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Mahāvīra's geometrical problems: traces of unknown links between Jaina and Mediterranean mathematics in the classical ages

Jens Høyrup

In various publications [Høyrup 1995; 1996; 2001] I have argued for the existence in (what Western Europe sees as) the Near East of a long-lived community of practical geometers – first of all surveyors – which was not or only marginally linked to the scribe school traditions, and which (with branchings) carried a stock of methods and problems from the late third millennium BCE at least into the early second millennium CE. The arguments for this conclusion constitute an intricate web, and I shall only repeat those of them which are of immediate importance for my present concern: the links between the geometrical section of Mahāvīra's *Gaṇita-sāra-saṅgraha* and the practical mathematics of the Mediterranean region in the classical ages.

Many of the mathematical methods of pre-Modern practical geometry are too generic to allow us to discriminate diffusion from independent creation – once area measures are based on length measures, for instance, there is only one reasonable way to determine the areas of rectangles, right triangles and right trapezia. Some formulae, it is true, are so complex and/or allow so many variations that agreement in detail appears to make accidental coincidence implausible, in particular if identical patterns turn up repeatedly in the same textual setting. The best evidence for transmission, however, is constituted by those mathematical riddles (often known as "recreational problems") that pre-Modern communities of mathematical practitioners used to define themselves cognitively and to demonstrate the professional valour of the members.

Communities which in pre-Modern times were linked only marginally or not at all to school institutions have evidently left no written evidence of their knowledge; the information we can gather about the tradition in question thus comes from comparative analysis of the written sources produced by the various literate traditions that borrowed from or were otherwise inspired by it.

The first of these is the Old Babylonian scribe school, whose mathematical texts were created between 1800 and 1600 BCE. Among the hundreds of quasi-algebraic problems of the second degree dealing with fields and their sides,[1]

[1] When speaking of these as "quasi-algebraic" I refer to two characteristics. Firstly, their technique is *analytic*, as analysis is defined by Viète, "the assumption of what is searched for as if it were given, and then from the consequences of this to arrive at the truly given" – *In artem analyticen isagoge*, Ch. I [ed. Hofmann 1970: 1]. Secondly, their steps may be mapped in symbolic algebra, even though the actual technique consisted of geometrical cut-and-paste procedures. Both attributes characterize the original surveyors' riddles no less than the school descendants. The school technique, however, was also

a small core can be identified as borrowings from a pre-existing non-school tradition.[2] Four of these treat of a single square with side s and area $\square(s)$ (here and in the following, $\square(s)$ stands for the square with side s, $\sqsubset\!\sqsupset(l,w)$ for the rectangle contained by the sides l and w; ${}_4s$ stands for "the four sides", Greek letters for given numbers):

$$s+\square(s) = \alpha$$
$$_4s+\square(s) = \beta$$
$$\square(s)-s = \gamma$$
$$s = \square(s)+\delta$$

Four others treat of two concentric squares (sides s_1 and s_2):

$$\square(s_1)+\square(s_2) = \alpha\ ,\ \ s_1\pm s_2 = \beta$$
$$\square(s_1)-\square(s_2) = \alpha\ ,\ \ s_1\pm s_2 = \beta$$

Further problems deal with a rectangle with sides l and w, area A and diagonal d:

$$A = \alpha\ ,\ \ l\pm w = \beta$$
$$A = l+w \text{ (alone or with } (l+w)+A = \alpha)$$
$$A+(l\pm w) = \alpha\ ,\ \ l\mp w = \beta$$
$$A = \alpha\ ,\ \ d = \beta$$

One problem, finally, deals with a circle with circumference c, diameter d and area A:

$$c+d+A = \alpha$$

Later evidence suggests but does not prove definitively that a few more single-square problems circulated in the pre-school environment without appearing in the extant Old Babylonian corpus:

$$_4s = \square(s)$$
$$d-s = 4$$

Combination of Old Babylonian and later evidence suggests that the following four problems on a rectangle with given area belonged together as a fixed sequence already before 1800 BCE:

$$l = \alpha$$
$$w = \beta$$
$$l+w = \gamma$$
$$l-w = \gamma$$

The shape in which we find the problems in the clay tablet is often slightly changed with regard to the original format (as the latter is revealed by traces in some of the Old Babylonian specimens that agree with formats that turn up

algebraic in a third sense: its lines and areas were used to *represent* entities belonging to other categories – men, workdays, and the bricks produced by the men during the workdays in question; numbers and their products; prices and profits; etc. The surveyors' riddles, in contrast, were riddles about the entities known from surveying everyday and nothing else; they did not serve representation.

