http://mathdl.maa.org/mathDL/19/?pa=reviews&sa=viewBook&bookId=68542 (accessed 24.11.2011).

Robson, Eleanor, 2004. "Mathematical Cuneiform Tablets in the Ashmolean Museum, Oxford". *SCIAMUS* **5**, 3–65.

Robson, Eleanor, 2008. *Mathematics in Ancient Iraq: A Social History*. Princeton & Oxford: Princeton University Press.

Said, Edward W., 1978. *Orientalism*. New York: Pantheon Books.

Soubeyran, Denis, 1984. “Textes mathématiques de Mari”. *Revue d’Assyriologie* **78**, 19–48.

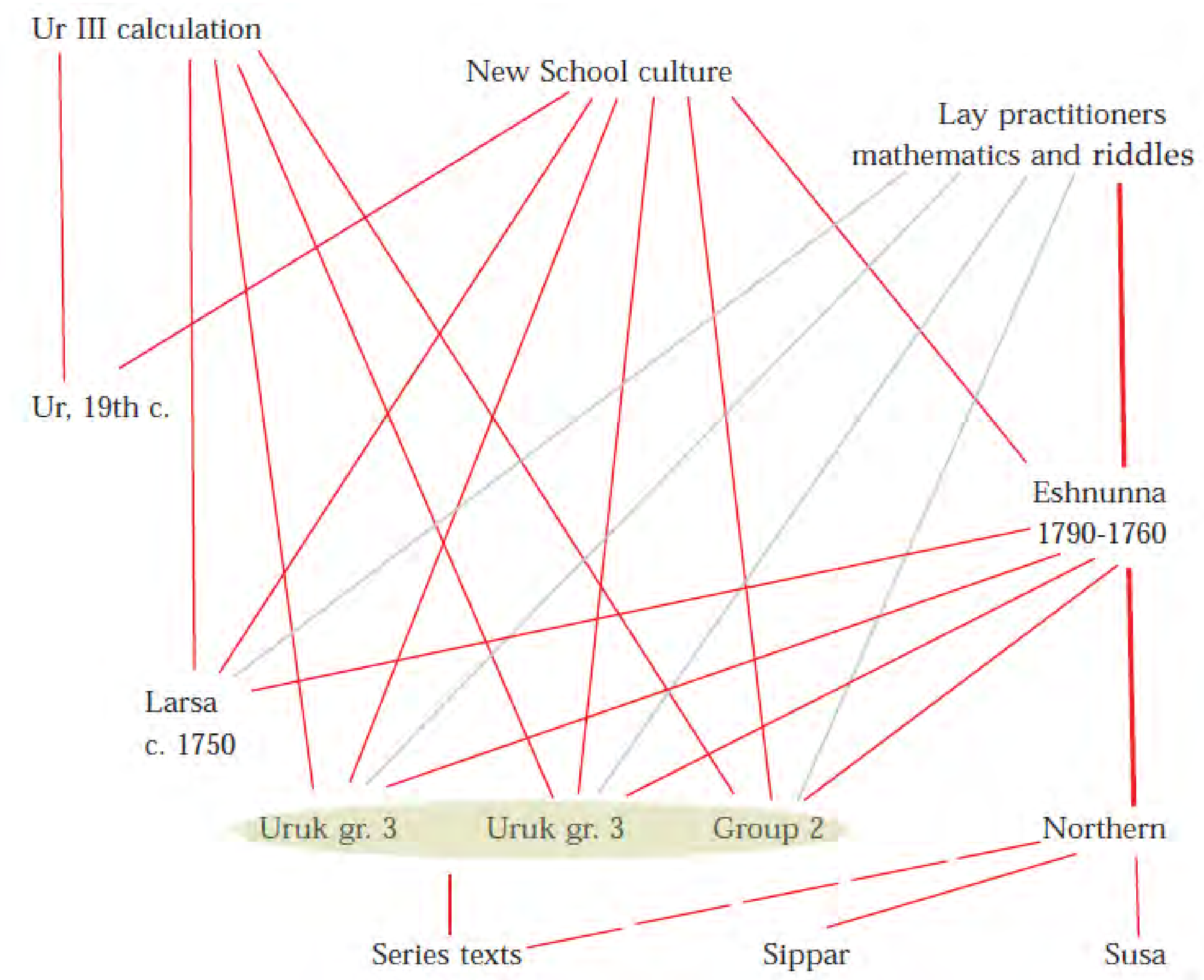
Thureau-Dangin, F., 1936. “L’Équation du deuxième degré dans la mathématique babylonienne d’après une tablette inédite du British Museum”. *Revue d’Assyriologie* **33**, 27–48.

