# The Herman-Swiatec theorem with applications

Petersen, Carsten Lunde

*Publication date:*
1998

*Document Version*
Publisher's PDF, also known as Version of record

*Citation for published version (APA):*
Petersen, C. L. (1998). *The Herman-Swiatec theorem with applications*. Roskilde Universitet. Tekster fra IMFUFA No. 352 http://milne.ruc.dk/ImfufaTekster/

**TEKST NR 352** **1998**

# The Herman-Świątec Theorem with applications

**Carsten Lunde Petersen**

**TEKSTER fra**

IMFUFA **ROSKILDE UNIVERSITETSCENTER**
INSTITUT FOR STUDIET AF MATEMATIK OG FYSIK SAMT DERES
FUNKTIONER I UNDERVISNING, FORSKNING OG ANVENDELSER

IMFUFA, ROSKILDE UNIVERSITY, POSTBOX 260, DK-4000 ROSKILDE
DENMARK.

# The Herman-Światec Theorem with applications

by

**Carsten Lunde Petersen**

IMFUFA-text no 352/98 14 pages ISSN 0106-6242

---

## Abstract

The paper provides a proof of the Herman-Światec Theorem [Her] that a $C^3$ critical circle homeomorphism with one non-flat critical point and irrational rotation number $\theta$ is quasi-symmetrically conjugate to the corresponding rigid rotation, if and only if $\theta$ is Diophantine of exponent 2. Moreover the Herman-Światec Theorem is applied to prove there exist quadratic Siegel polynomials, $P_\theta(z) = e^{i2\pi\theta}z + z^2$, with $\theta$ of unbounded type, for which the Julia set is locally connected.

# The Herman-Światec Theorem with applications

Carsten Lunde Petersen

**Abstract**

The paper provides a proof of the Herman-Światec Theorem [Her] that a $C^3$ critical circle homeomorphism with one non-flat critical point and irrational rotation number $\theta$ is quasi-symmetrically conjugate to the corresponding rigid rotation, if and only if $\theta$ is Diophantine of exponent 2. Moreover the Herman-Światec Theorem is applied to prove there exist quadratic Siegel polynomials, $P_\theta(z) = e^{i2\pi\theta}z + z^2$, with $\theta$ of unbounded type, for which the Julia set is locally connected.

## 1 Introduction

Let $f$ be a $C^3$, orientation preserving circle homeomorphism with irrational rotation number $\theta$. We suppose moreover that $f$ has exactly one critical point $a_0$, which is non flat. We shall identify the circle with $\mathbb{T} = \mathbb{R}/\mathbb{Z}$ and give $\mathbb{T}$ the induced orientation. Moreover by abuse of notation we shall not distinguish $f$ and lifts of $f$ to $\mathbb{R}$. Technically it suffices that :

**Hypothesis 1.1** *The homeomorphism $f$ is $C^1$, has a critical point $a_0$ and there exists a neighbourhood $W \subsetneq \mathbb{T}$ of $a_0$ such that :*

1. *$f$ is $C^3$ on $W$ and has negative Schwarzian derivative $Sf$ on $W \setminus a_0$.*
2. *There exists a constant $A > 0$ and $l \in \mathbb{N}$ such that*

$$\forall x \in W : A|x - a_0|^{2l} < f'(x) < 2A|x - a_0|^{2l}$$

3. *The variation of $\log f'$ on $\mathbb{T} \setminus W$ is bounded by some number, say $\rho$.*

4. *$f$ has irrational rotation number $\theta \in \mathbb{T}$*

In this article we shall prove the following Theorem:

**Theorem 1.2 (Herman-Swiatec, 86)** *Let $f$ be a circle homeomorphism with rotation number $\theta$ and satisfying the hypotheses above. Then $f$ is $c$-quasi symmetrically conjugate to the rigid rotation (of angle $\theta$), if and only if $\theta$ has constant type. Moreover the constant $c > 1$ depends only on the bound $N$ on the coefficients of the continued fraction expansion for $\theta$.*

The proof is based partially on a more extensive, but unpublished manuscript by J.-C. Yoccoz, [Yoc].

For $|\theta| = 1$ let $c_\theta$ be the complex number for which the quadratic polynomial $Q_{c_\theta}(z) = z^2 + c_\theta$ has a fixed point with multiplier $\exp(i2\pi\theta)$. Herman has proved that there exists irrational numbers $\theta$ not of constant type for which the quadratic polynomial $Q_{c_\theta}$ has a Siegel disk, whose boundary is a Jordan curve containing the critical point 0. Here we take his proof one step further and prove:

**Theorem 1.3** *There exist irrational $\theta$ not of constant type, such that the Julia set of the quadratic polynomial $Q_{c_\theta}$ is locally connected.*

Following the established conventions of the subject, we shall freely use the letter $c$ to denote any constant, which depends only on $Df$. The obtained constants are by no means optimal. It should be noted however that unless stated explicitly otherwise *the constants $c$ depend only on those macroscopic properties of $Df$, stipulated in the first three items of the above hypotheses.* In particular the constants do not depend on the irrational rotation number $\theta$ of $f$. Thus given some $f$ then for any $\eta \in \mathbb{T}$ such that $f_\eta = f + \eta$ have irrational rotation number, the constants only depend on $f$.

# 2 Notation and prerequisites

**Notation 2.1** *We shall use freely the following notations:*

1. *The sequence of irreducible rational numbers $\left\{\frac{p_n}{q_n}\right\}_{n \geq 0}$ will denote the convergents of $\theta$. Moreover the sequence $\{b_n\}_{n \in \mathbb{N}}$ denotes the coefficients of the continued fraction expansion of $\theta$, so that $p_n = b_n p_{n-1} + p_{n-2}$ and $q_n = b_n q_{n-1} + q_{n-2}$. The number $\theta$ is Diophantine of exponent 2*

*or equivalently, has constant type, if and only if the sequence $b_n$ is bounded.*

2. *For each $i \in \mathbb{Z}$ define $a_i = f^{-i}(a_0)$, (note the critical orbit is indexed time reversedly).*

3. *For $a < b < c < d < a+1$ in $\mathbb{R}$ we define the cross-ratio :*

$$[a,b,c,d] = \frac{b-a}{c-a}\frac{d-c}{d-b}$$

4. *For $n \geq 0$ we define*

$$I_n(x) = \begin{cases} [x, f^{-q_n}(x)], & n \text{ odd}, \\ [f^{-q_n}(x), x], & n \text{ even}. \end{cases}$$
$$m_n(x) = |f^{q_n}(x) - x|,$$
$$K_n = \{a_i \mid 0 \leq i < q_n\}, \qquad K_n^* = K_n - \{a_{q_n}\}$$

Let us recall some basic properties of circle homeomorphism with irrational rotation numbers in general and critical such circle homeomorphism in particular. The first property is the Poincaré semi-conjugation Theorem.

**Theorem 2.2 (Poincaré semi-conjugation Theorem)** *Let $f$ be any orientation preserving circle homeomorphism with irrational rotation number $\theta$. Then $f$ is semi conjugate to the rigid rotation $R_\theta(z) = z + \theta$ with angle $\theta$, i.e. there exists a continuous map $\phi : \mathbb{T} \longrightarrow \mathbb{T}$ such that $\phi \circ f = R_\theta \circ \phi$.*

The Poincaré semi-conjugation Theorem implies that the combinatorial orbit structure of any orientation preserving circle homeomorphism is the same as that for the rigid rotation.

