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As the Outsider Walked In
The Historiography of Mesopotamian Mathematics Until Neugebauer

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Abstract

Those who nowadays work on the history of advanced-level Babylonian mathematics do so as if everything had begun with the publication of Neugebauer's *Mathematische Keilschrift-Texte* from 1935–37 and Thureau-Dangin's *Textes mathématiques babyloniens* from 1938, or at most with the articles published by Neugebauer and Thureau-Dangin during the few preceding years. Of course they/we know better, but often that is only in principle. The present paper is a sketch of how knowledge of Babylonian mathematics developed from the beginnings of Assyriology until the 1930s, and raises the question why an outsider was able to create a breakthrough where Assyriologists, in spite of the best will, had been blocked. One may see it as the anatomy of a particular "Kuhnian revolution".

The background

In an obituary of Jules Oppert [Heuzey 1905: 73*f*] we find the following[1]:

> With Jules Oppert disappears the last and the most famous representative of what one may call the creation epoch Assyriology. When he entered the scene, Assyriological science had existed but a few years. The decipherment of the Persian texts, inaugurated by Grotefend in the beginning of the last century, had opened the way; the proper nouns common to the two Persian and Assyrian versions of the trilingual Achaemenid inscriptions provided a firm base for the determination of the value of a certain number of signs; Rawlinson recognized the polyphonic character of the Assyrian system, and Hincks justly defended the syllabic principle against Sauley. After a few works on Old Persian, Oppert brought his main effort to the Assyrian inscriptions. After having been entrusted together with Fresnel with a mission to the Babylonian area, he published in 1859, after his return, the second volume (actually the first in date) of his Expédition en Méslopotamie [*sic*] in which, by means of recently discovered sign collections or syllabaries, he established the principal rules of decipherment. This volume, Oppert's masterpiece, constitutes a turning point; it put an end to the gropings and established Assyriology definitively.

Similarly, Samuel Noah Kramer [1963: 15] states that

> Rawlinson, Hincks and Oppert – cuneiform's "holy" triad – non only put Old Persian on firm ground, but also launched Akkadian and Sumerian on the course to decipherment.

Kramer's whole description of the process of decipherment of the three languages (pp. 11–26) shows the importance during the initial phase of knowledge derived from classical and Hebrew sources (often very approximative knowledge, as it turned out, except for the Hebrew *language* and *terminology*) and of bi- and trilingual texts.[2]

So much concerning the conditions for the beginning of cuneiform scholarship. The conditions for initial work on matters connected to cuneiform *mathematics* (understood broadly, as numero-metrological practice) are reflected slightly later in Heuzey's obituary:

> Oppert's scientific activity pointed in very different directions: historical and religious texts, (Sumero-Assyrian) bilingual and purely Sumerian texts, juridical and divinatory texts, Persian and neo-Susian texts, there is almost no branch of the vast literature

[1] My translation, as everywhere in what follows when no translator is indicated.

[2] A more detailed description of the process, confirming this picture, is found in [Sayce 1908: 7–35]. Even more detailed is [Fossey 1907: 102–244].

> of cuneiform inscriptions he has not explored. The most particular questions – juridical, metrological, chronological – attracted his curiosity [...].

Administrative, economical and historiographic documents were indeed not only a main source for metrology; reversely they could only be understood to the full once the pertinent metrology itself was understood, for which reason they were also the main motive for understanding numeration and metrology.

This is illustrated by the earliest discovery of sexagesimal counting. In connection with work on calendaric material, Edward Hincks [1854a: 232] describes a tablet ("K 90") containing "an estimate of the magnitude of the illuminated portion of the lunar disk on each of the thirty days of the month"[3] without going into the question how its numbers were written; in a parallel publication [1854b] "On the Assyrian Mythology" concerned with the numbers attached to the gods he refers to the "use of the different numbers to express sixty times what they would most naturally do" and bases this claim on the numbers on the tablet just mentioned, where 240 is written iv (Hincks uses Roman numerals to render the cuneiform numbers), and where "iii.xxviii, iii.xii, ii.lvi, ii.xl, etc." stand for "208, 192, 176, 160, etc.". Henry Rawlinson's contribution to the topic in [1855] (already communicated to Hincks when the second paper of the latter was in print, in December 1854) consists of a long footnote (pp. 217–221) within an article on "The Early History of Babylonia", in which he states that the values ascribed by Berossos [ed. Cory 1832: 32] to σάρος (*šār*), νῆρος (*nēru*) and σώσσος (*šūši*), namely respectively 3600, 600 and 60 years, are "abundantly proved by the monuments" (p. 217), giving as further confirmation an extract of "a table of squares, which extends in due order from 1 to 60"(pp. 218–219), in which the place-value character of the notation is obvious but only claimed indirectly by Rawlinson.[4]

Oppert wrote a number of major papers on metrology [1872; 1885; 1894; etc.], which confirm the picture; the sources are archaeological measurements combined with evidence contained in written sources, mostly indicating concrete measures rather than dealing with metrology, and comparison with other metrologies.[5]

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[3] Archibald Henry Sayce, when returning to the text (now identified as K 490) in [1887: 337–340], reinterprets the topic as a table of lunar longitudes (Hincks seems to be right).

[4] That Rawlinson is anyhow also interested in the mathematics *per se* and not only as a means for chronology (after all, he was interested in *everything* Assyro-Babylonian) is however revealed by what comes next in the note, namely that "while I am now discussing the notation of the Babylonians, I may as well give the phonetic reading of the numbers, as they are found in the Assyrian vocabularies".

[5] Since Mesopotamian metrology varies much more over the epochs than Oppert had

The first of these draws, inter alia, on the "Esagila tablet", a copy from 229 BCE of an earlier text and described by Marvin Powell [1982: 107] as

> a key document for Babylonian metrology, because it 1) describes in metrological terms a monument that has been explored and carefully measured, 2) links the standard system of mensuration with the Kassite system, 3) makes it possible to identify the standard cubit with the NB [Neo-Babylonian / JH] cubit, and 5) enables us to calculate the absolute length of these units as well as the area of the i k u used in both Sumerian-OB [Old Babylonian / JH] and in Kassite-Early NB documents

– which means that it fits precisely into the general pattern of Oppert's and contemporary work on metrologies, even though the full exploitation of the document was not possible at a moment when the Esagila complex had not yet been excavated, and when relative chronologies preceding the neo-Assyrian epoch were still not firmly established.[6]

Over the following five decades, work with this focus was pursued by a number of scholars – beyond Oppert also Vincent Scheil, François Thureau-Dangin, Herman Hilprecht, Franz Heinrich Weißbach, Arthur Ungnad, François-Maurice Allotte de la Fuÿe, Louis Delaporte, Ernst Weidner and others.[7] The outcome was a fair understanding of the many different metrologies [Thureau-Dangin 1909; 1921] including brick metrology [Scheil 1915b]; of the place-value system and the function of tables of reciprocals [Scheil 1915a; 1916];[8] and of techniques for area determination [Allotte de la Fuÿe 1915] – all (as far as allowed

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imagined, it is obvious that the comparative method led him astray as often as to the goal. The task may be claimed only to have been brought to a really satisfactory end by Marvin Powell [1990].

6 This is well illustrated by the chapter "History and Chronology [of Chaldaea]" in the second edition of George Rawlinson's *Five Great Monarchies of the Ancient Eastern World* from [1871: I, 149–179]. The author can still do no better than his brother Henry had done in [1855] – all we find is a critically reflective combination of Berossos and Genesis, with a few ruler names from various cities inserted as if they were part of one single dynasty.

This was soon to change. In [1885: 317–790], Fritz Hommel was able to locate everything from Gudea onward in correct order; absolute chronologies before Hammurapi were still constructed from late Babylonian fancies (Hommel locates Sargon around 3800 BCE and Ur-Nammu around 3500 and lets the Ur, Larsa and Isin dynasties (whose actual total duration was c. 350 years) last from c. 3500 until c. 2000 BCE – pp. 167*f*).

7 See [Friberg 1982: 3-27].

8 Actually, Scheil's understanding was not broadly accepted: Meissner [1920: II, 387] from 1925 does not know about sexagesimal fractions. Meissner also mixes up the place-value and the absolute system.

by available sources) in contexts extending from the mid-third (occasionally the outgoing fourth) to the late first millennium BCE.

Hilprecht's discussion of "multiplication and division tables" from [1906] deserves special mention. It made available an important text group, but also cast long shadows: not understanding sexagesimal fractions and thus wishing all numbers occurring in the tables to be integers, he interpreted the table of reciprocals as a table of division of 12,960,000 – a number he then finds (p. 29) in an interpretation of Plato's *Republic* VIII, 546B–D (the notoriously obscure passage about the "nuptial number"). That allowed him to confirm a statement he quotes from Carl Bezold on p. 34:

> Mathematics was with the Babylonians, as far as we now know, first of all in the service of astronomy and the latter again in the service of a pseudo-science, astrology, which probably arose in Mesopotamia, spread from there and was inherited by the gnostic writings and the Middle Ages [...].

In this way, Babylonian mathematical thought was made much more numerological and linked much more intimately to esoteric wisdom than warranted.[9]

The earliest "properly mathematical" texts

All these insights built on the combination of archaeological measurement (of building structures and of metrological standards) with various kinds of written documents and (with gradually dwindling importance) comparative studies. None of this material except some tables of multiplication, reciprocals and powers belonged to genres which were soon to be considered as "properly mathematical" texts.[10]

A few such texts were published during the years 1900–1928. In 1900, hand copies without transliteration of the two extensive Old Babylonian problem collections BM 85194 and BM 85210 appeared in CT IX. However, since these

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[9] Esoteric numerology certainly left many traces in Mesopotamian sources – but not in sources normally counted as "mathematical"; the only exception is a late Babylonian metrological table starting with the sacred numbers of the gods (W 23273, see [Friberg 1993: 400]). Apart from that, even the text corpus produced by the Late Babylonian and Seleucid priestly environment kept the two interests strictly separate.

[10] I disregard the "metrological tables", which were not yet understood as mediators between the various metrologies and the place value system. I also disregard mathematical astronomy, where the extension of the place value system to fractions had been understood better [Epping 1889: 9*f*], [Kugler 1900: 12, 14], without this understanding being generalized, cf. [Scheil 1915a: 196].

texts could be judged by Ernst Weidner [1916: 257] to be “the most difficult handed down in cuneiform”, it is barely a wonder that no attempt was made to approach them for long.[11] In [1916: 258], Weidner announced to have lately “had the occasion to copy a whole sequence of similar texts”, and he gave a transliteration and an attempted translation of two sections from one of them, the tablet VAT 6598; Weidner’s contribution was immediately followed up by Heinrich Zimmern [1916] and Ungnad [1916], both of whom improved the understanding of the text and the terminology in general, drawing on the same text and on the texts published in CT IX, from which Ungnad transliterated and translated a short extract in [1916] and another one in [1918].

The next step was C. J. Gadd’s publication of a first fragment of BM 15285, a text about the subdivision of a square into smaller squares, smaller triangles, etc. This text was quite different from those published previously, but a few terms were shared, which confirmed readings proposed by Weidner and Zimmern.

Also in [1922: pl. LXI–LXII], Thureau-Dangin published hand copies of AO 6484, a major Seleucid problem text, but without seeing more in it than “arithmetical operations”.

Finally, Carl Frank published six mathematical texts from the Strasbourg collection in [1928], with transliteration and attempted translation.

By then, however, the study of cuneiform mathematical texts had also been taken up at Neugebauer’s Göttingen seminar. In 1985 Kurt Vogel told me about the immense astonishment when one morning Hans-Siegfried Schuster related that he had discovered solutions of second-degree equations in a cuneiform text. Vogel did not date the event, but it must have taken place in late 1928 or (most likely, see below, note 36) very early 1929.

