Fibonacci, Leonardo

Høyrup, Jens

Publication date:
2013

Fibonacci, Leonardo

Summary:
Leonardo Fibonacci (*1170–80, † after 1241) is known for mathematical writing, often submitting topics from the practical traditions to the approach of scholarly mathematics: *Liber abbaci*, *Pratica geometriae*, *Flos*, *Letter to master Theodorus*, *Liber quadratorum*. He makes us of Greek and Arabic sources translated into Latin as well as unidentified scholarly and "vernacular" Arabic, Byzantine and Ibero-Provençal material. His influence was probably much more modest than has been believed.

Leonardo was one of the transmitters of Arabic mathematical knowledge to Christian Europe, and the only transmitter of commercially tainted arithmetic who is known by name. He was born in Pisa, probably between 1170 and 1180 C.E., a son of Guglielmo *de filiis Bonaccii*, whence the modern surname Fibonacci; he was still active in 1241. Outside Pisa he referred to himself as *pisano*, while two documents from Pisa and several of his introductions identify him as *bigollo*, probably "the traveller" (Bonaini; Ulivi, 247–254). In boyhood he was brought to Bejaïa by his father, who served there as "public scribe for the Pisa merchants", so that "for some days" he might learn practical calculation – thus the preface to his *Liber abbaci* (on which below). Having become captivated by the Hindu-Arabic-numerals, the account continues, he studied (what was done with) them on business travels to Egypt, Syria, Constantinople, Sicily and Provence (*Scritti* I, 1).

In 1202, he wrote a first version of the *Liber abbaci*, which has been lost; its title does not refer to an abacus of any kind but apparently means "Book of practical computation" (unless it intends to render Arabic *Kitāb al-muʿāmalāt*). Around a dozen complete and incomplete copies exist of a second version from 1228, in which Fibonacci "added certain necessary and eliminated certain superfluous matters" (*Scritti* I,1). Another lost work to which the *Liber abbaci* refers once and which a mid-fifteenth-century writer knew by name is *Liber minoris guise*, "Book in a smaller manner" introducing to commercial arithmetic. Other works are *Pratica geometriae* (1220), *Flos*, a *Letter to Master Theodorus*, and *Liber quadratorum* – all in (*Scritti* II). *Liber quadratorum* is dated 1225, but claims to have been presented to the Emperor Frederick II von Hohenstaufen ("Holy Roman Emperor" but also King of Sicily) during his visit to Pisa, which took

place in 1226; the others are undated but probably close in date. The revised *Liber abbaci* is dedicated to Michael Scot, formerly a translator based in Toledo but now philosopher at Frederick II's Sicilian court. Some of the dedicatees of the other works (perhaps all of them) were also connected to Emperor Frederick's court; Fibonacci himself did not belong to the Emperor's circle.

The *Liber abbaci* starts by describing the Hindu-Arabic numerals and how to compute with them. These were known in Christian Europe since the Latin translations of the twelfth century; they are also used in late twelfth-century notarial documents from Perugia (Burnett V, 254). However, what Fibonacci describes is clearly derived from Maghreb habits, including notations for composite fractions such as $\frac{1\ 5\ 3}{2\ 8\ 11}$ (meaning $\frac{3}{11} + \frac{5}{8}$ of $\frac{1}{1} + \frac{1}{2}$ of $\frac{1}{8}$ of $\frac{1}{11}$); he also writes mixed numbers with the fraction to the left, $\frac{2}{5}4$ where we would write $4\frac{2}{5}$ (he does so in all extant works). The bulk of the book presents commercial arithmetic, including complicated "recreational" problems (three men finding a purse and dividing its contents according to a complicated prescription, etc.), but often, as he says, dealt with *magistraliter* ("in the manner of the schools") and distancing itself from "vernacular" ways (*vulgi modus*). It thus presents an attempt to submit its subject-matter to the norms of scholarly, not least Euclidean mathematics. Chapter 14 deals with roots and operations with bi- and trinomials involving radicals, with reference to Euclid's *Elements* II and X; chapter 15 mainly treats of algebra, in a style going back to al-Khwārizmī and Abū Kāmil but also using *Elements* II and proportion theory where these would apply purely algebraic arguments.

The *Pratica geometriae* represents a similar integration of levels, looking to theory rather than practice (*Scritti* I, 1). It introduces matters borrowed from the Arabic *misaḥah* (mensuration) tradition; actual Pisa metrology; advice for surveying sloping surfaces; basic trigonometry; extraction of square and cube roots – all within a Euclidean framework, and together with classical problems like the doubling of the cube and partition of figures.

