Barnabas B. Hughes (ed., trans.), Fibonacci's De practica geometrie. New York: Springer, 2008

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Published in:
Historia Mathematica

DOI:
10.1016/j.hm.2008.06.005

Publication date:
2009

Document Version
Publisher's PDF, also known as Version of record

Citation for published version (APA):
Høyrup, J. (2009). Barnabas B. Hughes (ed., trans.), Fibonacci's *De practica geometrie*. New York: Springer, 2008. *Historia Mathematica*, *36*(1), 73-77. https://doi.org/10.1016/j.hm.2008.06.005

Barnabas Hughes (ed.), Fibonacci's *De Practica Geometrie*. (Sources and Studies in the History of Mathematics and Physical Sciences). New York: Springer, 2008. Pp. xxxv+408. ISBN 978-0-387-72930-5.

Until recently, Fibonacci's major works were only accessible in Latin in Boncompagni's editions from [1857] (the *Liber abbaci*) and [1862] (the *Pratica geometrie*) – which means that only the happy few (mainly historians of mathematics) had access to them. In [2002], Springer then published Laurence Sigler's translation of Fibonacci's *Liber abbaci* posthumously, and now the same publisher has produced Barnabas Hughes' translation of his *Practica geometrie*. Thereby mathematicians have finally got access to a major predecessor whom many of them venerate from hearsay only.

Translation always implies choices on many levels, and the main choices of Hughes indeed orient his translation towards mathematicians and mathematics teachers. He has tried to render Fibonacci's language in familiar mathematical idiom, rather than in words which correspond to Fibonacci's own thought – we may say that the translation tacitly integrates elements of a basic mathematical explanation or commentary.[1] Sometimes, but not systematically (which might indeed be too cumbersome for the reader who is primarily a mathematician), such deviations from Fibonacci's own text and concepts are explained in notes. Fibonacci's chapter- and section-headings are also changed and extra headings inserted, but in this case the Latin text is given in footnotes when it exists. Sometimes, diagrams are also tacitly added or redrawn, or letters and numbers are omitted which Hughes must be supposed to find superfluous for the understanding.[2] In Hughes' words (p. xxx), "I saw no purpose in recording the changes; Boncompagni's edition can always be consulted for comparisons" – which alas will only be true for a minority of readers.

The translation is divided in the same chapters as Fibonacci's text. Each chapter is provided with an introduction, containing basic commentary and a generally well-informed discussion of Fibonacci's plausible sources for the chapter in question. An initial general introduction ("Background", pp. xvii–xxxv) discusses Fibonacci's possible knowledge of Arabic, his schooling and the basic resources on which he could draw; this introduction also discusses the principles of the translation and the resources that Hughes has consulted: ten Latin manuscripts and five Italian manuscripts from the 15th and 16th centuries. Even though the preface (p. ix) refers to the notion of a "critical translation", references to any resource beyond the Boncompagni text and the manuscript on which it is based (Vatican, Urbino 292) are rare.

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[1] For instance, when an *angulus rectus* / "right angle" becomes "90°" (p. 65). Later on (p. 154 and *passim*) "right angle" is used.

[2] This is rather unfortunate when letters are omitted to which the text or Hughes' own notes refer – as in Figures 3.64, 3.65, 4.89 (the latter belonging however with an incoherent pseudo-proof), 4.101 and 7.13.

At this point, the reviewer has to inform the reader that he read Hughes´ book proposal for the publisher, which at that stage contained the preface, the introduction, and translations of Chapters 4, 5 and 7 and by then carried the subtitle “A Critical Translation”, and that he recommended the proposal warmly. It is therefore with deep regret that he feels obliged to be quite critical in what follows.