Confronted readings

Before we shift our attention to this new phase, we may look at what had been achieved – and what not yet – up to then by confronting Weidner’s interpretation and the commentaries it called forth with that of Neugebauer of the same text in MKT.[12] Some of the differences, we should be aware, come

[11] Weidner mentions as the only exception “an occasional notice” by Hommel in a *Beilage* to the *Münchener Neuesten Nachrichten* 1908, Aug. 27, Nr. 49, p. 459, which I have not seen. He says nothing about its substance being in any way important, only that it interprets the final clause *ne-pé-šum* of problems as “quod erat demonstrandum”.

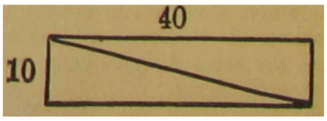
[12] This is certainly “whiggish” historiography – and it has to be, if our aim is to locate Neugebauer’s achievement in its historical context.

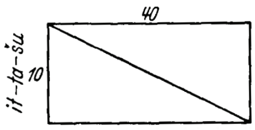
from the fact that Neugebauer’s transliterations follow the conventions of Thureau-Dangin’s *Syllabaire accadien*, which was only published in [1926].

This is Weidner’s transliteration and translation from [1916: 258*f*] (left) and Neugebauer’s treatment of the same text in MKT (I, 280, 282) (right):[13]

1. *2 ú da 40 ú šir zi-li-ip-tu-šu en-nam za-e 10 sag*
   2 Ellen (?) Seite (?), 40 Ellen Tiefe (?). Seine Diagonale berechne du. 10 (ist) die Höhe
2. *šá-ne 1 40 ta-mar ka-bi-rum 1 40 a-na 40 ú šir i-ši-ma*
   als Quadrat 1 40 erhältst du. Die Quadratfläche 1 40 auf 40 Ellen Tiefe (?) ist sie,
3. *1 6 40 ta-mar a-na tab-ba 2 13 20 ta-mar a-na 40 ú šir*
   1 6 40 erhältst du. Zu verdoppeln, 2 13 20 erhältst du. Zu 40 Ellen Tiefe (?)
4. *daḫ-ḫa 42 13 20 zi-li-ip-to ta-mar ne-pi-šum*
   hinzufügen, 42 13 20 als Diagonale erhälts du. (Also) ist es gemacht worden.

1. 2 kùš dagal 40 kùš sukud *sí-li-ip-ta-šu* en-nam za-e 10 sag
   2 Ellen Weite, 0;40 Ellen(sic) Höhe, Seine Diagonale (ist) was? Du? 0;10, die Breite
2. *šu-tam-ḫir* 1,40 *ta-mar qà-qá-rum* 1,40 *a-na* 40 kùš sukud *i-ši-ma* (?)
   quadriere. 0;1,40 siehst Du (als) Fläche. 0:1,40 mit 0;40 Ellen(sic) Höhe multipliziere und (?)
3. 1 6 40 *ta-mar a-na* tab-ba 2,13,20 *ta-mar a-na* 40 kùš sukud
   0;1,6,40 siehst Du. Mit ⟨2⟩ verdopple. 0;2,13,20 siehst Du. Zu 0;40 Ellen (sic) Höhe
4. daḫ-ḫa. 42,13,20 *ṣí-li-ip-ta ta-mar ne-pé-šum*
   addiere. 0;42,13,20 (als) Diagonale siehst Du. Verfahren.





The drawing on the tablet, as rendered by Weidner (left) and Neugebauer (right)

As already seen by Weidner [1916: 359] (the diagram, indeed, leaves little doubt), the text contains a “calculation of the diagonal of a rectangle whose sides are given”. If the given sides are *a* and *b*, Weidner states the diagonal to be

[13] Here and in what follows, when quoting transcriptions and transliterations (also of single words and signs), I follow the conventions of the respective originals. When speaking “from the outside”, on my own, I follow modern conventions. Since the delimitation is not always clear, some inconsistencies may well have resulted.

$a+\frac{2a\cdot b^2}{3600}$, whereas Neugebauer gives $a+2a\cdot b^2$;[14] Weidner's divisor 3600 is a symptom that he writes at a moment when he has certainly more or less understood the use of the place value system even for fractions but still writes in a spirit untouched by this understanding.[15]

Some of the other differences between the two transliterations hinge on different ways to render the same cuneiform character even though it is understood in the same way, as can be seen from the translations. For instance, Weidner has Ú, the sign name, where Neugebauer has kùš, the Sumerian reading of the sign when meaning a cubit ("Elle"), as it had been identified in the meantime.[16] Such changes are immaterial for our present concern.

Somewhat more pertinent is the disagreement in the first line concerning DA/dagal. These are different signs but rather similar in the Old Babylonian

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[14] Neugebauer tries to make sense of this impossible formula by interpreting it as an approximation to $a+\frac{2a\cdot b^2}{2a^2+b^2}$; [Neugebauer 1931a: 95–99] explains the origin of the guess, which he finds in the music theory of Nicomachos and Iamblichos – classical Antiquity remained a resource when other arguments were not available.

Difficulties in the handling of the sexagesimal system may be the reason that Weidner did not discover that the formula – adding a length and a volume – is impossible because a change of measuring unit would not change the two addends by the same factor (this is the gist of "dimension analysis").

[15] In detail: Weidner supposes the dimensions of the rectangle to be 10 and 40, even though the initial "2 cubits" should make him understand that 10 stands for 10´, and 40 in consequence (if the calculations are to be meaningful) for 40´ – both corresponding to the unit nindan (1 nindan = 12 cubits); instead he wonders (col. 259) what these 2 cubits may be. Weidner therefore supposes the product to be 4000, about which he says that "it is written in cuneiform as 1 6 40, i.e., 1 (3600) + 6 (·60) + 40. But this number can also be understood as $1+6(\cdot\frac{1}{60})+40(\cdot\frac{1}{3600}) = \frac{4000}{3600} = 1{,}11$".

A small remark on notations: the ´–˝ notation was used (and possibly introduced) by Louis Delaporte in [1911:132] (´ and ˝ only); Scheil [1916: 139], immediately followed by Ungnad [1916: 366], uses ´, ˝ and °, as does later Thureau-Dangin. Strangely, Neugebauer believed in [1932a: 221] that the °-´-˝ notation had been created "recently" by Thureau-Dangin (similarly MKT I, p. vii n. 5); I have not noticed references from his hand to [Scheil 1915a], but he had referred to Ungnad [1916] on several occasions – e.g., [Neugebauer 1928: 45 n.3]. Neugebauer's own notation goes back at least to [1929: 68, 71].

[16] See, e.g., [Thureau-Dangin 1921: 133].

period.[17] Weidner's mistake illustrates the difficulty of reading a cuneiform text whose genre and terminology is as yet unknown. Fortunately for him, the two words are more or less synonymous according to his dictionary [Delitzsch 1914: 130*f*], respectively "side" and "breadth".

Most significant are the cases where Weidner, as he states himself, had to guess at a meaning from the context – the context presented by the present text as well as that of the CT-IX texts, which Weidner had evidently studied intensely without getting to a point where he could make coherent sense of them.

This starts with ŠIR (now UZU = šir$_4$), which again is similar to the sign read by Neugebauer (SUKUD, meaning "height"[18]); since the sign is often found in CT IX, Weidner concludes that it must refer to a dimension, and he finds in Rudolph Brünnow's list from [1889: 200 #4558] that it may stand for *naqbu*, "depth".[19] This seems to make sense, after which the interpretation of *ziliptum* "follows by itself from the context". Neugebauer's spelling *ṣi-li-ip-ta* corresponds to modern orthography, but even he is not able to connect the word to the verb *ṣalāpum*, whose sense "cross out" was not yet established – at least still not in [Bezold 1926: 113, 238].

The next word *en-nam*, thus Weidner with many references to CT IX, "must mean 'calculate'". Neugebauer's "was" corresponds to what he had observed in [1932b: 8] – that en.nam stands where other texts have the interrogative particle *mīnûm*.[20]

Weidner's reading of *za-e* as "you" is correct, and conserved in MKT. However, Weidner connects it to his preceding presumed imperative; it was Ungnad [1916: 363*f*] who pointed out that this word, here and often in the CT-IX texts, marks the beginning of the calculation. Ungnad does not feel sure that

[17] For such similarity I rely on [Labat 1963].

[18] This reading goes back to [Zimmern 1916: 323].

[19] Now *nagbu*, interpreted "spring, fountain, underground water" ([CAD XI, 108], cf. [AHw 710]). The error was pointed out by Ungnad [1916: 363], who also proposed the reading sukud, "height".

[20] In [1929: 88], Neugebauer still accepted Weidner's interpretation. Arguments that a verbal imperative was most unlikely and an alternative orthography for *mīnûm* unsupported by other evidence were first given by Thureau-Dangin [1931: 195*f*]; the idea that it is a (pseudo-)Sumerogram for that word was first hinted at by Albert Schott, see [Neugebauer 1932b: 8 n. 18].

a Sumerian z a . e, "you", is meant, and as we see Neugebauer adopts his doubt.[21]

šá-ne is interpreted by Weidner as "square" simply because 1 40 is the square of 10; he confesses not to be able to explain it further; the correct reading of the sign group as *šu-tam-ḫir*, adopted by Neugebauer, was suggested by Zimmern [1916: 322*f*] and explained as the "imperative of a [verb] *šutamḫuru*, 'to raise to square' (literally let stand against, let correspond to each other)".

The ensuing *ka-bi-rum* is interpreted (reasonably, if only the reading had been correct) as "breadth", and Weidner then supposes that it refers to the square understood as a "broad rectangle". The proper reading (as given in MKT) means "ground" (in mathematical texts the basis of a prismatic volume).

Weidner does not comment upon his interpretation of *ta-mar* in line 2 (and again in line 3) as "erhältst du", but it is obviously derived from the context. Neugebauer's philologically correct reading "you see" goes back to Ungnad [1916: 364].

In the end of line 2, Weidner understands *i-ši-ma* as "it is", which forces him to understand *a-na* (translated "auf") as a multiplication (without specifying that this is what he does).[22] Zimmern [1916: 322] and Ungnad [1916: 364] point out that *i-ši* is the imperative of *našûm*, "to raise", and that this term (always "raising *to*", which explains *a-na*) is used repeatedly for multiplication in CT IX; this understanding (but not the translation) recurs in MKT.

In the next line, the interpretation of *tab-ba* as doubling is correct, and goes back to [Delitzsch 1914: 152]; only Neugebauer's familiarity with a much larger range of texts allows him to see that the scribe has omitted a number 2 – yet even he, trapped by the interpretation of the operation as just multiplication, does not see that *ana* should be taken in its ordinary sense "to" (doubling "until twice").

Also the interpretation of *daḫ-ḫa* as addition is correct.[23] The derivation of the closing phrase *ne-pí-šum* from the verb *epēšum*, "to do" / "to proceed" is correct too, even though the actual grammatical interpretation is mistaken, as

[21] No longer needed, since other texts have the Akkadian *atta*.

[22] I wonder whether Weidner was led to this conclusion by numerical necessity alone (1 6 40 being indeed the product of 40 and 1 40) or by the parallel use of ἐπί in Greek mathematics.

[23] Unfortunately, Weidner's commentary equates this Sumerian word with *eṣēpu*, building on a hint in Delitzsch [1914: 134]; Delitzsch's supportive examples are conjugated forms of *waṣābum*, also the actual equivalent in Old Babylonian mathematical texts.

pointed out by Zimmern [1916: 322] and Ungnad [1916: 364] ; they both correct to "Verfahren", "way to proceed", as taken over by Neugebauer.

