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APPLICATION AND THE IDENTITY OF MATHEMATICS

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In the paper a conceptual framework for discussing the identity of mathematics as a school subject is constructed with particular emphasize on application of mathematics. The framework is used to analyze the identity of mathematics, as it appears on two different kinds of domains: the political system and the teachers. At the end it is discussed whether this frameworks gives us new insights into mathematics teaching. It is concluded that the framework can articulates important aspects.

INTRODUCTION

This paper is a theoretical part of a larger research project on the *identity of mathematics* as a subject in the Danish general upper secondary school (the Gymnasium). The term *identity* is borrowed from the subject specific regulations. Every subject of the Gymnasium has its own regulation, starting with a paragraph named “identity”. An analysis of those paragraphs shows that an identity of a subject seems to consist of three aspects: 1) A general description of the objects studied in the subject, 2) specific descriptions of methods, theories, contents, etc. in the subject and 3) external justifications of the existence of the subject as an independent entity.

Those three aspects are used to compare different subjects, by highlighting their principal differences. They are also used to declare how the political system officially wants the subject to be identified. For many of the subjects, such identification may be uncontroversial. But in the case of mathematics, this is not so. The identity of mathematics as a discipline and especially as a school subject is in general disputed.

I define an *identity of mathematics* as a holistic view of what kinds of tasks, contents, knowledge, actions, etc. that can be recognized as belonging to the field of mathematics. It is my claim that an identity of mathematics as a school subject can be described as a vector in (at least) three dimensions: 1) A view on the role of theory, 2) a view on the role of application and 3) a view on the role of meta-issues. In short they can be named the *in-*, *with-* and *about dimension* (with inspiration from Jankvist 2008). A dimension can be described in several ways. For this purpose, a set of levels ordered by inclusion will be convenient.

Identities of a school subject are not well described animals, living in a well defined territory. It is blurred creatures living on qualitatively different domains and in different states. The domains can be the political system, the individual teacher, a textbook, a student, etc. The different kinds of states found on a domain depend on the characteristics of this.

A math teacher is an example of a domain. Here we can find an identity of mathematics as a school subject in different states, e.g. *intended identity*, *practiced identity* and *principal identity*. While the first two have to do with teaching, the last one is the persons more general identification of mathematics as a discipline and academic field.

My overall research question(s) is: »Which identities dominate the mathematics subject in the Danish gymnasium today and what consequences does it have for the possibilities of making general changes in the identity?«. By this I want to grab the struggle between identities focusing narrowly on mathematics as a theoretical field versus identities allowing mathematics to be a tool for application outside its own world.

To answer this question, I am investigating four categories of domains: 1) The political system, 2) the textbook systems, 3) the math teachers and 4) the academic environment around the subject of mathematics in the Gymnasium. In this paper I will present some of the theoretical considerations about the *with*-dimension. I will follow this up, by giving some examples on the analysis of two of those four mentioned domains, the political system and the math teachers.

I should underline here, that the discussion about the identity of mathematics as a subject in the Danish Gymnasium is relevant, because of a larger reform that was fully implemented in 2008. The reform has moved the mathematics subject toward a stronger role for application. This can be seen at two points:

Firstly, the reform has emphasized cooperation between different subjects. This change forces mathematics to think in terms of application. This change has been discussed in several articles (e.g. K. B. Jensen (2010) and Andresen & Lindenskov (2008)). I will not go deeper into this in the paper. Secondly, there are a larger emphasize on modeling and application in the new regulations of mathematics as a single subject. It is this change that I will discuss here. What role does application and modeling play, when mathematics is on its own? This role has earlier in the pre-reform era been discussed in e.g. T. H. Jensen (2007).

THE ROLE OF APPLICATION

As claimed above, the view on the role of application is an independent dimension in an identity of mathematics. The simplest “value” for this dimension would be zero, corresponding to the view, that application of math shall not play any role at all. This is not the same as saying that mathematics can’t be applied, but that the applications belong elsewhere. This viewpoint can be found among the math teachers in the Danish Gymnasium (that typically holds a master degree in mathematics from a university), but in this very radical form, it will probably be rare.

