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Publication date:
1988

Document Version
Publisher's PDF, also known as Version of record

Citation for published version (APA):
Ottesen, J. T. (1988). *The dirac equation with light-cone data*. Roskilde Universitet.

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TEKST NR 171

1988

The Dirac Equation with Light-Cone Data.

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TEKSTER fra

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FUNKTIONER I UNDERVISNING, FORSKNING OG ANVENDELSER

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IMFUFA tekst nr. 171/88

29 sider

ISSN 0106-6242

ABSTRACT

The Dirac equation with light-cone data ;

$$(i\gamma^0 \frac{\partial}{\partial t} + i\gamma \cdot \nabla - m\mathbb{1})u(t,x) = 0.$$

$$u(-|x|,x) = f(x), \quad x \in \mathbb{R}^3$$

is considered and explicitly solved in terms of a "Light-cone

Fourier-transform" under appropriate conditions on f .

1. Introduction.

We shall consider the characteristic Cauchy problem for the free Dirac equation

$$(i\gamma^0 \frac{\partial}{\partial t} + i\gamma^j \nabla_j - m\mathbb{1})u(t,x) = 0 \quad (1.1)$$

$$u(-|x|,x) = f(x) \quad (1.2)$$

under suitable conditions on f , in four space-time dimensions.

The analogous problem for the Klein-Gordon equation was considered in [1]. The wave-equation has been considered by Riesz [2] and Strichartz [3], but in a very different way. For some general results on characteristic Cauchy-problems see Hörmander [4].

We shall show that there exist a Hilbert-space \mathcal{H} of light-cone data such that (1.1) and (1.2) have a unique weak solution $u(t,x)$, with $u(t,\cdot) \in L^2(\mathbb{R}^3, \mathbb{C}^4)$ for f in \mathcal{H} and so that all " L^2 - solutions" of (1.1) have the property that $u(t-|\cdot|,\cdot)$ is in \mathcal{H} .

The Dirac equation (1.1) will be rewritten, as an evolution

equation $i \frac{\partial u}{\partial t} = Hu$ in \mathcal{H} and the generator H will be diagonalized

by a "light-cone Fourier transform", which then allows us to give an explicit formula for $u(t,x)$ in terms of f .

2. The Dirac Equation and Associated Evolution Equations.

The Dirac equation is a hyperbolic system of linear partial differential equations which can be written as

$$(i\gamma^0 \frac{\partial}{\partial t} + i\gamma \cdot \nabla - m\mathbb{1})u(t,x) = 0 \quad (2.1)$$

where $m \neq 0$ is the mass of the particle, $u: \mathbb{R} \times \mathbb{R}^3 \rightarrow \mathbb{C}^4$, γ^0 and the three components of the vector $\gamma = (\gamma^1, \gamma^2, \gamma^3)$ are 4×4 complex matrices fulfilling the anti-commutation relation

$$\gamma^\mu \gamma^\nu + \gamma^\nu \gamma^\mu = 2g^{\mu\nu} \mathbb{1} \quad (2.2)$$

where $g^{\mu\nu} = 0$ for $\mu \neq \nu$, $g^{00} = -g^{ii} = 1$, for $i = 1, 2, 3$, and $\mathbb{1}$ is the 4×4 unit matrix. An explicit representation of the γ^μ 's is given by

$$\gamma^0 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad \text{and} \quad \gamma^j = \begin{bmatrix} 0 & -\sigma^j \\ \sigma^j & 0 \end{bmatrix}, \quad \text{where } 1 \text{ in } \gamma^0 \text{ is the}$$

2×2 unit matrix and the σ^j 's are the familiar Pauli-matrices given by

$$\sigma^1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad \sigma^2 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, \quad \sigma^3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}. \quad \text{We shall use this representation}$$

in the following.

The Dirac equation (2.1) is often considered as an evolution equation with initial data on a space-like hyperplane, consider for exsampel a $t = \text{constant}$ hyperplane and write the equation as

$$i \frac{\partial u}{\partial t} = \gamma^0(-i\gamma \cdot \nabla + m1)u, \text{ which can be considered as an evolution}$$

equation in the Hilbert-space $L^2(\mathbb{R}^3, \mathbb{C}^4)$. The generator

$D = \gamma^0(-i\gamma \cdot \nabla + m1)$ defines a self-adjoint operator D with domain

