State, "Justice", Scribal Culture and Mathematics in Ancient Mesopotamia

Sarton Chair Lecture

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Published in:
Sartoniana

Publication date:
2009

Document Version
Publisher's PDF, also known as Version of record

Citation for published version (APA):
Høyrup, J. (2009). State, "Justice", Scribal Culture and Mathematics in Ancient Mesopotamia: Sarton Chair Lecture. *Sartoniana*, *22*, 13-45.

Sartoniana 22 (2009), 13-45

State, "Justice", Scribal Culture and Mathematics in Ancient Mesopotamia

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Abstract

The functioning of the modern state presupposes a variety of mathematical technologies – accounting, statistics, and much more. Mathematics, on its part, needs the institutions of the state (schools, universities, research institutions, etc.) to secure financing, recruitment and the rearing of competence. At a given moment, the state as well as mathematics largely take the partner "as it is", and none of them appears to the immediate view to depend for its essence on the other.

At the moment of pristine state formation, the situation was different. Most pristine state structures depended on organized violence, on religious institutions, etc., and mathematics did not enter. At least one major exception to this rule can be found, however: the earliest "proto-literate" state formation in Mesopotamia of the late fourth millennium, intimately connected to a system of accounting that seems to have guaranteed an apparent continuation of pre-state "just redistribution". Both for its functioning and its legitimization, the state depended on the mathematics of accounting. On its part, the kind of mathematics which was created was totally bound up with its administrative role.

The lecture follows the interaction of state, "justice", mathematics and scribal profession from the late fourth millennium over the "Ur III" period (21st century BCE, culmination and apparently end of the intertwinement of statal structure and legitimization with mathematics) until the Assyrian empire of the earlier first millennium.

State and mathematics

The functioning of the modern state presupposes a variety of mathematical technologies – accounting, statistics, and much more. Mathematics, on its part, needs the institutions of the state (schools, universities, research institutions, etc.) to secure financing, recruitment and the rearing of competence. At a given moment, the state as well as mathematics largely take the partner "as it is", and none of them appears to the immediate view to depend for its essence on the other.[1]

At the emergence of the state as a type of social organization, the situation was different. Most statal systems have originated in complex processes, either as "pristine states" via expanding chiefdoms or as "secondary states" in interaction with (often, indeed, as military protection against) existing states. As a rule, the involvement of anything than can be considered as mathematics in such processes has been peripheral, if not totally absent.

In a few exceptional cases, however, mathematical technologies have played a major role in the shaping of the state (and have, in consequence, themselves become more sophisticated in the process, developing into recognizable *mathematics*).

One instance of such an intimate bond is that between the Inca state and its accounting. I know too little about the matter to go into details – I suspect, moreover, that available evidence on the topic is insufficient to trace the connections between the development of the state and that of the *quipu* system.

> Possibly, another instance is constituted by the relation (which, however, may be less pivotal) between the Maya states and their "chrono-theology"; even here I abstain from further discussion for lack of deeper knowledge. In any case, the Maya state formation was not pristine.

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1 Second thoughts should force us to admit that this "immediate view" may not correspond to the actual situation of the latest four decades or so: without information technology, the immense increase of administrative control of citizens (to mention but that) would never have been possible. Only by discarding computer science from what we perceive as "mathematics" can we claim that the global mathematical enterprise has not been transformed in the same process.
But this is not my topic here. I shall leave it to the reader to ponder after finishing the paper whether, paradoxically, the late fourth millennium BCE offers an illuminative model of our own lifetime. That possibility is indeed one of my reasons for choosing the subject I do deal with.

A third instance, perhaps the most indisputable case and at least the one which is best reflected in the sources (though still indirectly), is offered by the formation of states in southern Mesopotamia from the late fourth millennium BCE onward.

Prolegomena

Before approaching the subject-matter itself, something must be said about what I mean here by "mathematics", and about the notion of a "state".

For the present purpose, the transition to "recognizable mathematics" may be characterized as

> the point where *pre-existent but previously independent* mathematical practices are coordinated through a minimum of at least intuitive understanding of formal relations.

Political anthropologists have discussed the emergence of *statal organization* of society in different terms, not necessarily as mutually exclusive as often assumed in the debate. According to Morton Fried's classic *The Evolution of Political Society* [1: 235], the state arises as

> a collection of specialized institutions and agencies, some formal and others informal, that maintain an order of stratification,

"stratified society" being a society [1: 186]

> in which members of the same sex and equivalent age status do not have equal access to the basic resources that sustain life.

This stratification may come about in several steps: in brief, from "big man" practice to chiefdom spurred by warfare, leading to a three-class division *slave owners – commoners – slaves*.

Elman Service's emphasis in the equally classic *Origins of the State and Civilization* was different, seeing [2: 305] statal organization as the end result of a quantitative and often gradual development from

> relatively simple hierarchical-bureaucratic chiefdoms, under some unusual conditions, into much larger, more complex bureaucratic empires.

The chiefdom itself was understood by Service as a hierarchical organization legitimized by social *functions* wielded by the chief for common benefit[2] in a theocratic frame of reference, where

> economic and political functions were all overlaid or subsumed by the priestly aspects of the organization.

A number of other, less abstract discussions of the early state have been regionally focused (either explicitly or implicitly). In an article on "Population, Exchange, and Early State Formation in Southwestern Iran", Henry T. Wright and Gregory A. Johnson tried to base themselves "on the total organization of decision-making activities rather than on any list of criteria", describing the state [3: 267]

> as a society with specialized administrative activities. By `administrative' we mean `control', thus including what is commonly termed `politics' under administration. In states as defined for purposes of this study, decision-making activities are differentiated or specialized in two ways. First, there is a hierarchy of control in which the highest level involves making decisions about other, lower-order decisions rather than about any particular condition or movement of material goods or people. Any society with three or more levels of decision-making hierarchy must necessarily involve such specialization because the lowest or first-order decision-making will be directly involved in productive and transfer activities and second-order decision-making will be coordinating these and correcting their material errors. However, third-order decision-making will be concerned with coordinating and correcting these corrections. Second, the effectiveness of such a hierarchy of control is facilitated by the complementary specialization of information processing activities into observing, summarizing, message-carrying, data-storing, and actual decision-making. This both enables the efficient handling of masses of information and decisions moving through a control hierarchy with three or more levels, and undercuts the independence of subordinates.

2 According to Service mostly functions of a redistributive nature; but if we include functions of military leadership the contrast with Fried can be seen to be far from absolute.

Though meant to be generally useful, the description *was* specifically geared to what happened when statal systems emerged in southern Mesopotamia and southwestern Iran, for which reason I shall adopt it here.

The West-Asian "token system"

Central to the "control" which Wright and Johnson spoke about is the "token system", an accounting system based on small and less small cones, spheres, discs, tetrahedra, rods etc. made of burnt clay – often (though at first only rarely) provided with markings that define sub-types. The system turns up in Syria and Western Iran around 8000 BCE, concomitantly with the agricultural revolution, spreading over the following millennia to a region reaching from south-eastern Anatolia and Palestine to the Iranian plateau, and remaining alive at least until the early third millennium BCE. Though some suggestions had been made in discussions of late fourth-millennium Iranian material, the discovery of the system and of its chronological and geographical range is unambiguously the merit of Denise Schmandt-Besserat. Her first publication on the topic [4] is from 1977; a complete survey of her results and interpretations is the double volume *Before Writing* [5].

According to their use in the fourth millennium and to continuity with proto-cuneiform writing, the various tokens served to represent quantities (presumably standard containers) of grain, oil, etc., and heads of livestock – perhaps also quantities of work.

For a number of reasons, the original social function of the system cannot have been inter-community trade (which did exist, as documented by the spread of obsidian). First of all, any use of quasi-monetary symbols without tangible value (paper money, bills of exchange) presupposes banks and police forces which can enforce the obligations they represent. Moreover, the tokens were simply thrown out once they had been used, which excludes even a local monetary function.

Instead, the use of prestige versions (made from marble, alabaster, etc.) as grave-goods in high-status graves [6] and the presence of tokens in communal storehouse areas suggest that the tokens functioned as means of accounting in a redistribution system, and that management of this redistribution system carried very high social prestige – cf. Elman Service as quoted above.