**Remark 2.3** *Recall the following properties of circle homeomorphisms:*

1. *For even $n$ (respectively for odd $n$) the point of $K_n^*$ following (respectively preceding) a point $a_i$ for $0 \leq i < q_n$ is $a_j$ with*

$$j = \begin{cases} i + q_{n-1}, & \text{if } 0 \leq i < q_n - q_{n-1} \\ i + q_{n-1} - q_n, & \text{if } q_n - q_{n-1} < i < q_n \end{cases}$$

2. *For any $x \in \mathbb{T}$ and any $n \geq 1$ all the intervals $I_n(f^j(x))$ for $0 \leq j < q_{n+1}$ and $I_{n+1}(f^j(x))$ for $0 \leq j < q_n$ have mutually disjoint interiors.*

3. *For any $x \in \mathbb{T}$*

$$\mathbb{T} = \bigcup_{0 \leq j < q_{n+1}+q_n} I_n(f^j(x)) = \bigcup_{0 \leq j < q_{n+1}} I_n(f^j(x)) \cup \bigcup_{0 \leq j < q_n} I_{n+1}(f^j(x))$$

4. *Let $a < b < c < d < a+1$ in $\mathbb{R}$ be arbitrary and let $N \geq 1$. If the open interval $]a,d[$ does not contain any of the points $a_j$, $0 \leq j < N$ and if $Sf^N \leq 0$ on $]a,d[$, then*

$$[f^N(a), f^N(b), f^N(c), f^N(d)] \leq [a,b,c,d].$$

*Here as elsewhere $Sf$ denotes the Schwarzian derivative of $f$ and is given by*

$$\begin{aligned} Sf &= D^2 \log Df - \frac{1}{2}(D \log Df)^2 \\ &= (D^3 Df - \frac{3}{2}(D^2 f)^2)(Df)^{-2} \\ &= -2(Df)^{-\frac{1}{2}} \cdot D^2(Df)^{-\frac{1}{2}}. \end{aligned}$$

**Theorem 2.4 (Światec)** *There exists a constant $c > 1$ such that for any quadruple $a < b < c < d < a+1$ in $\mathbb{R}$ and any $N > 0$ one has*

$$[f^N(a), f^N(b), f^N(c), f^N(d)] \leq c^m \cdot [a,b,c,d],$$

*where $m$ is the covering number :*

$$m = \max_{x \in \mathbb{T}} \#\{j | 0 \leq j < N \text{ and } x \in ]f^j(a), f^j(d)[\}.$$

**Proof :** This is the *Cross-Ratio Inequality* of Światec [Swi, p.112]. **q.e.d.**

Combining Remark 2.3.4. and Theorem 2.4 we have

**Corollary 2.5** *Suppose $f$ is $C^3$ and $Sf(x) \leq 0$, $\forall x \in \mathbb{T} - \{a_0\}$. Then there exists a constant $c > 1$ such that for any $N > 0$ and any $a < b < c < d < a+1$ in $\mathbb{R}$ one has*

$$[f^N(a), f^N(b), f^N(c), f^N(d)] \leq c^m \cdot [a,b,c,d],$$

*where $m = \#\{j | 0 \leq j < N$ and $a_0 \in ]f^j(a), f^j(d)[\}$.*

# 3 The a priori bounds

Let us fix $n \geq 0$, say even to fix the ideas. Set $q = q_n$, $m(x) = m_n(x)$, $I(x) = I_n(x)$ for $x \in \mathbb{T}$ and define

$$\Delta(x) = [f^{-2q}(x), f^{-q}(x), x, f^q(x)] \qquad \text{for } x \in \mathbb{T}.$$

**Lemma 3.1** *There exists a constant $c > 1$ such that $\forall x \in \mathbb{T}$ and for all $z \in I(x)$*

$$c^{-1} \cdot \min\{m(z), m(f^q(z))\} \leq m(x) \leq m(z) + m(f^q(z)).$$

**Proof :** The right-hand inequality is a useful but trivial observation. Towards the left hand inequality, let us prove that, there exists a constant $c_1 > 1$ such that

$$m(x) \geq c_1^{-1} \cdot \min\{m(f^q(x)), m(f^{-q}(x))\} \qquad \forall x \in \mathbb{T}. \tag{1}$$

For any $x \in \mathbb{T}$ the inequality $m(x) \leq \min\{m(f^q(x)), m(f^{-q(x)})\}$ implies $\Delta(x) \geq \frac{1}{4}$. Write $r(x) \cdot m(x) = \min\{m(f^q(x)), m(f^{-q}(x))\}$, and assume $r(x) > 1$ then

$$\begin{aligned}\Delta(f^{-q}(x)) &= \frac{m(f^{-2q}(x))}{m(f^{-2q}(x)) + m(f^{-q}(x))} \cdot \frac{m(x)}{m(x) + m(f^{-q}(x))} \\ &\leq \frac{m(x)}{m(x) + m(f^{-q}(x))} \leq \frac{1}{1 + r(x)}.\end{aligned}$$

On the other hand the intervals $f^j(]f^{-3q}(x), x[)$, $0 \leq j < q$ cover any point

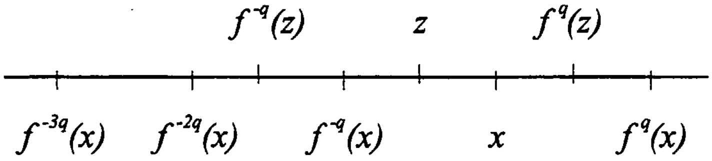

Figure 1:

$y \in \mathbb{T}$ at most 3 times. Let $c_0 > 1$ be the constant of the Światec Theorem (2.4), then

$$\frac{1}{4} \leq \Delta(x) \leq c_0^3 \cdot \Delta(f^{-q}(x)) \leq \frac{c_0^3}{1 + r(x)},$$

thus $r(x) \leq 4 \cdot c_0^3 - 1$.

Let $c = 4 \cdot c_0^3$, so that the constant $c_1 = c - 1$ applies in (1), by arbitrariness of $x \in \mathbb{T}$. Let $x \in \mathbb{T}$ and $z \in I(x)$ be arbitrary. Then by (1) either

$$m(z) < m(f^{-q}(x)) + m(x) \leq (c_1 + 1) \cdot m(x) = c \cdot m(x) \qquad \text{or}$$
$$m(f^q(z)) < m(f^q(x)) + m(x) \geq (c_1 + 1) \cdot m(x) = c \cdot m(x).$$

**q.e.d.**

**Proposition 3.2** *There exists a constant $c > 1$ such that for all $x \in \mathbb{T}$ and $z \in I(x)$ one has*

$$c^{-1} \cdot m(x) \leq m(z) \leq c \cdot m(x).$$

**Proof :** By the above Lemma 3.1 it suffices to prove that there exists $c_1 > 1$ such that for all $z \in \mathbb{T}$

$$c_1^{-1} \cdot m(z) \leq m(f^{-q}(z)) \leq c_1 \cdot m(z) \tag{2}$$

Let $y \in \mathbb{T}$ be a point where the continuous function $z \mapsto m(z)$ attains its minimum, and let $y_j = f^{-j}(y)$ for all $j \in \mathbb{Z}$. Given any $z \in \mathbb{T}$ let $j$ be the minimal non negative integer with $y_j \in I_n(f^{2q}(z))$. Then $0 \leq j < q_{n+1} + q_n$ by Remark 2.3.2. Thus by the Światec Theorem there exists $c_1 > 1$ such that $c_1^{-1} \leq \Delta(y_j)$, because the intervals $]y_{i+2q}, y_{i-q}[$, $0 < i \leq j$ cover any point of the circle at most three times. Furthermore again by the Światec Theorem we can suppose $\Delta(y_{j+q}), \Delta(j + 2q) \geq c_1^{-1}$ increasing $c_1$ if necessary. Writing out the definitions of $\Delta(y_j), \Delta(y_{j+q}), \Delta(y_{j+2q})$ we obtain for $c_2 = c_1 + 1$ that:

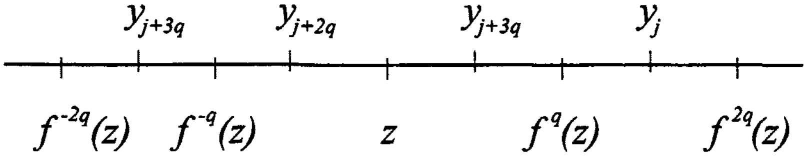

Figure 2:

$$c_1^{-1} \leq \Delta(y_j) \Rightarrow m(y_j) \leq c_2 m(y_{j+q})$$
$$c_1^{-1} \leq \Delta(y_j) \Rightarrow \begin{cases} m(y_{j+q}) \leq c_2 m(y_j) \\ m(y_{j+q}) \leq c_2 m(y_{j+2q}) \end{cases}$$
$$c_1^{-1} \leq \Delta(y_{j+2q}) \Rightarrow m(y_{j+2q}) \leq c_2 m(y_{j+q})$$

So that

$$c_2^{-1} m(y_{j+q}) \leq m(y_j), m(y_{j+2q}) \leq c_2 m(y_{j+q}) \tag{3}$$

Let $c_3$ be the constant from Lemma 3.1 then

$$c_3^{-1} \min\{y_{j+2q}, y_{j+q}\} \leq m(z) \leq y_{j+2q} + y_{j+q} \tag{4}$$
$$c_3^{-1} \min\{y_j, y_{j+q}\} \leq m(f^q(z)) \leq y_j + y_{j+q} \tag{5}$$

Combining the estimates (3) with the pair of estimates (4) and (5) we obtain (2), and the Proposition follows. **q.e.d.**

**Corollary 3.3** *The circle homeomorphism $f$ is minimal, i.e every orbit is dense and any Poincaré semi conjugacy is a homeomorphism and true conjugacy.*

**Proof :** Suppose to the contrary that $f$ is a Denjoy counter example with invariant Cantor set $L$. Let $z$ be a left endpoint of some interval $I$ of the complement of $L$. Then

$$\lim_{n\to\infty} m_{2n}(z) = 0,$$
$$\lim_{n\to\infty} m_{2n}(f^{q_{2n}}(z)) = |I| > 0,$$

which contradicts Proposition 3.2. Here $|I|$ denotes the length of the interval $I$. **q.e.d.**

**Theorem 3.4** *There exists a constant $c > 1$ such that*

$$\forall x \in K_n : m_{n-1}(x) \leq c \cdot m_n(x).$$

**Proof :** To fix the ideas let us suppose $n$ is even and define for $i \in \mathbb{Z}$

$$\begin{aligned} \Sigma_i &= [a_{i+q_n}, a_i, a_{i+q_{n-1}}, a_{i+2q_{n-1}}] \\ &= \frac{m_n(a_i)}{m_n(a_i) + m_{n-1}(a_i)} \cdot \frac{m_{n-1}(a_{i+2q_{n-1}})}{m_{n-1}(a_{i+q_{n-1}}) + m_{n-1}(a_{i+2q_{n-1}})} \end{aligned}$$

and

$$u_i = \frac{m_n(a_i)}{m_{n-1}(a_i)} = \frac{a_i - a_{i+q_n}}{a_{i+q_{n-1}} - a_i}.$$

Figure 3:

then by Proposition 3.2 there exists $c_1 > 1$ such that

$$c_1^{-1} \frac{u_i}{1+u_i} \leq \Sigma_i \leq u_i \tag{6}$$

Thus it suffices to prove that there exists $c > 1$ such that

$$c^{-1} \leq \Sigma_i \qquad \text{for } 0 \leq i \leq q_n. \tag{7}$$

Moreover by the Światec-Theorem it suffices to prove (7) in the case $i = 0$, because the intervals $]a_{i+q_n}, a_{i+2q_{n-1}}[$, $0 < i \leq q_n$ cover any point of the circle at most three times.

Comparing interval by interval and using Proposition 3.2 we find that there exists $c_2 > 1$ such that

$$\Sigma_0 \leq c_2 \cdot \Sigma_{-q_n}. \tag{8}$$

Increasing $n$ if necessary we can suppose $I_{n-1}(a_0), I_n(a_0) \subset W$ so that (by integrating $f'$) there exists $c_3 > 1$ with

$$\Sigma_{-1} \leq u_{-1} \leq c_3 u_0^{2l+1}.$$

Invoking Theorem 2.4 again (from $\Sigma_{-1}$ to $\Sigma_{-q_n}$) we can suppose, increasing $c_3$ if necessary that

$$\Sigma_{-q_n} \leq c_3 \cdot u_0^{2l+1}. \tag{9}$$

Suppose $u_0 < 1$ we then obtain from (6), (8) and (9)

$$\frac{u_0}{2c_1} \leq \Sigma_0 \leq c_2 \cdot c_3 \cdot u_0^{2l+1}$$

This shows that both $u_0$ and $\Sigma_0$ are bounded from below and completes the proof. **q.e.d.**

The above Theorem/or the following Corollary is often referred to as a-priori real bounds for critical circle maps. They are used as a bootstrap to control the geometry and topology of holomorphic maps, which restricts to critical circle homeomorphisms. See for example [dF], [dFdM], [dFaWdM], [GS], [Pet] and [Yam]. The papers [GS] and [dFdM] also contain proofs of the a-priori real bounds, not much different from the one presented here.

**Corollary 3.5 (a-priori real bounds)** *There exists a constant $c > 1$ such that*

$$\forall\, n \geq 2 \ : \ |a_0 - a_{q_{n-1}}| \leq c \cdot |a_{q_n} - a_0|.$$

# 4 Quasi-symmetric conjugacies

**Corollary 4.1** *Suppose $\theta$ has constant type. Then there exists a constant $c > 1$ depending only on $Df$ and on the bound $N$ on the coefficients $b_n$ so that*

$$\forall z \in \mathbb{T}, \forall n \in \mathbb{N} : \frac{m_n(z)}{m_{n+1}(z)} \leq c.$$

**Proof :** Let $z \in \mathbb{T}$ be arbitrary. Given $n \in \mathbb{N}$ there exists $x \in K_n$ such that either $z \in I_{n-1}(z)$ or $z \in I_{n-1}(f^{-q_{n-1}}(z))$ by Remark 2.3.2. For this $x$ there exists $k \leq N+1$ such that $z \in I_n(f^{\pm k q_n})$. Let $c_1$ be the constant from Proposition 3.2 and let $c_2$ be the constant from Proposition 3.4 then we obtain

$$\frac{m_{n-1}(z)}{m_n(z)} \leq c_2 c_1^{k+1} \leq c_2 c_1^{N+2}.$$

**q.e.d.**

For $n \in \mathbb{N}$ let $M_n = d(\mathbb{Z}, q_n\theta) = |z - R_\theta^{q_n}(z)|$, where the last equality holds for any $z \in \mathbb{T}$ exactly because $R_\theta$ is a rigid rotation. Then for any $n \in \mathbb{N}$

$$a_{n+1} M_n < M_{n-1} < (1 + a_{n+1}) M_n.$$

**Corollary 4.2** *Suppose $\theta$ has constant type. Then there exists a constant $c > 1$ depending only on $Df$ and on the bound $N$ on the coefficients $b_n$ so that $f$ is c-quasi-symmetrically conjugate to the rigid rotation $R_\theta$.*