$\mathcal{D}(D) = H^1(\mathbb{R}^3, \mathbb{C}^4)$, the direct sum of the four identical Sobolev

spaces $H^1(\mathbb{R}^3)$. The associated spectral representation of a solution

$e^{-iDt}f$ can be written

$$u(t,x) = (2\pi)^{-3/2} \sum_{\pm} \sum_{s=1,2} \int_{\mathbb{R}^3} \frac{d^3k}{\omega_k} e^{\mp i\omega_k t + ik \cdot x} u_{\pm}(k,s) \hat{f}_{\pm}(k,s) \quad (2.3)$$

for $f(\cdot) \in \mathcal{S}(\mathbb{R}^3, \mathbb{C}^4)$, the direct sum of the four identical Schwartz-spaces $\mathcal{S}(\mathbb{R}^3)$, where $u_{\pm}(k, s)$, $s = 1, 2$, are four orthogonal eigenvectors of $\hat{D} = \gamma^0(\gamma \cdot k + m\mathbb{1})$ normalized such that $u_{\pm}^* u_{\pm} = \omega_k = (|k|^2 + m^2)^{1/2}$. The "Fourier-components" $\hat{f}_{\pm}(\cdot, s)$ is essentially given by the ordinary Fourier-Plancherel transformation,

since $L^2(\mathbb{R}^3, \mathbb{C}^4) = \bigoplus_{n=1}^4 L^2(\mathbb{R}^3)$ it follows that the ordinary Fourier-

Plancherel transform lifts in a canonical way to a unitary operator $F_0 : L^2(\mathbb{R}_x^3, \mathbb{C}^4) \rightarrow L^2(\mathbb{R}_k^3, \mathbb{C}^4)$ such that

$F_0 D F_0^* = \hat{D} = \gamma^0(\gamma \cdot k + m\mathbb{1})$ and $\hat{f} = F_0 f$. For $f(\cdot) \in \mathcal{S}(\mathbb{R}^3, \mathbb{C}^4)$ we have the following explicit formula

$$(F_0 f)_{\pm}(k, s) = \langle v_{\pm}(\cdot, k, s), f(\cdot) \rangle_{L^2(\mathbb{R}_x^3, \mathbb{C}^4)} \quad (2.4)$$

Where

$$v_{\pm}(x, k, s) = (2\pi)^{-3/2} e^{ik \cdot x} u_{\pm}(k, s) \quad (2.5)$$

The associated Parseval's formula reads

$$\int_{\mathbb{R}^3} d^3x \, u^*(t,x)u(t,x) = \sum_{\pm} \sum_{s=1,2} \int_{\mathbb{R}^3} \frac{d^3k}{\omega_k} \left| \hat{f}_{\pm}(k,s) \right|^2 \quad (2.6)$$

for every $t \in \mathbb{R}$. Consider a solution $u(t,x)$ such that $\hat{f}_{\pm} \in \mathcal{S}(\mathbb{R}^3)$ and construct the current $J^{\mu} = \bar{u} \gamma^{\mu} u$, where $\bar{u} = u^* \gamma^0$. It follows from the Dirac equation (2.1) that J^{μ} is conserved, i.e.

$$\nabla_{\mu} J^{\mu} \equiv \frac{\partial}{\partial t} J^0 + \nabla \cdot J = 0. \text{ Intergration of this conservation equation}$$

over the region in $\mathbb{R} \times \mathbb{R}^3$ between a $t=t_0$ hyperplane, $t_0 \in \mathbb{R}$, and the ligh-cone $\mathfrak{C} = \{ (t,x) : t=t'-|x|, t' < t_0 \}$ together with Gauss's theorem and a decay estimate, given in [1], gives

$$\begin{aligned} & \int_{\mathbb{R}^3} d^3x \, \bar{u}(t'-|x|,x) (\gamma^0 + \gamma \cdot n) u(t'-|x|,x) \\ &= \sum_{s=1,2} \int_{\mathbb{R}^3} \frac{d^3k}{\omega_k} \left(\left| \hat{f}_{+}(k,s) \right|^2 + \left| \hat{f}_{-}(k,s) \right|^2 \right) \end{aligned} \quad (2.7)$$

where $n = x|x|^{-1}$, equation (2.7) resembles a Parseval's formula.

This indicates that there might exist a Hilbert-space \mathcal{H} of lighth-cone data $u(t-|\cdot|, \cdot)$ with the left-hand side of (2.7) as scalar product and such that the map $u(-|\cdot|, \cdot) \rightarrow u(t-|\cdot|, \cdot)$ defines a unitary strongly continuous one-parameter group $t \rightarrow U(t)$ on \mathcal{H} . This will be shown to be the case in the next section.

Let $u(0, x)$ have support in the region $|x| \leq R$ and consider $t \geq R$. It then follows from the finite propagation velocity that the support of $u(t-|x|, x)$ is in the region $|x| \leq (R+t)/2$. This means that there is a subspace $\mathcal{H}_0 \subset \mathcal{H}$ of lighth-cone data with compact support.

It follows again from the finite propagation velocity, that $U(t)\mathcal{H}_0 \subset \mathcal{H}_0$, for $t \geq 0$. This need not be the case for $t < 0$, which means that compact support of lighth-cone data, is in general, only a semi-group property.