In this connection, two observations should be made:

– Redistribution within the community is very common in pre-state societies, but redistribution built on detailed accounting is rather unique. If Inuit hunters kill a walrus and give others access to the meat, this is done from an expectation of reciprocity, and on the part of the more skilled hunters in expectation of prestige; but in neither respect is detailed accounting involved, nor possible.
– Accounting by means of tokens can doubtlessly be characterized as a *mathematical technique*. But we have no evidence for numerically standardized bundling of units (actually there is some counter-evidence from the fourth millennium, cf. below). It is therefore most likely that (e.g.) a small cone corresponded to a specific customary basket containing grain and a small sphere to some larger equally customary container, and that the ratio between the two was not numerically but physically (that is, not precisely) fixed. In other words, the mensuration inherent in the token system appears not to have been coordinated neither with the bundling levels of an oral counting system nor with any other numerical bundling principle; if this is so, the system is hardly an instance of (integrated) *mathematics* in the above sense.

Fourth-millennium developments

In the earlier fourth millennium, the city Susa in a river valley in the Zagros area in southwestern Iran became the centre of a wider settlement system; in this context the redistribution system developed into what looks most of all as payment of tribute or taxes to the central temples of Susa. The tokens were put to new use: enclosed in hollow clay envelopes ("bullae"), they appear to have served as bills of lading for goods delivered from the periphery to the centre. This goes hand in hand with the development and refinement of other bureaucratic devices and procedures – not least the use of cylinder seals as "certifiable signatures" of particular officials or offices. Since the contents of bullae could only be "read" if they were broken (after which they could no longer be controlled), impressions (or representative pictures) of the tokens to be put into them began to be made on their surfaces before they were closed and sealed.

A somewhat similar social development may have started slightly later in Uruk in the Mesopotamian South, but it soon went much further. The background was that a climatic change and lowering of the water level in the Gulf opened the possibility for irrigation agriculture in the future Sumerian area, allowing a violent growth of agricultural output as well as population – see, e.g., [7: 58-61].

Probably in an initial phase, it was realized that impression or depiction of the tokens on the surface of bullae made it possible to dispense with the contents, and that the bulla itself could then be replaced by a flattened piece of clay as carrier of the impressions/depictions.[3] Very soon (c. 3200 BCE[4]), writing was also invented – *invented* indeed, in one leap or at least in a very speedy process (no "primitive" precursor steps are known).[5]

The "proto-literate" script was ideographic, and used composition in a way that is quite similar to what is found in pidgin and creole languages.[6] Most signs (traced by means of a pointed stylus) were directly pictographic, showing for instance a jar, a head, the mountains to the east, the sun rising between these, etc.[7] Some, however, depict *tokens representing the thing* instead of the thing itself. Quite striking, and enigmatic until the discovery of the token system, is the sign for a sheep: a circle marked by a cross. Indeed, it does not depict the animal but the token standing for the animal.

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3 These "numerical tablets" provide the evidence referred to above that no arithmetically defined bundling system was yet in existence around the mid-fourth millennium BCE.

4 From this point onward, I follow the "middle chronology", as used, e.g., in [8]. It should be pointed out that dates, even when they can be given exactly *within* this chronology, are not fixed absolutely before the first millennium BCE.

5 Except for what will be said about the possible existence of a creole language, most of what is said in the following about early writing and accounting and their function is explained in much greater depth in [9]. In general, I draw heavily on the works of Hans Nissen, Peter Damerow and Robert Englund.
The reconstruction of underlying cognitive type and mathematical conceptions are on the whole of my own responsibility.

6 This statement should not be taken as a claim that the inventors of the "proto-literate" script spoke a pidgin – the patterns in sacred architecture shows cultural continuity over about 2000 years preceding the invention, and thus continuity of the culturally hegemonic stratum of the area. But the principles of the invention *may* have been inspired by familiarity with a pidgin spoken by enslaved populations – cf. imminently.

7 In the third millennium, the drawings were no longer traced but made by oblique impression of a prismatic stylus; this gave the script its characteristic "cuneiform" character.
The early as well as the developed forms are shown in [10].

In contrast to these drawings of things or tokens, metrological and numerical units were *impressed* by a different stylus, as representations of tokens. This stylus was cylindrical, thick in one end and thin in the other. Impressed vertically it might produce a large or a small circle, oblique impression could represent a large or a small cone.

The proto-literate script did not attempt to render the sentences of spoken language – it was not "glottographic". Some 85% of the surviving texts are accounts made in fixed formats, rather to be likened to a statistical table or a ledger than to literary texts; what was written could of course be *spoken of* or *told* in words but it could not be *read*. The remaining 15% are "lexical lists" which served to teach the script.

Whereas writing was thus (to all we know) invented in Uruk, the idea and the bureaucratic use (not the script itself) were soon borrowed into Susa and a number of other Iranian localities which formed a shared cultural system. Until some decades ago the earliest known evidence for Egyptian writing was a century or two later than the earliest Uruk script, and contemporary with artefacts inspired or imported from Mesopotamia (e.g., cylinder seals). It therefore seemed a good guess that even the Egyptian script was inspired by knowledge of the *possibility* to write (whatever that may mean precisely); now, as the earliest beginnings of Egyptian writing has moved back a century, this is much more doubtful [11]; independence seems more likely, but partial inspiration going either way cannot be excluded.

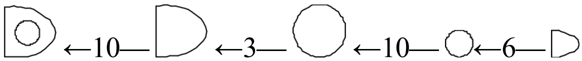
The proto-literate Uruk metrologies

In the numerical and metrological sequences of the Uruk writing system, bundling was numerically determined.[8]

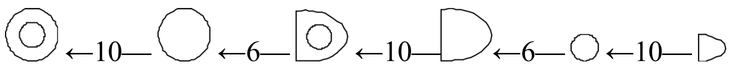
One sequence was used for the measurement of grain, and may reasonably be considered a continuation of the traditional use of tokens. The "basic unit" in this system, depicting a small cone, was . 6 of these became , the picture of a small sphere. 10 small spheres were , the picture of a

8 For the following description of the metrological and numerical sequences I build on [12].

large sphere. 3 large spheres were bundled as , the picture of a large cone. 10 large cones, finally, became , possibly a representing a punched large cone (an existing token), but perhaps a new construction made in parallel with the number sequence. In a notation due to Jöran Friberg, the sequence as a whole looks as follows



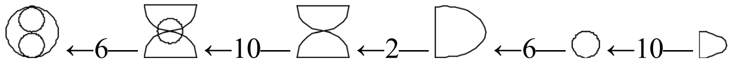
Another sequence was used for counting most types of discrete items, and may be regarded as a "number sequence". Whereas the grain sequence is likely to continue an old system in a new medium (though now with arithmetical bundling), the number sequence can be supposed to be new – the representation of pure numbers (that is, numbers abstracted from the quantity they count) by tokens will have had no purpose, at least not before their inclusion in bullae (here, the external official's seal might *in principle* determine which kind of goods was involved). The corresponding diagram is



This sequence, in contrast to the preceding one, is highly systematic, and therefore almost certainly represents a deliberate transformation of the grain sequence made so as to fit an existing oral number system, and perhaps extending it beyond existing spoken numerals. As we see, the signs for 600 ( ) and 36000 ( ) are produced by superposition of 10 ( ) on 60 ( ) and 3600 ( ), respectively, while 60 ( ) is chosen as an "enlarged" unit ( ).

The latter feature suggests that the spoken numeral system treated the step 1→10 differently than the step 10→60 (if not, there would be no reason to invert the order of and in the grain system); 60 must in some way have been understood as a "return of the unit". Evidently, the "second return" of the unit as 3600 could not repeat the visual trick, the "number-and-measure" stylus having only two ends, each of which could be impressed vertically or obliquely. In consequence, the written system gives no clues as to whether 3600 was already a unit in the spoken system.

For specific counting purposes – apparently the counting of bread or grain rations, perhaps also portions of dairy products – a particular "bi-sexagesimal system" with the following structure was in use:

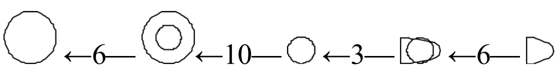


The agreement with the lower orders of the "general" counting system suggests the bisexagesimal system to have been shaped so as to fit particularly bureaucratic procedures or habits. Such an adaptation recalls our counting sheets of paper in units of 500, bottles of wine in dozens, etc., sometimes but not always corresponding to standard packages – such adaptations are amply present in the later Mesopotamian record.