So to describe the existing viewpoints as parts of identities, we would need to formulate a suitable number of levels. The levels should be inclusive ordered, i.e. that

a lower level should be contained in a higher level. In order to construct those levels, it will be necessary to choose a set of notions about application of mathematics.

Application is a matter of working with models in a more or less unfolded way. A model is an object, that can be described as a triple (S, M, R) , where S is a real-world situation, M a collection of mathematical objects and R a relation between S and M (Blum and Niss, 1991). Modelling is a process, in which a model is constructed. The modelling process can be described as six sub-processes, as in Blomhøj and Kjeldsen (2006). The sub-processes are shown in figure 1.

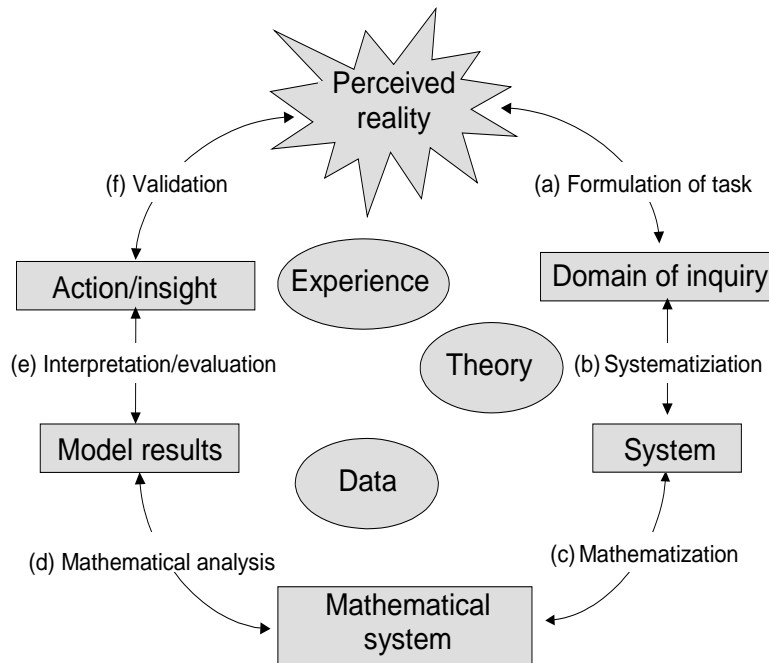


Figure 1: The modelling cycle consisting of six sub-processes

A good entrance to the discussion of the role of application is to talk about different kinds of tasks based on the application of mathematics. Based on the modelling-cycle, five kinds of such tasks can be defined:

- *Modelling task.* A task that can only be solved by going through sub-process (b), (c), (d) and (e), and eventually also (a) and (f).
- *Model task.* A task involving sub-process (c), (d) and (e).
- *Mathematization task.* A task involving sub-process (c) and eventually (d).
- *Interpretation task.* A task involving sub-process (e) and eventually (d).
- *Wrapped task.* A task that in any practical sense only involves sub-process (d).

I will then define roles of application on five levels:

1. *Illustration.* The role of application is narrowly to illustrate the pure theory.

2. *Motivation*. The role of application is to motivate work with the pure theory.
3. *Service function*. Math has a service function in other subjects and areas.
4. *Personal tool*. Math is a tool, one carries around to use in the real world.
5. *Critical inquiry*. Math is a field for investigating a wide range of problems.

Level 1 and 2 will be found in identities that see mathematics as a field for only pure theoretical activities. Applications must serve theory. On the illustration level, it is not important that the application has anything to do with the real world, while on the motivation level the cases must have some sort of real world relevance. That gives the order of the two levels. On these two levels *wrapped tasks* are sufficient.

On level 3, mathematics is basically recognized as a field for pure theory, but it's also important to activate this theory in situations, where other fields needs it. For instance when biologists and physicists needs a differential equation to be set up, solved and interpreted, or a carpenter needs to calculate the angles of a roof. The service function solves real world problems, but it does not cope with their background. On this level *mathematization tasks*, *interpretation tasks* and *model tasks* are introduced.