3. The light-cone evolution equation.

Consider the Dirac equation (2.1) and make the coordinate exchange $t \rightarrow t' = t + |x|$ and $x \rightarrow x' = x$ such that a $t' = \text{constant}$ is the backward light-cone with its apex at $(t, 0)$. The new coordinates will be called light-cone (LC) coordinates. In the LC coordinates the Dirac

equation becomes $i(1 + \alpha \cdot n') \frac{\partial u'}{\partial t'} = (-i\alpha \cdot \nabla' + m\gamma^0)u'$ where

$u'(t', x') = u(t' - |x'|, x')$, $\alpha = \gamma^0 \gamma$ and $n' = x' |x'|^{-1}$, note that ∇' is associated with the light-cone $\mathfrak{C} = \{(t, x) : t = -|x|\}$ in the original coordinates and $\mathfrak{C} = \{x' \in \mathbb{R}^3\}$ in the new coordinates, i.e. the light-cone can be identified with \mathbb{R}^3 which will be done in the following. We shall only use the LC coordinates from now on and can therefore drop the primes. Hence the Dirac equation becomes

$$i(1 + \alpha \cdot n) \frac{\partial u}{\partial t} = (-i\alpha \cdot \nabla + m\gamma^0)u \quad (3.1)$$

where $n = x |x|^{-1}$ and $\alpha = \gamma^0 \gamma$.

Definition 3.1. Let $L^2 = L^2(\mathfrak{E}, \mathbb{C}^4)$ where the measure on \mathfrak{E} is the Lebesgue measure on \mathbb{R}^3 . Put

$$Q = \mathbb{1} + \alpha \cdot n \quad \text{and} \quad D = -i\alpha \cdot \nabla + \gamma^0 m \quad (3.2)$$

It then follows that $2^{-1}Q$ is an orthogonal projection in L^2 and that the Dirac operator D defines a self-adjoint operator in L^2 with spectrum $\sigma(D) = (-\infty, -m] \cup [m, \infty)$, $m > 0$.

The Dirac equation (3.1) can now be considered as an equation in L^2

$$iQ \frac{du}{dt}(t) = Du(t) \quad (3.3)$$

which can be written as

$$iD^{-1}Q \frac{du}{dt}(t) = u(t) \quad (3.4)$$

since $0 \notin \sigma(D)$. From equation (3.4) it follows that if $\frac{du}{dt} \in L^2$

then $u \in D^{-1}QL^2$.

Definition 3.2. Let $B = D^{-1}Q$ and $\mathcal{H}_0 = BL^2$. Furthermore put

$$\langle f, g \rangle_{\mathcal{H}_0} = \langle f, Qg \rangle_{L^2} \quad (3.5)$$

for $f, g \in \mathcal{H}_0$.

It will be proved that \mathcal{H}_0 is a pre-Hilbert space, but before doing that we have to mention two lemmas, both proved in [1].

Lemma 3.3. Let $S_0 = -i(n \cdot \nabla + \nabla \cdot n)$ with domain $\mathcal{D}(S_0) = C_0^\infty(\mathbb{R}^3)$

in $L^2(\mathbb{R}^3)$, then the following inequality holds

$$\|S_0 f\|_{L^2(\mathbb{R}^3)} \leq 4 \int_{\mathbb{R}^3} d^3x |\nabla f(x)|^2 \quad (3.6)$$

for all $f \in \mathcal{D}(S_0)$.

Lemma 3.4. The range $\mathcal{R}(S_0)$ of S_0 is dense in $L^2(\mathbb{R}^3)$.

Due to the inequality (3.6) S_0 can be extended continuously to the Sobolev space $H^1(\mathbb{R}^3)$, let this extension be denoted S , it also follows that S is symmetric in $L^2(\mathbb{R}^3)$, since S_0 is. Due to lemma 3.4. also S have dense range $\mathcal{R}(S_0)$ in $L^2(\mathbb{R}^3)$, and since S is symmetric in $L^2(\mathbb{R}^3)$ it follows that S is one-to-one, i.e. S has a densely defined inverse in $L^2(\mathbb{R}^3)$, in fact it is easy to show that

$$(S_0^{-1}g)(x) = \frac{i}{2r} \int_{-\infty}^r \rho g(\rho, x|x|^{-1}) d\rho \quad (3.7)$$

for $g \in \mathcal{R}(S_0)$.

Proposition 3.5. \mathcal{H}_0 is a pre-Hilbert space.