We might be tempted to conclude from the divergence of the two counting systems after the level of 60 that the level 3600 did not exist in the spoken number system but was a product of the new bureaucratic device; the existence of the medieval "hundredweight" and the Germanic *Großhundert*, both deviating from the pre-existing 100 for similar reasons, shows that such a conclusion is not warranted.

Two other metrological sequences exemplify the converse process, the adjustment of administrative procedures to mathematical structures. One is the area system, the other the administrative calendar.

The structure of the area system in itself shows little mathematical system:



Such lack of mathematical system is in itself an indication that the system is a normalization of a pre-existing system of "natural" (irrigation, seeding or similar) measures – a conclusion which is supported by linguistic arguments [13, *passim*]. There is no direct proof of it, but it is a fair assumption that the system (which coincides with what is still known and well documented in much later periods) was already geared to the length metrology (based on the unit nindan or "rod" of c. 6 m) – not least since it is almost certain that the area of slightly irregular rectangular fields was already determined as average length times average width (the "surveyors' formula"), which would make no point if area units were not derived from length units.  (the iku of later times) would then be the square on 10 nindan,  a rectangle contained by 10 and 60 nindan.[9] On this foundation we may conclude that the area metrology presents us with a deliberate coordination of several mathematical techniques and with integration of the result in the administrative procedures concerned with the allotment of land in arithmetically determined proportion (which, without this new tool, could not be made, and hardly imagined).

Alongside the true luni-solar calendar with its months of variable length and its insertion of intercalary months when such turned out to be needed (which remained in use for ritual and time-keeping purposes until the first millennium BCE), an administrative calendar was introduced, which counted each month as if it consisted of 30 days, and each year as 12 months[10]. It served for the calculation of fodder to be allocated to herds and, at least in later times, of the work which overseers were to press out of their crew each month irrespective of its length. Even in this case, only the introduction of a mathematical tool made possible the system of intense administrative control of subordinate staff.

Still other metrological sequences were in use – most of them derived from those already mentioned by means of various kinds of extra marks (similar to those that had served in the token system), and serving, for instance, to count malted instead of ordinary grain. There is no need to describe them in detail.

One common feature of all sequences which is worth mentioning is the way they were provided with subunits below . In all cases, the first level of sub-units was obtained by rotating either this sign or a shortened  90°

9 The definition of area units in terms of linear metrology presupposes a conceptualization of area linked to square and rectangular shapes with measured sides; that this conceptualization was at hand, however, is not subject to doubt. Firstly, a number of prestige buildings in the area from this and earlier periods exhibit clearly rectangular layout – a number of specimens are rendered in [14]; secondly, the dimensions at least of certain buildings from the proto-literate period are determined in terms of an identifiable length unit – see [15].
This modular-orthogonal architecture represents a kind of "integrated" mathematics beyond the one represented by the state-accounting complex.

10 This calendar and its use until the outgoing third millennium BCE was analyzed in depth by Robert Englund [16].

clockwise,  and , respectively –  standing apparently for a halving (except when a day is seen as a sub-unit of an administrative month),  for a division into 5 parts.

It is possible that *one* of these subdivisions precedes writing –  could well be a depiction of a hemisphere, one of the old tokens. But , a mere rotation of , can hardly correspond to a particular token, nor can a rotation possibly correspond to any feature of the token system. Globally, the way sub-units are formed thus reflects an underlying general idea of "forming sub-units".

Another general feature to be observed has to do with the function of the counting sequence. As observed above, freely movable tokens had to represent both the *kind* of thing they stood for and the quantity involved. In writing, it became possible to separate the two, combining, e.g., the ideogram for a sheep with the number "2" – which was indeed done. The mental habit involved in this splitting of quality and quantity also underlies the way the "lexical lists" were constructed from which the script was learned: in Luria's terminology [17: 48*ff*], it reflects "categorical classification" and not "situational thinking". A plough will thus appear in a list of wooden objects, not together with the ploughman or the grain. In one list – the "profession list" – the Cartesian product is not only an external condition but also involved in the structure of the list itself, which confronts field of activity with the hierarchy of positions. Even the orderly formats of bureaucratic accounting reflects the same mental habit.[11]

The splitting into a Cartesian product of quantity and quality was not followed rigidly: quality, if determined unambiguously by context, was routinely left implicit – if a number stood for the length or width quality of a field, the unit nindan was thus omitted. This should not be understood as an indication of "primitivity" but as an instance of economical flexibility of thought: exactly the same thing happened, for instance, when Stevin's

11 According to Mogens Trolle Larsen [18: 211], the format of the lists "points to a special logic that is additive and aggregative rather than subordinative and analytic", and whatever hierarchy occurs in the list simply reflects "the surrounding highly stratified society". In [19: 337] I endorsed this view, but second thoughts now suggest to me that the hierarchy of the "profession list" is too regular to represent a spontaneous historical development: it looks like a construction inspired by categorical thought, perhaps fully implemented in real social life, perhaps in part a theoretical construction reflecting how future officials were meant to perceive social reality.

decimal fractions came in common use, and his 375⓪7①2② was reduced to 375.72.[12]

Even this principle of economy can be seen in the light of Luria's dichotomy: Situational thinking is the habit of those whose world is largely made up of fixed situations, categorical classification is needed by those whose existence is less predictable – but in situations that *are* predictable, there is no reason they (indeed *we*) should not resort to the simpler pattern. True mental flexibility encompasses the possibility to switch when it is adequate to subordinate patterns which, *if* they were hegemonic, would not be flexible.

Uruk: A "mathematical state"

If the emergence of mathematics proper is understood as the coordination of "*pre-existent but previously independent* mathematical practices [...] through a minimum of at least intuitive understanding of formal relations", there is hence no doubt that mathematics had started its career, if not before, then at least in late preliterate or proto-literate Uruk (and Susa) – nor that it were primarily the needs of the administration of the new social system that asked for the creation or further unfolding of mathematics.

More interesting is perhaps the converse observation. The use of the mathematical tool was no instance of pure "technical rationality", the creation and implementation of means for an already established end which itself is not touched. If we compare the Uruk and subsequent Mesopotamian state formations with other early states, the end itself (the Mesopotamian state) can be seen to have been shaped by the means, no less than the successful appeal to military means may lead to the transformation of the state that appealed to it.[13]

A rash statement of this kind must evidently be explained. Redistributive systems are found in many pre-state societies; they correspond to the need for mutual support, and may thus be said to correspond to a notion of social justice. However, this notion of justice cannot easily be carried over to the proto-statal situation. In Robert Carneiro's words [22: 58], "what a chief

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12 This theme is further explored in my [20].

13 Keeping aloof from real politics we may think of Kästner's nightmarish poem about what would have happened to Germany after WW1 "Wenn wir den Krieg gewonnen hätten" [21: 102*f*].

gets from redistribution proper is esteem, not power"; further on (p. 61) Carneiro observes that

> As long as a chief merely returns everything he has been handed, he gains nothing in wealth or power. Only when he begins to keep a large part of it, sharing with his retainers and supporters but not beyond that, does his power begin to augment.
>
> But the power of a chief to appropriate and retain food does not flow automatically from his right to collect and redistribute it. Villagers freely allow a chief to equalize each family's share of meat or fish or crops through redistribution because they benefit from it. But they will not willingly suffer the same chief to keep the lion's share of food for himself. Before doing this, he must acquire additional power, and that power must come from some other source.

Since power only results when redistribution proper (where the chief retains only a small percentage of what passes through his hands) is transformed into *tribute* or *taxation*, where he keeps a large part for himself and for the "core of officials, warriors, henchmen, retainers, and the like who will be personally loyal to him and through whom he can issue orders and have them obeyed" [22: 61], neither the commoners nor the chief and his circle have any immediate reason to conceptualize the new situation in terms of social justice.

In the Susa-Uruk area, matters were probably perceived differently (at least by upper and middle strata), even though realities may have been similar. As shown by the use of *bullae* and by the accounting tablets, taxation and allocation of resources – be it the fields apportioned to high-ranking temple officials, be it the rations of grain distributed to workers – were made according to mathematically determined rules. In this way, statal power was structured around "just measure" and thus, apparently, legitimized by a transformed concept of social "justice". Since accounts and lexical lists constitute our only written sources, we have no direct evidence for how the situation was conceptualized at the time; but literary evidence from a time when lexical lists from the proto-literate period were still in use indicates that at least the higher literate stratum thought of statal power in such terms.