Weidner's article deals not only with this but also with another section of the same tablet, in which a different approximation to the same diagonal is found, namely $a+\frac{b^2}{2a}$ – much better, both by being meaningful and by being more precise even with the actual numbers and unit. On the tablet, this section comes first, and with hindsight it seems a reasonable assumption that the second method (the one Weidner presents first) is a second approximation gone awry.[24]

In connection with the first approximation, Weidner makes only two new terminological observations, one wrong and one slightly problematic. Firstly, he translates the passage *½ 2 30 dùg-bi 1 15 ta-mar* as "Die Hälfte von 2 30, als seinen Quotienten 1 15 erhältst du", believing from his inspection of the CT-IX texts (probably from parallels to the present passage) that DÙG stands for the result of a division, and taking *bi* to be the Sumerian possessive suffix (thus "its quotient"). Zimmern [1916: 322] corrected this Sumerographic reading, replacing it by phonetic Akkadian *ḫi-pí* "break" (*viz* "break off ½") – cf. also [Ungnad 1916: 364 n.5] (the signs read by Weidner and Zimmern are the same).

Secondly, Weidner [1916: 261] states that "*igi-dú-a* with a number enclosed between *igi* and *dú* means substantially [*sachlich*] that the ensuing higher power of 60 is divided by the enclosed number". Zimmern [1916: 324] specifies that *igi* must be understood as "part", and *dú* (d u $_8$ since Thureau-Dangin's *Syllabaire*) as "to split", while Ungnad [1916: 366] suggests an interpretation that comes close to the determination of the reciprocal of the enclosed number[25] – clearly the understanding of the Old Babylonian calculators, as was soon to be known with certainty, whereas that of the Ur III inventors had probably been the corresponding fraction of 60 (cf. [Scheil 1915a] and [Steinkeller 1979]) – closer indeed to Weidner's understanding without being identical.

Beyond the attempted "substantial" and philological interpretation of the mathematics of the text, Weidner [1916: 259] also speaks about its purpose:

> Oriental science was never undertaken for its own sake but was always science with a purpose [*Tendenzwissenschaft*], and therefore the present piece of text was of course not written down by the Akkadian in order to show how right triangles were

24 A possible interpretation is offered in [Høyrup 2002: 271*f*]

25 He does not use the term "reciprocal" but speaks about the operation of dividing 1 by the number in question.

> calculated in his times, but it must have had a very real background. It is probably the calculation of an architect or a surveyor, who has then executed his task in agreement with the calculation.

Later, as a commentary to the only approximate character of the calculations, Weidner continues thus:

> However, if we take into account, as already pointed out, that this is nothing but applied mathematics in the service of the architect and the surveyor, then we arrive at a milder judgement. We know sufficiently well, indeed, that these gentlemen do not always insist on maximal precision in their work.

The insight that the text might be a school problem had to wait.

Beyond objections and direct commentaries to Weidner, some further important observations concerning the terminology are made by Zimmern and Ungnad. Zimmern [1916: 323] notices that two different terms express addition, *daḫ-ḫa*[26] and UL.GAR. Ungnad [1916: 367] points out hat there are also two ways to express subtraction, the operation BA.ZI (Akkadian *nasāḫum*, “to tear out”) and the observation that one entity exceeds (DIR) another one by so and so much; he also mentions KIL.KIL (NIGIN) (col. 366*f*) as a term for squaring and reminds of the already known use of ÍB.DI (í b . s i 8) for “square root”.

Both also end their articles by hoping for new texts and new insights in the area. Apart from a transliteration and translation of another problem from CT IX (namely, BM 85194, obv. III, 23–30) produced by Ungnad in [1918], it lasted quite a while before this wish was fulfilled.

As already mentioned, the next text to be discussed was a large fragment of BM 15285, a text about the subdivision of a square in various smaller figures [Gadd 1922]. It contains drawing of these together with verbal descriptions, and even though drawing and description are only conserved together in a few cases, Gadd was able to make new observations [1922: 151] on the terminology – not least to identify *mitḫartum* with a square, which, as he states, agrees perfectly with Zimmern’s reading *šutamḫurum*, and to show that ÍB.DI (í b . s i 8) was used ideographically for *mitḫartum*. He was also able to confirm the interpretation of *kippatum*[27] as “ring” / “circle”, as derived already by Thureau-Dangin on the basis of non-mathematical texts, and to read SAG.KAK as “triangle”.

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26 To this he links Akkadian *ruddûm* instead of *waṣābum* – a mistake in the context of the mathematical texts, as it was to turn out when more of these became known.

27 Gadd says *kibbatum*, but that orthography has already disappeared in [Bezold 1926: 147].

Also in 1922, Thureau-Dangin published a hand copy of AO 6484, a fairly long mathematical text from the Seleucid era, no. 33 of 58 texts from the collections of the Louvre and the Musée du Cinquantenaire. However, all he has to say about it is that it contains "arithmetical operations (fragmentary tablet). Probably from the first half of the second Seleucid century". In spite of his interest in anything that had to do with mathematics, evident since his astute analysis of a field plan from Ur III in [1897], he did not return to the text in the following years, which can probably be taken as evidence that he understood no more than what he had already said in 1922.

Then we come to Frank's edition from [1928][28] of 50 texts from the Strasbourg collection, six of which were mathematical. Frank offers hand copies, transliteration and German translation of some of the texts, partial transliteration mixed with explanatory translation of the others – and a short general commentary (pp. 19*f*). In this commentary it is stated that

> the following texts, like those close to them in CT IX, cannot yet be understood in all details. More intensive work than what is intended here, and indeed on all "mathematical" texts, would in itself be most welcome.

The quotes around "mathematical", however, point back to an important insight, entirely missed in 1916: that these texts are *Rechenaufgaben*, that is, school texts.

Neugebauer enters the game

Very soon – as a matter of fact almost immediately – more intensive work was indeed taken up. Neugebauer had already published a paper in [1927] about the origin of the sexagesimal system, and in [1928] a short note from his hand pointed out that the approximation discussed second in [Weidner 1916] might be meant as an approximation to the exact value predicted by the Pythagorean theorem; he also suggested that both Greek geometry and the Indian śulba-sūtras might have borrowed from the Babylonians. The watershed was [Neugebauer 1929], appearing in the first issue of the *Quellen und Studien* B[29] and dealing

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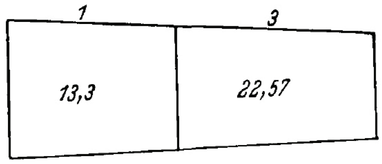
[28] According to what is written on p. 6, the hand copies were made in 1914, after which Frank had no more access to the tablets; he only received his old copies and notes in 1925, after which he could resume working on them. Actually, what Frank received through the mediation of a friend were only draft hand copies; what he had originally prepared for an edition arrived too late [Waschow 1932a: 211], cf. [Thureau-Dangin 1934].

[29] Full title *Quellen und Studien zur Geschichte der Mathematik, Astronomie und Physik*. Abteilung B: *Studien*.

with the mathematical Strasbourg texts. How much had happened can be illustrated by a confrontation of Frank's text of no. 10 with Neugebauer's new translation [1929: 67*f*] and transliteration (MKT I, pp. 259*f*) (the article from 1929 brings a translation only and a handful of notes correcting Franks transliteration). The figure in Neugebauer's first line is taken from his transliteration, but corresponds to what is found in his translation.



	Frank		Neugebauer
1	Oben Zahlen: 1, 3; 783, 1377.	1	
2	sag-gi-gud(!) (so wohl, nicht bi) *ina libbi* 2 íd-meš 783 a-šà(g) [sa]g(?) ein Viereck (*ummatu*), darinnen 2 'Flüsse', 738 das erste Feld,	2	SAG-KI-GUD *i-na libbi* 2 id-meš 13,3 a-šà an Ein Viereck, darinnen zwei Flüsse, 13,3 (= 783) die obere Fläche,
3	1377 a-šà(g) *šanū*^{u} ... 3 (?) gál uš-ki ... 1377 das zweite Feld ... $^{1}/_{3}$ untere Länge ...	3	22,57 a-šà ⟨ki-⟩2 i[gi] 3 gál uš ki *i-n*[*a*] 22,57 (=1377) die zweite Fläche [und] ein Drittel der unteren Länge für
4	uš-an-na-ta sag(!)-an-na *eli* RI dirig-a von der oberen Länge an die obere Breite größer als RI	4	uš an-na *ša* sag an-na u-gù RI dirig die obere Länge, die obere Breite größer als die Trennungslinie
5	*ù* RI eli sag-ki-ta dirig gar-gar ... igi (?) und RI größer als die untere Breite ...	5	*ú* RI u-gù sag ki-ta dirig gar-gar $_{[}$36$_{]}$ und die Trennungslinie größer als die untere Breite, zusammen
6	uš-ne-ne sag-meš *ù* RI en-nam Die Längen, Breiten und RI berechne.	6	uš-ne-ne sag-meš *ù* RI en-nam Die Längen, Breiten und die Trennungslinie berechne.
7	za-e ak-da-zu-de 1 *ù* 3 ḫe-ga[r] Wenn du dabei (so) verfährst: 1 und 3 (seien) angesetzt(?);	7	za-e ki-da-zu-dè 1 *ù* 3 ḫé-gar Du verfährst so: 1 und 3 lege (?)
8	1 *ù* 3 gar-gar 4 igi 4 dù-ma 15 1 u. 3 addiert = 4; (60) durch 4 dividiert = 15;	8	1 *ù* 3 gar-gar 4 igi 4 du$_{8}$-ma 15 1 und 3 zusammen (ist) 4. Das Reziproke von 4 (ist) 0;15 (= 1/4) und
9	15 *a-na* 36 nim 540 in-se 540 *a-n*[*a*] 15 auf 36 erhöht gibt 540; 540	9	15 *a-na* 36 nim 9 in-sum 9 *a-na* 0;15 (= 1/4) mit 36 erhöht gibt 9. 9 mit
10	1 nim 540 in-se 540 *a-na* 3 nim 1620 auf 1 erhöht gibt 540; 540 auf 3 erhöht = 1620;	10	1 nim 9 in-sum 9 *a-na* 3 nim 27 1 erhöht gibt 9. 9 mit 3 erhöht 27.
11	540-ta sag-an-na *eli* RI dirig um 540 ist die obere Breitseite größer als RI;	11	9 *ša* sag an-na u-gù RI dirig Um 9 ist die obere Breite über die Trennungslinie größer,
12	1620 ta (?) RI *eli* sag-ki-ta dirig um 1620(?) ist RI größer als die untere Breitseite.	12	27 *ša* RI u-gù sag ki-ta dirig um 27 ist die Trennungslinie gegen die untere Breite größer:
13	igi 1 dù 1 *a-na* 783 nim Divisor 1. 1 auf 783 erhöht	13	igi 1 du$_{8}$ 1 *a-na* 13,3 nim Das Reziproke von 1 ist 1. Mit 13,3 (= 783) erhöht

14 783 in-se igi 3 dù 20 *a-na*
gibt 783. Divisor 3 (d. h. 60: 3) 20 auf

15 1377 nim 27540 in-se
1377 erhöht macht 27540.

Rs.

1 783 *eli* 459 en-nam dirig
783 ist größer als 459: berechne die Differenz.

2 324 dirig 1 *ù* 3 gar-gar 4
324 ist die Differenz. 1 und 3 addiert:=4;

3 bar(!) (= *mišil*) 4 (!) QU 2 igi 2 dù 30 *a-na* 324
die Hälfte von 4 geteilt: 2; (60) durch 2 dividiert = 30, auf 324

4 9720 in-(se)-*ma* nu- GIR 9720 nu-dù
gibt 9720, nicht ... ; 9720 nicht teilbar.

5 en-nam *a-na* 9720 ḫe-gar *ša* 540 in-se
berechne. Zu 9720 soll gelegt werden, ‘daß, was 540 gibt’.