In “Background”, Hughes argues (in my opinion convincingly) that Fibonacci knew Arabic well enough to be able to use Arabic manuscripts freely. In a translation of Fibonacci’s short autobiographical notice from the *Liber abbaci* he assumes that “pursuing *studio abbaci*” means that Fibonacci frequented an “abacus school”; personally I tend to believe this controversial claim may be right, and that something like the institution which we know from late 13th-c. Italy was indeed present on both sides of the western Mediterranean, although we have absolutely no sources for that; but if so, the list of authors which Fibonacci is presumed to have studied under “a wonderful teacher” (a mistranslation, should be “wonderful instruction”) – namely al-Khwārizmī, al-Ḥaṣṣār, al-Karajī, Ibn al-Yāsamīn – is likely to be misleading; at least Italian abbacus school students did not study books, they were trained on problems.

The editor or translator of a scholarly text tends to fall in love with it. This may be the reason that Hughes’ commentary sometimes overstates Fibonacci’s merits. On p. xxix Fibonacci is thus claimed to use in his algebra “not [...] the verb *balances* as is found in earlier tracts, such as *Liber augmentis et diminutionis* [but] either the verb or adjective for *equals*”. Actually, both the tract which is cited and Gherardo’s translation of al-Khwārizmī use *equari*, “to be equated with”, just as does Fibonacci; Robert of Chester’s translation uses *coequari*; Hughes must know, since he has made editions of all three texts. Hughes also states that (pp. xxix, 361) that Fibonacci is the first to use the concept of an *equation*, once in the *Liber abbaci* [ed. Boncompagni 1857: 407] and once in the *Pratica* [ed. Boncompagni 1862: 210]. In both cases, however, the word *equationem* may rather stand as a verbal noun referring to the action of equating that for the object which we refer to under that name. The concept of the equation as a mathematical *object* probably has to await Dardi of Pisa, in whose *aliabraa argibra* from 1344[3] it occurs abundantly (as *adequation*).

Infatuation may also be the reason that Hughes cannot accept the idea that Fibonacci himself should announce in the heading of Chapter 7 all the traditional topics of *altimetria* but actually not touch at planetary longitudes in a treatise dedicated to a good friend. He therefore moves the chord table and the text explaining its use from Chapter 3 to Chapter 7, even though there are no references at all in this piece of text to its astronomical use. Since the beginning of the chord passage refers to what precedes it in Chapter 3, and the next part of Chapter 3 refers to the calculation of chords from their arcs and vice versa as preceding, this suggestion is certainly to be ruled out.

[3] The best manuscript is Vatican, Chigi M.VIII.170.

Hughes may have been under time pressure after the book proposal was accepted. In any case, whereas Chapters 4 and 5 (division of figures and the finding of cube roots, respectively) are quite satisfactory,[4] Chapters 1–3 are unfortunately not.

First of all, there are problems of consistency. For instance, on p. xxxii Hughes tells how his "initial attempt at translation" of the terms for various trapezia (understood by Fibonacci as *caput abscisa*, "[triangles with the] head cut off") "ended in simply leaving the Latin words mixed with the English text"; then he decided to use instead "right (angled) trapezoid" etc. However, in Chapter 4, p. 211, we find the Latin words with a note "These names do not translate well". The new translations are only used in Chapter 3.[5] Similarly, whereas *posse* (as equivalent of Greek *dýnasthai*) in the sense of "be equal in square" (*viz* one segment to two segments) is translated on p. 171 in a way which no reader without knowledge of the Greek terminology would understand ("a pentagonal side can be over a hexagonal side and a decagonal side"), it is translated perfectly on p. 292 ("the squares on the lines *ds* and *za* equal the square on line *db*); even here, no harmonization is thus attempted. Similarly, the notion that the magnitude *A* "adds *d* over" a magnitude *B* (that is, exceeds it by *d*) is misunderstood completely on pp. 114 and 117;[6] on pp. 131 and 133, Hughes has discovered the meaning, but does not correct what was already written.

