Convergence of rational rays in parameter spaces

Petersen, Carsten Lunde; Ryd, Gustav

Publication date:
1998

Document Version
Publisher's PDF, also known as Version of record

Citation for published version (APA):
Petersen, C. L., & Ryd, G. (1998). *Convergence of rational rays in parameter spaces*. Roskilde Universitet. Tekster fra IMFUFA No. 335 <http://milne.ruc.dk/ImfufaTekster/>

General rights

Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain.
- You may freely distribute the URL identifying the publication in the public portal.

Take down policy

If you believe that this document breaches copyright please contact rucforsk@kb.dk providing details, and we will remove access to the work immediately and investigate your claim.

**Convergence of rational rays
in parameter spaces**

Carsten Lunde Petersen and Gustav Ryd

TEKSTER fra

IMFUFA

ROSKILDE UNIVERSITETSCENTER
INSTITUT FOR STUDIET AF MATEMATIK OG FYSIK SAMT DERES
FUNKTIONER I UNDERVISNING, FORSKNING OG ANVENDELSER

IMFUFA, ROSKILDE UNIVERSITY, POSTBOX 260, DK-4000 ROSKILDE
DENMARK.

Convergence of rational rays in parameter spaces

by

Carsten Lunde Petersen, IMFUFA and

**Gustav Ryd ¹, Dept. of Mathematics, KTH, 100 44 Stockholm,
Sweden**

IMFUFA-text no 355/98

10 pages

ISSN 0106-6242

Abstract

We give an elementary proof of the landing Theorem for rational external rays of the Mandelbrot set and related connectedness loci for the one-parameter families of polynomials $\{P_c(z) = z^d + c\}_{c \in \mathbb{C}}$, $d \geq 2$.

¹present address : Research & Trade, 103 95 Stockholm, Sweden.

Convergence of rational rays in parameter spaces

Carsten Lunde Petersen and Gustav Ryd

Abstract

We give an elementary proof of the landing Theorem for rational external rays of the Mandelbrot set and related connectedness loci for the one-parameter families of polynomials $\{P_c(z) = z^d + c\}_{c \in \mathbb{C}}$, $d \geq 2$.

Throughout this paper the integer $d \geq 2$ will be arbitrary but fixed, and used without further notice. It will be the degree of our polynomials. Let $P_c(z) = P_c = z^d + c$, $c \in \mathbb{C}$, denote the family of monic degree d polynomials with a degree $(d - 1)$ critical point at the origin. For each c let J_c denote the Julia set for P_c and define the domain of attraction to infinity

$$\Lambda_c = \Lambda_c(\infty) = \{z \in \mathbb{C} \mid P_c^n(z) \longrightarrow \infty, \text{ as } n \rightarrow \infty\}.$$

Let $\phi_c : \tilde{\Lambda}_c \longrightarrow (\mathbb{C} \setminus \overline{\mathbb{D}})_c$ be the maximal univalent ‘radial’ (on the image side) extension of the unique Bötcher coordinate tangent to the identity at infinity.

Define $M_d = \{c \in \mathbb{C} \mid c \notin \Lambda_c\}$ and let $\Phi_d : \overline{\mathbb{C}} \setminus M_d \longrightarrow \overline{\mathbb{C}} \setminus \overline{\mathbb{D}}$ denote the unique Riemann map tangent to the identity at infinity, then $\Phi_d(c) = \phi_c(c)$. Given $\theta \in \mathbb{T} = \mathbb{R}/\mathbb{Z}$ and $c \in \mathbb{C}$: The dynamical (external) ray $R_c(\theta)$ of J_c is the analytic arc $\phi_c^{-1}(\{\exp(s + i2\pi\theta) \mid s > 0\} \cap (\overline{\mathbb{C}} \setminus \overline{\mathbb{D}})_c)$. It starts at ∞ and ends either at a precritical point $z_0 \in \bigcup_{n \geq 0} P_c^{-n}(c)$ or on the Julia set J_c . The parameter (external) ray $R_{M_d}(\theta)$ of M_d is the analytic arc $\Phi_d^{-1}(\{\exp(s + i2\pi\theta) \mid s > 0\})$. A ray is called a rational ray if θ is rational, i.e. $\theta \in \mathbb{Q}/\mathbb{Z}$. A ray R is said to land or converge if $\overline{R} \setminus R$ is a singleton subset of J (if it is a dynamical ray) or M_d (if it is a parameter ray).

On the boundary of M_d we distinguish two particular and different types of parameters, parabolic parameters and Misiurewicz parameters. A parameter c_0 is called parabolic, if P_{c_0} has a parabolic cycle, i.e., there exists a positive integer n and a periodic point p such that $P_{c_0}^n(p) = p$ and $(P_{c_0}^n)'(p) = 1$.

Figure 1: The Mandelbrot set with a periodic and a strictly preperiodic ray.

A parameter c_0 is called a Misiurewicz parameter, if the orbit of c_0 is finite and ends in a repelling periodic orbit.