[2] I restrict myself to problems of clear riddle character; this eliminates, for instance, the finding of the area and the diagonal of a square from its side – problems which are anyhow too simple to serve as argument for any borrowing.

later).

In the original format (the "riddle format", as I shall call it) sides are referred to before the area – all riddles, indeed, tend to mention first the familiar and the active before the derived or the passive, and the lengths of sides are certainly what is immediately given to the surveyor, whereas areas are calculated and thus derived. The only coefficients of which the riddles make use are "natural" and thus not really to be understood as coefficients: *the* side or *all four* sides of a square, *the* length, *the* width or *the* sides (length and width, perhaps both lengths and both widths) of a rectangle, etc. Within the tradition of which we are speaking, the preferred value of the sides of squares or other regular polygons, moreover, is 10.

The school format, in contrast, will preferentially speak of the area before the side, anticipating the method of solution (in which areas are drawn first, and sides drawn or imagined as "broad lines" with breadth 1, to be joined to or cut out from the areas). The circle riddle $c+d+A = \alpha$ thus is changed by the school into $A+d+c = \alpha$. The infatuation of schools with drilling also calls for systematic variation of coefficients – "$^1/_3$ of the side", "$^2/_3$ of the area", "the width plus $^1/_{17}$ of the sum of 3 lengths and 4 widths", etc. – whereas the reference to "all four sides" of the square is eliminated. Finally, the compliance with Sumerian numeration and metrological tradition in the school makes 30 (meant as 30´ = $^{30}/_{60}$) the standard side of the square (and of regular polygons in general).

For the solution of rectangle problems involving the area and the side, the school as well as the surveyors made use of the semi-sum and semi-difference of the sides, more precisely of the fact that $\square(\frac{l+w}{2}) = A+\square(\frac{l-w}{2})$. Problems about squares and their sides were solved in analogous ways. The rectangle problem with given area and diagonal was reduced by means of the identity $\square(d)-2A = \square(l-w)$ to the problem $A = \alpha$, $l-w = \beta$.

In 1600 BCE, the Hittites made a raid against Babylon, which turned out to be the final blow to the Old Babylonian social system. A consequence of the ensuing breakdown was the disappearance of the scribe school and of its sophisticated mathematics. We do not know the precise channels through which basic mathematical techniques survived, but they have plausibly been several: scribes trained within "scribal families" may have been taught something, scribal schools in the Syrian and Hittite periphery may have been involved too, but a further transmission within a non-scribal surveyors' environment of oral cultural type (though possibly not quite illiterate) is next to indubitable. A restricted number of quasi-algebraic problems turn up again in a tablet from Late Babylonian but pre-Seleucid times (perhaps c. 500 BCE) – but only the basic riddle types, without coefficients beyond the natural ones.[3] This and kindred tablets are written by scholar-scribes, but discontinuities in the Sumerian translation of Akkadian words

[3] The rectangle with given area and given w, $l+w$, or $l-w$; two concentric squares, for which $\square(s_1)-\square(s_2) = \alpha$, $s_1-s_2 = \beta$. The tablet is W 23291, ed., trans. [Friberg 1997].

show that the riddles have survived in an environment where Sumerian was not learned.