A striking contrast is offered by early Pharaonic Egypt, the "nearest neighbour" in terms of state formation. All evidence suggests that the Pharaonic state was legitimized by conquest, and (at least in the view of the literate) by a religious guarantee of cosmic order. Already during the First Dynasty, it is true, the yearly level of the Nile was recorded, in all probability in order to allow calculation of the taxation level of the year to come, and a biennial "counting of the riches of the land" was introduced.[14] But a biennial counting certainly does not allow any specific determination of dues and rights, nor is there any evidence that the measured Nile height served such purposes. Social "justice" has no place in the picture of early Pharaonic Egypt.

"Real justice"

"Real socialism" did not coincide too well with what had been proclaimed in programmes, and the real feudalism of the Middle Ages was conspicuously different both from Charlemagne's blueprint and from Fulbert of Chartres' theory of the respective roles of the praying, the warring and the labouring order. Likewise, mathematical social "jubstice" (however much unequal) was certainly not the whole truth about the Uruk state. But it remains an essential part of the truth, and it conditioned Mesopotamian statal structures at least until the mid-second millennium BCE.

That it was only *part* of the truth, belonging rather on the level of hegemonic ideology than on that of social realities, can be seen from the preferred motif of the seals of high officials (found on no less than half of all known early Uruk seals; two specimens are reproduced in [9: 16]): A high official or priest looking on while overseers beat up pinioned prisoners. It is probable that the vehement increase in population did not result from local breeding alone but also from enslavement of significant populations from the mountain areas to the east – the pictograms for male and female slaves are indeed composed of an indication of sex (of a person) with a picture of the mountains[15] – and that this was brought about by the same climatic change as had made possible the irrigation revolution in the lowlands.

Such a hypothesis is supported by linguistics: many features of Sumerian look like those of languages that over some centuries have developed from

[14] Nile observations as well as countings are documented on the Palermo Stone – translated, e.g., in [23: 67–95].

[15] No. 50 and 558, respectively, in Labat's sign list [10].

pidgins and creoles.[16] Enslaved workers are likely to have had different languages – also in later times, many languages are found in the region. Like the slaves in the West Indian plantations (who were in the same linguistic situation), they can therefore be supposed to have created a pidgin (based largely on the vocabulary of the masters' language but losing its grammar) which the next generation transformed into a creole.[17] In the absence of a metropole conserving their original language, new generations of masters influenced as children by lower-class nurses and servants will also have adopted the creole over some generations (probably without perceiving the shift as a change of language) – the final outcome (after centuries) being Sumerian.[18]

The Early Dynastic and Sargonic periods

The proto-literate period may have lasted from c. 3200 BCE to c. 2900 BCE (falling in two distinct sub-periods, "Uruk IV", 3200-3000, and "Uruk III" or "Jemdet Nasr", 3000-2900). It was followed in the Sumerian area (now without doubt Sumerian) by the "Early Dynastic Phase" (subdivided into ED I, ED II and ED III), c. 2900-2750-2600-2350 BCE.

In this phase, what appears to have been a social system with one major centre (Uruk) changed (collapsed?) into one consisting of competing city states; and what looks like a state centred around a staff of high temple officials developed into states rules by a king (though still heavily influenced by the temple institution).

From ED I we have no written sources, and from ED II very few. In ED III, their number proliferates; the continued use of the old lexical lists demonstrates continuity not only of the writing system but also of the school tradition. In the 26th century, however, a new phenomenon can be observed. Writing was now in wider use, serving also, e.g., for the stipulation of private contracts; at the same time, and in consequence, the circle of the

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16 This theme is explored in depth in my [24].

17 For this process, see for instance [25] or [26].

18 Since the proto-literate script was ideographic and indicated neither grammar not the word order of full sentences, we have no means to identify the language spoken by its inventors. One or two cases of possible use of homophones corresponding to later Sumerian ("rebus principle" writing) decide nothing, since a pidgin and the creole it engenders borrow most of their vocabulary from the language of the masters – cf. the derivation of the language name *Tok pisin* from (the pronunciation of) *talk pidgin*.

literate became broader; in John Baines' terms [27], a transition from "very restricted" to "restricted literacy" took place. The group of scribes (dub.sar) turns up for the first time as a distinct *profession* in the city-state Shuruppak [28: 4, 12-23].

Also at the same time, and in all probability as a further consequence of this, the script was put to new uses. We find the first literary texts – a proverb collection and a hymn – and the first instances of "supra-utilitarian" mathematical school problems (problems that are not directly connected to practice even though they are formulated as if they were). In contrast, all mathematical texts from the proto-literate period that can be identified as school texts are "model documents", distinguishable from real administrative texts only by the absence of an office seal and by the occurrence of numbers that are suspiciously round or nice and at times suspiciously large.

Literary texts as well as supra-utilitarian mathematics were probably meant to probe and make manifest the reach of the two professional tools – writing and computation – and thus as expressions of professional pride. This agrees well with the appearance of many of the so-called "school-texts" from Shuruppak (edition in [29]): empty corners may be filled out by nice drawings, and according to the judgement of Aage Westenholz (personal communication) the tablets may indeed be *de luxe* versions made for mature scribes looking back at the real or imagined pleasure of their school time, emblem of their present professional identity and social position.[19]

Rising city walls show clearly that warfare was an endemic condition of the ED-period, and that the king was a military leader; Shuruppak itself was completely devastated in a military attack, following upon a general mobilization [30: 144*f*]. The many killed servants that followed their master to the underworld in the Royal Cemetery of Ur (initial ED III) also demonstrate that the king had given up any idea of being the servant of society – he was its overlord, and society a means for his greatness. None the less, only the very end of the ED period gives us written evidence, if not of the ritual slaughter of servants then at least of military activities; until then, even royal inscriptions show the king solely as the benefactor of temples

[19] Cf. Giuseppe Visicato's work on third-millennium scribes [30].

and provider of agricultural prosperity (in strong contrast to early Pharaonic documents). Literacy, so it appears, only reflects the functional and pseudo-just characteristics of the state; those features of the state which had been irrelevant for the invention of writing and bookkeeping remained outside the perspective of writing. In this respect, ED Sumer was a *dual society*, one of whose faces was still “mathematical”.

From c. 2350 to c. 2200 BCE, the Sumerian area (and soon the whole of Mesopotamia and even more) was united into a single territorial state; after an initial short-lived centralization around a Sumerian city-king, the centre was the Akkadian “Sargonic” state[20] (Akkadian is a Semitic language, of which the later Babylonian and Assyrian languages are dialects, Sargon the founder of the dynasty; the school language remained Sumerian).

“Literature”, at first apparently a free creation of the scribe school and a means for scribes to probe and demonstrate their professional identity, was soon taken over by the Sargonic rulers as propaganda (hymns being written so as to serve the new dynasty [31: 186]). While mathematical administration certainly expanded [32], the utilization of supra-utilitarian problems in mathematics teaching was continued; there is no reason to presume that they fulfilled, or could fulfil, any role outside the school.

Already during the ED phase (documented in ED III) but accelerating during the Sargonic period, metrologies were adjusted with concern for mathematical regularity as well as administrative convenience. The former concern (mathematical regularity) is especially visible in the weight system, apparently a fresh development of the ED phase, where the step factor 60 was given a prominent position (only one factor had to be 3×60 in order to accommodate the “natural” measure of a barleycorn). But other metrologies too were extended upwards and downwards with this step factor.

The concern for administrative convenience, at times but not always in conflict with the former, asked for the adaptation to administrative procedures or technical practice, for instance in the definition of a Sargonic “royal” gur (“tun” – the largest capacity unit) and in the creation of particular brick metrologies geared to the various standard bricks; cf. [28: 5] [33].

[20] Given the travelling times and the plurality of languages it is even justified to speak of an “empire”, as indeed often done.

All in all, the relation between the state and its mathematics seems to have developed during the later ED and the Sargonic period along lines known from other societies provided with an accounting or otherwise mathematically organized administration: mathematics was taught in a way which was needed by future staff, but it was also allowed a certain autonomy in the school. It was certainly not taught by "mathematicians" – but even when teachers are supposed to teach for practice, teaching will normally be affected by the fact that the practice which teachers are *really* familiar with is the practice of teaching. Thus also here, according to the meagre evidence at our disposition.

The Janus-faced innovations in metrology correspond to this tension in the situation of mathematics: sexagesimalization is likely to have been driven by a preference for intra-mathematical coherence, the other innovations by the links to extra-mathematical practice, in particular to the administrative procedures of the state.