Proof. The non-trivial part is to prove that if $\langle f, f \rangle_{\mathcal{H}_0} = 0$ then $f = 0$

and since $\langle f, f \rangle_{\mathcal{H}_0} = \langle f, Qf \rangle_{L^2}$ this is to prove that $Qf = 0$ implies

$f = 0$ in L^2 . Let $f \in \mathcal{H}_0 \setminus \{0\}$. From the anti-commutation relation (2.2)

it follows by a direct computation that

$$(\alpha \cdot n)(\alpha \cdot \nabla) + (\alpha \cdot \nabla)(\alpha \cdot n) = (n \cdot \nabla + \nabla n) \mathbf{1} \quad (3.8)$$

on $C_0^\infty(\mathbb{R}^3, \mathbb{C}^4)$. In section 2 it was shown that the Dirac operator D in

L^2 is self-adjoint with domain $\mathcal{D}(D) = H^1(\mathbb{R}^3, \mathbb{C}^4)$. Now the above

defined operator S can be lifted in a canonical way to an operator $S \cdot \mathbf{1}$

on $\mathcal{D}(D)$, where $\mathbf{1}$ here denotes the 4×4 unit matrix. Hence

$$DQ = (2 - Q)D + S \cdot \mathbf{1} \quad (3.9)$$

on $\mathcal{D}(D)$, since the left-hand side and the right-hand side agree on the

dense subspace $C_0^\infty(\mathbb{R}^3, \mathbb{C}^4)$. Since $0 \notin \sigma(D)$ then D have a bound invers

D^{-1} defined on $\mathcal{R}(D)$, which is dense in L^2 , hence D^{-1} can be extended

to a continuous operator on L^2 also denoted by D^{-1} . Acting on (3.9)

with D^{-1} from both left and right gives

$$QD^{-1} = D^{-1}(2-Q) + D^{-1}S \cdot 1D^{-1} \quad (3.10)$$

on $\mathcal{R}(D)$. By acting with Q from both left and right equation (3.10) gives

$$QB = D^{-1}S \cdot 1B \quad (3.11)$$

on $\{f \in L^2 : Qf \in \mathcal{R}(D)\}$, since $Q^2 = 2Q$. Now $f = Bg \in \mathcal{H}_0 \setminus \{0\}$ so it follows from equation (3.11) that $Qf = QBg = D^{-1}S \cdot 1Bg$ and since D^{-1} is a bounded operator and the kernel for S is trivial in $L^2(\mathbb{R}^3)$ then $D^{-1}S \cdot 1Bg \neq 0$, i.e. if $f \in \mathcal{H}_0 \setminus \{0\}$ then $Qf \in L^2 \setminus \{0\}$, or in other words if $Qf = 0$ in L^2 then $f = 0$ in \mathcal{H}_0 .

Definition 3.6. Let \mathcal{H} be the completion of \mathcal{H}_0 .

It follows from the definition that $2^{-1/2}Q$ defines an isometry from \mathcal{H} into L^2 , and that B defines a bounded self-adjoint operator on \mathcal{H} .

Proposition 3.7. The range $\mathcal{R}(B)$ of $B : \mathcal{H} \rightarrow \mathcal{H}$ is dense in \mathcal{H} .

Proof. Let $\psi \in (B\mathcal{H})^\perp$, i.e. ψ is in the orthogonal complement of $B\mathcal{H}$,

so $\langle \psi, Bf \rangle_{\mathcal{H}} = 0$ for all $f \in \mathcal{H}$. Consider $f \in \mathcal{H}_0$, then $f = Bg$ for some

$g \in L^2$. Then $\langle \psi, Bf \rangle = 0$ for all $f \in \mathcal{H}$ implies that

$\langle \psi, QD^{-1}QD^{-1}Qg \rangle_{L^2} = 0$ for all $g \in L^2$. Since $QD^{-1}QD^{-1}Q = (D^{-1}S \cdot 1D^{-1})^2Q$

and the commutator $[Q, D^{-1}S \cdot 1D^{-1}] = 0$ then

$$\begin{aligned} \langle (D^{-1}S \cdot 1D^{-1})^2Q\psi, g \rangle_{L^2} &= \langle QD^{-1}QD^{-1}Q\psi, g \rangle_{L^2} \\ &= \langle \psi, QD^{-1}QD^{-1}Qg \rangle_{L^2} \\ &= \langle \psi, D^{-1}QD^{-1}Qg \rangle_{L^2} = 0 \end{aligned} \quad (3.12)$$

for all $g \in L^2$, i.e. $(D^{-1}S \cdot 1D^{-1})^2Q\psi = 0$. Hence $Q\psi = 0$ in L^2 , since D^{-1}

is invertible and the kernel for S is trivial in $L^2(\mathbb{R}^3)$, i.e. $\psi = 0$ in \mathcal{H}

(which is proved in proposition 3.5.)

Proposition 3.7. shows that B is one-to-one since it is self-adjoint.