TMS: Evert M. Bruins & Marguerite Rutten, *Textes mathématiques de Suse*. Paris: Paul Geuthner.

Van De Mieroop, Marc, 2007. *A History of the Ancient Near East: ca. 3000–323* BC. Malden, Mass., & Oxford: Blackwell.

Veldhuis, Niek, 1997. “Elementary Education at Nippur. The Lists of Trees and Wooden Objects”. *Proefschrift ter verkrijging van het doctoraat in de Letteren*. Groningen: Rijksuniversiteit Groningen.

von Soden, Wolfram, 1952. “Zu den mathematischen Aufgabentexten vom Tell Harmal”. *Sumer* **8**, 49–56.



MAX-PLANCK-INSTITUT FÜR WISSENSCHAFTSGESCHICHTE

Max Planck Institute for the History of Science

Preprints since 2010 (a full list can be found at our website)

391 Horst-Heino v. Borzeszkowski & Renate Wahsner **Die Fassung der Welt unter der Form des Objekts und der philosophische Begriff der Objektivität**

392 Ana Barahona, Edna Suarez-Díaz, and Hans-Jörg Rheinberger (eds.) **The Hereditary Hourglass. Genetics and Epigenetics, 1868–2000**

393 Luis Campos and Alexander von Schwerin (eds.) **Making Mutations: Objects, Practices, Contexts**

394 Volkmar Schüller **Some Remarks on Prop. VIII Probl. II of Newton's Opticks Book I Part I**

395 Tamás Demeter **Hume's Experimental Method**

396 Fynn Ole Engler, Björn Henning und Karsten Böger **Transformationen der wissenschaftlichen Philosophie und ihre integrative Kraft – Wolfgang Köhler, Otto Neurath und Moritz Schlick**

397 Frank W. Stahnisch **«Der Rosenthal'sche Versuch» oder: Über den Ort produktiver Forschung – Zur Exkursion des physiologischen Experimentallabors von Isidor Rosenthal (1836–1915) von der Stadt aufs Land**

398 Angela Matyssek **Überleben und Restaurierung. Barnett Newmans *Who's afraid of Red, Yellow, and Blue III* und *Cathedra***

399 Susanne Lehmann-Brauns, Christian Sichau, Helmuth Trischler (eds.) **The Exhibition as Product and Generator of Scholarship**

400 Fynn Ole Engler und Jürgen Renn **Wissenschaftliche Philosophie, moderne Wissenschaft und Historische Epistemologie**

401 M. J. Geller **Look to the Stars: Babylonian medicine, magic, astrology and *melothesia***

402 Matthias Schemmel **Medieval Representations of Change and Their Early Modern Application** (TOPOI – Towards a Historical Epistemology of Space)

403 Frank W. Stahnisch **German-Speaking Émigré Neuroscientists in North America after 1933: Critical Reflections on Emigration-Induced Scientific Change**

404 Francesca Bordogna **Asceticism and Truth: The Case of 'Magic Pragmatism'**

405 Christoph Hoffmann and Alexandre Métraux (eds.) **Working with Instruments – Three Papers of Ernst Mach and Ludwig Mach** (Translated by Daniel Bowles)

406 Karin Krauthausen **Paul Valéry and Geometry: Instrument, Writing Model, Practice**

407 Wolfgang Lefèvre **Picturing the World of Mining in the Renaissance The *Schwazer Bergbuch* (1556)**

408 Tobias Breidenmoser, Fynn Ole Engler, Günther Jirikowski, Michael Pohl and Dieter G. Weiss **Transformation of Scientific Knowledge in Biology: Changes in our Understanding of the Living Cell through Microscopic Imaging**