6 200 ḫe-gar igi 200 dù 18 in-še
200 sei gelegt, durch 200 dividiert gibt 18(?);

7 18 *a-na* 1 nim 18 uš-an(!?)-na 18
18 auf 1 erhöht: 18 die obere Langseite; 18

8 *a-na* 3 nim 54 uš-ki us-ki-ta
auf 3 erhöht: 54 die untere Langseite; von (?) der unteren Langseite

9 *mišil*(!) 36 sag(?)-ne 18 (statt 17!) *a-na* 72 nim
die Hälfte von 36 die Breiten(?) 18(!), auf 72 erhöht

10 1296 *i-na* 36 a-šà(g) dù 864
1296; durch 36 Felder(?) teilbar;

11 igi 72 ba-dù 50 *a-na* 864 nim
864 durch 72 teilbar; 50 auf 864 erhöht

12 43 200 (!) in-se 22 4*a-na* 26(!) daḫ-ḫi-*ma* 48 GAB(?)
gibt 43200 (!); 22 zu 26 (!) hinzugefügt = 48, teilbar (?),

13 48 sag-an-na 12 *a-na* 27 daḫ
48 obere Breitseite; 12 zu 27 hinzugefügt

14 13,3 in-sum igi 3 du$_8$ 20 *a-na*
gibt 13,3(=783). Das Reziproke von 3 (ist) 0;20 (= 1/3). Mit

15 22,57 nim 7,39 in-sum
22,57 (=1377) erhöht gibt 7,39 (=459).

Rs.

1 13,3 u-gù 7,39 en-nam dirig
13,3 (= 783) gegen 7,39 (= 459) berechne den Überschuß.

2 5,24 dirig 1 *ù* 3 gar-gar 4
5,24 (= 324) ist der Überschuß. 1 und 3 zusammen (ist) 4.

3 1/2 4 gaz 2 igi 2 du$_8$ 30 a-na 5,24
Halbiere 4 (das ist) 2. Das Reziproke von 2 (ist) 0;30 (= 1/2). Mit 5,24(=324)

4 2,42 in⟨-sum⟩-ma nu-GÌR 2,42 nu-du$_8$
gibt 2,42 (= 162), nicht .... . 2,42 (= 162) nicht teilbar.

5 en-nam *a-na* 2,42 ḫé-gar *ša* 9 in-sum
Berechne mit 2,42 (= 162) gelegt, was 9 gibt.

6 3,20 ḫé-gar igi 3,20 du$_8$ 18 in-sum
0;03,20 (= 1/18) gelegt. Das Reziproke von 0;03,20 (= 1/18) gibt 18.

7 18 *a-na* 1 nim 18 uš an-na 18
18 auf 1 erhöht (ist) 18. Die obere Länge (ist) 18.

8 *a-na* 3 nim 54 uš ki {uš ki-ta}
Mit 3 erhöht: 54 (ist) die untere Länge von der [oberen] Länge aus

9 ½ 36 gaz ne 17(sic) *a-na* 1,12 nim
Halbiere die Breite 36. 18 mit 1,12 (= 72)

10 21,36 *i-na* 36 a-šà du$_8$ 14,24
(ist) 21,36 (=1296). Von 36,00 (= 2160) subtrahiert (ist) 14,24 (= 864).

11 igi 1,12 uš du$_8$ 50 *a-na* 14,24 nim
Das Reziproke von 1,12 (= 72), der Länge, ist 0;00,50 (= 1/72). Mit 14,24 (= 864) erhöht

12 12 in-sum 12 *a-na* 36 daḫ-ma 48
gibt 12. 12 mit 36 addiere. 48 [ist es.]

13 48 sag an-na 12 *a-na* 27 daḫ
48 die obere Breite, 12 mit 27 addiert:

14 39 RI 12 sag-ki-ta in-se 39 RI, gibt 12 von der unteren Breitseite aus.	14 39 RI 12 sag ki-ta in-sum 39, die Trennungslinie, von 12, der unteren Breite, gibt es.

The most striking difference between the two translations is probably that Neugebauer conserves the sexagesimal place value notation (though still, probably as help to readers not accustomed to it, translating parenthetically into decimal notation). This is in any case the reason he gives to make a revised translation instead of just copying Frank, and we see indeed that Frank time and again locates the numbers in a wrong sexagesimal order of magnitude, which did not facilitate his understanding of this very complicated procedure.[30] Once this was corrected, Neugebauer was also able to correct a number of Frank's readings – but this was, if we are to believe his words, at least in the main a secondary effect of getting the numbers right.[31]

Of particular importance was Neugebauer's insight that igi *n* should be understood as the reciprocal of *n*. As we have seen, this almost coincides with what Ungnad had said in 1916 (but not fully, cf. below, note 43). However, Neugebauer's explanation was much more transparent, and from now on it was generally accepted.[32]

From 1929 to 1935 there were few important but a number of less decisive changes in Neugebauer's translation. In obv. 2, the quadrangle becomes a trapezium, and the rivers become strips – but both in agreement with the commentary from 1929, there is no change in the interpretation. Obv. 4 becomes clearer, "die obere Länge. Was die obere Breite über die Trennungslinie hinausgeht", and the beginning of obv. 5 and a number of similar passages are modified correspondingly. In obv. 5 and elsewhere, "zusammen" becomes "addiert, and in obv. 6 and elsewhere the imperative "berechne" for en.nam becomes "was", in agreement with the understanding of this term as a logogram for *mīnûm*. In obv. 7, "Du verfährst so" becomes "Du bei deinem Verfahren", in better agreement with the Sumerian expression and indeed a perfect translation

[30] In MKT I, p. 263, Neugebauer characterizes it as *umwegig*, "roundabout". A possible understanding of the underlying idea, based on a proposal by Jöran Friberg, is in [Høyrup 2002: 241–244]. The procedure itself was perfectly understood by Neugebauer

[31] The interpretation of RI as "Trennungslinie", the parallel transversal dividing the trapezoidal quadrangle into two strips, is probably an exception to this rule; according to p. 70, n. 14 it was due to V. V. Struve.

[32] As I have experienced several times, this does not mean that today's Assyriologists are generally familiar with the place value system. Indeed, unless they work on astronomical texts or mathematical school texts (very few do), they never see it in use.

of the corresponding Akkadian phrase *atta ina epēšika*, with which Neugebauer was now familiar; further, "lege" becomes "mögest du nehmen", in better agreement with the precative prefix ḫe but less close to the semantics of g̃ar; similarly elsewhere. In obv. 8, "Das Reziproke von 4 (ist) 0;15" becomes "Das Reziproke von 4 gebildet und 0;15 (ist es)"; this at least renders the presence of a verb du$_8$, even though it semantics ("split" / "detach", correctly described by Zimmern, cf. above) is not respected[33] (nor the imperative found in parallel syllabic texts); similarly elsewhere. In obv. 9, a change from literal to "substantial" translation takes place, and "erhöht" becomes "multipliziert". In rev. 3, on the other hand, "halbiert" becomes "abgebrochen" – here, the "substantial" translation is replaced by a literal one. Rev. 5 becomes "Was mit 2,42 sollst du nehmen, das 9 gibt", both clearer and closer to the original (apart from the semantics of g̃ar) than the 1929 version. In rev. 8, MKT understands that the repetition in the end is a dittography, and the attempted repair from 1929 disappears. In rev. 9, "subtrahiert" becomes "brich ab", an attempted return from "substantial" to literal translation – not successful, "brich ab" is used in the preceding line and elsewhere for gaz / *ḫepûm*, while du$_8$ elsewhere designates the "detaching" or "splitting off" of a reciprocal (rendered "substantially" in MKT by "gebildet").[34] In rev. 12, "mit 36 addiere" becomes "zu 36 addiere", which fits the preposition *ana* better but still conflates the symmetrical operation g̃ar.g̃ar, connected with *u* ("and"), and the asymmetric operation daḫ, connected with *ana*; similar rev. 13.

In the programmatic statement for MKT (I, p. viii) it is said that "in principle, the translation is obviously literal",[35] but then explains why this principle cannot always be respected – a dilemma every translator knows too well. As we see, the 1929 version followed the same rule – but not in the same way; sometimes, MKT becomes more literal than the early translation, sometimes less. For the purpose of understanding what Neugebauer saw as the mathematical structure of the texts, this was immaterial.

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33 dù, we remember, had become du$_8$ in Thureau-Dangin's reform.

34 Footnote 5a in [Neugebauer 1930a: 122] reveals that "subtrahiert" was chosen originally because Neugebauer had mistakenly believed to improve Frank's reading *a-šà(g) dù* by changing it into *usuḫ*. The same note shows that Neugebauer is now perfectly aware that the correct literal translation would be "abgespalten"; we may perhaps presume the deviating translation in MKT to be nothing but a slip.

35 "Die Übersetzung ist selbstverständlich im Prinzip eine wörtliche".

The 1929-article also dealt with Frank's text no. 8, in front of which Frank had given up, offering no transliteration and only translation of small isolated bits. The text is indeed very difficult, firstly because it is badly broken, secondly because it gives only problem statements (fortunately illustrated by diagrams) but no indication of the procedure. Also fortunately they can be arranged in groups that belong together. Taking advantage of this, Neugebauer succeeded in reconstructing the problems, and showed that they presuppose the ability to solve mixed second-degree equations; in the final paragraph (pp. 79*f*) he summarized the outcome of the analysis:

> One may legitimately say that the present text presents us with a fair piece of Babylonian mathematics that enriches our all too meagre knowledge of this field with essential features. Quite apart from the use of formulas for triangle and trapezium we see that complex linear equation *systems* were drawn up and solved, and that the Babylonians drew up systematically problems of *quadratic* character and certainly also knew to solve them – all of it with a computational technique that is wholly equivalent to ours. When this was the situation already in Old Babylonian times, in future one will have to learn to look at the later development with different eyes.

In a note added after the proofs were finished (that is, in March 1929), Neugebauer points out that the solution of a problem from CT IX (namely BM 85194, rev. II, 7–21) shows how to solve quadratic equations, and acknowledges the decisive contribution of Schuster.[36] Schuster himself published an analysis of the second-degree *igûm-igibûm* problems from the Seleucid text AO 6484 in the following issue of *Quellen und Studien* B [1930].

In the first issue, Neugebauer and Struve [1929] had published an article purportedly dealing with the Babylonian treatment of the geometry of the circle, actually also with the truncated cone as well as with other configurations that allowed Neugebauer to establish UR.DAM as a term for the height in a plane or solid figure.[37] Apart from establishing which mathematical insights, method and "formulae"[38] were used in the texts, this and subsequent publications in *Quellen und Studien* B[39] (and one in Weidner's *Archiv für Orientforschung*,

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[36] This is why Schuster's discovery should probably be dated in early 1929.

[37] Actually, verbal forms of *warādum* ("to descend") are involved, but for the immediate technical purpose this was not decisive, as observed by Thureau-Dangin [1932b: 80] in the note where he made the grammatical analysis of the term.

[38] In the sense of "standard schemes" – no symbolic writing was of course intended as far as the Babylonians were concerned.

[39] For instance [Schuster 1930], [Neugebauer 1932b], [Waschow 1932a].

namely [Waschow 1932b]) thus established the meaning of a number of technical terms while giving more precision to earlier proposals or putting them on firmer ground. Thureau-Dangin [1931: 195] was thus mistaken when believing in a kind of division of labour, where he was going to take care of terminology and grammar and Neugebauer of the substance.[40] As it turned out, he was also mistaken on his own account, from [1932a] onward he too was to take up both aspects of the texts – and in a note from [1933a: 310], Neugebauer could justly point out that a philological disagreement between the two was due to a "substantial" disagreement about the construction of a fortification.

Beyond mathematical substance and terminology, Neugebauer and the other contributors to *Quellen und Studien* also elucidated the historical setting of the mathematical texts, to the very limited extent it could at all be done at the moment.[41]

In [1932b: 6*f*], Neugebauer made a first (fully adequate) division of the Old Babylonian material into two groups, represented respectively by the Strasbourg texts and the CT IX-texts. He further correctly suggests that the former are slightly older and the latter slightly younger, and even (probably also correct, see [Goetze 1945: 149]) that the Strasbourg texts are from Uruk, and that AO 8862, though not properly a member of the Strasbourg group, is still likely to be related to it.