On level 4, it is important to be able to solve real world problems, when you meet them. Math should be a tool that you carry with you to use on appropriate problems. An economist must know how to handle problems like »what is the actual taxation as a function of income tax and VAT«, and a physicist must know how to handle problems like »with what speed does a parachute land«. Therefore those two questions are relevant to ask in mathematics. On level 4, the modeling tasks are introduced.

On level 5, mathematics is recognized as a field where a lot of open questions can be examined critically. It can be tasks like »what is the best means of transport«, »how early does Venus rise«, »how many elevators are needed in a warehouse with many floors« and »should we trust the polls«.

Those five levels will be a part of my framework to analyze what identities are dominating in the Danish Gymnasium. In the two following sections, I will give examples on the use on two different categories of domains.

THE POLITICAL SYSTEM

In the political system, the identity of mathematics as a subject in Danish Gymnasium lives primarily in different kinds of documents. Therefore document analysis is the most important method to describe it. The political system is an aggregation of many peoples' individual viewpoints and interests, so it is not expected to find a clear well defined identity.

The documents to analyze, is first of all the *regulation*, especially its appendices on mathematics. Secondly it is the *guidelines* following the regulation. And then it is the annual written examinations. The written examination is taken by all students with

To demonstrate knowledge about application of mathematics means, that you in a reflected way can present some content that you have worked with. In that do not lay the idea that students independently can take care of a mathematical problem and modeling of a material or problem, which has not been prepared. (UVM 2008, p.22., my translation)


In this text, the talking about unassisted applications is laid dead. Instead application is something taking place in continuation of work done by others. So this text places the role of application around the level of *motivation* or *service function*.

The four pieces of text draws together a blurred picture of the systems *declared identity*. The relation between theory and application is unclear. According to the general declarations of aims and purposes, application should be very central. But if one looks at the list of mandatory contents, it is the pure theory that is in focus. It is also unclear on what level applications are to be presented. But again, the general parts draw up, the concrete parts draw down.

It is therefore my claim that the regulation leaves the teacher with a broad range of choices of what identity he or she will practice in the daily teaching. Therefore it is very important to examine the identities held by the individual teachers. But the system has one important tool left: tasks for the written examination. Even though the rules are unclear, the teacher still has to take the written examination into account.

The written examination is a collection of typically 16-17 tasks, that every student must answer in 5 hours. The first five tasks must be answered and handed in, in the first hour, without any aid. The remaining tasks must be answered with the use of calculators, tables of formulas, computer programs and other means, not including communication with others. The tasks are very similar from year to year. They are therefore a clear message to teacher and students about what kind of task they should be training to answer.

Out of the 11-12 tasks with aids, 5-6 are formulated in an applied way (i.e. by referring to some extra-mathematical context). It is my assumption, that these tasks are the strongest declaration from the system about, what role application should play. Here I will just give two examples of typically applied tasks from written examination on the highest level (UVM 2009, *my translation*):

<p>Example A</p> <p>In a model, the weight of a certain fish as a function of the fish's age, is given by:</p> <p>Where w is the weight (measured in kg), and t is the age (measured in years).</p> <p>a) Use the model to decide the fish's weight, when it is 3 years.</p>	<p>Example B</p> <p>In a garden a flower bed is landscaped with the shape of a circle sector (se the figure).</p> <p>It is informed, that the area of the flower bed as a function of the angle ν (measured in radians) is:</p> 
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b) Determine the age of the fish, when the fish's weight is 13 kg.	a) Determine v , so the area of the flowerbed becomes as big as possible
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Both of these tasks are of the *wrapped* kind. In both tasks are given an explicit expression and asked questions that it is very simple to unwrap as pure mathematical questions. Said in another way, it is possible to formulate both tasks as pure theoretical tasks. Therefore the term “wrapped”, because it is pure tasks wrapped in an extra-mathematical context.

Example A is a task of the kind *given $y = f(x)$, find $f(x_0)$ and x so that $f(x) = y_0$* . Example B is of the kind *given $y = f(x)$, find the x where $f(x)$ has its maximum at the interval $[a;b]$* . It is my claim, that those tasks focuses mathematics on the theoretical dimension, while it places what we could call the systems *assessed identity* on the lower levels of the application dimension. The tasks do illustrate and motivate theory through application, but they do not train how to apply math.