Definition 3.8. Put $H = B^{-1}$ with $\mathcal{D}(H) = \mathcal{R}(B)$ in \mathcal{H} .

Note that H is self-adjoint since B is. Equation (3.4) can then be written as

$$i \frac{du}{dt}(t) = Hu(t) \quad (3.13)$$

with the solution $u(t) = e^{-iHt}u(0)$, $u(0) \in \mathcal{D}(H)$, i.e. $U_t = e^{-iHt}$ is the time-evolution operator for light-cone data. From definition 3.8 it follows that

$$\langle f, Hf \rangle_{\mathcal{H}} = \langle f, Df \rangle_{L^2} \quad (3.14)$$

and

$$\langle Hf, Hf \rangle_{\mathcal{H}} = \langle Df, Df \rangle_{L^2} \quad (3.15)$$

for all $f \in \mathcal{D}(H)$, i.e. $\mathcal{D}(H) \subset \mathcal{D}(D) = H^1(\mathbb{R}^3, \mathbb{C}^4)$.

4. The light-cone Fourier transform.

The self-adjointness of H implies, due to the spectral theorem, that

there exist a spectral family $\{E(\lambda)\}_{\lambda \in \mathbb{R}}$ such that $H = \int_{\mathbb{R}} \lambda dE(\lambda)$.

The following well-known formula allows $E(\lambda)$ to be expressed in terms of the resolvent $R(z) = (H-z)^{-1}$, for z in the resolvent set $\rho(H)$

$$\begin{aligned} & \langle f, 1/2[(E(\beta)+E(\beta-0))-(E(\alpha)+E(\alpha-0))]f \rangle_{\mathcal{H}} \\ &= \lim_{\varepsilon \searrow 0} \frac{1}{2\pi i} \int_{\alpha}^{\beta} d\mu \langle f, [R(\mu+i\varepsilon)-R(\mu-i\varepsilon)]f \rangle_{\mathcal{H}} \end{aligned} \quad (4.1)$$

In the following the right-hand side of equation (4.1) will be evaluated.

Let f belong to $C_0^{\infty}(\mathbb{R}^3, \mathbb{C}^4)$ and put $g = R(z)f$ for $\text{Im}(z) \neq 0$.

It then follows that g belongs to $\mathcal{D}(H)$ and fulfills the equation

$$(H-z)g = f \quad (4.2)$$

which also can be written as $(B^{-1}-z)g = f$. Now applying B to this equation gives $(1-zB)g = Bf$. Consider this equation in L^2 , then the fact that $\mathcal{D}(H) \subset \mathcal{D}(D)$ allows us to act with D , which gives

$$(D-zQ)g = Qf \tag{4.3}$$

since $B = D^{-1}Q$.

Definition 4.1. Let V_z be defined in L^2 by

$$(V_z\varphi)(x) = e^{-iz|x|}\varphi(x) \tag{4.4}$$

on its maximal domain.

Proposition 4.2. If φ is in L^2 such that $V_z^{-1}\varphi$ is in $\mathcal{D}(D)$ then

$$(D-zQ)V_z^{-1}\varphi = V_z^{-1}(D-z)\varphi \tag{4.5}$$

Proof. Equation (4.5) follows from a straightforward computation.

Rewriting equation (4.3) as $(D-zQ)V_z^{-1}V_zg = Qf$ and using

proposition 4.2 gives

$$V_z^{-1}(D-z)V_z g = f \quad (4.6)$$

for $\text{Im}(z) < 0$, such that V_z is bounded. Equation (4.6) can be solved for g , which gives

$$g = V_z^{-1}(D-z)^{-1}V_z Qf \quad (4.7)$$

and then

$$\langle f, R(z)f \rangle_{\mathcal{H}} = \langle V_z^{-1}Qf, (D-z)^{-1}V_z Qf \rangle_{L^2} \quad (4.8)$$

for $\text{Im}(z) < 0$. The left-hand side of (4.8) is analytic for z not in the spectra $\sigma(H)$ of H , which then shows that the right-hand side has an analytic continuation to $\text{Im}(z) > 0$.

As described in section 2 the spectral theory for the Dirac operator D in L^2 , is given by a unitary operator F_0 on L^2 , such that

$F_0 D F_0^* = \hat{D} = \alpha \cdot k + \gamma^0 m$, $k \in \mathbb{R}^3$, with eigenvalues $\pm \omega_k$. For $f \in L^2$

put for short $\hat{f} = F_0 f$. Equation (4.8) can then be written as

$$\langle f, R(z)f \rangle_{\mathcal{H}} = \langle \widehat{V_{\bar{z}} Q f}, (\hat{D} - z)^{-1} \widehat{V_z Q f} \rangle_{L^2} \quad (4.9)$$

where $(\hat{D} - z)^{-1}$ has the following explicit representation

$$(\hat{D} - z)^{-1} = \omega_k^{-1} \sum_{\pm} \sum_{s=1,2} (\pm \omega_k - z)^{-1} u_{\pm}(k,s) u_{\pm}^*(k,s) \quad (4.10)$$

and $u(\cdot, s)$ are four orthogonal eigenvectors of \hat{D} normalized such

that $u_{\pm}^*(k,s) u_{\pm}(k,s) = \omega_k$ also described in section 2.