Negatively, Neugebauer points out in the conclusion of the same paper (p. 24) that the Old Babylonian mathematical texts are wholly unconnected to astronomy, and that they go far beyond the practical concerns of surveying and accounting. This was a rebuttal of opinions held by many Assyriologists at the time, expressed for instance by Bruno Meissner [1920: II, 380] – cf. also [Weidner 1916: 259] as quoted above, and Hilprecht quoting Bezold in note 7. Already in [1929: 73] Neugebauer had pointed out that the Strasbourg problems were

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[40] "[...] les études d'O. Neugebauer, qui ont pour objet plutôt le fond que la forme des textes, apportent au philologue d'utiles données"

[41] This limitation was emphatically pointed out by Neugebauer in [1932b: 24]. In [1934a: 204], he was perhaps even more emphatic when pointing out that "we still know practically nothing about how Babylonian mathematics was situated within the overall cultural framework".

We may take it as an expression of the same explicitly agnostic attitude that Neugebauer never spoke of Babylonian "mathematicians". We may recognize *mathematics* in the texts, but nothing was known about the social role of their authors, in particular, whether any social role or identity (even a part-time role or an aspect of identity) would allow this characterization.

constructed in such a way that they produced neat solutions – which implies that they were *constructed*, and thus that they were school texts and not a surveyor's working notes. This had already been understood by Frank [1928: 19] (cf. above), but Frank had underplayed his insight even more than Neugebauer did here.

A last insight into the cultural embedding of Babylonian mathematics – in this case, of the Seleucid period – was due to Schuster. In [1930: 194] he observes that the colophon of AO 6484, like that of other tablets published in [Thureau-Dangin 1922], shows it to have been written by "a representative of a large family of priests known since long from other texts from the Seleucid epoch".

The sexagesimal system

The understanding of the sexagesimal place value system was mentioned several times above, but some aspects of it deserve separate discussion.

I shall not go into the speculations of Thureau-Dangin, Neugebauer and others concerning its origin: before the metrological and numerical notations of the protoliterate period were deciphered,[42] all such attempts had to remain speculations – some of them sensible, some of them definitely not sensible, but never more than speculations.

Until this point, [Ungnad 1917] was not mentioned, even though this publication was often referred to during the critical years. It was important both for the information it gave and for the problematic traces it left.

Ungnad discussed (pp. 41*f*) the boasting of Assurbanipal that he was able to "*u-pa-ṭar I.GI A.DU.E itguruti*". He pointed out that *itguru* (<*egērum*, "to twist", "to be(come) twisted / confused / ...") meant "complicated", and took *paṭārum* to mean "solve" (that is, solve problems). Since A.DU can be read a . r á , a term familiar from tables of multiplication as well as lexical lists, it had to mean "multiplication"; finally, concerning I.GI, a phonetic writing of i g i , he explained with reference to [Hilprecht 1906: 21*ff*] that *[x] IGI y GÁL.BI* = *z* means that *[x]* : *y* = *z*, that is, that the term refers to division – which of course seemed to make beautiful company with the multiplication.[43] On the whole, Assurbanipal was thus supposed to have boasted that he was able to solve complicated

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[42] That is, until [Friberg 1978; 1979] and the definitive analysis in [Damerow & Englund 1987].

[43] Ungnad's failure to take his own article from [1916] into account indicates that he had not yet fully realized that i g i designates the reciprocal, not a quotient in general – cf. Neugebauer's remark in [1930b: 187 n. 8].

problems of division and multiplication. This interpretation of the quotation was still repeated by Adam Falkenstein in [1953: 126].

In [1929], Neugebauer had already translated i g i as "reciprocal". However, in an editorial note to Schuster's analysis of what is now known as *igûm-igibûm*-problems [Schuster 1930: 196 n.1], he cites Ungnad for the insight that i g i may mean reciprocal, but also (in the Assurbanipal-passage) "division" simply / *schlechthin*. Written with sign names, the two unknown quantities dealt with in the problems are ŠI and ŠI.BU.Ú. ŠI may also be IGI, for which reason Schuster called them *igû* and *šipû*. Inspired by Ungnad Neugebauer now feels tempted to translate the former term "divisor", and since the two quantities turn out to be each other's reciprocals, this seems to him to suggest that the latter term should be translated "multiplier". Since *šipû* could not in any way be connected to a known term for multiplication, he ended up by opting for *Nenner* ("denominator") and *Zähler* ("numerator"), though characterizing the choice as "disputable".

When MKT was published, ŠI.BU.Ú had become *igi-bu-ù*. Yet Neugebauer still uses the same "disputable" translation of the two terms, in the absence of more adequate words; he is quite aware and explains (MKT I, p. 349) that they constitute a pair of reciprocal numbers (already Schuster had assumed that this was meant by the text). The first to recognize that the two terms were Akkadianized forms of Sumerian i g i and i g i . b i, "igi" and "its igi" was Thureau-Dangin [1933: 183*f*].[44] This insight was then taken over in MCT (p. 130) by Neugebauer and Abraham Sachs.

However, the story did not end here. In H. Goetsch's "Die Algebra der Babylonier" [1968: 83], a problem supposedly dealing with *Nenner* and *Zähler* is quoted – but without Neugebauer's explanation that these names are used in the absence of better alternatives. Nor is it revealed that they stand for a pair of reciprocals – perhaps because this is told by Neugebauer in connection with a different problem.

More rectilinear was the progress in the understanding of how the sexagesimal place value system works. In [1930b: 188–193], Neugebauer described the *system* constituted by tables of reciprocals and multiplication (not yet being aware that this system is Old Babylonian and thus does not concern the large Seleucid table of reciprocals AO 6456) – in particular that those numbers that occur as multiplicands (Neugebauer's *Kopfzahlen*) are those that turn up as

[44] In [1932a: 52], Thureau-Dangin still speaks of *igû* and *šibû*.

reciprocals,[45] the multiplicand 7 being the only exception – in Neugebauer's later terminology, today in general use, *regular numbers*. In [1931b], these results were presented in a more systematic way and on the basis of a larger text material; but now the irrelevance of the Seleucid material was recognized.

The two articles develop the idea that the system of tables was originally meant as a way to express general fractions,[46] and only accidentally became a system based on place value – in particular due to the presence of the table with multiplicand 7, because of which the tables contained everything needed for any multiplication. This idea (as well as the idea that creation of the place value system was inspired by metrology, which Neugebauer had maintained since [1927]) was made possible by neglect of the fact that more than a millennium of sexagesimal absolute value counting precedes the place value notation.[47] Given the apparently very sudden implementation during Ur III (a process of which no hints were known in the 1930s), an only accidental development is now implausible, and the development from weight metrology (now known to be created much later than the absolute sexagesimal system) impossible.[48]

Two more articles in *Quellen und Studien* B deal with the place value system: [Neugebauer 1931c] is a mathematical analysis centred upon the notion of regular / irregular numbers, [Neugebauer 1932c] proposes how AO 6456, the big Seleucid table of reciprocals, might have been constructed (suggesting also that the same method was used for the Old Babylonian standard table with its 30 or fewer entries). None of them are of specific interest for the present investigation.

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45 This observation had already been made by Hilprecht [1906: 21], but did not make much sense in his context of "Plato's number".

46 This idea could possibly explain his otherwise not obvious translation of igi / igi.bi as "Nenner" / "Zähler".

47 The absolute sexagesimal system is described in [Thureau-Dangin 1898: 81*f*]. That it goes back to the fourth millennium BCE was not known in 1898, nor in 1930, but in any case it precedes every hint of use of the place value notation by many centuries; besides, the original curviform character of its signs shows them to belong with the earliest phase of writing.

Neugebauer does discuss the absolute system in [1927: 8–13], but mixing it up with speculations that thwart his understanding.

48 Since this is peripheral to my topic, I shall not document these claims, just refer to [Powell 1976] as a seminal publication.

Neugebauer's contributions concerned the internal structure of the place value system. Thureau-Dangin's *Esquisse d'une historie du système sexagésimal* from [1932a] is very different in approach. It deals with other aspects of Sumerian and Akkadian numeration too, including spoken numerals as well as the absolute sexagesimal system and the absolute notations for fractions, and shows that metrological systems, though compatible with sexagesimality, cannot be the starting point of the sexagesimal system, whether place-value or absolute (while recognizing on p. 33 that the use of g i n in the generalized sense of a sixtieth is probably borrowed from metrology). It also points out very explicitly that the place value system was introduced as an *instrument de calcul* (p. 51) and shows how metrological tables served to translate between this "abstract system" and the measurement of concrete quantities in the appurtenant metrologies. This publication can thus be considered a culmination and completion of the development of the preceding eight decades, and gives much more insight into the overall numerical culture of ancient Mesopotamia than Neugebauer's papers on the topic from 1930–32. However, even though a strongly revised version appeared in English translation in *Osiris* in [1939], and even though it also reveals its author's broad knowledge of relevant aspects of the mathematics of other pre-Modern cultures (from ancient Egypt and Greece to Fibonacci and Stevin), this study never had much impact on the historiography of mathematics.

Neugebauer's project

The preceding two sections concerned what Neugebauer did concretely to the understanding of Babylonian mathematics. This, however, was part of a programme, which is expressed in the inaugural statements from the first issue of *Quellen und Studien* B [Neugebauer, Stenzel & Toeplitz 1929: 1–2]. Here we read:

> Through the title *Quellen und Studien* we want to express that we see in the constant reference to original sources the necessary condition for every serious historical research. It shall thus be our first aim to make accessible *sources*, that is, to offer them inasfar as possible in a form which not only may meet the demands of modern philology but also, through translation and commentary, will enable the non-philologist to check for himself the words of the original in any moment. To fulfil the legitimate requests of *both* groups, philologists and mathematicians, will only be possible if we succeed in producing close collaboration between them. To open the road for that will be one of the main purposes of our undertaking.

The *Quellen und Studien* were to appear in two sections:

> One, A, *Quellen*, will contain the actual large editions, containing the text in its original language, a philological apparatus and as literal a translation as possible, which makes the text as accessible also for the non-philologist as can be done. [...]. The issues of section B, *Studien*, will collect articles that are closely or less closely associated with the material that can be drawn from the sources.
>
> The *Quellen und Studien* will offer contributions to the history of mathematics. However, they do not address specialists of the history of science alone. They will certainly propose their material in a form which may *also* be useful for the specialist. But beyond that they address all those who feel that mathematics and mathematical thought are not only concerns of a particular science but profoundly connected to the totality of our culture and its historical development, and that a bridge can be found between the so-called *Geisteswissenschaften* and the apparently so ahistorical "exact sciences". Our final aim is to participate in the building of such a bridge.

Unfortunately, as Neugebauer had to observe in [1934a: 204], "we still know practically nothing about how Babylonian mathematics was situated within the overall cultural framework" – cf. above, p. 18. The bridge he was able to build was thus one between mathematics and highly technical Assyriological philology. No doubt, even the general educated public (not to speak of historians of mathematics) would find the latter field much more arcane than the former.

Another kind of programmatic statement is found in [Neugebauer 1933b: 316*f*][49] – a kind of elaboration of the negative conclusions of [1932b: 24]. Neugebauer starts by summing up polemically the picture of Mesopotamian mathematics that had been derived from field plans and tables: "the level of purely empirical mensuration, loaded with all kinds of number-mystical ballast" – "chaldaeic wisdom" which was then supposed (cf. above, note 7) to be

> continued in Pythagorean wisdom, from which by pure miracle exact Platonic mathematics grew out: indeed a miracle, this almost unmediated transition from Pythagorean number mysticism to a rigorous theory of irrational numbers operating with the class separation of "Dedekind's cut"

Thanks to "the work of Junge, Vogt, E. Sachs, Frank and others", he goes on, this construction had been deprived of one of its main pillars, the Pythagoras legend. The destruction of the other main pillar, the belief in purely empirical and numerological Babylonian mathematics, was now to be accompanied by the introduction of a new understanding of Old Babylonian mathematics: not at all in the style of [Greek Euclidean] geometry but rather of "pure formal-

[49] The main theme of this article is the link between, on one hand, tables of cubes and cube roots (known since Rawlinson) and a recently discovered tabulation of n^3+n^2, on the other the third-degree problems of a tablet now known as BM 8200+VAT6599.

algebraic character”. In terms of a later epoch, the programm is thus anti-Orientalist, anti-new-wave in spirit. In 1933, readers may have observed implicit anti-Spenglerianism.