409 Werner Kogge **Schrift und das Rätsel des Lebendigen.** Die Entstehung des Begriffssystems der Molekularbiologie zwischen 1880 und 1950

410 Florentina Badalanova Geller ***2 (Slavonic Apocalypse of) Enoch*: Text and Context**

411 Henning Schmidgen & Urs Schoepflin (eds.) **Hans-Jörg Rheinberger : a Bibliography**

412 Renate Wahsner & Horst-Heino v. Borzeszkowski **Erkenntnis statt Erbauung: Hegel und das naturwissenschaftliche Denken der Moderne**

413 Mirjam Brusius **From photographic science to scientific photography: Photographic experiments at the British Museum around 1850**

414 Viktor J. Frenkel **Professor Friedrich Houtermans – Arbeit, Leben, Schicksal. Biographie eines Physikers des zwanzigsten Jahrhunderts.** Herausgegeben und ergänzt von Dieter Hoffmann, unter Mitwirkung von Mary Beer

415 Ilana Löwy (ed.) **Microscope Slides – Reassessing a Neglected Historical Ressource**

416 André L. Blum, John Michael Krois und Hans-Jörg Rheinberger (Hrsg.) **Verkörperungen**

417 Pietro Daniel Omodeo **Sixteenth Century Professors of Mathematics at the German University of Helmstedt.** A Case Study on Renaissance Scholarly Work and Networks

418 Peter Schöttler & Hans-Jörg Rheinberger (éds.) **Marc Bloch et les crises du savoir**

419 Albert Presas i Puig (ed.) **A Comparative Study of European Nuclear Energy Programs**

420 Mathias Grote & Max Stadler (eds.) **Membranes Surfaces Boundaries**
Interstices in the History of Science, Technology and Culture

421 Frank W. Stahnisch **The emergence of *Nervennahrung*: Nerves, mind and metabolism in the long eighteenth century**

422 Pietro Daniel Omodeo, Irina Tupikova **Aristotle and Ptolemy on Geocentrism: Diverging Argumentative Strategies and Epistemologies** (TOPOI – Towards a Historical Epistemology of Space)

423 Han F. Vermeulen **Linguistik und Völkerkunde – der Beitrag der historisch-vergleichenden Linguistik von G.W. Leibniz zur Entstehung der Völkerkunde im 18. Jahrhundert**
[Leicht erweiterte Fassung des Working Papers No. 133 aus dem MPI for Social Anthropology]

424 Alfred Gierer **Mit Schiller gegen den „Egoismus der Vernunft".** Zeitübergreifende Gedanken zur Natur des Menschen

425 Annette Vogt **Die Berliner Humboldt–Universität von 1945/1946 bis 1960/1961**

426 Klaus Geus, Martin Thiering (eds.) **Common Sense Geography and Mental Modelling**

427 Renate Wahsner **Kann eine moderne Naturphilosophie auf Hegelsche Prinzipien gegründet werden?** Spekulatives und naturwissenschaftliches Denken

428 Stefano Bordoni **Widening the Scope of Analytical Mechanics** Duhem's third pathway to Thermodynamics

429 Pietro Daniel Omodeo **Copernicus in the Cultural Debates of the Renaissance: Reception, Legacy, Transformation** [Part I & II]

430 Mark Geller & Klaus Geus (eds.) **Productive Errors: Scientific Concepts in Antiquity**
(TOPOI – Dahlem Seminar for the History of Ancient Sciences)

431 Klaus Gottstein **The Amaldi Conferences. Their Past and Their Potential Future**

432 Mikuláš Teich **The Scientific Revolution Revisited**

433 Lorraine Daston & Jürgen Renn (Hrsg.) **Festkolloquium für Hans-Jörg Rheinberger**
Beiträge zum Symposium am 24. 1. 2011 im Max-Planck-Institut für Wissenschaftgeschichte

434 Conference **Epistemology and History. From Bachelard and Canguilhem to Today's History of Science**

435 Jens Høyrup **Sanskrit-Prakrit interaction in elementary mathematics as reflected in Arabic and Italian formulations of the rule of three – and something more on the rule elsewhere**