The Neo-Sumerian state

Around 2200, the Akkadian territorial state or "empire" lost most – in the end all – of its territory, and smaller states reemerged, of which only Gudea's Lagash (2141-2122 BCE) has left sources that might be considered relevant for our topic – inscriptions telling in meticulous accounting what he has given to the temple, and how he laid out the geometric plan for sacred buildings (texts with translation in [34: 69-101], see in particular pp. 72-82). From 2112 BCE onward, however, the Third Dynasty of Ur established a new "Neo-Sumerian" territorial state or empire, mostly referred to as "Ur III".

The early decades of this dynasty present us with nothing spectacular. In 2074 BCE, however, king Šulgi undertook a military reform, which was immediately followed by an administrative reform. From this point onward and until the collapse of the empire, scores and scores of thousands of accounting tablets inform us about the details of the administration (and, indirectly, about its governing principles).

At least in the Sumerian South, the larger part if not the overwhelming majority of the working population in both agriculture and handicraft production seems to have been submitted to conditions close to those of slavery,[21] working in crews under scribal overseers who were responsible for the work performed, reckoned in units corresponding to $^1/_{60}$ of a working day (i.e., 12 minutes).

The accounts of the overseers are extremely meticulous, converting all outputs into a common unit,[22] taking illness, death and absence as well as workers lent to or borrowed from other overseers into account. The old administrative calendar was still in use – Ur III is the epoch in which sources show that the overseer scribes were to press out of their crew 30 days' work each month irrespective of its actual length. As shown by Robert Englund [35: 46*f* and *passim*], the yearly deficits of an overseer scribe were accumulated, and at his death the family was held responsible for it (if needed by being drawn into the enslaved crews) – at least in private discussion, Englund would speak of the system as a *Kapo* economy.

For use in this immensely expanded accounting, two decisive mathematical innovations appear to have been introduced.

One is the accounting system itself, with built-in automatic controls (in this respect an analogue of what was brought about in the later Middle Ages by the introduction of double-entry bookkeeping). This was taken over in the subsequent "Old Babylonian" period, during which it was also used for private large-scale accounting – after which it was forgotten.

The other was the sexagesimal place-value system. This was a floating-point system, serving equally well for integers and for fractions. It was used for intermediate calculations, of which relatively few traces remain; in mathematical school texts, where orders of magnitude could be presupposed, could be remembered, or were immaterial; and in the late astronomical tables, where the tabular format helped to determine orders of magnitude.

Neither school texts nor astronomical tables can have been the original purpose for which the system was introduced – the latter already for chronological reasons. Nor did it ease additive and subtractive computations

[21] A survey of the debate about how to interpret the sources on this account is given by Robert Englund [35: 63–68]. The system appears to have been established during what was originally a state of emergency declared at the same occasion as the military reform and which was soon made permanent [35: 57].

[22] Often weight of silver, but barley was another possibility – see [35: 18–20].

(which anyhow appear to have been performed on some abacus-like device [36] [37]). What it did facilitate was multiplication and division – but only if multiplication tables and tables of reciprocal numbers were available or learned by heart, along with tables permitting the translation of metrological units into sexagesimal multiples of a standard unit.[23] The production and teaching of such tables, on the other hand, had no point before the place-value system was in use.

This observation leads to a striking conclusion: The important step was not the *invention* of the new notation – which, by the way, was in the air since centuries, as shown by Marvin Powell [33], and may even have been invented well before Ur III without leaving any traces in tablets that happen to have survived and to have been read by Assyriologists. What was decisive will have been a *political decision to implement it* – a decision which could only be effectual in a centralized system like Ur III.

We have no direct evidence for the taking of such a decision nor for where it was taken;[24] but we may safely assume that the planning was made in a scribe school environment that was closely connected to the royal administration. Similarly, F. R. Kraus [39: 24-27] concludes that official year names, royal inscriptions and royal hymns were produced in the subsequent Old Babylonian period (see presently) in an institution which at one and the same time served as "palace school" and as "court chancery", and that this institution went back to some similar Ur III institution.

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23 I borrow the following explanatory example from [38: 18], adapting it slightly:
If a platform had to be built to a certain height and covered by bricks and bitumen, a "metrological table" could be used to transform the different units of length into sexagesimal multiples of the nindan and kùš ("cubit", $^{1}/_{12}$ of a nindan), allowing the determination of the surface and the volume in the basic units sar [square nindan] and [volume] sar [an area sar provided with a height of one kùš]. A list of "constant coefficients" (igi.gub) would give the amount of earth carried by a worker in a day over a particular distance, the number of bricks to an area or volume unit, and the volume of bitumen needed per area unit – all expressed in basic units (if no transformation into basic units had taken place, different coefficients for the bitumen would have had to be used for small platforms whose dimensions were measured in kùš and for large ones measured in nindan). With these values at hand the number of bricks and the amount of bitumen as well as the number of man-days required for the construction could be found by means of sexagesimal multiplications and divisions – once again facilitated by recourse to tables, this time tables of multiplication and of reciprocal values. Finally, renewed use of metrological tables would allow the calculator to translate the results of the calculations into the units used in technical practice.

24 Until recently, direct evidence for use of the notation during Ur III was itself extremely scarce (and not fully compelling), in particular because of the uncertainty of palaeographic dating of tablets containing only numbers (that is, of mathematical tables and scratch pads for computation). A few years ago, however, Eleanor Robson [personal communication] discovered tables of reciprocals found in dated contexts.

That king Šulgi himself (or at least those who produced propaganda in his name) saw the school as an essential tool for his project is obvious from one of the so-called Šulgi hymns,[25] according to which the king was taught from an early age in the "tablet-house", learning the art of writing together with addition, subtraction, counting and accounting under the protection of the scribal goddess Nisaba; later we hear that his praise is song in the same tablet house.

Considering the marvellous feats of which Šulgi boasts elsewhere in this and other hymns we may wonder at the level of his mathematical curriculum, far below the actual level of mathematical competence of which the texts of the Old Babylonian age bear witness – even multiplication goes unmentioned, at most it may perhaps be presupposed as an auxiliary technique in accounting (but why then mention addition?). Actually, however, this fits what can be derived from the absence of all mathematical school texts apart from model documents, in particular when viewed in the light of evidence offered by the terminology of the Old Babylonian period. It appears that *problems*, well represented in the (meagre) corpus of mathematical texts surviving from ED III and the Sargonic period, were banished from the Ur III school: it looks as if even the modicum of independent thought needed when students have to find and not just follow a prescribed way was considered a threat to their docility.[26]

If any ruler ever *was* the state, the deified Šulgi was. The various Šulgi hymns and the prologue of the law-code he produced[27] are therefore informative about the official ideology of the state. Šulgi is not only a potent military leader and pitiless avenger of wrongs (which, conveniently, permits him to provide slaves) but also a "good shepherd" and exceedingly just (dual society, passed away in late ED III, had not been resurrected). However, only one feature of his "social justice"[28] goes beyond verbatim repetition of the trite commonplaces of the preceding centuries (protection

[25] Hymn B, l. 13–19 [40: 31*f*]. Castellino's translation and commentary miss the mathematical points.

[26] The full argument for this in unfolded in [41].

[27] At first ascribed by Assyriologists to his father Ur-Nammu and hence known as the Ur-Nammu laws. The law-code is published with translation in [42: 14–21], hymns B and C in [40], hymns A, D and X in [43].

[28] Social justice should be distinguished from "judicial justice", punishing enforcement of the laws which follow after the prologue.

of orphans from wealthy and widows from mighty men), and only one thus rings true: metrological reform.

All in all, Ur III enhances features which already appeared to characterize proto-literate Uruk: the management of the state was meticulously planned and controlled. This meticulous planning and control had several effects:

– In mathematics, important innovations were introduced – one of them still important for us, given that the sexagesimal place-value system may possibly have provided part of the inspiration for the Indian invention of the decimal place-value system and was certainly the direct inspiration for the introduction of decimal fractions. Free supra-utilitarian developments, on the other hand, appear to have been blocked.
– Socially and ideologically, the fact that the extremely oppressive policies of the system were metered out according to mathematical rules permitted that these policies could be seen by those in power – and probably even by the overseer scribes – as embodiments of *justice*.