Definition 4.3. If $f \in C_0^{\infty}(\mathbb{R}^3, \mathbb{C}^4)$ then $f(\cdot, s)$ is defined by

$$f_{\pm}(k,s) = u_{\pm}^*(k,s) (\widehat{V_{\pm \omega_k} Q f})(k) \quad (4.11)$$

for $s = 1, 2$.

With proper interpretation formula (4.11) also can be written as

$$\widehat{f}_{\pm}(k,s) = \langle u_{\pm}(\cdot, k, s), f(\cdot) \rangle_{\mathcal{H}} \quad (4.12)$$

where

$$u_{\pm}(x, k, s) = (2\pi)^{-3/2} e^{\pm i\omega_k |x| + ik \cdot x} u_{\pm}(k, s) \quad (4.13)$$

Now the right-hand side of equation (4.1) can be evaluated. Let

$f \in C_0^{\infty}(\mathbb{R}^3, \mathbb{C}^4)$ and put

$$g_{\pm}(k, s, z) = (u_{\pm}^*(k, s) \widehat{(V_{\bar{z}} Q f)}(k))^* (u_{\pm}^*(k, s) \widehat{(V_z Q f)}(k)) \quad (4.14)$$

then formula (4.9) becomes

$$\langle f, R(z)f \rangle_{\mathcal{H}} = \sum_{\pm} \sum_{s=1,2} \int_{\mathbb{R}^3} \frac{d^3 k}{\omega_k} (\pm \omega_k - z)^{-1} g_{\pm}(k, s, z) \quad (4.15)$$

Due to the fact that g_{\pm} is analytic in z and continuous in k

Fubini's theorem can be used on equation (4.15) which gives

$$\begin{aligned} & \lim_{\epsilon \searrow 0} \frac{1}{2\pi i} \int_{\alpha}^{\beta} d\mu \langle f, [R(\mu+i\epsilon) - R(\mu-i\epsilon)] f \rangle_{\mathcal{H}} \\ &= \sum_{\pm} \sum_{s=1,2} \lim_{\epsilon \searrow 0} \int_{\mathbb{R}^3} \frac{d^3 k}{\omega_k} \frac{1}{2\pi i} \int_{\alpha}^{\beta} d\mu h_{\pm}(k, s, \mu, \epsilon) \end{aligned} \quad (4.16)$$

where

$$\begin{aligned} h_{\pm}(k, s, \mu, \epsilon) &= (\pm\omega_k - (\mu+i\epsilon))^{-1} g_{\pm}(k, s, \mu+i\epsilon) \\ &\quad - (\pm\omega_k - (\mu-i\epsilon))^{-1} g_{\pm}(k, s, \mu-i\epsilon) \end{aligned} \quad (4.17)$$

Since g_{\pm} is analytic in z Taylor's formula can be used and together

with Lebesgue's dominated convergence theorem equation (4.16)

becomes

$$\begin{aligned} & \lim_{\epsilon \searrow 0} \frac{1}{2\pi i} \int_{\alpha}^{\beta} d\mu \langle f, [R(\mu+i\epsilon) - R(\mu-i\epsilon)] f \rangle_{\mathcal{H}} \\ &= \sum_{\pm} \sum_{s=1,2} \int_{\mathbb{R}^3} \frac{d^3 k}{\omega_k} : \pi^{-1} \lim_{\epsilon \searrow 0} \int_{\alpha}^{\beta} d\mu \Psi_{\pm}(k, s, \mu, \epsilon) \frac{\epsilon}{(\mu \mp \omega_k)^2 + \epsilon^2} \end{aligned} \quad (4.18)$$

where

$$\begin{aligned}
 \Psi_{\pm}(k,s,\mu,\epsilon) = & g_{\pm}(k,s,\mu) - g_{\pm}'(k,s,\mu)(\mu \mp \omega_k) \\
 & - \epsilon(1/2[\operatorname{Re}(g_{\pm}''(\xi_{\pm})) + \operatorname{Im}(g_{\pm}''(\eta_{\pm}))](\pm \omega_k - (\mu - i\epsilon)) \\
 & - 1/2[\operatorname{Re}(g_{\pm}''(\xi_{\pm}')) + \operatorname{Im}(g_{\pm}''(\eta_{\pm}'))](\pm \omega_k - (\mu - i\epsilon)))
 \end{aligned}
 \tag{4.19}$$

here g_{\pm}' denote the partial derivative of g_{\pm} , g_{\pm}'' the second partial derivative of g_{\pm} in the z -variable and $0 \leq \xi_{\pm}, \xi_{\pm}', \eta_{\pm}, \eta_{\pm}' \leq \epsilon$.