We have no sources from Babylonia for the discovery of how the area of a scalene triangle may be calculated from the sides, but combination of Greek and medieval (mostly Arabic, but also Hebrew and Latin) practical geometries shows that the computation of the (inner) height in scalene triangles is pre-Greek and almost certainly pre-fourth century BCE.[4] The formula makes use of semi-sum and semi-difference (see Figure 1):

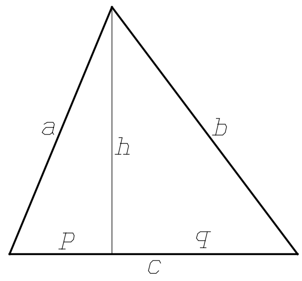

Figure 1.

$$\frac{q-p}{2} = \frac{b^2-a^2}{2}\div c \ , \quad \frac{q+p}{2} = \frac{c}{2}$$

whence

$$q = \frac{c}{2} + \frac{b^2-a^2}{2}\div c \ , \quad p = \frac{c}{2} - \frac{b^2-a^2}{2}\div c$$

The probable argument behind this formula runs as follows:

$$\square(b)-\square(a) = \{\square(q)+\square(h)\}-\{\square(p)+\square(h)\} = \square(q)-\square(p)$$

$\square(q)-\square(p)$, however, is the difference between two squares, most likely to be understood as the band between concentric squares (see Figure 2):

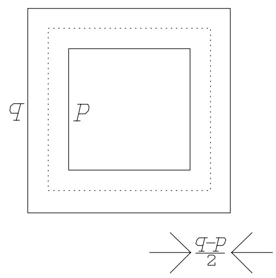

Figure 2

$$\square(q)-\square(p) = \sqsubset\!\sqsupset(\tfrac{q-p}{2}, 2(q+p)) = \sqsubset\!\sqsupset(\tfrac{q-p}{2}, 2c)$$

(This argument, a “naive” version of *Elements* II.8, is found in ibn Thabāt’s *Reckoners’ Wealth* [ed., trans. Rebstock 1993] and in Hero’s *Metrica* I.xxvi [ed., trans. Schöne 1903], and suggested in the two-square problem of W 23291 just mentioned). Therefore,

$$\frac{q-p}{2} = \frac{b^2-a^2}{2}\div c$$

Other innovations turn up more or less simultaneously in a couple of Seleucid texts and in a papyrus from Demotic Egypt;[5] the new problems and methods are not identical in the three texts, but the overlap is sufficient to show that they represent a single development – see [Høyrup 2000a; 2000b].

One innovation (only attested in the Demotic papyrus) is a new version of the rectangle with known area and diagonal: $\square(l+w)$ and $\square(l-w)$ are both found, as $\square(d)+2A$ and $\square(d)-2A$, respectively, and l and w from these without use of average and deviation ($2l = [l+w]+[l-w]$). In BM 34568, the rectangle problem with known area and $l+w$ is similarly solved from $\square(l-w) = \square(l+w)-4\mathrm{A}$, whence $2l = (l+w)+(l-w)$, etc. (similarly with known $l-w$ and A).

Also in the Seleucid material, we find rectangle problems where the data are $d+l$ and w; $l+w$ and d; $d-l$ and w (dressed as a “reed against a wall”-problem);

[4] See [Høyrup 1997: 81–85].

[5] AO 6484, ed. [Neugebauer 1935: I, 96–99]; BM 34568, ed. [Neugebauer 1935: III, 14–17]; Pap. Cairo J.E. 89127–30, 89137–42, ed. [Parker 1972: 41–43]. The papyrus will be from the third century BCE, the two tablets may be slightly later.

$d+l$ and $d+w$; $l+w+d$ and A. Several of the geometric solutions are given by Fibonacci in his *Pratica geometrie*.

The traces we find of the tradition in Greek theoretical mathematics all point to the pre-Seleucid-Demotic phase:

Elements II, 1–10 can be read as "critiques" of the pre-Old-Babylonian ways to solve for instance rectangle problems (given A and $l \pm w$, II.5 and II.6) and two-square problems with given sum of the areas (II.9–10) and given difference (II.8) – that is, as investigations of *why* and under which conditions the traditional solutions work; mostly the proofs copy the traditional procedures. II.13 is a reformulation of the fundament for the determination of the inner height in the scalene triangle which connects it to the Pythagorean theorem (I.47), while II.12 is a parallel result for the external height (almost certainly a contribution of the Greek geometers – the practical tradition seems to have considered inner heights only). In all cases where the distinction is relevant, the method is based on average and deviation.