**Proof :** By Corollary 3.3 there exists a homeomorphism $h : \mathbb{T} \longrightarrow \mathbb{T}$ conjugating the rigid rotation $R_\theta$ to $f$. Let $z \in \mathbb{T}$ and $0 < \delta < \frac{1}{2}$ be arbitrary. Choose $n \in \mathbb{N}$ such that $M_{n+1} < \delta \leq M_n$. Let us suppose $n$ is even to fix the ideas ($n$ odd being similar) then

$$m_{n+1}(h(z)) \leq |h([z, z+\delta])| \leq m_n(f^{q_n}(h(z))) \tag{10}$$
$$m_{n+1}(f^{q_{n+1}}(h(z))) \leq |h([z-\delta, z])| \quad \leq m_n(h(z)). \tag{11}$$

Let $c_1$ be the constant from Proposition 3.2 and let $c_2$ be the constant from Proposition 4.1 then we obtain

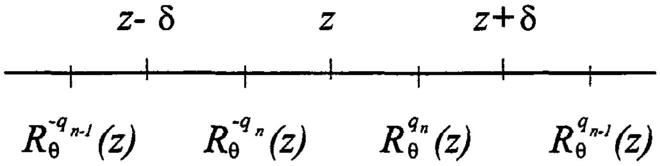

Figure 4:

$$\frac{1}{c_2} \leq \frac{|h([z, z+\delta])|}{|h([z-\delta, z])|} \leq c_1^2 c_2.$$

**q.e.d.**

**Corollary 4.3** *Suppose $\theta$ does not have constant type. Then $f$ is not quasi symmetrically conjugate to the rigid rotation.*

**Proof :** It suffices to prove that a homeomorphism $h : \mathbb{T} \longrightarrow \mathbb{T}$ conjugating $f$ to the rigid rotation $R_\theta$ is not quasi symmetric. To this end choose a subsequence $\{b_{n_k}\}_{k \in \mathbb{N}}$ diverging to $\infty$. Then

$$\frac{h(I_{n_k-1}(a_0))}{h(I_{n_k-2}(a_0))} = \frac{M_{n_k-1}}{M_{n_k-2}} \leq \frac{1}{a_{n_k}},$$

which contradicts that $h$ is quasi symmetric, because $I_{n_k-1}(a_0)$ and $I_{n_k-2}(a_0)$ are commensurable by Corollary 3.4. **q.e.d.**

# 5 Local connectivity of Julia sets

Let $f_0 : \overline{\mathbb{C}} \longrightarrow \overline{\mathbb{C}}$ denote the Blaschke product

$$f_0(z) = z^2 \frac{z-3}{1-3z}.$$

For each irrational $\theta \in ]0,1[$ let $\lambda_\theta$ be the unique unimodular constant for which the restriction $f_\theta = \lambda_\theta \cdot f_0 : \mathbb{S}^1 \longrightarrow \mathbb{S}^1$ has rotation number $\theta$. Let $J_{f_\theta}$ denote the Julia set of $f_\theta$ and let $J_\theta \subset J_{f_\theta}$ denote the boundary of the immediate attracted basin $\Lambda_\theta(\infty)$ for $\infty$. It was proved in [Pet] that

**Theorem 5.1** *For every irrational $\theta$ the subset $J_\theta$ and the full Julia set $J_{f_\theta}$ are locally connected.*

We shall prove the following Theorem which combined with Theorem 5.1 implies Theorem 1.3:

**Theorem 5.2** *There exists irrational $\theta$ of unbounded type for which the Julia set $J_{c_\theta}$ for $Q_{c_\theta}$ is homeomorphic to $J_\theta$.*

The equivalent of Theorem 5.2 in the case of constant type $\theta$ was proven simultaneously by Douady, Ghys, Herman, Hubbard and Shishikura, who all used the following procedure:
Suppose the number $\theta$ is of constant type. Let $h : \mathbb{S}^1 \longrightarrow \mathbb{S}^1$ denote the conjugacy, between $f_\theta$ and the rigid rotation $R_\theta$. As $\theta$ has constant type, the Herman-Światec Theorem 1.2 implies that the map $h$ is $c$-quasi-symmetric, with a constant $c > 1$, which only depends on the bound $N$ on the coefficients of the continued fraction expansion for $\theta$. Let $\psi_\theta : \overline{\mathbb{D}} \longrightarrow \overline{\mathbb{D}}$ be a $K$-quasiconformal extension of $h$ (with a $K > 1$ which only depends on $c$ and hence on $N$) and define $F_\theta : \overline{\mathbb{C}} \longrightarrow \overline{\mathbb{C}}$ by

$$F_\theta(z) = \begin{cases} f_\theta(z) & \text{if } z \in \overline{\mathbb{C}} \setminus \mathbb{D} \\ \psi_\theta^{-1} \circ R_\theta \circ \psi_\theta & \text{if } z \in \overline{\mathbb{D}}. \end{cases}$$

Let $\sigma_0$ denote the standard almost complex structure on $\overline{\mathbb{C}}$, and let $\sigma_\theta$ denote the $F_\theta$ invariant almost complex structure given by

$$\sigma_\theta(z) = \begin{cases} \psi^*(\sigma_0)(z) & \text{if } z \in \mathbb{D} \\ ((\psi \circ F_\theta^n)^*(\sigma_0)(z) & \text{if } F^n(z) \in \mathbb{D} \\ \sigma_0(z) & \text{othervice.} \end{cases}$$

Finally let $\phi_\theta : \overline{\mathbb{C}} \longrightarrow \overline{\mathbb{C}}$ be the integrating map, the quasi-conformal homeomorphism for which $\sigma_\theta = \phi_\theta^*(\sigma_0)$, normalized so that the conjugate map $\phi_\theta \circ F_\theta \circ \phi_\theta^{-1}$ equals the quadratic polynomial $Q_\theta(z) = z^2 + c_\theta$, with an indifferent fixed point of multiplier $e^{i2\pi\theta}$.

The conjugacy $h : \mathbb{S}^1 \longrightarrow \mathbb{S}^1$ between $f_\theta$ and $R_\theta$, normalized by $h(1) = 1$ depends continuously on $\theta$ in the $C^0$-topology, because $f_\theta$ and $R_\theta$ depend continuously on $\theta$ in the $C^0$-topology.

Thus by choosing the quasi-conformal extension $\psi_\theta$ in some canonical way, say by using the Ahlfors-Beurling extension, [LV, Th. 6.3] or the Douady-Earle extension, [DE] we can suppose that also the quasi-conformal extensions $\psi_\theta$ depend continuously on $\theta$ in the $C^0$-topology. To fix the ideas we choose to use say the Douady-Earle extension for every $\theta$.

**Lemma 5.3** *Let $(\theta_n)$ be a sequence of irrationals converging to some irrational $\theta_0$. Suppose the $\theta_n$ have constant type with a uniform constant type bound $N$. Then $\theta_0$ has constant type with bound $N$ and the two sequences $(\phi_{\theta_n})$ and $\phi_{\theta_n}^{-1}$ converges to $\phi_{\theta_0}$ respectively $\phi_{\theta_0}^{-1}$ in the $C^0$-topology.*

**Proof :** The coefficients of $\theta_n$ converge to those of $\theta_0$ so that $\theta_0$ has constant type with bound $N$. We shall prove that $\phi_{\theta_n}$ converges uniformly $(C^0)$ to $\phi_{\theta_0}$, from which the Lemma follows.