Finally since $f_{\epsilon}(y) = \pi^{-1}\epsilon(y^2 + \epsilon^2)^{-1}$ is a delta family (as $\epsilon \searrow 0$) equation (4.18) gives

$$\begin{aligned}
 \lim_{\epsilon \searrow 0} \frac{1}{2\pi i} \int_{\alpha}^{\beta} d\mu \langle f, [R(\mu + i\epsilon) - R(\mu - i\epsilon)] f \rangle_{\mathcal{H}} \\
 = \sum_{\pm} \sum_{s=1,2} \int_{\alpha < \pm \omega_k < \beta} \frac{d^3 k}{\omega_k} \Psi_{\pm}(k,s,\pm \omega_k, 0)
 \end{aligned}
 \tag{4.20}$$

i.e.

$$\lim_{\epsilon \searrow 0} \frac{1}{2\pi i} \int_{\alpha}^{\beta} d\bar{\mu} \langle f, [R(\bar{\mu} + i\epsilon) - R(\bar{\mu} - i\epsilon)] f \rangle_{\mathcal{H}}$$

$$= \left\{ \begin{array}{ll} \sum_{s=1,2} \int_{\alpha < \omega_k < \beta} \frac{d^3 k}{\omega_k} \quad | \hat{f}_+(k,s) |^2, & m < \alpha < \beta \\ \sum_{s=1,2} \int_{-\beta < \omega_k < -\alpha} \frac{d^3 k}{\omega_k} \quad | \hat{f}_-(k,s) |^2, & \alpha < \beta < -m \\ \sum_{s=1} \int_{\omega_k < \beta} \frac{d^3 k}{\omega_k} \quad | \hat{f}_+(k,s) |^2 \\ + \sum_{s=1,2} \int_{\omega_k < -\alpha} \frac{d^3 k}{\omega_k} \quad | \hat{f}_-(k,s) |^2, & \alpha < -m, \beta > m \\ 0 & , \text{ otherwise} \end{array} \right. \quad (4.21)$$

duing to the fact that $\Psi_{\pm}(k,s,\pm\omega_k,0) = g_{\pm}(k,s,\pm\omega_k) = | \hat{f}_{\pm}(k,s) |^2$.
From equation (4.1) and (4.17) it then follows that

$$\langle f, [E(\beta) - E(\alpha)]f \rangle_{\mathcal{H}}$$

$$= \begin{cases} \sum_{s=1,2} \int_{\alpha < \omega_k < \beta} \frac{d^3k}{\omega_k} |\hat{f}_+(k,s)|^2, & m < \alpha < \beta \\ \sum_{s=1,2} \int_{-\beta < \omega_k < -\alpha} \frac{d^3k}{\omega_k} |\hat{f}_-(k,s)|^2, & \alpha < \beta < -m \\ \sum_{s=1,2} \int_{\omega_k < \beta} \frac{d^3k}{\omega_k} |\hat{f}_+(k,s)|^2 \\ + \sum_{s=1,2} \int_{\omega_k < -\alpha} \frac{d^3k}{\omega_k} |\hat{f}_-(k,s)|^2, & \alpha < -m, \beta > m \\ 0 & , \text{otherwise} \end{cases} \quad (4.22)$$

Letting $\alpha \rightarrow -\infty$ and $\beta \rightarrow \infty$ formular (4.23) gives

$$\|f\|_{\mathcal{H}}^2 = \sum_{\pm} \sum_{s=1,2} \int_{\mathbb{R}^3} \frac{d^3k}{\omega_k} |\hat{f}_{\pm}(k,s)|^2 \quad (4.24)$$

for $f \in C_0^\infty(\mathbb{R}^3, \mathbb{C}^4)$. Now the map defined in definition 4.3 can be extended by continuity to an isometry.

Definition 4.4. Let $\widehat{\mathcal{H}} = \widehat{\mathcal{H}}_+ \oplus \widehat{\mathcal{H}}_-$, $\widehat{\mathcal{H}}_\pm = L^2(\mathfrak{M}_\pm, \mathbb{C}^2)$ where \mathfrak{M}_\pm are the mass-hyperboloids and the measure is $\frac{d^3k}{\omega_k}$. Define

$$F : \mathcal{H} \rightarrow \widehat{\mathcal{H}} \text{ by } Ff = (\widehat{f}_+, \widehat{f}_-).$$

It follows from formula (2.7) that the map $F : \mathcal{H} \rightarrow \widehat{\mathcal{H}}$ in fact is onto, i.e. F is unitary.