THE TEACHERS

In the case of a teacher, the identity lives as a collection of viewpoints, believes, habits, abilities, etc. inside the teachers mind. Therefore more sophisticated methods than document analysis are needed, to uncover it. In the singular case, the best method would probably be a combination of deep conversations and observation of teaching practice. But if we want an overview of the entire population of teachers, we need to do a survey on a well chosen sample.

Here the main problem is how to screen a person's mind. If the person is asked directly about the identity, you will probably not get complete and accurate answers. Therefore it is necessary to ask questions where the answer builds more or less unconsciously on the teacher's identity of mathematics. Examples of such a method in this case, are to present a number of different kinds of tasks to the teacher, and ask him or her to place it in relation to their teaching (e.g. central, supplementary or not belonging), and to show the teacher four proposals for the identity-paragraph in the regulation and ask which one the person would vote for in a referendum.

It is not the purpose of this paper to justify my methodology, present results or draw any conclusions. But I will exemplify the use of the identity concept on the domains of teachers, by referring to interviews made with four math teachers, as a pilot study before designing a survey.

The teachers were asked to comment on the task »How early does Venus rise?«. This task can be answered in many different ways. Venus is placed between the Earth and the Sun. Therefore it must rise relatively close to sunrise. One way to answer the question, is to estimate the greatest time difference between the rise of Venus and sunrise. By making a plane geometric model one can convince him- or herself, that

this happens when the Sun-Venus line is perpendicular to the Earth-Venus line. In that case the time difference T is given as:

$$\frac{\dots}{\dots}$$

The interviewed teachers were not presented for this or any other ideas to a solution. Two of the teachers refused the task, claiming that it does not belong to the field of mathematics. One of these two teachers explained this in the following way:

T1 [...] as the question stands, you can't calculate it unless you have some preknowledge, unless you have been taught astronomy. And of course there is some mathematics in it [...] It would be a good fourth question in a report, where a lot of other questions leads up to it.

According to this teacher, the question is first relevant, when another subject has been on work. Then there will be something to do for mathematics. This indicates a *service function*-level. The other refusing teacher says:

T4 It depends on where you see it from. And it requires different astronomy software at your disposal... I really don't think that has something to do with mathematics.

This teacher doesn't recognize the task as something where mathematics can play a role. This does as well indicate that we are not at the highest level of application. A teacher at the two highest levels would be expected to spontaneously being open towards investigating the possibilities of a question like this.

The two other teachers didn't refuse the task, but declared the task to be "supplementary", i.e. useful but not central.

T2 [...] the description of the celestial bodies is traditionally handled by physics. But fundamentally it is also a mathematical question and can be modeled with mathematics [...] so of course it can be supplementary, but as the task stands here... it demands to much additional knowledge.

Here the teacher does not refuse the task. Instead he talks about modeling. He is though critical to the missing informations. So to him, mathematics is not a field of *critical inquiry*, where open questions are examined. On the other hands, it is more a lack of information, than a lack of another discipline that bothers the teacher. This indicates a teacher on the *personal tool*-level.

So what I try to indicate here is that by asking those kinds of questions, it is possible to state a first impression of a teacher's position on the *with*-dimension of identity.

DISCUSSION

In this paper I have presented a conceptual framework of the identity of mathematics as a school subject. The part of the framework describing the application of mathematics was particularly developed. The framework has been used to discuss the

identity of mathematics living on two different kinds of domains: the political system and the teachers. Examples have been given on how to use the conceptual framework on those two domains.

Whether a conceptual framework is useable or not, must be decided in its ability to articulate relevant problems from reality. In this case it is the overall discussion on the conflict between a pure theoretical approach to mathematics versus an applied approach and the discussion between different approaches to application.

The identity concept adds a way of addressing the disputes on what should characterize the mathematics subject. The concept describes different holistic views on the subject. Differences in such views can be used to address special challenges, when the aims and contents of the subjects are changing. At the same time it can be used to give an overall quantitative description of “the state of the art”, though this is methodological complicated. So the conceptual framework presented, seems to be useful to the Danish context, but may very well be so for other countries as well.

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