There exists $K(N) \geq 1$, such that each map $\phi_{\theta_n}$ is $K$-quasi conformal. Thus extracting a subsequence, if necessary we can assume the sequence converge $C^0$ to a $K$ quasi-conformal homeomorphism $\phi_0$. We shall prove that

$$\phi_0 = \phi_{\theta_0}. \tag{12}$$

As $(F_{\theta_n})$ converges $C^0$ to $F_{\theta_0}$ and $(Q_{c_{\theta_n}})$ converges $C^0$ to $Q_{c_{\theta_0}}$ the map $\phi_0$ conjugates $F_{\theta_0}$ to $Q_{\theta_0}$ and the restriction of $\phi_0$ to the immediate attracted basin of $\infty$ is biholomorphic. Thus (12) holds on the closure of the immediate attracted basin of $\infty$, by uniqueness of the holomorphic conjugacy. Moreover (12) also holds on the 'Siegel-disk' $\mathbb{D}$, because the Douady-Earle extension depends continuously on the boundary data $h_\theta$. Finally it holds on the grand orbit of $\mathbb{D}$, because both $\phi_0$ and $\phi_{\theta_0}$ conjugates dynamics. Thus any $(C^0)$ limit function of the $\phi_{\theta_n}$ equals $\phi_{\theta_0}$. Combining this with the precompactness of the sequence $(\phi_{\theta_n})$ and the compactness of $\overline{\mathbb{C}}$ completes the proof. **q.e.d.**

**Proof of Theorem 5.2:** Let $(\epsilon_n)$ be a summable sequence of strictly positive numbers. Let $(d_n)$ be any unbounded sequence of natural numbers and let $\theta_1$ be any irrational of constant type. Given a natural number $k_1$ let for each $n \geq 1$ the irrational number $\theta_{1,n}$ be obtained from $\theta_1$ by replacing the $(n+k_1)$-th coefficient $b_{n+k_1}$ in the continued fraction expansion of $\theta_1$ with $d_1$. Then the sequence $(\theta_{1,n})$ converges to $\theta_1$. Hence by Lemma 5.3 there exists $n$ such that

$$d_{C^0}(\phi_{\theta_1}, \phi_{\theta_{1,n}}) \leq \epsilon_1 \qquad \text{and} \qquad d_{C^0}(\phi_{\theta_1}^{-1}, \phi_{\theta_{1,n}}^{-1}) \leq \epsilon_1$$

Let $\theta_2 = \theta_{1,n}$ and let $k_2 = k_1 + n$. Then we may restart the process using $\theta_2$, $k_2$, $d_2$ and $\epsilon_2$ to obtain an irrational $\theta_3$ of constant type, but with $b_{k_2} = d_1$ and $b_{k_3} = d_2$. Proceeding inductively we find a sequence $(\theta_n)$ of irrational numbers of constant type converging to some irrational of non constant type $\theta_0$ and a Cauchy sequence (in the $C^0$-norm) of (quasi-conformal) homeomorphisms $(\phi_{\theta_n})$, such that also the the sequence of inverse maps is a Cauchy sequence. Let $\phi_0$ denote the limit of the sequence $(\phi_{\theta_n})$, then $\phi_0$ is a homeomorphism conjugating $F_{\theta_0}$ to $Q_{c_0}$. In particular $J_{c_{\theta_0}} = \phi_0(J_{\theta_0})$. **q.e.d.**

# References

[dF] E. de Faria. Assymptotic rigidity of scaling ratios for critical circle mappings. *Ergod. TH. & Dynam. Sys.* **17**(1998), 227-260.

[dFdM] E. de Faria and W. de Melo. Rigidity critical circle mappings I. preprint IMS#1997/16, Institute for Mathematical Sciences, StonyBrook, November 1997.

[dFaWdM] E. de Faria an W. de Melo. Rigidity critical circle mappings II. preprint IMS#1997/17, Institute for Mathematical Sciences, StonyBrook, November 1997.

[DE] A. Douady and C. J. Earle. Conformally natural extension of homeomorphisms of the circle. *Acta Math.* **157**(1986), 25–48.

[GS] J. Graczyk and G. Swiatec. Critical circle maps near bifurcation. *Commun. Math. Phys.* **176**(1996), 227-260.

[Her] M. R. Herman. Conjugaison quasi symétrique des homéomorphismes analytiques du cercle à des rotations. Le Version très très préliminaire, 19 pages manuscript, 1987.

[LV] O. Lehto and K. I. Virtanen. *Quasiconformal mappings in the plane.* Springer Verlag, 1973.

[Pet] C .L. Petersen. Local connectivity of some Julia sets containing a circle with an irrational rotation. *Acta Math.* **177**(1996), 163–224.

[Swi] G. Swiatec. Rational Rotation Numbers for Maps of the Circle. *Commun. Math. Phys.* **119**(1988), 109–128.

[Yam] M. Yampolsky. Complex bounds for critical circlemaps. preprint IMS#1995/12, Institute for Mathematical Sciences, StonyBrook, December 1995.

[Yoc] J.-C. Yoccoz. Structure des orbites des homéomorphismes analytiques possédant un point critique. 30 pages manuscript, 1987.

*Liste over tidligere udkomne tekster tilsendes gerne. Henvendelse herom kan ske til IMFUFA's sekretariat*

tlf. 46 74 22 63

---

217/92 "Two papers on APPLICATIONS AND MODELLING IN THE MATHEMATICS CURRICULUM"
by: Mogens Niss

218/92 "A Three-Square Theorem"
by: Lars Kadison

219/92 "RUPNOK - stationær strømning i elastiske rør"
af: Anja Boisen, Karen Birkelund, Mette Olufsen
Vejleder: Jesper Larsen

220/92 "Automatisk diagnosticering i digitale kredsløb"
af: Bjørn Christensen, Ole Møller Nielsen
Vejleder: Stig Andur Pedersen

221/92 "A BUNDLE VALUED RADON TRANSFORM, WITH APPLICATIONS TO INVARIANT WAVE EQUATIONS"
by: Thomas P. Branson, Gestur Olafsson and Henrik Schlichtkrull

222/92 On the Representations of some Infinite Dimensional Groups and Algebras Related to Quantum Physics
by: Johnny T. Ottesen

223/92 *THE FUNCTIONAL DETERMINANT*
*by: Thomas P. Branson*

224/92 *UNIVERSAL AC CONDUCTIVITY OF NON-METALLIC SOLIDS AT LOW TEMPERATURES*
*by: Jeppe C. Dyre*

225/92 *"HATMODELLEN" Impedansspektroskopi i ultrarent en-krystallinsk silicium*
*af: Anja Boisen, Anders Gorm Larsen, Jesper Varmer, Johannes K. Nielsen, Kit R. Hansen, Peter Bøggild og Thomas Hougaard*
*Vejleder: Petr Viscor*

226/92 *"METHODS AND MODELS FOR ESTIMATING THE GLOBAL CIRCULATION OF SELECTED EMISSIONS FROM ENERGY CONVERSION"*
*by: Bent Sørensen*

227/92 "Computersimulering og fysik"
af: Per M.Hansen, Steffen Holm, Peter Maibom, Mads K. Dall Petersen, Pernille Postgaard, Thomas B.Schrøder, Ivar P. Zeck
Vejleder: Peder Voetmann Christiansen

228/92 "Teknologi og historie"
Fire artikler af:
Mogens Niss, Jens Høyrup, Ib Thiersen, Hans Hedal

229/92 "Masser af information uden betydning"
En diskussion af informationsteorien i Tor Nørretranders' "Mærk Verden" og en skitse til et alternativ basseret på andenordens kybernetik og semiotik.
af: Søren Brier

230/92 "Vinklens tredeling - et klassisk problem"
et matematisk projekt af
Karen Birkelund, Bjørn Christensen
Vejleder: Johnny Ottesen

231A/92 "Elektrondiffusion i silicium - en matematisk model"
af: Jesper Voetmann, Karen Birkelund, Mette Olufsen, Ole Møller Nielsen
Vejledere: Johnny Ottesen, H.B.Hansen