The undernourished workers, however, fell ill or ran away the best they could – even this can be read from the accounting texts; after all, they had not been brought up in the scribal school and may have had other opinions about social justice if at all caring about such questions.[29] This is likely to be one of the reasons that the Ur III state did not outlast the third millennium. All in all, this early instance of immoderate Taylorism seems to have provoked a reaction similar to what British trade union activist of the twentieth century CE responded to the "scientific management" of their own days: "time and motion studies means that motion stops and time is wasted".

[29] It appears that some of them did. An Old Babylonian epic which seems to reflect Ur III experience and not Old Babylonian conditions (*Atra-hasīs*, ed., trans. [44]) transposes a strike into the realm of the gods. After the creation of the world, An takes possession of the heavens, Enlil of the earth, and Enkidu of the waters below the earth – and the minor gods are put to work, digging Euphrates and Tigris. After toiling for forty years they revolt, set fire to their spades and prepare an attack on Enlil's abode. So much in the account reflects the psychology of real wildcat strikes (Enlil asking who is the instigator, the mutinees answering that everyone is the instigator) that we may safely assume that the story builds on historical experience.

The Old Babylonian period and the culmination of Mesopotamian mathematics

Already around 2025, the periphery rebelled, and the Ur III state lost its character of an empire. A few decades later, even the centre dissolved into small states. Gradually, some of these absorbed the others, and in the eighteenth century BCE Hammurapi of Babylon managed to subdue the whole Mesopotamian south and centre. From then on, this region can be spoken of as "Babylonia"; the centuries from 2000 BCE to 1600 BCE are known as the "Old Babylonian period"; it produced the most sophisticated mathematics we find in ancient Mesopotamia.

This culmination arrived when the mathematical Taylorism of Ur III had disappeared. The period is characterized by individualism, both in the economic structure (even though it would be a mistake to speak of a general market economy) and on the level of ideology or culture [45]. Land, even when owned by the Crown, was often rented under contract. Private correspondence turns up (a large number of letters are published with translation in [46]). The letters were often written by free lance scribes (a category we do not know from Ur III). The Ur III accounting system was now used in private business, handled by privately employed scribes. The seal, so far a symbol of office, now belonged to the individual. We may speak of the rise of an ideology of *personal identity*.

This ideology also affected scribal culture, in a way which is reflected in the texts used in school to inculcate understanding of what should characterize a *real* scribe (the so-called "examination texts" – cf. [47] [48] [49]).

The Sumerian language was dead by now, and Babylonian could be written adequately with a phonetic syllabary of 70 signs or less; a *true* scribe, however, would also use a large number of word signs, borrowed from the Sumerian script but now meant to be pronounced in Babylonian. This, however, was not a sufficient demonstration that the scribe was somebody special. He should also be able to read, write *and speak* Sumerian – a feat only other scribes would be able to appreciate.[30] He should know every-

[30] A real feat: Sumerian and Babylonian are as different as, for instance, Basque and Spanish. Quite apart from the vocabulary, the scribes should thus understand a grammar based on principles totally different from those of their mother tongue. Without the lexical lists and explanations prepared by the Babylonian masters for their students, nineteenth-century scholars would have been unable to decipher the Sumerian texts.

thing about bilingual texts, he should be familiar with all the significations of the cuneiform signs (each single sign would have one or several phonetic and one or several logographic meanings – to which comes further occult meanings which we do not understand). He should know about music, and about mathematics. He whole complex was called “humanism” (true! – namely nam-lú-ulu, Sumerian for “the condition of being human”). Quite adequately, lú corresponding to Latin *vir*, another literal but still adequate translation would be “virtuosity”.

The texts from which we know this do not specify *which* kind of mathematics would count as “humanist”. Training tablets which carry a Sumerian proverb on the obverse often have quite simple calculations on the reverse. Elementary mathematics was thus taught at a rather advanced level, and most scribes presumably never went further. On the other hand, however, very sophisticated supra-utilitarian mathematics was also produced, and it is a fair guess that this (as useless as spoken Sumerian) was the really “humanist” level of mathematics.

What we find together with Sumerian proverbs are simple numerical multiplications, area determinations and such things. Before that, future scribes copied metrological tables and tables of reciprocals and multiplication – probably so often that they learned them by heart. All of this was useful training for future professional practice, and hence not supra-utilitarian.

At the sophisticated, supra-utilitarian level we still find numerical problems – for instance, an intricate technique for finding reciprocals of numbers not listed in the standard table nor easily derived from it by successive halving and doubling. The favourite genre, however, was what has been interpreted as “algebra” of the second (at times the third) degree. Nominally, these “algebraic” problems deal with areas of rectangles or volumes of excavations and their sides, at times combined for instance with the wage to be paid for the excavation; the substance of the problems, however, is entirely artificial, and the “algebraic” technique that is taught is completely useless for professional practice. This “algebra” is thus truly supra-utilitarian.[31]

[31] Not only the terminology but also the technique of the “algebra” in question is geometric – see my [38].

Its inspiration had probably come from a riddle tradition carried by "lay", that is, non-scribal (whence fully or almost illiterate), Akkadian-speaking surveyors [50]. These riddles (as they can be reconstructed from consideration also of their appearance in much later surveying texts) were of this kind:

> "I have added together the side of a square and its area, and the outcome was 110".
> "I have added together the four sides of a square and its area, and the outcome was 140".
> "I have added together the length and the width of a rectangle, and the outcome was 14, while its area is 48".
> "the diagonal of a rectangle is 10, and its area is 48".
> "I have added together the perimeter, the diameter, and the area of a circle, and the outcome was 115".

Others probably concerned differences between square area and one or all four sides, the sum of or difference between areas and sides of two squares. The total number of the riddles will not have exceeded ten to fifteen.

As mentioned above, mathematical *problems*, and *a fortiori* supra-utilitarian problems, appear to have been totally absent from the Ur III school. As the Old Babylonian scribe school developed, its "humanist" pretensions appear to have induced it to adopt these riddles; in the context of the school, however, a handful of standard riddles could not do: the riddles became the starting point for a genuine mathematical discipline, with rich variation and exploration of the possibilities offered by the technique – for instance, letting the sides of a rectangle represent a number or a price, or even a square area or the volume of a cube (the latter in a problem of the eighth degree, resolvable as a bi-biquadratic). Rich variation had the added advantage of allowing copious training of sexagesimal arithmetic.

Beyond their "humanism", scribes were (supposed to be) proud, if not of being leading officials of the state (few of them of course were), then of belonging to a group from which leading officials came. This state was still supposed to represent social justice, and serving it could hence be a reason for pride. That can be seen in one of the texts used to form the self-image of scribes in the Old Babylonian school, known as "Lipit-štar, King of

Justice, Wisdom and learning".[32] The king was taught the scribal art by Nisaba, the goddess of scribal wisdom – consisting, the text reveals, in writing and use of "the measuring rod, the gleaming surveyor's line", and she bestowed upon him "the cubit ruler which gives wisdom". The praise goes on

> [...] you are Enlil's son;
> Truth and justice you make manifest;
> Lord, your goodness covers even the horizon.
> King Lipit-eštar, councellor of great judgment,
> (Whose) word never falters, wise one (whose) decision provides justice for the people;
> Great mind, knowing all things deeply,
> In order to lay down the law for all foreign countries [...
> ...] you rage against the enemies,
> From evil and oppression you know how to save people
> From sin and destruction you know how to free them.
> The mighty do not perpetrate robbery,
> And the strong do not make the weaker ones into hirelings –
> Thus you established justice in Sumer and Akkad.

The mathematical scribal arts and justice are neighbours, as we see, but the only link beyond this vicinity is indirect, the common reference to generic wisdom.

One step further, the statal social justice of which Hammurapi proclaims himself the supreme protector in the introduction of his famous "law-code"[33] is not mathematical at all but a continuation of commonplaces going back to the outgoing Early Dynastic epoch (Hammurapi is still the protector of orphans and widows); beyond that his justice is judicial (some of his legal decisions, however, concern metrology and punish metrological fraud). One of his successors also issued a decree "re-establishing justice to the country", prescribing a debt cancellation [53] [54: 151-153], reminiscent of the Old Testament jubilee (Leviticus 25:11-15) but apparently a once-only measure meant to palliate the threat to general econom-

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32 Lipit-eštar had been king of Isin, one of the smaller states emerging from the collapse of Ur III. The text was published with translation by H. L. J. Vanstiphout [51].