Neugebauer still published a number of articles on Babylonian mathematics in *Quellen und Studien* B and other journals during the next few years.[50] In Vol. **4** of *Quellen und Studien* B from 1937–38, however, he has five articles on ancient astronomy but nothing more on non-astronomical mathematics, Mesopotamian or otherwise. By then we may say that his work on Babylonian mathematics had come to an end, apart from the volume he prepared with Sachs in 1945 (MCT), which can be seen as a mandatory supplement to MKT, necessitated or at least invited by the new texts to which he had got access by then. Neither the discovery and publication of a number of texts from Eshnunna [Baqir 1950a; 1950b; 1951; 1962] nor the problematic edition of the mathematical Susa texts (nor E. M. Bruins’s venomous slander) ever provoked him to take up the topic again.

MKT is thus at the same time a marvellous culmination and a farewell.[51] Whatever programmatic statement we find here may therefore be considered definitive.

Actually, we find very little. MKT appeared in three volumes in *Quellen und Studien* A in 1935–37; with due respect for Struve’s edition of the Moscow Papyrus [1930], it was certainly the weightiest contribution to this section. As we remember, section A was to “contain the actual large editions, containing the text in its original language, a philological apparatus and as literal a translation as possible, which makes the text as accessible also for the non-philologist as can be done”. In agreement with this description of the section,

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50 Among these, I shall mention in particular [Neugebauer 1934b], the first description of the mathematical series texts.

51 It can hence be considered a paradox that Assyriologists, after the appearance of MKT, tended to put aside any tablet containing too many numbers in place-value notation as “at matter for Neugebauer” (as formulated to me with regret by Hans Nissen at one of the Berlin workshops on “Concept Formation in Mesopotamian Mathematics” in the 1980s). As we have seen, the fathers and giants of Assyriology, from Hicks, Rawlinson and Oppert to Thureau-Dangin, considered anything mathematical as very important. Even after the revival of active work on Mesopotamian mathematics during the last three decades and many new insights, an *Encyclopedia of Ancient History* planned by Blackwell and Wiley in 2009 suggested 500 words for “Mathematics, Mesopotamian” – the same as was dedicated to Mesopotamian hairstyles (I succeeded in raising the limit to 700 words).

the *Vorwort* of vol. I (p. v) starts by stating that the purpose of the work "from the very beginning [in 1929] was to procure a complete collection of all mathematical cuneiform texts", and that this aim had been achieved in the sense that probably no essential published material had escaped notice, while all unpublished material to which Neugebauer had had access had been included.

As mentioned above (p. 16), there is also a programmatic statement (p. viii) that "in principle, the translation is obviously literal", but that this principle cannot always be respected. But that is all.

In the end of vol. III (pp. 76–80) we then find a *Rückblick*, a retrospect on the three volumes. It mostly contains tentative conclusions and delineations of open questions, but one passage (p. 79) confirms the apparently restrictive programme formulated in the *Vorwort*:

> It does not belong among the tasks that I have proposed for myself in this edition to develop the consequences which can be drawn from this text material. I have outlined them within a broader framework in my *Vorlesungen* [Neugebauer 1934a / JH], and sketched the connections to Greek mathematics in a work "Zur geometrischen Algebra" [Neugebauer 1936 / JH]; I hope to finish in the not too distant future a detailed investigation of all questions pertaining to the history of terminologies [which never appeared / JH].

Still, the following page – apart from indexes and reproductions of tablets the final page of the work – draws some general conclusions. These pertain not least to the nature of and conditions for the development of early mathematics (p. 80):

> Since our knowledge of these things is of relatively recent date, and current datings had to be pushed considerably, there is an obvious danger to overestimate the mathematics of the Babylonians. In order to somehow gloss over the lack of a basis in sources, many familiar books change elementary mathematical things into "propositions" and "discoveries" that must be ascribed to great men. It seems to me that we should not stamp the Babylonians as such discoverers. What is often overlooked and cannot be sufficiently emphasized is the terrible difficulty and slowness of the development of the very simplest fundamental mathematical concepts, first of all of a genuine computational technique. This, however, is not the achievement of a single person; it can only be understood within a historical process, inextricably attached to the emergence of a whole culture.

So, the broader programme of the *Quellen und Studien* had not been forgotten – only the limits imposed by available sources (and by the lack of relevant sources) prevented Neugebauer from filling it out.

Why Neugebauer, why Göttingen?

As we have seen, many outstanding Assyriologists had been interested in everything mathematical they could get their hands on. Gradually, they had come to understand the many different metrologies well (except those of the proto-literate period). Assyriologists' attempts to understand the two CT IX texts and the Strasbourg texts had yielded important insights into the mathematical terminology; actually, most of the basic vocabulary for mathematical operations was already acceptably well understood in 1928, thanks to Weidner, Zimmern, Ungnad and Frank. When it came to understanding such texts, however, progress was blocked.

On the other hand, once the breakthrough had been effectuated by Neugebauer, Struve and Schuster, even Thureau-Dangin was able to participate in the new development. What was so special, we may ask, about Neugebauer and his Göttingen circle, which allowed the opening of a road which even the most eminent of Assyriologist had not been able to find on his own?

Other Assyriologists may have been blocked by their expectation that the Babylonians could have engaged in nothing but "empirically based" practical calculation. As cited (above, p. 18), this was the opinion expressed by Meissner in his survey from 1925. Some, like Frank, may have been stopped by the habit to translate all numbers into Arabic numerals, sometimes mistaking orders of magnitude (and, in general, by not understanding to the full the floating-point nature of the sexagesimal place-value notation). Many will surely have had a mathematical training that did not suffice as support for the mathematical fantasy required for the task.

None of this is valid for Thureau-Dangin, except perhaps the low expectations concerning the level of the Babylonians[52] – nor indeed for Allotte de la Fuÿe, ancient *polytechnicien* who, though born in 1844, was still quite active, but mostly interested in third-millennium documents.[53]

Assyriologists, including Thureau-Dangin, were of course interested in many other topics than mathematics. As long as nothing beyond practical calculation was expected to exist, they may simply not have looked for it; once it was known there was something to be found, that situation may have changed. After all,

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52 His characterization of AO 6484 as "arithmetical operations" (above, p. 12) might suggest exactly such low expectations.

53 In 1930 he published an article on protoliterate (Jemdet Nasr) metrology and mensuration, in 1932 another one on AO 6456, the Seleucid table of reciprocals. On his mathematical interest and competence, see [de Genouillac 1939].

however, it only changed radically in the case of Thureau-Dangin, as illustrated by Wolfram von Soden's case. Von Soden was certainly interested in mathematics: he wrote extensive and thorough reviews of MKT in [1937], of TMB in [1939] and of TMS in [1964]; I also experienced his interest in the mathematical area personally in correspondences I had with him during the 1980s. He was even (as far as I am aware of) the first to suspect publicly that the picture of Babylonian mathematics constructed by Neugebauer and Thureau-Dangin was too modernizing [von Soden 1937: 189–191], which might well have spurred him to pursue this particular interest. Apart from the reviews, though, only two publications from his industrious hand deal with mathematics as such:[54] an analysis of a number of problem texts from Eshnunna from [1952], and a collaborative work [Gundlach & von Soden 1963] treating a problem text from Eshnunna and one from Susa. Even in Thureau-Dangin's case his *full* concentration of matters mathematical only lasted some five years, from 1932 to 1936. This can be seen in his "Notes assyriologiques", containing miscellaneous observations on the material he worked on: during these years, almost everything in these notes concerns mathematics and its applications; before 1932 and after 1936, that is not the case.

In any case, Neugebauer and his collaborators initiated a breakthrough where nobody else had succeeded. It is important not to leave out from this observation the collaborators and participants in the seminar: the contributions of Schuster, Struve, Waschow and Schott, all fully trained and active Assyriologists, are very visible in *Quellen und Studien* B, and explicitly acknowledged by Neugebauer in MKT and elsewhere.[55]

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[54] I disregard publications where general *a priori* ideas about the nature of Mesopotamian mathematics enter as part of a broader argument, such as [von Soden 1936].

[55] It may perhaps be adequate to recapitulate some elements of what these four Assyriologists did later in connection with Mesopotamian mathematics.

Schuster published oft-cited works on Sumero-Babylonian bilingual texts in 1938 and on Hatto-Hittite bilinguals in 1974 and 2002; he lived until 2002, but seems not to have worked on mathematical texts after 1930.

Struve, as curator of the cuneiform collection of the Ermitage in Leningrad, analyzed its corpus of Ur III accounts, which induced him to draw a very grim picture of the social system that implemented the place value system in its social engineering [Struve 1934] – a picture that has now been amply confirmed by Robert Englund [1990]. He lived until 1965 but seems never to have published more on "mathematics proper".

Waschow prepared an edition of the important Seleucid problem text BM 34568, published in MKT III (pp. 14–22). In his dissertation from [1936], an edition of letters from the Kassite period, he states in the (unpaginated) CV that he had entered active

Neugebauer's personal stamina and competence may have been very important – we are dealing with the statistics of very small numbers, where personalities count for very much and become primary facts allowing no full explanation from or reduction to general factors. But it was probably important for this stamina and competence to come into play, *both* that Neugebauer himself was not primarily an Assyriologist but a historian of mathematics (in the dichotomy of [Neugebauer, Stenzel & Toeplitz 1929], not primarily "a philologist" but "a mathematician"), *and* that he was able to enter into close collaboration with and inspire a number of Assyriologists.[56] As a non-Assyriologist, he could concentrate (at least until 1937) on Babylonian *mathematics* alone, and thereby come to know the totality of the corpus much better than anybody had done before 1929; The preface of MKT I (p. v) lists how few higher-level texts were at all known by then. However, his deep respect for sources, as reflected in the programme for *Quellen und Studien*, caused him to seek philological collaboration and advice, and kept him free of the danger of rational reconstruction based on what the Babylonians *might* have done, if only they had been more or less Greek or more or less modern mathematicians. In Leopold von Ranke's words (in the sense von Ranke really used them in 1824 [von Ranke 1885: vii], against lazy

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army service in 1934 and was at the moment serving as a non-commissioned anti-aircraft officer while intending to continue scholarly activity in parallel. In 1938 he published *4000 Jahre Kampf um die Mauer*, about siege techniques since Old Babylonian times. I can find no later traces of him and assume that he is one of those collaborators of Neugebauer who according to Vogel (private communication) fell in the war.

According to Neugebauer (MKT I, p. ix), Schott contributed intensely to MKT. He was also one of Neugebauer's intended collaborators in the publication of the corpus of Babylonian astronomical texts, planned around 1935 (see the description in [Neugebauer 1938]) – not realized immediately because of the war. Schott died at the end of the war in 1945 [Thompson 2010]. He had also collaborated with the astronomer Paul Neugebauer on other aspects of Mesopotamian astronomy, and he translated the Gilgameš-epic in 1934 (eventually published with revisions by von Soden in 1958).

As we see, "mathematics proper" did not stay central to those three who had the possibility to make Assyriological work after 1936. Though also engagin in other matters, Thureau-Dangin was actually more tenacious as regards mathematics, as expressed in his [1940a; 1940b].

[56] As Neugebauer tells with gratitude in [1927: 5], he has also been well counselled and trained by Anton Deimel during a fairly long stay at the Pontificium Institutum Biblicum in Rome, as his initial interest in Mesopotamian mathematics (as a parallel elucidating the foundations of Egyptian mathematics) had first been stimulated by works of Thureau-Dangin [1898 ; 1921] and Deimel [1922].

invention and too hasty generalization), Neugebauer's proudly modest aim was to find out *wie es eigentlich gewesen*.