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[4] Apart from the problems with two diagrams mentioned in note 2, I only noticed a few minor problems that I had overlooked in my first reading:

- P. 205, Fibonacci's "the diameter *ac*" is corrected into "diameters *ac* and *bd*", and it is claimed in a note that "the context requires two" diameters. This is simply not true, and *bd* is in fact absent from the ensuing proof.
- P. 246, n. 146 it is claimed that no "helpful figure" assists in understanding a particular construction. The diagram is in [Boncompagni 1862: 144] (belonging with the preceding paragraph), but Hughes has eliminated essential parts when producing Fig. 4.89 and appears to have consulted his own redrawing instead of the original.
- P. 250, "To divide a semicircle in two at a given point" should merely be "To divide a semicircle in two".
- P. 252, Hughes interprets a *trigonum* delimited by a circular arc and two straight segments as a sector, even though it is clear from the proof (also as translated by Hughes) that the two segments may neither radial nor equal.
- P. 257, "$(ag)^2+(ag)(bg)^2$" should be "$(ag)^3+(ag)(bg)^2$".
- P. 267, "make a transition rule for moving as line" should be "make a ruler move".
- P. 270, "therefore if you wish" should be "therefore you wish".

There may of course be more, but they should be rare.

[5] Not quite consistently, however. What is named "scalene trapezoid" on p. xxxii becomes "trapezoid" *simpliciter* on p. 144.

[6] "If 6 the diameter of a square is added to one of the sides of the same quadrilateral" instead of "If the diameter of a square adds 6 over one of the sides of the same quadrilateral" (i.e., $d–s = 6$); and "add the measures of the longer and shorter sides [of a rectangle]" instead of "the longer side adds in quantity 2 over the shorter side"; the missing parameter 2 is of course used in the solution of the problem.

Haste is also the only explanation I can find that many references to the *Elements* are mistaken;[7] that the translation regularly destroys the argumentative structure of Fibonacci's text or otherwise misrepresents it[8] and that the headings which Hughes

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[7] P. 26, II.1 should be "generalization of II.2); p. 27, II.2 should be II.1; p. 32, VI.13 should be VI.17; p. 36, II,13 should be VI.13; p. 74, I.45 should be VI.2; p. 130, II.6 should be II.11 (correct in n. 150); p. 302, VI.20 should be VI.19. There may be more instances, but I stopped semi-systematic checking around p. 100.

[8] For instance:

– P. 32, where "*If three numbers or quantities are proportional, then the first is to the second as the second is to the third*. Then the product of the first number by the third equals the product of the second by itself" should have been "*If three numbers or quantities are proportional, so that the first is to the second as the second is to the third, then the product of the first number by the third equals the product of the second by itself*".
– P. 50, speaking of square roots, "their sum is either a rational number or the root of another number. When they cannot be added, then either a number arises from their sum or another root" should be "their sum is either a rational number or the root of another number. And sometimes they cannot be added in such a way that either a number arises from their sum or another root". And further, p. 51, "When we wish to join squares, then a number results from their sum" should be "When we wish to join roots of squares, then a number results from their sum", whereas "When we wish to add roots that have among themselves the ratio of their squares" should be "When we wish to add roots of numbers that have among themselves the ratio of squares".
– P. 69, "among right triangles, some are obtuse and others are isosceles. Still others are scalene. Among the right oxigonal triangles, some are equilateral, others scalene" should be "among right ⟨and⟩ obtuse triangles, some are isosceles, others are scalene. Among the oxigonal [slightly later translated "acute" / JH] triangles, some are equilateral, some isosceles, some indeed scalene".
– P. 88, "When two sides of a triangle are known together with a line drawn through them equidistant from the remaining line, then we know the parts of one line, the sections of another, and the length of the drawn line" should be "When two sides of a triangle are known and a line is drawn through them equidistant from the remaining line, and we know the parts of one line, then the sections of the other and the length of the drawn line are known".
– P. 106, the non-rectangular quadrilaterals are divided into four groups: "rhombi, rhomboids, trapezoids, and those with unequal, equidistant sides", which should be "rhombi, rhomboids, trapezoids, which have two equidistant sides, and diversilaterals, of which none of the sides is equidistant from the others".
– P. 278, the explanation "POLYHEDRA classified according to their faces are of many kinds, among which those with 8 faces, 12 faces, and 20 faces" is abbreviated illegitimately into "POLYHEDRA classified according to their faces are of many kinds: those with 8 faces, 12 faces, and 20 faces". The same page abounds with misunderstandings.
– P. 396, n. 5, the Latin introductory clause to an appendix containing indeterminate number problems is rendered "Et incipiunt questiones, quorum solutiones non sunt terminate [...]" / "And problems begin, whose solution is indeterminate". Hughes omits its first part, "Expliciunt questiones geometricales" / "The geometrical problems are finished", and thus obscures Fibonacci's awareness of leaving the geometrical genre.