33 Text in [42: 76–140]. Actually, the text represents itself not as a law-code but as Hammurapi's (presumable paradigmatic) judicial verdicts, cf. [52: 228*f*].

ical stability resulting from a debt crisis and crushing interest rates – in any case a cancellation of the very idea of that "mathematical justice" where everyone receives and contributes his exactly calculated due (indeed the kind of "justice" which had led to the crisis).

Accounting, as mentioned, was still around, but even when done for the state its role was that of a subservient tool. The relation between the state and mathematics had become accidental, not constitutive for either part. Mathematical "humanism" should probably be understood as an *alternative* legitimation rather than as a continuation of the ancient pattern.

Disappearance of a pattern

The final dissolution of the pattern state—social justice—accounting mathematics arrived with the collapse of the Old Babylonian state. After a Hittite conquest of Babylon and ensuing social chaos, power was taken by the Kassite tribes, already present in Babylonia as mercenary soldiers. The ratio between town and countryside dwellers fell to fifth-millennium levels, and the role of scribal administration and culture – always the carriers of ideas of the just state – was not only strongly reduced but also appears (to the extent the extremely meagre written evidence from the period allows us to distinguish) to have lost its ideological hegemony. As writing once again became copious in the late second and the first millennium with the expansion of the Assyrian city-state into a territorial state and finally an empire, we find scribes in central positions at court or somehow working for the court – but now as producers of an ideology emphasizing the king and the empire as creators and upholders of order [55] [56] [57] [58], and as omen priests and astrologers protecting the king [57] [59];[34] the huge libraries of the Assyrian royal palaces are also evidence of the activity of learned court librarians and copyists. These

[34] The letters from these scholar-scribes collected in [60], apart from giving technical advice, mostly wish the king good health and vigour. One exorcist needing to flatter Assurbanipal – who, in contrast to his predecessors, was pleased to take up themes from earlier epochs – praises him for having brought prosperity to the land (Assurbanipal does boasts of that himself, as Hammurapi had done 1100 years before) and for distributing particular favours; the exorcist also states [60: 91] that "the King, my lord, has revived the one who was guilty (and) condemned to death; you have released the one who was imprisoned for many years" (metaphorically, no actual event is meant). Even when trying to appear in the light of age-old traditions, the Assyrian king could only taint Iron-Age despotism with commonplaces of (mainly judicial) justice in Old Babylonian style.

scribes working for or corresponding with the court were certainly also proud of their professional status, but even those of them who may have worked on incipient mathematical astronomy identified themselves as “writers” of omen series, exorcists etc.; mathematics was peripheral to their professional self-esteem.[35] Ordinary daily administration was probably taken care of in Aramaic alphabetic writing, and not in cuneiform on clay tablets, for which reason the evidence has disappeared together with traces of the clerks who took care of it.[36]

To sum up: During the late fourth and the third millennium, “writing” was in power; but “writing” in this respect was first of all accounting and management of resources, somehow connected to the pre-historic redistributive structures. However, during ED III we find the first evidence of literary writing and supra-utilitarian mathematics as evidence of professional self-esteem of scribes, and soon afterwards the use of literature as state propaganda.

During the Old Babylonian period, the role of professional self-esteem becomes much more conspicuous in scribal culture; concomitantly, the legitimization of the state, though still referring to “justice”, is decoupled from accounting.

After the Kassite interlude (the “Babylonian Middle Ages”), “justice” however meant does not characterize the role of the state; activities of importance for professional self-esteem of cuneiform scribes were predominantly literary, divinatory and theurgical.

Bibliography

[1] Fried, Morton, *The Evolution of Political Society. An Essay in Political Anthropology*. New York: Random House, 1967.

[2] Service, Elman R., *Origins of the State and Civilization. The Process of Cultural Evolution*. New York: W. W. Norton, 1975.

[35] This is no less true in the Seleucid era (third and second century BCE), the epoch where mathematical astronomy attained maturity.

[36] Contracts on clay tablets are revealing in this respect. Belonging to a legal genre, they were mostly written in cuneiform Assyrian; but often they carried a resume of some lines in Aramaic – see the specimens in [61].

Some of the contracts, though legal stuff, are in Aramaic, and carry no resume in Assyrian. Contracts on parchment or papyrus, if they existed, will also have been in Aramaic only, the support not being suitable for cuneiform.

[3] Wright, Henry T., & Gregory A. Johnson, “Population, Exchange, and Early State Formation in Southwestern Iran”. *American Anthropologist* **77** (1975), 267-289.

[4] Schmandt-Besserat, Denise, “An Archaic Recording System and the Origin of Writing”. *Syro-Mesopotamian Studies* **1**:2 (1977).

[5] Schmandt-Besserat, Denise, *Before Writing*. I. *From Counting to Cuneiform*. II. *A Catalog of Near Eastern Tokens*. Austin: University of Texas Press, 1992.

[6] Schmandt-Besserat, Denise, “Tokens as Funerary Offerings”. *Vicino Oriente* **7** (1988), 3-9, Tav. I-V.

[7] Nissen, Hans J., *The Early History of the Ancient Near East*. Chicago & London: University of Chicago Press, 1988.

[8] Liverani, Mario, *Antico Oriente. Storia, società, economia*. Roma & Bari: Laterza, 1988.

[9] Nissen, Hans J., Peter Damerow & Robert Englund, *Archaic Bookkeeping: Writing and Techniques of Economic Administration in the Ancient Near East*. Chicago: Chicago University Press, 1993.

[10] Labat, René, *Manuel d’épigraphie akkadienne (signes, syllabaire, idéogrammes)*. 4^{e} édition. Paris: Imprimerie Nationale, 1963. ^{1}1948.

[11] Baines, John, “The Earliest Egyuptian Writing: Development, Context, Purpose”. *Max-Planck-Institut für Wissenschaftsgeschichte. Preprint* 180 (2001).

[12] Damerow, Peter, & Robert K. Englund, “Die Zahlzeichensysteme der Archaischen Texte aus Uruk”, Kapitel 3 (pp. 117-166) *in* M. W. Green & Hans J. Nissen, *Zeichenliste der Archaischen Texte aus Uruk*, Band II (ATU 2). Berlin: Gebr. Mann, 1987.

[13] Powell, Marvin A., Jr., “Sumerian Area Measures and the Alleged Decimal Substratum”. *Zeitschrift für Assyriologie und Vorderasiatische Archäologie* **62** (1972-73), 165-221.

[14] Aurenche, Olivier, *La maison orientale: L’Architecture du Proche Orient ancien des origines au milieu du quatrième millénaire*. Paris: Paul Geuthner, 1981.

[15] Beale, T. W., & S. M. Carter, “On the Track of the Yahya Large Kuš: Evidence for Architectural Planning in the Period IVC Complex at Tepe Yahya”. *Paléorient* **9**:1 (1983), 81-88.

[16] Englund, Robert K., “Administrative Timekeeping in Ancient Mesopotamia”. *Journal of the Economic and Social History of the Orient* **31** (1988), 121-185.

[17] Luria, Aleksandr R., *Cognitive Development. Its Cultural and Social Foundations.* Edited by Michael Cole. Cambridge, Mass., & London: Harvard University Press, 1976. 1st ed. Moskva: Nauka, 1974.

[18] Larsen, Mogens Trolle, “The Mesopotamian Lukewarm Mind. Reflections on Science, Divination and Literacy”, pp. 203-225 *in* F. Rochberg-Halton (ed.), *Language, Literature, and History: Philological and Historical Studies presented to Erica Reiner*. (American Oriental Series, vol. 67). New Haven, Connecticutt: American Oriental Society, 1987.

[19] Høyrup, Jens, “Algebra and Naive Geometry. An Investigation of Some Basic Aspects of Old Babylonian Mathematical Thought”. *Altorientalische Forschungen* **17** (1990), 27-69, 262-354.

[20] Høyrup, Jens, "Embedding: Multi-purpose Device for Understanding Mathematics and Its Development, or Empty Generalization?" *Filosofi og Videnskabsteori på Roskilde Universitetscenter.* 3. Række: *Preprints og Reprints* 2000 Nr. 8.

[21] Kästner, Erich, *Bei Durchsicht meiner Bücher*. Zürich: Atrium Verlag, 1946.

[22] Carneiro, Robert L., "The Chiefdom: Precursor of the State", pp. 37-79 *in* G. D. Jones & R. R. Kautz 1981.

[23] Clagett, Marshall, *Ancient Egyptian Science. A Source Book.* Volume I: *Knowledge and Order*. 2 tomes. (Memoirs of the American Philosophical Society, 184 A+B). Philadelphia: American Philosophical Society, 1989.