Correspondingly, it was probably decisive for the way in which Thureau-Dangin could contribute when the parallel work of the two began, that *his* starting point was that of the philologist, a reader and interpreter of texts, also in his approach to the history of mathematics (where he is much more akin to for instance Kurt Vogel than to Neugebauer). Even his aim is covered by von Ranke's maxim.

Thureau-Dangin's starting point had been the classical stance of Assyriologists: in order to understand Mesopotamian sources and civilization, it was mandatory to understand metrology and mathematics. Reversely, for Neugebauer, the Göttingen seminar and the *Quellen und Studien* programme, understanding Babylonian mathematics was necessary for understanding *mathematics* as the product of an ongoing historical process. However, it was essential for the outcome that both had left behind these motivations (or at least behaved as if they had), and took up "Babylonian mathematics" as a research project that was of major interest in itself and needed no further excuse.

For the fruitful outcome of the race it was also essential that the two, in spite of the unmistakeable animosity which gradually developed between them,[57] in general remained respectful when citing each other and constructive in their mutual criticism, and even allowed each other access to whatever material was needed.[58] Great moral models for all scholars, and giants on whose shoulders it was always a pleasure to stand.

References

AHw: Wolfram von Soden, *Akkadisches Handwörterbuch*. Wiesbaden: Otto Harrassowitz, 1965–81.

Allotte de la Fuÿe, François-Maurice, 1915. "Mesures agraires et formules d'arpentage à l'époque présargonique". *Revue d'Assyriologie* **12** (1915), 117–146.

Baqir, Taha, 1950a. "An Important Mathematical Problem Text from Tell Harmal". *Sumer* **6**, 39–54.

[57] "They hated each other", I was told by Olaf Schmidt, Neugebauer's assistant during his stay in Copenhagen. Schmidt, too gentle to hate anybody as far as I can judge, may have mistaken animosity for genuine hatred.

[58] Given the general unreliability of Evert Bruins, I permit myself to disregard what he claimed in a letter to me: that Thureau-Dangin took care that Neugebauer should not get access to the mathematical texts from Susa, which were found in 1933.

Baqir, Taha, 1950b. “Another Important Mathematical Text from Tell Harmal”. *Sumer* **6**, 130–148.

Baqir, Taha, 1951. “Some More Mathematical Texts from Tell Harmal”. *Sumer* **7**, 28–45.

Baqir, Taha, 1962. “Tell Dhiba’i: New Mathematical Texts”. *Sumer* **18**, 11–14, pl. 1–3.

Bezold, Carl, 1926. *Babylonisch-Assyrisches Glossar*. Heidelberg: Carl Winter.

Brünnow, Rudolph E., 1889. *A Classified List of All Simple and Compound Cuneiforn Ideographs Occurring in the Texts Hitherto Published, with Their Assyro-Babylonian Equivalents, Phonetic Values, etc.* Leiden: E. J. Brill.

CAD: *The Assyrian Dictionary of the Oriental Institute of Chicago*. 21 vols. Chicago: The Oriental Institute, 1964–.

Cory, Isaac Preston, 1832. *Ancient Fragments of the Phoenician, Chaldæan, Egyptian, Tyrian, Carthaginian, Indian, Persian, and Other Writers*. Second Edition. London: William Pickering.

CT IX: *Cuneiform Texts from Babylonian Tablets, &c., in the British Museum*. Part IX. London: British Museum, 1900.

Damerow, Peter, & Robert K. Englund, 1987. “Die Zahlzeichensysteme der Archaischen Texte aus Uruk”, Kapitel 3 (pp. 117–166) *in* M. W. Green & Hans J. Nissen, *Zeichenliste der Archaischen Texte aus Uruk*, Band II (ATU 2). Berlin: Gebr. Mann.

de Genouillac, Henri, 1939. “Allotte de la Fuÿe (1844–1939). *Revue d’Assyriologie et d’Archéologie Orientale* **36**, 41–42.

Deimel, Anton, 1922. *Die Inschriften von Fara*. I, *Liste der archaischen Keilschriftzeichen*. (Wissenschaftliche Veröffentlichungen der Deutschen Orient-Gesellschaft, 40). Leipzig: J.C. Hinrichs’sche Buchhandlung.

Delaporte, Louis, 1911. “Document mathématique de l’époque des rois d’Our”. *Revue d’Assyriologie et d’Archéologie Orientale* **8**, 131–133.

Delitzsch, Friedrich, 1914. *Sumerisches Glossar*. Leipzig: J. C. Hinrichs’sche Buchhandlung.

Englund, Robert, 1990. *Organisation und Verwaltung der Ur III-Fischerei*. (Berliner Beiträge zum Vorderen Orient, 10). Berlin: Dietrich Reimer.

Epping, Josef, unter Mitwirkung von P. J. N. Strassmaier, 1889. *Astronomisches aus Babylon*. Freiburg im Breisgau: Herder.

Falkenstein, Adam, 1953. “Die babylonische Schule”. *Saeculum* **4**, 125–137.

Fossey, Charles, 1907. *Manuel d’assyriologie*. Tôme premier. *Explorations et fouilles, déchiffrement des cunéiformes, origine et histoire de l’écriture*. Paris: Leroux.

Frank, Carl, 1928. *Straßburger Keilschrifttexte in sumerischer und babylonischer Sprache*. (Schriften der Straßburger Wissenschaftlichen Gesellschaft in Heidelberg, Neue Folge, Heft 9). Berlin & Leipzig: Walter de Gruyter.

Friberg, Jöran, 1978. “The Third Millennium Roots of Babylonian Mathematics. I. A Method for the Decipherment, through Mathematical and Metrological Analysis, of Proto-Sumerian and proto-Elamite Semi-Pictographic Inscriptions”. *Department of Mathematics, Chalmers University of Technology and the University of Göteborg* No. 1978–9.

Friberg, Jöran, 1979. “The Early Roots of Babylonian Mathematics. II: Metrological Relations in a Group of Semi-Pictographic Tablets of the Jemdet Nasr Type, Probably from Uruk-Warka”. *Department of Mathematics, Chalmers University of Technology and the University of Göteborg* No. 1979–15.

Friberg, Jöran, 1982. “A Survey of Publications on Sumero-Akkadian Mathematics, Metrology and Related Matters (1854–1982)”. *Department of Mathematics, Chalmers University of Technology and the University of Göteborg* No. 1982–17.

Friberg, Jöran, 1993. “On the Structure of Cuneiform Metrological Table Texts from the -1st Millennium”. *Grazer Morgenländische Studien* **3**, 383–405.

Gadd, C. J., 1922. “Forms and Colours”. *Revue d’Assyriologie et d’Archéologie Orientale* **19**, 149–159.

Goetsch, H., 1968. “Die Algebra der Babylonier”. *Archive for History of Exact Sciences* **5** (1968–69), 79–153.

Goetze, Albrecht, 1945. “The Akkadian Dialects of the Old Babylonian Mathematical Texts”, pp. 146–151 *in* O. Neugebauer & A. Sachs, *Mathematical Cuneiform Texts*. (American Oriental Series, vol. 29). New Haven, Connecticut: American Oriental Society.

Gundlach, Karl-Bernhard, & Wolfram von Soden, 1963. “Einige altbabylonische Texte zur Lösung »quadratischer Gleichungen«”. *Abhandlungen aus dem mathematischen Seminar der Universität Hamburg* **26**, 248–263.

Heuzey, Léon, 1906. “À la mémoire de Jules Oppert”. *Revue d’Assyriologie et d’Archéologie Orientale* **6** (1904–07), 73–74.

Hilprecht, Herman V., 1906. *Mathematical, Metrological and Chronological Tablets from the Temple Library of Nippur*. (The Babylonian Expedition of the University of Pennsylvania. A: Cuneiform Texts, XX,1). Philadelphia: Department of Archaeology, University of Pennsylvania, 1906.

Hincks, Edward, 1854a. “Cuneiform Inscriptions in the British Museum”. *Journal of Sacred Literature*, New Series **13** (October 1854), 231–234, reprint after *The Literary Gazette* **38** (1854), 707.

Hincks, Edward, 1854b. “On the Asssyrian Mythology”- *Transactions of the Royal Irish Academy* **22**:6, 405–422.

Hommel, Fritz, 1885. *Geschichte Babyloniens und Assyriens*. (Allgemeine Geschichte in Einzeldarstellungen, 2). Berlin: Grote’sche Verlagsbuchhandlung.

Høyrup, Jens, 2002. *Lengths, Widths, Surfaces: A Portrait of Old Babylonian Algebra and Its Kin*. (Studies and Sources in the History of Mathematics and Physical Sciences). New York: Springer.

Kramer, Samuel Noah, 1963. *The Sumerians: Their History, Culture, and Character*. Chicago: Chicago University Press.

Kugler, Franz Xaver, 1900. *Die babylonische Mondrechnung. Zwei Systeme der Chaldäer über den Lauf des Mondes und der Sonne*. Freiburg im Breisgau: Herder.

Labat, René, 1963. *Manuel d’épigraphie akkadienne (signes, syllabaire, idéogrammes)*. [4]Paris: Imprimerie Nationale.

MCT: O. Neugebauer & A. Sachs, *Mathematical Cuneiform Texts*. (American Oriental Series, vol. 29). New Haven, Connecticut: American Oriental Society, 1945.

Meißner, Bruno, 1920. *Babylonien und Assyrien*. 2 vols. Heidelberg: Carl Winther, 1920, 1925.

MKT: O. Neugebauer, *Mathematische Keilschrift-Texte*. 3 vols. (Quellen und Studien zur Geschichte der Mathematik, Astronomie und Physik. Abteilung A: Quellen. 3. Band, erster-dritter Teil). Berlin: Julius Springer, 1935, 1935, 1937.

Neugebauer, Otto, 1927. “Zur Entstehung des Sexagesimalsystems”. *Abhandlungen der Gesellschaft der Wissenschaften zu Göttingen. Mathematisch-Physikalische Klasse*, Neue Folge **13**,1.

Neugebauer, Otto, 1928. “Zur Geschichte des pythagoräischen Lehrsatzes”. *Nachrichten von der Gesellschaft der Wissenschaften zu Göttingen, Mathematisch-physikalische Klasse* 1928, 45–48.

Neugebauer, Otto, 1929. “Zur Geschichte der babylonischen Mathematik”. *Quellen und Studien zur Geschichte der Mathematik, Astronomie und Physik.* Abteilung B: *Studien* **1** (1929–31), 67–80.

Neugebauer, Otto, 1930a. “Beiträge zur Geschichte der Babylonischen Arithmetik”. *Quellen und Studien zur Geschichte der Mathematik, Astronomie und Physik.* Abteilung B: *Studien* **1** (1929–31), 120–130.

Neugebauer, Otto, 1930b. “Sexagesimalsystem und babylonische Bruchrechnung”. *Quellen und Studien zur Geschichte der Mathematik, Astronomie und Physik.* Abteilung B: *Studien* **1** (1929–31).

Neugebauer, Otto, 1931a. “Über die Approximation irrationaler Quadratwurzeln in der Babylonischen Mathematik”. *Archiv für Orientforschung* **6** (1931–32), 90–99.

Neugebauer, Otto, 1931b. “Sexagesimalsystem und babylonische Bruchrechnung II”. *Quellen und Studien zur Geschichte der Mathematik, Astronomie und Physik.* Abteilung B: *Studien*. **1**, 452–457.

Neugebauer, Otto, 1931c. “Sexagesimalsystem und babylonische Bruchrechnung III”. *Quellen und Studien zur Geschichte der Mathematik, Astronomie und Physik.* Abteilung B: *Studien* **1** (1929–31), 458–463.