Elements VI.28–29 and *Data* 84–85 also point to the rectangle problems where A and $l \pm w$ are given (in similar treatment).

Euclid's *Division of Figures* contains as one of the simple cases a problem that was already solved in the 23d century BCE: the bisection of a trapezium by a parallel transversal

Diophantos *Arithmetic* I is a collection of pure-number versions of a wide range of "recreational" problems – "finding a purse", "purchase of a horse", etc. This context leaves little doubt that prop. 27–30 are arithmetical versions of the rectangle problems $A = \alpha$, $l \pm w = \beta$, and the two-square problems $\square(s_1) \pm \square(s_2) = \alpha$, $s_1 + s_2 = \beta$. The solution is based on average and deviation, in contrast to all other problems of the book.

Totally absent is, not only influence from the Seleucid-Demotic innovations (the extant sources for which are contemporary with or later than Euclid) but also everything that might point to the particular contributions of the Old Babylonian school.

The situation is different if we look at the "low" tradition of Greek practical mathematics. The sources for this tradition – carried by culturally unenfranchised strata – are meagre, but not totally absent.

Firstly, there is the Neo-Pythagorean and similar evidence, produced by philosophers whose understanding of mathematics may not have allowed them to grasp the works of the theoreticians, or whose appreciation of mathematics may simply have been derived from what will have been closer at hand than the utterly few mathematicians of renown. The pseudo-Nichomachean *Theologumena arithmeticae* mentions[6] that the square $\square(4)$ is the only square that has its

[6] In II.11, and again in IV.29, ed. [de Falco 1922: 11^{11-13}, 29^{6-10}].

area equal to the perimeter, and Plutarch[7] tells that the Pythagoreans knew 16 and 18 to be the only numbers that might be both perimeter and area of a rectangle – namely □(4) and ⊏⊐(3,6), respectively. The first is obviously the old "all four sides equal area" square problem, and the second an "all four sides" variant of the $l+w = A$ rectangle problem. Finally, both Theon of Byzantium[8] and Proclos[9] refer to the side-and-diagonal-number algorithm, which may also be an inheritance from the Near Eastern tradition (and may even be reflected in Old Babylonian texts and have to do with the square problem $d–s = \alpha$).

Secondly, a few texts belonging to the practical tradition itself have survived which contain identifiable borrowings.

One such text is Heiberg's conglomerate *Geometrica* [ed. Heiberg 1912], one component of which (ch. **24**) contains the problem "square area plus perimeter equals 896", and two of which (ch. **24** and mss A+C) contain the circle problem $d+c+A$ (in "riddle order", but now with the diameter as the basic parameter instead of the circumference).

It is worth noticing that the *Geometrica* manuscripts share certain standard phrases with the Near Eastern tradition, two of which (the idea of "separating" for instance circular diameter, circumference and area, and the directive "always" to make a step which is independent of the actual parameters) are also found in a few Old Babylonian texts.

The Greek Papyrus Genève 259 [ed. Sesiano 1999], probably from the second century CE, has the rectangle problem (formulated about a triangle) $l+w$ and d given, and solves it in a way that is related to (though not identical with) that of the Demotic papyrus;[10] it also has the "Seleucid" problem where $w+d$ and l are given.

A Latin *Liber podismi* [ed. Bubnov 1899: 511*f*], whose very title shows it to be of Greek origin, contains a short collection of problems about right triangles. Most of the problems are too simple to tells us much. One of them, however, repeats the old rectangle problem where d and A are given. The solution follows the same pattern as the Cairo papyrus (without referring to average and deviation), and is thus in the new Demotic-Seleucid style.

This finally brings us to the point where we may approach Mahāvīra's 9th-century *Gaṇita-sāra-saṅgraha* [ed., trans. Raṅgācārya 1912].