supplies do not always correspond to what actually comes.[9] Presumably, haste also explains that Hughes sometimes has not thought through the mathematics of the text or done so wrongly[10] and that the metrological table shown on p. 2 states the “grain” to be one half of the “point” (whence 0,7 mm), against the text which is quoted (p. 1 n. 4) and which explains it to be its double (whence 2.8 mm).

Haste may finally be the reason that Hughes does not always think through the consequences of his free modernizing translation and the inconsistencies it produces

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The list could be continued.

[9] For instance p. 6, where “Constructions” introduces a section listing Euclidean constructional propositions, postulates and theorems. The word “construction” is also in the translated text, while the Latin speaks adequately of “many [things] which are clearly shown in Euclid”.

[10] For instance:

- P. 61, concerning Fibonacci’s presentation of the Pisan way to calculate the circular area – square the diameter, divided by 7 [should be “divide the outcome by 7” / JH], and you will have the area of the outcome in panes”, Hughes adds that Fibonacci “assumed that the reader knew that the diameter also had to be measured in panes” – the *panis* being an *area* measure equal to 5½ square rods. This is wrong, both because the diameter cannot be measured in *panes* and because the formula holds if it is measured in rods.
- P. 129*f*, the impossible argument “because *fd* and *de* are in the same ratio as the squares on *fd* and *de*, there is a square number, 256, whose root is 16 for one of the sides” should have been “because *fd* and *de* have the ratio of squares, from *fd* and *de* comes forth a square number, namely 256, whose root, namely 16, will be one side”. The error is induced by Boncompagni’s interpunctuation [1862: 72], which has been accepted without mathematical second thoughts.
- P. 280, “If two planes are described by all the sides of two cubes with opposite sides divided through the middle cutting the cube itself, then their common section cuts the diameter of the cube in half”. This is already meaningless from the grammatical point of view, “two cubes” becoming suddenly “the cube”. Hughes must have read the Latin “Si duarum oppositarum cubi superficierum cunctis lateribus per medium divisis ...” / “If, when of two opposite sides of a cube all the edges are divided through the middle” [Boncompagni 1862: 161] too rapidly, overlooking the case endings, and not tried to figure out which geometrical situation is thought of; strangely, a correct reference to *Elements* XI.38 follows.
- P. 282 refers to the mathematically impossible erection of “a perpendicular on a given surface from a given point above to another given point above”. Here, what has mislead Hughes is a repetition in the Latin text, “á dato punctum in altum á puncto in alto designato” / “from a given point upwards from a designated point above”.
- P. 283, an algebraic problem is constructed about a 10×10×10 cube: the sum of the square on the diameter and the side is 310. At one point Hughes miscopies $10^{1}/_{6}$ (by Fibonacci written $\frac{1}{6}10$) as $10^{1}/_{3}$ (not checking that $\sqrt{103^{13}/_{36}} = 10^{1}/_{6}$ and not $10^{1}/_{3}$); later, when Fibonacci subtracts $^{1}/_{6}$ and finds 10, Hughes claims that a rounding is made, overlooking that the outcome 10 is the exact result.

in his text.[11]

Haste, on the other hand, is hardly behind two misinterpretations of the text at higher levels. The first is on p. 87, where Fibonacci presents the surveyors' method to measure the height of a triangle in the terrain. A rope is stretched from the vertex so as to reach a point at the base, and then moved to the other point where "accident may have it" (*ubi sors dediderit*) to touch the base again; this Latin phrase is read as "where the partner has chosen", which is mathematically as well as linguistically impossible. Slightly later a method is told to circumvent the difficulty which arises if the triangle is planted with vines or trees that prevent the movement of the rope; this becomes "if [...] the area of the triangle were to be measured in ells, or it were an orchard".