The *Flos* solves difficult problems that are all algebraic from a modern point of view but had no such connection at the time. It shows that a particular cubic equation can have no solution within the domain of Euclidean irrationals and finds an approximate root (not telling the method); next it deals with intricate recreational problems (of types "purchase of a horse" etc.) that translate into indeterminate linear problems in several variables; in one of them Fibonacci makes use of two unknowns, *causa* (pseudo-Latinization of contemporary Iberian or Italian for "thing") and *res* (Latin with the same meaning); another problem works with three unknowns, *res*, *dragma* and *bursa* (the unknown contents of

a “purse”) (*Scritti* II, 236, 238).

The *Letter* takes up similar matters. *Liber quadratorum* explores the problem to find three square numbers such that (in modern terminology) $x^2–y^2 = y^2–z^2 = 5$, which leads Fibonacci to investigate more general properties of “congruent numbers” (Oort).

Euclid, as mentioned, was a very important authority for Fibonacci. Sometimes he uses the Greco-Latin translation of the *Elements*, sometimes he quotes from memory, apparently from an Arabic or an Arabo-Latin version (Folkerts, IX). He cites and uses Ptolemy, Menelaus and Theodosius, and uses the Latin translations of the 9th-10th-century astronomer-mathematicians Banu Mūsā and of Aḥmad ibn Yūṣuf, citing only the latter by name.

He also knew and drew verbatim upon Gerard of Cremona’s translations of al-Khwārizmī’s *Algebra* and of a *Liber mensurationum* (“Book on Mensuration”) attributed to an otherwise unidentified Abū Bakr “called Heus”. Material borrowed from Abū Kāmil and al-Karajī is taken from memory or indirectly. His explanation of *algebra et almuchabala* as “proportion and restoration” (*Scritti* I, 406) raises doubts about the depth of his familiarity with Arabic, but he must have had access to Arabic knowledge that was not diffused in the Latin-Christian world. His familiarity with practical arithmetic beyond numerals and computation was acquired at least in part in Constantinople and in the Ibero-Provençal area. In the latter area he is also likely to have learned about 12th-century investigations of theoretical arithmetic which (like other late developments in al-Andalus) have not been transmitted in Arabic.

His influence in Italy has probably been overrated. The early Italian “abbacus school” (a school type emerging in the later 13th century and teaching a two-year course in practical arithmetic for merchant and artisan youth) appears to have been primarily inspired by the same “vernacular” practices which Fibonacci had cited. One school tradition in Florence, from Paolo dell’Abbaco to Benedetto of Florence, held Fibonacci in high honour, but made its own work within the normal abbacus tradition.

Very few medieval university scholars knew his work: Jordanus de Nemore around Fibonacci’s own times perhaps, Jean de Murs a hundred years later certainly.

Bibliography:

F. Bonaini 1858. “Memoria unica sincrona di Leonardo Fibonacci, novamente scoperta”. *Giornale Storico degli Archivi toscani* **1** (1858), 3–10; C. Burnett, *Numerals*

and arithmetic in the Middle Ages. Farnham, Surrey, & Burlington, Vermont, 2010; M. Folkerts, *The development of mathematics in medieval Europe: The Arabs, Euclid, Regiomontanus*. Aldershot: Ashgate, 2006; F. Oort, "Congruent numbers in the tenth and in the twentieth century", pp. 77–97 *in* Arnoud Vrolijk & Jan P. Hogendijk (eds), *O Ye Gentlemen: Arabic Studies on Science and Literary Culture*. Leiden & Boston, 2007; *Scritti* di Leonardo Pisano matematico del secolo decimoterzo, ed. B. Boncompagni. I. Il *Liber abbaci*. II. *Practica geometriae* ed *Opusculi* Roma, 1857, 1862; E. Ulivi, "Su Leonardo Fibonacci e sui maestri d'abaco pisani dei secoli XIII–XV". *Bollettino di Storia delle Scienze Matematiche* **31** (2011), 247–286.

The above small article was written at request for the *Encyclopaedia of Islam*, Third edition, and then in the end rejected because of "changing priorities" by the executive editor Everett Rowson – whose personal research and teaching priorities concentrate on "gender and sexuality", more specifically on "homosexuality in medieval Islamic cultures as treated in literary, legal, medical, philosophical, and mystical texts" – see http://meis.as.nyu.edu/object/EverettKRowson.html, accessed 23.11.2013).