[24] Høyrup, Jens, "Sumerian: The Descendant of a Proto-Historical Creole? An Alternative Approach to the 'Sumerian Problem'". ΑΙΩΝ. *Annali del Dipartimento di Studi del Mondo Classico e del Mediterraneo Antico. Sezione linguistica. Istituto Universitario Orientale, Napoli* **14** (1992; publ. 1994), 21-72, Figs. 1-3.

[25] Mühlhäusler, Peter, *Pidgin and Creole Linguistics*. (Language in Society, 11). Oxford & New York: Blackwell.

[26] Romaine, Suzanne, *Pidgin and Creole Languages*. London & New York: Longman, 1988.

[27] Baines, John, "Literacy, Social Organization, and the Archaeological Record: The Case of Early Egypt", *in* Gledhill, Bender & Larsen (eds) 1988: 192-214.

[28] Visicato, Giuseppe, *The Power and the Writing. The Early Scribes of Mesopotamia*. Bethesda, Maryland: CDL Press, 2000.

[29] Deimel, Anton, *Die Inschriften von Fara*. II, *Schultexte aus Fara*, in Umschrift herausgegeben und bearbeitet (Wissenschaftliche Veröffentlichungen der Deutschen Orient-Gesellschaft, 43). Leipzig: J.C. Hinrichs'sche Buchhandlung, 1923.

[30] Visicato, Giuseppe, *The Bureaucracy of Šuruppak. Administrative Centres, Central Offices, Intermediate Structures and Hierarchies in the Economic Documentation of Fara*. (Abhandlungen zur Literatur Alt-Syrien-Palästinas und Mesopotamien, 10). Münster: Ugarit-Verlag, 1995.

[31] Hallo, William W., "Toward a History of Sumerian Literature", *in Sumerological Studies ...* 1976: 181-203.

[32] Foster, Benjamin R., "Archives and Record-Keeping in Sargonic Mesopotamia". *Zeitschrift für Assyriologie und Vorderasiatische Archäologie* **72** (1982), 1-27.

[33] Powell, Marvin A., "The Antecedents of Old Babylonian Place Notation and the Early History of Babylonian Mathematics". *Historia Mathematica* **3** (1976), 417-439.

[34] Edzard, Dietz Otto, *Gudea and His Dynasty*. Toronto: Toronto University Press, 1997.

[35] Englund, Robert, *Organisation und Verwaltung der Ur III-Fischerei*. (Berliner Beiträge zum Vorderen Orient, 10). Berlin: Dietrich Reimer, 1990.

[36] Høyrup, Jens, "A Note on Old Babylonian Computational Techniques". *Historia Mathematica* **29** (2002), 193-198.

[37] Proust, Christine, "La multiplication babylonienne: la part non écrite du calcul". *Revue d'Histoire des Mathématiques* **6** (2000), 293-303.

[38] Høyrup, Jens, *Lengths, Widths, Surfaces: A Portrait of Old Babylonian Algebra and Its Kin*. (Studies and Sources in the History of Mathematics and Physical Sciences). New York: Springer, 2002.

[39] Kraus, F. R., *Vom mesopotamischen Menschen der altbabylonischen Zeit und seiner Welt.* (Mededelingen der Koninklijke Nederlandse Akademie van Wetenschappen, Afd. Letterkunde. Nieuwe Reeks – Deel 36 – No. 6). Amsterdam & London: North-Holland, 1973.

[40] Castellino, G. R., *Two Šulgi Hymns (BC)*. (Studi semitici, 42). Roma: Istituto di studi del Vicino Oriente, 1972.

[41] Høyrup, Jens, "How to Educate a Kapo, or, Reflections on the Absence of a Culture of Mathematical Problems in Ur III", pp. 121-145 *in* John M. Steele & Annette Imhausen (eds), *Under One Sky. Astronomy and Mathematics in the Ancient Near East*. (Alter Orient und Altes Testament, 297). Münster: Ugarit-Verlag, 2002.

[42] Roth, Martha, *Law Collections from Mesopotamia and Asia Minor*. Atlanta, Giorgia: Scholars Press, 1995.

[43] Klein, Jacob, *Three Šulgi Hymns. Sumerian Hymns Glorifying King Šulgi of Ur*. Ramat-Gan, Israel: Bar-Ilan University Press, 1981.

[44] Lambert, W. G., & A. R. Millard, *Atra-hasīs: The Babylonian Story of the Flood*. Oxford: Oxford University Press, 1969.

[45] Klengel, Horst, "Zur Rolle der Persönlichkeit in der altbabylonischen Gesellschaft", pp. 109-117 *in Humanismus und Menschenbild im Orient und in der Antike*. Konferenzvorträge, Herausgegeben von der Sektion Orient und Altertumswissenschaften der Martin-Luther-Universität Halle-Wittenberg. (Martin-Luther-Universität Halle-Wittenberg, Wissenschaftliche Beiträge 1977/28 (I 2)). Halle (Saale), 1977.

[46] Kraus, F. R., *Altbabylonische Briefe in Umschrift und Übersetzung*. 10 vols. Leiden: Brill, 1964-1985.

[47] Sjöberg, Åke W., "In Praise of the Scribal Art". *Journal of Cuneiform Studies* **24** (1971-72), 126-129.

[48] Sjöberg, Åke W., "Der Examenstext A". *Zeitschrift für Assyriologie und vorderasiatische Archäologie* **64** (1975), 137-176.

[49] Sjöberg, Åke W., "The Old Babylonian Eduba", *in Sumerological Studies* ... 1976: 159-179.

[50] Høyrup, Jens, "On a Collection of Geometrical Riddles and Their Role in the Shaping of Four to Six \`Algebras'". *Science in Context* **14** (2001), 85-131.

[51] Vanstiphout, H. L. J., 1978. "Lipit-Eštar's Praise in the Edubba". *Journal of Cuneiform Studies* **30** (1978), 33-61.

[52] Renger, Johannes, "Hammurapi's Stele \`König der Gerechtigkeit´. Zur Frage von Recht und Gesetz in der altbabylonischen Zeit". *Die Welt des Orients* **8** (1975/76), 228-235.

[53] Bottéro, Jean, "Désordre économique et annulation des dettes en Mésopotamie à l'époque paléo-babylonienne". *Journal of the Economic and Social History of the Orient* **4** (1961), 113-164.

[54] Edzard, Dietz Otto, "'Soziale Reformen' im Zweistromland bis nach 1600 v. Chr.: Realität oder literarischer Topos?" *Acta Antiqua Academiae Scientiarum Hungaricae* **22** (1974), 145-156.

[55] Liverani, Mario, "The Ideology of the Assyrian Empire", pp. 297-317 *in* Mogens Trolle Larsen (ed.), *Power and Propaganda. A Symposium on Ancient Empires*. (Mesopotamia, 7). København: Akademisk Forlag, 1979.

[56] Garelli, Paul, "L'État et la légitimité royale sous l'empire assyrien" pp. 319-328 *in* Mogens Trolle Larsen (ed.), *Power and Propaganda. A Symposium on Ancient Empires*. (Mesopotamia, 7). København: Akademisk Forlag, 1979.

[57] Reade, Julian, "Ideology and Propaganda in Assyrian Art", pp. 329-343 *in* Mogens Trolle Larsen (ed.), *Power and Propaganda. A Symposium on Ancient Empires*. (Mesopotamia, 7). København: Akademisk Forlag, 1979.

[58] Parpola, Simo, "Sons of God – the ideology of Assyrian Kingship". *Archaeology Odissey Archives*, December 1999.

[59] Koch-Westenholz, Ulla, *Mesopotamian Astrology: an Introduction to Babylonian and Assyrian Celestial Divination*. (CNI Publications, 19). København: Carsten Niebuhr Institute of Near Eastern Studies/Museum Tusculanum, 1995.

[60] Parpola, Simo, *Letters from Assyrian Scholars to the Kings Essarheddon and Assurbanipal*. 2 vols. (Alter Orient und Altes Testament, 5/1-2) Neukirchen-Vluyn: Butzon & Bercher Kevelaer, 1970, 1971.

[61] Fales, Federico Mario, *Aramaic Epigraphs on Clay Tablets of the Neo-Assyrian Period*. (Studi Semitici, NS 2). Roma: Università degli Studi "La Sapienza", 1986.