The second is the interpretation of the measurement of the horizontal extension of slopes (pp. 174*ff*). Even though Fibonacci states that they are measured by means of a *pertica*, a "rod" (also used as a unit of measurement, c. 3 m, cf. note 10), Hughes claims it to be performed by means of a measuring tape (which he supposes carries the name *pertica* because it measures in this unit). The mistake is obvious when Fibonacci asks (p. 175) for an archipendulum, a wooden instrument of a certain weight, to be put (in Hughes' words) "atop the tape" in order to ensure that it is kept horizontally. Slightly earlier it is also clear that the length of "tape" should be exactly 1 *pertica*, which would eliminate any reason to replace the rod by a piece of string.

Not quite a misinterpretation but an easily falsifiable hypothesis is formulated on p. 345, namely that Fibonacci "constructed a huge semicircle of diameter 42 [*perticae*], divided it into 66 equal parts, constructed the required chords, and measured them to create the [chord] table". If one takes the trouble to calculate the chords (two hours'

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[11] I shall restrict myself to three examples (more could be given):

– On p. 93, the prescription "complete the figure" is replaced by "complete the triangle". Since this completion consists in adding points and lines which are *not* part of the triangle, Hughes adds a note that "Leonardo uses this phrase several times as though his reader is expected to know that additional points and lines are necessary".

– P. 109, Fibonacci asks for the "separation" of the sides from the area of a square, for which we know the sum of the area and the four sides to be 140 – that is, to split the sum into its constituents. The term belongs to a tradition going back at least to late Antiquity (it is abundantly used in the pseudo-Heronian *Geometrica*); Hughes obscures this link, and requires "to evaluate the sides from the area", which is hardly good mathematical terminology (neither medieval nor contemporary) and also mathematically mistaken, since side as well as area are found *from the sum* (first indeed the side, then the area from the side). However, on p. 118 the notion of "separation" is used, since this time it cannot be avoided.

– As observed in the introduction to chapter 6 (p. 274 n. 1), Fibonacci uses *piramis* for pyramids as well as cones (for cones he specifies that the base is a circle). None the less, a note to the text (p. 306 n. 60) claims that "Inasmuch as the context focuses on the truncated cone, the use of *totius pyramidis]* [...] is incorrect and out of place" – but Fibonacci speaks explicitly of a *piramis curta* [...] *cuius basis sit circulus.*

work with a pocket calculator), this is easily seen to be impossible. 48 of the 66 chords err by less than 8 points, i.e., 11 mm,[12] the diameter being 126 m – all but two by 3 points (4 mm) or less; 6 then err by a few inches[13] – and 13, in a sequence with a single interruption, err by excess by exactly 1 *pertica* plus or minus 3 points or less. This error distribution is far from the more or less normal distribution of errors that would follow from measurement (and for 48 of the chords much more precise than could be measured with ropes or rods); but it could arise from calculation if values were somehow determined sequentially.

There are numerous other errors and clumsy (or outright misleading) translations, but I shall abstain from listing more than already done. The conclusion I am forced to draw is that the volume is still relatively adequate for a mathematician who wants to get an impression of the contents, the scope and the style of Fibonacci's work (for the style preferably the well-polished chapters 4 and 5); but readers should unfortunately be aware that logical and mathematical slips in the text are mostly (not always) to be ascribed to the translator and not to Fibonacci.

When recommending the project, I suggested to the publisher to make the Latin text available in electronic format (a CD already circulates privately among historians), and I was told that Springer might ponder to put it on its website. Even though this has not happened yet, it is highly desirable; the possibility to check the translation and the figures against the original would enhance the value of the volume immensely (and, from the perspective of the publisher, enhance its commercial value). As mentioned above, this possibility of checking is also presupposed by Hughes ("Boncompagni's edition can always be consulted for comparisons").

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April 2008

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[12] The *pertica* consists of 6 "feet", the "foot" of 18 inches, and the inch of 20 points.

[13] Only one is likely to represent a copying error, 6 inches instead of 16 inches, but the points exact.