231B/92 "Elektrondiffusion i silicium - en matematisk model" Kildetekster
af: Jesper Voetmann, Karen Birkelund, Mette Olufsen, Ole Møller Nielsen
Vejledere: Johnny Ottesen, H.B.Hansen

232/92 "Undersøgelse om den simultane opdagelse af energiens bevarelse og især deles om de af Mayer, Colding, Joule og Helmholtz udførte arbejder"
af: L.Arleth, G.I.Dybkjær, M.T.Østergård
Vejleder: Dorthe Posselt

233/92 "The effect of age-dependent host mortality on the dynamics of an endemic disease and
Instability in an SIR-model with age-dependent susceptibility
by: Viggo Andreasen

234/92 "THE FUNCTIONAL DETERMINANT OF A FOUR-DIMENSIONAL BOUNDARY VALUE PROBLEM"
by: Thomas P. Branson and Peter B. Gilkey

235/92 OVERFLADESTRUKTUR OG POREUDVIKLING AF KOKS
- Modul 3 fysik projekt -
af: Thomas Jessén

---

236a/93 INTRODUKTION TIL KVANTE HALL EFFEKTEN

af: Anja Boisen, Peter Bøggild

Vejleder: Peder Voetmann Christiansen
Erland Brun Hansen

236b/93 STRØMSSAMMENBRUD AF KVANTE HALL EFFEKTEN

af: Anja Boisen, Peter Bøggild

Vejleder: Peder Voetmann Christiansen
Erland Brun Hansen

237/93 The Wedderburn principal theorem and Shukla cohomology

af: Lars Kadison

238/93 SEMIOTIK OG SYSTEMEGENSKABER (2)

Vektorbånd og tensorer

af: Peder Voetmann Christiansen

239/93 Valgsystemer - Modelbygning og analyse

Matematik 2. modul

af: Charlotte Gjerrild, Jane Hansen, Maria Hermannsson, Allan Jørgensen, Ragna Clauson-Kaas, Poul Lützen

Vejleder: Mogens Niss

240/93 Patologiske eksempler. Om sære matematiske fisks betydning for den matematiske udvikling

af: Claus Dræby, Jørn Skov Hansen, Runa Ulsøe Johansen, Peter Meibom, Johannes Kristoffer Nielsen

Vejleder: Mogens Niss

241/93 FOTOVOLTAISK STATUSNOTAT 1

af: Bent Sørensen

242/93 Brovedligeholdelse - bevar mig vel

Analyse af Vejdirektoratets model for optimering af broreparationer

af: Linda Kyndlev, Kare Fundal, Kamma Tulinius, Ivar Zeck

Vejleder: Jesper Larsen

243/93 TANKEEKSPERIMENTER I FYSIKKEN

Et 1.modul fysikprojekt

af: Karen Birkelund, Stine Sofia Korremann

Vejleder: Dorthe Posselt

244/93 RADONTRANSFORMATIONEN og dens anvendelse i CT-scanning

Projektrapport

af: Trine Andreasen, Tine Guldager Christiansen, Nina Skov Hansen og Christine Iversen

Vejledere: Gestur Olafsson og Jesper Larsen

245a+b /93 Time-Of-Flight målinger på krystallinske halvledere
Specialerapport

af: Linda Szkotak Jensen og Lise Odgaard Gade

Vejledere: Petr Viscor og Niels Boye Olsen

246/93 HVERDAGSVIDEN OG MATEMATIK - LÆREPROCESSER I SKOLEN

af: Lena Lindenskov, Statens Humanistiske Forskningsråd, RUC, IMFUFA

247/93 UNIVERSAL LOW TEMPERATURE AC CONDUCTIVITY OF MACROSCOPICALLY DISORDERED NON-METALS

by: Jeppe C. Dyre

248/93 DIRAC OPERATORS AND MANIFOLDS WITH BOUNDARY

by: B. Booss-Bavnbek, K.P.Wojciechowski

249/93 Perspectives on Teichmüller and the Jahresbericht Addendum to Schappacher, Scholz, et al.

by: B. Booss-Bavnbek

With comments by W.Abikoff, L.Ahlfors, J.Cerf, P.J.Davis, W.Fuchs, F.P.Gardiner, J.Jost, J.-P.Kahane, R.Lohan, L.Lorch, J.Radkau and T.Söderqvist

*250/93 EULER OG BOLZANO - MATEMATISK ANALYSE SET I ET VIDENSKABSTEORETISK PERSPEKTIV*

*Projektrapport af: Anja Juul, Lone Michelsen, Tomas Højgård Jensen*

*Vejleder: Stig Andur Pedersen*

***251|93 Genotypic Proportions in Hybrid Zones***

***by: Freddy Bugge Christiansen, Viggo Andreasen and Ebbe Thue Poulsen***

***252|93 MODELLERING AF TILFÆLDIGE FÆNOMENER***

***Projektrapport af: Birthe Friis, Lisbeth Helmgaard, Kristina Charlotte Jakobsen, Marina Mosbæk Johannessen, Lotte Ludvigsen, Mette Hass Nielsen***

***253/93 Kuglepakning***

***Teori og model***

***af: Lise Arleth, Kåre Fundal, Nils Kruse***

***Vejleder: Mogens Niss***

***254/93 Regressionsanalyse***

***Materiale til et statistikkursus***

***af: Jørgen Larsen***

***255/93 TID & BETINGET UAFHÆNGIGHED***

***af: Peter Harremoës***

***256/93 Determination of the Frequency Dependent Bulk Modulus of Liquids Using a Piezo-electric Spherical Shell (Preprint)***

***by: T. Christensen and N.B.Olsen***

***257/93 Modellering af dispersion i piezoelektriske keramikker***

***af: Pernille Postgaard, Jannik Rasmussen, Christina Specht, Mikko Østergård***

***Vejleder: Tage Christensen***

***258/93 Supplerende kursusmateriale til "Lineære strukturer fra algebra og analyse"***

***af: Mogens Brun Heefelt***

***259/93 STUDIES OF AC HOPPING CONDUCTION AT LOW TEMPERATURES***

***by: Jeppe C. Dyre***

***260/93 PARTITIONED MANIFOLDS AND INVARIANTS IN DIMENSIONS 2, 3, AND 4***

***by: B. Booss-Bavnbek, K.P.Wojciechowski***

261/93 OPGAVESAMLING
Bredde-kursus i Fysik
Eksamensopgaver fra 1976-93

262/93 Separability and the Jones Polynomial
by: Lars Kadison

263/93 Supplerende kursusmateriale til "Lineære strukturer fra algebra og analyse" II
af: Mogens Brun Heefelt

264/93 FOTOVOLTAISK STATUSNOTAT 2
af: Bent Sørensen

**265/94 SPHERICAL FUNCTIONS ON ORDERED SYMMETRIC SPACES**
**To Sigurdur Helgason on his sixtyfifth birthday**
**by: Jacques Faraut, Joachim Hilgert and Gestur Olafsson**

266/94 Kommensurabilitets-oscillationer i laterale supergitre
Fysikspeciale af: Anja Boisen, Peter Bøggild, Karen Birkelund
Vejledere: Rafael Taboryski, Poul Erik Lindelof, Peder Voetmann Christiansen

267/94 Kom til kort med matematik på Eksperimentarium - Et forslag til en opstilling
af: Charlotte Gjerrild, Jane Hansen
Vejleder: Bernhelm Booss-Bavnbek

268/94 Life is like a sewer ...
Et projekt om modellering af aorta via en model for strømning i kloakrør
af: Anders Marcussen, Anne C. Nilsson, Lone Michelsen, Per M. Hansen
Vejleder: Jesper Larsen