Neugebauer, Otto, 1932a. “Zur Transkription mathematischer und astronomischer Keilschrifttexte”. *Archiv für Orientforschung* **8** (1932–33), 221–223.

Neugebauer, Otto, 1932b. “Studien zur Geschichte der antiken Algebra I”. *Quellen und Studien zur Geschichte der Mathematik, Astronomie und Physik.* Abteilung B: *Studien* **2** (1932–33), 1–27.

Neugebauer, Otto, 1932c. “Sexagesimalsystem und babylonische Bruchrechnung IV”. *Quellen und Studien zur Geschichte der Mathematik, Astronomie und Physik.* Abteilung B: *Studien* **2** (1932–33), 199–410.

Neugebauer, Otto, 1933a. “Babylonische »Belagerungsrechnung«”. *Quellen und Studien zur Geschichte der Mathematik, Astronomie und Physik. Abteilung B: Studien* **2** (1932–33), 305–310.

Neugebauer, Otto, 1933b. “Über die Lösung kubischer Gleichungen in Babylonien”. *Nachrichten von der Gesellschaft der Wissenschaften zu Göttingen, Mathematisch-physikalische Klasse* 1933, 316–321.

Neugebauer, Otto, 1934a. *Vorlesungen über Geschichte der antiken mathematischen Wissenschaften.* I: *Vorgriechische Mathematik.* (Die Grundlehren der mathematischen Wissenschaften in Einzeldarstellungen, 43). Berlin: Julius Springer.

Neugebauer, Otto, 1934b. “Serientexte in der babylonischen Mathematik”. *Quellen und Studien zur Geschichte der Mathematik, Astronomie und Physik.* Abteilung B: *Studien* **3** (1934–36), 106–114.

Neugebauer, Otto, 1936. “Zur geometrischen Algebra (Studien zur Geschichte der antiken Algebra III)”. *Quellen und Studien zur Geschichte der Mathematik, Astronomie und Physik.* Abteilung B: *Studien* **3** (1934–36), 245–259.

Neugebauer, Otto, 1937. “Untersuchungen zur antiken Astronomie I”. *Quellen und Studien zur Geschichte der Mathematik, Astronomie, und Physik.* Abteilung B: *Studien* **4** (1937–38), 29–33.

Neugebauer, Otto, Julius Stenzel & Otto Toeplitz, 1929. “Geleitwort”. *Quellen und Studien zur Geschichte der Mathematik, Astronomie und Physik.* Abteilung B: *Studien* **1** (1929–31), 1–2.

Neugebauer, Otto, & W. Struve, 1929. “Über die Geometrie des Kreises in Babylonien”. *Quellen und Studien zur Geschichte der Mathematik, Astronomie und Physik.* Abteilung B: *Studien* **1** (1929–31), 81–92.

Oppert, Jules, 1872. “L’étalon des mesures assyriennes fixé par les textes cunéiformes”. *Journal asiatique*, sixième série **20** (1872), 157–177; septième série **4** (1874), 417–486.

Oppert, Jules, 1885. “Les mesures assyriennes de capacité et de superficie”. *Revue d’Assyriologie et d’Archéologie Orientale* **1** (1884–85), 124–147.

Oppert, Jules, 1894. “Les mesures de Khorsabad”. *Revue d’Assyriologie et d’Archéologie Orientale* **3** (1993–95), 89–104.

Powell, Marvin A., 1976. “The Antecedents of Old Babylonian Place Notation and the Early History of Babylonian Mathematics”. *Historia Mathematica* **3**, 417–439.

Powell, Marvin A., 1982. “Metrological Notes on the Esagila Tablet and Related Matters”. *Zeitschrift für Assyriologi* **72**, 106–123.

Powell, Marvin A., 1990. “Maße und Gewichte”. *Reallexikon der Assyriologie und Vorderasiatischen Archäologie* VII, 457–516. Berlin & New York: de Gruyter.

Rawlinson, George, 1871. *The Five Great Monarchies of the Ancient Eastern World*. 3 vols. London: John Murray.

Rawlinson, Henry, 1855. “Notes on the Early History of Babylonia”. *Journal of the Royal Asiatic Society of Great Britain and Ireland* **15**, 215–259.

Sayce, Archibald Henry, 1887. “Miscellaneous Notes”. *Zeitschrift für Assyriologie und verwandte Gebiete* **2**, 331–340.

Sayce, Archibald Henry, 1908. *The Archaeology of Cuneiform Inscriptions*. Second edition, revised. London: Society for Promoting Christian Knowledge.

Scheil, Vincent, 1915a. “*Les tables igi x gal-bi*, etc.”. *Revue d’Assyriologie et d’Archéologie Orientale* **12**, 195–198.

Scheil, Vincent, 1915b. “Le calcul des volumes dans un cas particulier à l’époque d’Ur”. *Revue d’Assyriologie et d’Archéologie Orientale* **12**, 161–172.

Scheil, Vincent, 1916. “Notules. XX.–Le texte mathématique 10201 du Musée de Philadelphie”. *Revue d’Assyriologie et d’Archéologie Orientale* **13**, 138–142.

Schuster, Hans-Siegfried, 1930. “Quadratische Gleichungen der Seleukidenzeit aus Uruk”. *Quellen und Studien zur Geschichte der Mathematik, Astronomie und Physik.* Abteilung B: *Studien* **1** (1929–31), 194–200.

Steinkeller, Piotr, 1979. “Alleged GUR.DA = u g u l a - g é š - d a and the Reading of the Sumerian Numeral 60”. *Zeitschrift für Assyriologie und Vorderasiatische Archäologie* **69**, 176–187.

Struve, Vasilij Vasil'evič, 1930. *Mathematischer Papyrus des Staatlichen Museums der Schönen Künste in Moskau*. Herausgegeben und Kommentiert. (Quellen und Studien zur Geschichte der Mathematik. Abteilung A: Quellen, 1. Band). Berlin: Julius Springer.

Struve, Vasilij Vasil'evič, 1934. "The Problem of the Genesis, Development and Disintegration of the Slave Societies in the Ancient Orient". Abbreviated translation of an article from 1934, pp. 17–67 *in* I. M. Diakonoff (ed.), 1969. *Ancient Mesopotamia, Socio-Economic History. A Collection of Studies by Soviet Scholars*. Moskva: «Nauka» Publishing House, Central Department of Oriental Literature.

Thomson, Gary D., 2010. "The Recovery of Babylonian Astronomy. (9) The Pinches Era – Otto Neugebauer and Abraham Sachs (and Theophilus Pinches)". http://members.westnet.com/Gary-David-Thompson/babylon9.html.

Thureau-Dangin, François, 1897. "Un cadastre chaldéen". *Revue d'Assyriologie* **4** (1897–98), 13–27.

Thureau-Dangin, François, 1898. *Recherches sur l'origine de l'écriture cunéiforme*. 1re partie+Supplément à la 1re partie. Paris: Leroux, 1898–99.

Thureau-Dangin, François, 1909. "L'u, le qa et la mine". *Journal asiatique*, 13ième série **13**, 79–111.

Thureau-Dangin, François, 1921. "Numération et métrologie sumériennes". *Revue d'Assyriologie et d'Archéologie Orientale* **18**, 123–142.

Thureau-Dangin, François, 1922. *Tablettes d'Uruk à l'usage des prêtres du Temple d'Anu au temps des Séleucides*. (Musée de Louvre – Département des Antiquités Orientales. Textes cunéiformes, 6). Paris: Paul Geuthner.

Thureau-Dangin, François, 1926. *Le syllabaire accadien*. Paris: Paul Geuthner.

Thureau-Dangin, François, 1931. "Notes sur la terminologie des textes mathématiques". *Revue d'Assyriologie et d'Archéologie Orientale* **28**, 195–198.

Thureau-Dangin, F., 1932a. *Esquisse d'une histoire du système sexagésimal*. Paris: Geuthner.

Thureau-Dangin, François, 1932b. "Notes assyriologiques. LXIV. – Encore un mot sur la mesure du segment de cercle. LXV. – BAL=«raison (arithmétique ou géométrique)». LXVI. – *Warâdu* «abaisser un perpendiculaire»; *elû* «élever un perpendiculaire». LXVII. – La mesure du volume d'un tronc de pyramide". *Revue d'Assyriologie et d'Archéologie Orientale* **29**, 77–88.

Thureau-Dangin, François, 1933. "Notes Assyriologiques. LXIIV. – *Igû* et *igibû*. LXXV. – La tablette de Strasbourg n° 11. LXXVI. – Le nom du «cercle» en babylonien". *Revue d'Assyriologie et d'Archéologie Orientale* **30**, 183–188.

Thureau-Dangin, François, 1934. "La tablette de Strasbourg n° 11". *Revue d'Assyriologie et d'Archéologie Orientale* **31**, 30.

Thureau-Dangin, F., 1939. "Sketch of a History of the Sexagesimal System". *Osiris* **7**, 95–141.

Thureau-Dangin, F., 1940a. "Notes sur la mathématique babylonienne". *Revue d'Assyriologie* **37**, 1–10.

Thureau-Dangin, F., 1940b. "L'Origine de l'algèbre". *Académie des Belles-Lettres. Comptes Rendus* 1940, 292–319.

TMB: F. Thureau-Dangin, *Textes mathématiques babyloniens*. (Ex Oriente Lux, Deel 1). Leiden: Brill, 1938.

TMS: Evert M. Bruins & Marguerite Rutten, *Textes mathématiques de Suse*. (Mémoires de la Mission Archéologique en Iran, XXXIV). Paris: Paul Geuthner, 1961.

Ungnad, Arthur, 1916. "Zur babylonischen Mathematik". *Orientalistische Literaturzeitung* **19**, 363–368.

Ungnad, Arthur, 1917. "Lexikalisches". *Zeitschrift für Assyriologie und Vorderasiatische Archäologie* **31** (1917–18), 38–57.

Ungnad, Arthur, 1918. "Lexikalisches". *Zeitschrift für Assyriologie und verwandte Gebiete* **31** (1917–18j), 248–276.

von Ranke, Leopold, 1885. *Geschichten der romanischen und germanischen Völker von 1494 bis 1514*. [3]Leipzig: Duncker & Humblott

von Soden, Wolfram, 1936. "Leistung und Grenze sumerischer und babylonischer Wissenschaft". *Die Welt als Geschichte* **2**, 411–464, 507–557.

von Soden, Wolfram, 1937. [Review of MKT]. *Zeitschrift der Deutschen Morgenländischen Gesellschaft* **91**, 185–203.

von Soden, Wolfram, 1939. [Review of TMB]. *Zeitschrift der Deutschen Morgenländischen Gesellschaft* **93**, 143–152.

von Soden, Wolfram, 1952. "Zu den mathematischen Aufgabentexten vom Tell Harmal". *Sumer* **8**, 49–56.

von Soden, Wolfram, 1964. [Review of TMS]. *Bibliotheca Orientalis* **21**, 44–50.

Waschow, Heinz, 1932. "Verbesserungen zu den babylonischen Dreiecksaufgaben S.K.T. 8". *Quellen und Studien zur Geschichte der Mathematik, Astronomie und Physik*. Abteilung B: *Studien* **2** (1931–32), 211–214.

Waschow, Heinz, 1932b. "Angewandte Mathematik im alten Babylonien (um 2000 v. Chr.). Studien zu den Texten CT IX, 8–15". *Archiv für Orientforschung* **8** (1932–33), 127–131, 215–220.

Waschow, Heinz, 1936. *Babylonische Briefe aus der Kassitenzeit*. Inaugural-Dissertation. Gräfenhainichen: A. Heine.

Weidner, Ernst F., 1916. "Die Berechnung rechtwinkliger Dreiecke bei den Akkadern um 2000 v. Chr." *Orientalistische Literaturzeitung* **19**, 257–263.

Zimmern, Heinrich, 1916. "Zu den altakkadischen geometrischen Berechnungsaufgaben". *Orientalistische Literaturzeitung* **19**, 321–325.