At first we may simply list the features which the geometrical chapter VII of this work (but no other Indian work I have looked at) shares with the Near

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[7] *Isis et Osiris* 42, ed. [Froidefond 1988: 214*f*].

[8] *Expositio* I.XXXI, ed. [Dupuis 1892: 70–74].

[9] *In Platonis Rem publicam*, ed. [Kroll 1899: II, 24*f*]; and *In primum Euclidis Elementorum librum*, ed. [Friedlein 1873: $427^{21–23}$].

[10] $\square(l–w)$ is found as $2\square(d)–\square(l+w)$.

Eastern tradition.[11] Taken singly, some of the sharings might be accidental, others cannot be explained away in this manner; taken as a whole, the cluster is convincing evidence of a connection:

- the rectangle problem with given area and $l+w$ is solved in Demotic-Seleucid manner (VII.129½);
- the problem "area = sum of the sides" is found in square as well as rectangle version (VII.113½ and 115½);
- the rectangle in which the area and the diagonal are given is solved in the Demotic way (VII.127½);
- the rectangle problem $2l+2w = \alpha$, $d = \beta$ is solved as in the Geneva papyrus (VII.125½);
- the circle problem turns up in the shape $c+d+A = \alpha$, and the three entities are to be "separated" (VII.30);
- the inner height of a scalene triangle is determined as described above; the argument that was suggested above is also outlined (VII.49). The two-tower problem[12] is solved by reference to this procedure (VII.201½–203½), which also presupposes the same argument.

Mahāvīra is a contemporary of al-Khwārizmī, or slightly younger. One may therefore ask whether the borrowings should be located in the 9th century CE or in an earlier epoch. All the evidence speaks in favour of the latter possibility. This is illustrated by Mahāvīra's treatment of the circle problem $c+d+A = \alpha$.

Firstly, his solution presupposes that $\pi = 3$. A borrowing from Arabic mathematics without simultaneous borrowing of the approximation $3^1/_7$ is not very likely. Moreover, the problem is normalized as a second-degree problem about c. Even if Mahāvīra would have introduced a venerated π-value instead

[11] Many of the arithmetical problems are certainly also shared with the Islamic tradition (and its European descendants), but these are mostly so widespread (also within India) that they tell us nothing specifically about borrowings or their direction.

It may be noted, however, that Mahāvīra describes the system of ascending continued fractions, which to my knowledge is not found in other Indian sources, and that this type of composite fractions even has a particular name (*Bhāgānubandha* or "associated" fractions, III.113–125). This is likely to be a borrowing from a Semitic-speaking area; given the full integration into the treatment the borrowing will have taken place long before Mahāvīra's times. All in all, the ascending continued fractions may well have been taken over in the same process as that in which Seleucid-Demotic quasi-algebra was imported (see below).

Chapter VI contains a number of formulae for the summation of series, which as they stand may or may not be related to analogous formulae found Demotic-Seleucid sources. Comparison with similar formulae given by Brahmagupta and Bhaskara II (trans. [Colebrooke 1817: 290–294] and [Colebrooke 1817: 51–57], respectively) makes a link more plausible, and suggests that the greater sophistication of Mahāvīra's treatment of the topic is due to further development of an original inspiration to which Brahmagupta was closer (and of whose geographical location we can say nothing).

[12] To find the point on the ground between two towers of unequal height which is equidistant from the two peaks.

of the unhandy $3^1/_7$, he would not have made this choice, given that his own basic circle parameter (VII.19) is the diameter. Finally, Mahāvīra gives the members in riddle order with c as the basic parameter. The *Geometrica* version and the Arabic version in ibn Thabāt's *Reckoners' Wealth* are in riddle order with the diameter as the basic parameter, $d+c+A$; the Old Babylonian specimen is in school order, $A+d+c$. Mahāvīra would have had no motive to introduce the order he uses if he had depended on Arabic or late Greek sources.