269/94 Dimensionsanalyse en introduktion metaprojekt, fysik
af: Tine Guldager Christiansen, Ken Andersen, Nikolaj Hermann, Jannik Rasmussen
Vejleder: Jens Højgaard Jensen

270/94 THE IMAGE OF THE ENVELOPING ALGEBRA AND IRREDUCIBILITY OF INDUCED REPRESENTATIONS OF EXPONENTIAL LIE GROUPS
by: Jacob Jacobsen

271/94 Matematikken i Fysikken.
Opdaget eller opfundet
NAT-BAS-projekt
vejleder: Jens Højgaard Jensen

272/94 Tradition og fornyelse
Det praktiske elevarbejde i gymnasiets fysikundervisning, 1907-1988
af: Kristian Hoppe og Jeppe Guldager
Vejledning: Karin Beyer og Nils Hybel

273/94 Model for kort- og mellemdistanceløb
Verifikation af model
af: Lise Fabricius Christensen, Helle Pilemann, Bettina Sørensen
Vejleder: Mette Olufsen

274/94 MODEL 10 - en matematisk model af intravenøse anæstetikas farmakokinetik
3. modul matematik, forår 1994
af: Trine Andreasen, Bjørn Christensen, Christine Green, Anja Skjoldborg Hansen. Lisbeth Helmgaard
Vejledere: Viggo Andreasen & Jesper Larsen

275/94 Perspectives on Teichmüller and the Jahresbericht
2nd Edition
by: Bernhelm Booss-Bavnbek

276/94 Dispersionsmodellering
Projektrapport 1. modul
af: Gitte Andersen, Rehannah Borup, Lisbeth Friis, Per Gregersen, Kristina Vejrø
Vejleder: Bernhelm Booss-Bavnbek

277/94 PROJEKTARBEJDSPÆDAGOGIK - Om tre tolkninger af problemorienteret projektarbejde
af: Claus Flensted Behrens, Frederik Voetmann Christiansen, Jørn Skov Hansen, Thomas Thingstrup
Vejleder: Jens Højgaard Jensen

278/94 The Models Underlying the Anaesthesia Simulator Sophus
by: Mette Olufsen(Math-Tech), Finn Nielsen (RISØ National Laboratory), Per Føge Jensen (Herlev University Hospital), Stig Andur Pedersen (Roskilde University)

279/94 Description of a method of measuring the shear modulus of supercooled liquids and a comparison of their thermal and mechanical response functions.
af: Tage Christensen

280/94 A Course in Projective Geometry
by Lars Kadison and Matthias T. Kromann

281/94 Modellering af Det Cardiovaskulære System med Neural Pulskontrol
Projektrapport udarbejdet af:
Stefan Frello, Runa Ulsøe Johansen, Michael Poul Curt Hansen, Klaus Dahl Jensen
Vejleder: Viggo Andreasen

282/94 Parallelle algoritmer
af: Erwin Dan Nielsen, Jan Danielsen, Niels Bo Johansen

283/94 Grænser for tilfældighed
(en kaotisk talgenerator)

af: Erwin Dan Nielsen og Niels Bo Johansen

284/94 Det er ikke til at se det, hvis man ikke lige ve' det!

Gymnasiematematikkens begrundelsesproblem

En specialerapport af Peter Hauge Jensen
og Linda Kyndlev

Vejleder: Mogens Niss

285/94 Slow coevolution of a viral pathogen and its diploid host

by: Viggo Andreasen and
Freddy B. Christiansen

286/94 The energy master equation: A low-temperature approximation to Bässler's random walk model

by: Jeppe C. Dyre

287/94 A Statistical Mechanical Approximation for the Calculation of Time Auto-Correlation Functions

by: Jeppe C. Dyre

288/95 PROGRESS IN WIND ENERGY UTILIZATION

by: Bent Sørensen

289/95 Universal Time-Dependence of the Mean-Square Displacement in Extremely Rugged Energy Landscapes with Equal Minima

by: Jeppe C. Dyre and Jacob Jacobsen

290/95 Modellering af uregelmæssige bølger

Et 3.modul matematik projekt

af: Anders Marcussen, Anne Charlotte Nilsson,
Lone Michelsen, Per Mørkegaard Hansen

Vejleder: Jesper Larsen

291/95 1st Annual Report from the project
LIFE-CYCLE ANALYSIS OF THE TOTAL DANISH ENERGY SYSTEM

an example of using methods developed for the OECD/IEA and the US/EU fuel cycle externality study

by: Bent Sørensen

292/95 Fotovoltaisk Statusnotat 3

af: Bent Sørensen

293/95 Geometridiskussionen - hvor blev den af?

af: Lotte Ludvigsen & Jens Frandsen

Vejleder: Anders Madsen

294/95 Universets udvidelse - et metaprojekt

Af: Jesper Duelund og Birthe Friis

Vejleder: Ib Lundgaard Rasmussen

295/95 A Review of Mathematical Modeling of the Controled Cardiovascular System

By: Johnny T. Ottesen

296/95 RETIKULER den klassiske mekanik

af: Peder Voetmann Christiansen

297/95 A fluid-dynamical model of the aorta with bifurcations

by: Mette Olufsen and Johnny Ottesen

298/95 Mordet på Schrödingers kat - et metaprojekt om to fortolkninger af kvantemekanikken

af: Maria Hermannsson, Sebastian Horst,
Christina Specht

Vejledere: Jeppe Dyre og Peder Voetmann Christiansen

299/95 ADAM under figenbladet - et kig på en samfundsvidenskabelig matematisk model

Et matematisk modelprojekt

af: Claus Dræby, Michael Hansen, Tomas Højgård Jensen

Vejleder: Jørgen Larsen

300/95 Scenarios for Greenhouse Warming Mitigation

by: Bent Sørensen

301/95 TOK Modellering af træers vækst under påvirkning af ozon

af: Glenn Møller-Holst, Marina Johannessen, Birthe
Nielsen og Bettina Sørensen

Vejleder: Jesper Larsen

302/95 KOMPRESSORER - Analyse af en matematisk model for aksialkompressorer

Projektrapport sf: Stine Bøggild, Jakob Hilmer,
Pernille Postgaard

Vejleder: Viggo Andreasen

303/95 Masterlignings-modeller af Glasovergangen

Termisk-Mekanisk Relaksation

Specialerapport udarbejdet af:

Johannes K. Nielsen, Klaus Dahl Jensen

Vejledere: Jeppe C. Dyre, Jørgen Larsen

304a/95 STATISTIKNOTER Simple binomialfordelingsmodeller

af: Jørgen Larsen

304b/95 STATISTIKNOTER Simple normalfordelingsmodeller

af: Jørgen Larsen

304c/95 STATISTIKNOTER Simple Poissonfordelingsmodeller

af: Jørgen Larsen

304d/95 STATISTIKNOTER Simple multinomialfordelingsmodeller

af: Jørgen Larsen

304e/95 STATISTIKNOTER Mindre matematisk-statistisk opslagsværk indeholdende bl.a. ordforklaringer, resuméer og tabeller

af: Jørgen Larsen

305/95 The Maslov Index:
A Functional Analytical Definition
And The Spectral Flow Formula

By: B. Booss-Bavnbek, K. Furutani

306/95 Goals of mathematics teaching

Preprint of a chapter for the forth-
comming International Handbook of
Mathematics Education (Alan J.Bishop, ed)

By: Mogens Niss

307/95 Habit Formation and the Thirdness of Signs

Presented at the semiotic symposium

The Emergence of Codes and Intensions as
a Basis of Sign Processes

By: Peder Voetmann Christiansen

308/95 Metaforer i Fysikken

af: Marianne Wilcken Bjerregaard,
Frederik Voetmann Christiansen,
Jørn Skov Hansen, Klaus Dahl Jensen
Ole Schmidt