Theorem 4.5. The unitary map $F : \mathcal{H} \rightarrow \widehat{\mathcal{H}}$, called the "light-cone Fourier transform", diagonalizes H , i.e.

$$FHf = (\omega_k \widehat{f}_+ , -\omega_k \widehat{f}_-) \quad (4.25)$$

for $f \in \mathcal{D}(H)$.

Proof. Let $f \in C_0^\infty(\mathbb{R}^3, \mathbb{C}^4)$ then

$$\begin{aligned} \widehat{Hf}_\pm(k,s) &= \langle u_\pm(\cdot, k, s), Hf(\cdot) \rangle_{\mathcal{H}} \\ &= \langle Du_\pm(\cdot, k, s), f(\cdot) \rangle_{L^2} \\ &= \pm \omega_k \langle u(\cdot, k, s), Qf(\cdot) \rangle_{L^2} \\ &= \pm \omega_k \widehat{f}_\pm(k, s) \end{aligned} \quad (4.26)$$

where Du_\pm is understood in weak sense. Since H is closed and F is continuous, formula (4.25) holds for all $f \in \mathcal{D}(H)$.

The light-cone Fourier transform defines a kind of "duality" between the light-cone and the mass-hyperboloids.

Recall that formula (2.3) gives rise to a unitary map from $\hat{\mathcal{H}}$ onto $L^2(\mathbb{R}^3, \mathbb{C}^4)$ of $t = \text{constant}$ data, i.e. there is a one-to-one correspondance between weak solutions $u(t, x)$ of the Dirac equation (2.1) with $u(t, \cdot) \in L^2(\mathbb{R}^3, \mathbb{C}^4)$ and the solution $e^{-iHt}f$, $f \in \mathcal{H}$, of equation (3.13).

5. Conclusion.

We have shown that the characteristic Cauchy problem for the Dirac equation

$$(i\gamma^0 \frac{\partial}{\partial t} + i\gamma \cdot \nabla - m\mathbf{1})u(t,x) = 0$$

$$u(-|x|, x) = f(x) \quad , \quad x \in \mathbb{R}^3 \quad (5.1)$$

has a unique weak solution with $u(t, \cdot) \in L^2(\mathbb{R}^3, \mathbb{C}^4)$ for all $f \in \mathcal{H}$ and that all "L²-solutions" are obtained in this way.

The Dirac equation was written as an evolution equation for light-cone data

$$i \frac{du}{dt} = Hu \quad (5.2)$$

and the spectral theory for H was developed including a "light-cone Fourier transform" which diagonalized H .

I want to acknowledge Department of mathematics, University of Copenhagen, Denmark where this letter is done and Department of mathematics, University of California at Santa Barbara where the final form has taken place. Especialy I want to thank Lars-Erik Lundberg Department of mathematics, University of Copenhagen, Denmark for suggesting the problem and for giving me lots of advice.

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Projektrapport af: Niels Jørgensen og Mikael Klintorp.
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Projektrapport af: Lis Eilertzen, Kirsten Habekost, Lill Røn og Susanne Stender.
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"A SIMPLE MODEL OF AC HOPPING CONDUCTIVITY".
Af: Jeppe C. Dyre.
Contributions to the Third International Conference on the Structure of Non - Crystalline Materials held in Grenoble July 1985.
- 106/85 "QUANTUM THEORY OF EXTENDED PARTICLES".
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- flodblindhed som eksempel på matematisk modellering af et epidemiologisk problem.
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Fysiklærerforeningen, IMFUFA, RUC.
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Samtlige opgaver stillet i tiden 1974-jan. 1986.
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Projektrapport af: Birger Lundgren.
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Projektrapport af: Pernille Sand, Heine Larsen & Lars Frandsen.
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Vejledere: Jens Høyrup, Jørgen Vogelius, Jens Højgaard Jensen.
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Vejleder: Jesper Larsen
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167a/88 "BASISSTATISTIK 1. Diskrete modeller"

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167b/88 "BASISSTATISTIK 2. Kontinuerte
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168/88 "OVERFLADEN AF PLANETEN MARS"

Laboratorie-simulering og MARS-analoger
undersøgt ved Mössbauerspektroskopi.

Fysikspeciale af:

Birger Lundgren

Vejleder: Jens Martin Knudsen
Fys.Lab./HCØ

169/88 "CHARLES S. PEIRCE: MURSTEN OG MØRTEL
TIL EN METAFYSIK."

Fem artikler fra tidsskriftet "The Monist"
1891-93.

Introduktion og oversættelse:

Peder Voetmann Christensen

170/88 "OPGAVESAMLING I MATEMATIK"

Samtlige opgaver stillet i tiden
1974 - juni 1988