The *Gaṇita-sāra-saṅgraha* as a whole contains numerous references to the tradition. For instance, VI.1 refers to "the Jinas who have gone over to the [other] shore of the ocean of Jaina doctrines, and are the guides and teachers of [all] born beings". VI.2 goes on with "Those who have gone to the end of the ocean of calculation".

The meaning of the ocean metaphor (which turns up time and again) as well as the appurtenance of Mahāvīra's mathematical masters to the group of Jaina guides and teachers becomes clear in I.17–19, where the author tells that with "the help of the accomplished holy sages, who are worthy to be worshipped by the lords of the world, and of their disciples and disciples' disciples, who constitute the well-known jointed series of preceptors, I glean from the great ocean of the knowledge of numbers a little of its essence, in the manner in which gems are [picked up] from the sea". The end of chapter I (I.70) also ascribes the whole mathematical terminology to "great sages".

Similarly, the explanation of the calculation of the height of the scalene triangle is ascribed to "learned teachers". This is hardly how Mahāvīra would refer to recent foreign inspiration.

It should be also remembered that the Jaina mathematical tradition was often very conservative by deliberate choice – Mahāvīra and other Jainas still stuck to $\pi = \sqrt{10}$ as the "precise" alternative to 3, well after the adoption of more precise approximations in Jaina as well as non-Jaina astronomy .

Socially, the Jaina community of the first millennium BCE – with its strong representation of artisans, merchants and officials [Thapar 1966: 65] – is of course the best possible candidate for a channel through which foreign practical mathematics might be adopted.

Mahāvira's chapter VII on plane geometry is divided into three sections. "Approximate measurement (of areas)" is VII.7–48; "minutely accurate calculation of the measure of areas" is VII.49–111½; "devilishly difficult problems", are treated in VII.112–232½.

This division turns out to correspond to the periodization that can be derived from the Near Eastern material – a fact which suggests imports to have been made also at different moments and in different contexts. The circle problem $c+d+A = \alpha$, clearly pre-Old-Babylonian, thus is in the first section. The determination of the height in the scalene triangle is in the second. All the rest is in the section of "devilishly difficult problems", which means that the main trunk of the import is not likely to antedate 300 BCE – a limit which might rather be 200 BCE.

We notice that the import as a whole corresponds to what is found in the Greek "low" tradition, including what is reported in Neopythagorean and related writings. In contrast, Arabic writings that draw on the ancient Near Eastern tradition do not include problems of the type "area = circumference"; they make preferential use of semi-sum and semi-difference; and they tend to think of "the two", not "the four sides" of a rectangle. If the Jaina borrowing is not directly from the Mediterranean civilization, it is at least from somewhere we do not know about but which has also affected the level of practical geometry in the Mediterranean.

One might ask whether the "Seleucid-Demotic" innovations might have arisen *in* India, for instance within the Jaina community. If not totally excluded, this seems very improbable. In Mahāvīra's work, material that is familiar in the Near East and the Mediterranean region is mixed with much more conspicuous interests that are not reflected outside India. Moreover, a text like the Seleucid "rectangle" text BM 34568 exhibits an inner coherence which makes it unlikely that this should be an elaboration of a quite restricted range of problems taken over from India; eastward diffusion of part of the Demotic-Seleucid material is much more plausible.

So far, no positive evidence has suggested that the development of Indian *algebra* was inspired by the Near Eastern ("Babylonian") geometrical tradition. Is this changed by the evidence that (pre-) Old Babylonian geometry did reach India and was remembered among the Jaina's?

It cannot be excluded, but no piece of positive evidence seems to support the hypothesis. Mahāvīra's work does contain an appreciable amount of second-degree algebra, but *if* a Near-Eastern geometric inspiration had once been of importance, then everything was already reshaped beyond recognition when Mahāvīra found the material. Moreover, Mahāvīra's second-degree problems are of the type that involves an unknown quantity and its [square] root (as are the fundamental Arabic *al-jabr* problems), not a quantity and its square. The kind of insights by which quadratic problems are solved in the *Gaṇita-sāra-saṅgraha* may well have been gained at an earlier moment from the solution of the surveyors' riddles, since these were actually present – but we have no means to decide.

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