Vejledere: Peder Voetmann Christiansen og
Petr Viscor

309/95 Tiden og Tanken
En undersøgelse af begrebsverdenen Matematik
udført ved hjælp af en analogi med tid

af: Anita Stark og Randi Petersen

Vejleder: Bernhelm Booss-Bavnbek

---

310/96 Kursusmateriale til "Lineære strukturer fra
algebra og analyse" (E1)

af: Mogens Brun Heefelt

311/96 2nd Annual Report from the project
LIFE-CYCLE ANALYSIS OF THE TOTAL DANISH
ENERGY SYSTEM

by: Hélène Connor-Lajambe, Bernd Kuemmel,
Stefan Krüger Nielsen, Bent Sørensen

312/96 Grassmannian and Chiral Anomaly

by: B. Booss-Bavnbek, K.P.Wojciechowski

313/96 THE IRREDUCIBILITY OF CHANCE AND
THE OPENNESS OF THE FUTURE
The Logical Function of Idealism in Peirce's
Philosophy of Nature

By: Helmut Pape, University of Hannover

314/96 Feedback Regulation of Mammalian
Cardiovascular System

By: Johnny T. Ottesen

315/96 a + b "Rejsen til tidens indre" - Udarbejdelse af
et manuskript til en fjernsynsudsendelse
+ manuskript

af: Gunhild Hune og Karina Goyle

Vejledere: Peder Voetmann Christiansen og
Bruno Ingemann

316/96 Plasmaoscillation i natriumklynger

Specialerapport af: Peter Meibom, Mikko Østergård

Vejledere: Jeppe Dyre & Jørn Borggreen

317/96 Poincaré og symplektiske algoritmer

af: Ulla Rasmussen

Vejleder: Anders Madsen

318/96 Modelling the Respiratory System

by: Tine Guldager Christiansen, Claus Dræby

Supervisors: Viggo Andreasen, Michael Danielsen

319/96 Externality Estimation of Greenhouse Warming
Impacts

by: Bent Sørensen

320/96 Grassmannian and Boundary Contribution to the
-Determinant

by: K.P.Wojciechowski et al.

321/96 Modelkompetencer - udvikling og afprøvning
af et begrebsapparat

Specialerapport af: Nina Skov Hansen,
Christine Iversen, Kristin Troels-Smith

Vejleder: Morten Blomhøj

322/96 OPGAVESAMLING

Bredde-Kursus i Fysik 1976 - 1996

323/96 Structure and Dynamics of Symmetric Diblock
Copolymers

PhD Thesis

by: Christine Maria Papadakis

324/96 Non-linearity of Baroreceptor Nerves

by: Johnny T. Ottesen

325/96 Retorik eller realitet ?
Anvendelser af matematik i det danske
Gymnasiums matematikundervisning i
perioden 1903 - 88

Specialerapport af Helle Pilemann

Vejleder: Mogens Niss

326/96 Bevisteori
Eksemplificeret ved Gentzens bevis for
konsistensen af teorien om de naturlige tal

af: Gitte Andersen, Lise Mariane Jeppesen,
Klaus Frovin Jørgensen, Ivar Peter Zeck

Vejledere: Bernhelm Booss-Bavnbek og
Stig Andur Pedersen

327/96 NON-LINEAR MODELLING OF INTEGRATED ENERGY
SUPPLY AND DEMAND MATCHING SYSTEMS

by: Bent Sørensen

328/96 Calculating Fuel Transport Emissions

by: Bernd Kuemmel

329/96 The dynamics of cocirculating influenza strains conferring partial cross-immunity
and
A model of influenza A drift evolution
by: Viggo Andreasen, Juan Lin and Simon Levin

330/96 LONG-TERM INTEGRATION OF PHOTOVOLTAICS INTO THE GLOBAL ENERGY SYSTEM
by: Bent Sørensen

331/96 Viskøse fingre
Specialerapport af:
Vibeke Orlien og Christina Specht
Vejledere: Jacob M. Jacobsen og Jesper Larsen

---

332/97 ANOMAL SWELLING AF LIPIDE DOBBELTLAG
Specialerapport af:
Stine Sofia Korremann
Vejleder: Dorthe Posselt

333/97 Biodiversity Matters
an extension of methods found in the literature on monetisation of biodiversity
by: Bernd Kuemmel

334/97 LIFE-CYCLE ANALYSIS OF THE TOTAL DANISH ENERGY SYSTEM
by: Bernd Kuemmel and Bent Sørensen

335/97 Dynamics of Amorphous Solids and Viscous Liquids
by: Jeppe C. Dyre

336/97 PROBLEM-ORIENTATED GROUP PROJECT WORK AT ROSKILDE UNIVERSITY
by: Kathrine Legge

337/97 Verdensbankens globale befolkningsprognose
- et projekt om matematisk modellering
af: Jørn Chr. Bendtsen, Kurt Jensen,
Per Pauli Petersen
Vejleder: Jørgen Larsen

338/97 Kvantisering af nanolederes elektriske ledningsevne
Første modul fysikprojekt
af: Søren Dam, Esben Danielsen, Martin Niss,
Esben Friis Pedersen, Frederik Resen Steenstrup
Vejleder: Tage Christensen

339/97 Defining Discipline
by: Wolfgang Coy

340/97 Prime ends revisited - a geometric point of view -
by: Carsten Lunde Petersen

341/97 Two chapters on the teaching, learning and assessment of geometry
by Mogens Niss

342/97 LONG-TERM SCENARIOS FOR GLOBAL ENERGY DEMAND AND SUPPLY
A global clean fossil scenario discussion paper
prepared by Bernd Kuemmel
Project leader: Bent Sørensen

343/97 IMPORT/EKSPORT-POLITIK SOM REDSKAB TIL OPTIMERET UDNYTTELSE AF EL PRODUCERET PÅ VE-ANLÆG
af: Peter Meibom, Torben Svendsen, Bent Sørensen

344/97 Puzzles and Siegel disks
by Carsten Lunde Petersen

---

345/98 Modeling the Arterial System with Reference to an Anestesia Simulator
Ph.D. Thesis
by: Mette Sofie Olufsen

346/98 Klyngedannelse i en hulkatode-forstøvningsproces
af: Sebastian Horst
Vejledere: Jørn Borggren, NBI, Niels Boye Olsen

347/98 Verificering af Matematiske Modeller
- en analyse af Den Danske Eulerske Model
af: Jonas Blomqvist, Tom Pedersen, Karen Timmermann,
Lisbet Øhlenschlæger
Vejleder: Bernhelm Booss-Bavnbek

348/98 Case study of the environmental permission procedure and the environmental impact assessment for power plants in Denmark
by: Stefan Krüger Nielsen
Project leader: Bent Sørensen

349/98 Tre rapporter fra FAGMAT - et projekt om tal og faglig matematik i arbejdsmarkedsuddannelserne
af: Lena Lindenskov og Tine Wedege

350/98 OPGAVESAMLING - Bredde-Kursus i Fysik 1976 - 1998
Erstatter teksterne 3/78, 261/93 og 322/96

351/98 Aspects of the Nature and State of Research in Mathematics Education
by: Mogens Niss

352/98 The Herman-Swiatec Theorem with applications

by: Carsten Lunde Petersen

353/98 Problemløsning og modellering i en almendannende matematikundervisning

Specialerapport af: Per Gregersen og Tomas Højgaard Jensen

Vejleder: Morten Blomhøj

354/98 A GLOBAL RENEWABLE ENERGY SCENARIO

by: Bent Sørensen and Peter Meibom

355/98 Convergence of rational rays in parameter spaces

by: Carsten Lunde Petersen and Gustav Ryd