In this paper we give an elementary proof of the landing Theorem for rational external rays of M_d , for any integer $d \geq 2$:

Theorem 1 *Given $\theta \in \mathbb{T}$ with $d^l \theta \equiv d^{l+q} \theta \pmod{1}$, for some minimal integers $l \geq 0$ and $q \geq 1$. Then the parameter ray $R_{M_d}(\theta)$ lands on a parameter $c_0 \in \partial M_d$. Furthermore,*

1. *if $l = 0$ then c_0 is a parabolic parameter, the (unique) parabolic orbit for P_{c_0} has one cycle of immediate attracted basins. This cycle contains both 0 and c and its exact period is q . Moreover the dynamic ray $R_{c_0}(\theta)$ lands on the parabolic point to which c_0 is attracted under iteration by $P_{c_0}^q$.*
2. *if $l > 0$ then c_0 is a Misiurewicz parameter, $P_{c_0}^l(c_0) = P_{c_0}^{l+q}(c_0)$ and the dynamic ray $R_{c_0}(\theta)$ land on c_0 .*

This Theorem is well-known, at least for $d = 2$, since [DH]. The original proof is however rather involved. There are other proofs, such as [HS], which

Figure 2: The Julia set for a quadratic polynomial with a period-2 ray landing at a parabolic fixed point.

uses iteration on Teichmüller spaces, and [Mil], [Sch], which both depend on global counting arguments and a priori analysis of the sets M_d . In [CG] this theorem is claimed, but there seems to be a part missing. This paper has evolved from [Pet] and [Ryd], which both were extending Theorem 1 to different settings but using similar techniques.

Our proof differs from the original proof in [DH] mainly in our direct approach to convergence and uses only elementary analytical means. Moreover it has easy generalizations to many other settings. We want to make our proof available to the public, because we believe that our elementary approach will be of use also to others.

We will frequently use $g_c : \mathbb{C} \rightarrow \mathbb{R}_+$, the Green's function for J_c with pole at ∞ , which is the subharmonic function which is harmonic on Λ_c , coincides with $\log |\phi_c(z)|$ on $\tilde{\Lambda}_c$ and which is identically zero on $\mathbb{C} \setminus \Lambda_c$. Similarly, the Green's function for M_d with pole at ∞ , $G_{M_d} : \mathbb{C} \rightarrow \mathbb{R}_+$ is the subharmonic function, which is 0 on M_d and which coincides with $\log |\Phi_{M_d}|$ on $\mathbb{C} \setminus M_d$. Note that an external ray is a gradient line for the appropriate Green's function.

We first prove that the end of the external ray gives rise to a sequence of

Figure 3: The Julia set for a quadratic polynomial with a preperiodic ray landing at a Misiurewicz point.

limit maps which relates the dynamics in the filled Julia set, K_{c_0} , with the dynamics in Λ_{c_0} .

Given $c_0 \in \mathbb{C}$ and $\phi \in \mathbb{T}$, for $0 < \epsilon$ and $w \in R_c(\phi)$ we define a ‘rectangle’

$$V(w, \epsilon) = \{\exp(t + i2\pi\eta) | g_{c_0}(w) - \epsilon < t < d^q g_{c_0}(w) + \epsilon \text{ and } |\eta - \phi| < \epsilon\}.$$

Proposition 1.1 *Given $\theta \in \mathbb{T}$ with $d^l \theta \equiv d^{l+q} \theta \pmod{1}$, for some minimal $l \geq 0$ and $q \geq 1$. Let $c_0 \in \partial M_d$ be a limit point of the ray $R_{M_d}(\theta)$. Then there exists $w \in R_{c_0}(d^l \theta)$, $\epsilon > 0$ and a sequence of maps $\{\psi_n : V \rightarrow K_{c_0}\}_{n \geq 0}$, $V = V(w, \epsilon)$ such that*

1. *For all $0 \leq n$: $\psi_n(w) = P_{c_0}^n(c_0)$ and $\psi_{n+1} = P_{c_0} \circ \psi_n$.*
2. *For all $l \leq n$: $\psi_n \circ P_{c_0}^q = P_{c_0}^q \circ \psi_n$, wherever defined and in particular $\psi_n(P_{c_0}^q(w)) = P_{c_0}^{n+q}(c_0) = \psi_{n+q}(w)$.*
3. *One of the following mutually exclusive cases occur*
 - (a) *The ψ_n are univalent and for all $0 \leq n \neq m$ with either $n < l$ or $|n - m| \neq q$: $\psi_n(V) \cap \psi_m(V) = \emptyset$.*

(b) All ψ_n are constants.

Proof: Let $\phi = d^l \theta \pmod 1$ be the first periodic angle in the orbit of θ . And let $\{c_j\}_{j \in \mathbb{N}} \subset R_{M_d}(\theta) \cap \{c | G_{M_d}(c) < d^{-l}\}$ be a sequence converging to c_0 . For each j define N_j by $P_{c_j}^{N_j}(c_j) := w_j \in R_{c_j}(\phi) \cap \{z | 1 \leq g_{c_j}(z) < d^q\}$. Passing to a subsequence if necessary, we can suppose by relative compactness that $P_{c_j}^{N_j}(c_j) \rightarrow w \in R_{c_0}(\phi)$ with $1 \leq g_{c_0}(w) \leq d^q$.

Fix $0 < \epsilon < 1/2$ and define a simply connected domain $V = V(w, \epsilon)$. We can assume that $w_j \in V$. For each j and $0 \leq n \leq N_j$ let $\psi_{n,j} : V \rightarrow \mathbb{C}$ be the unique branch of $P_{c_j}^{-N_j+n}$ which maps w_j to $P_{c_j}^n(c_j)$. Then for each fixed

Figure 4: The domain V and the image domains $\psi_{n,j}(V)$ viewed in the Böttcher coordinate at infinity.

j the uniformly bounded univalent maps $\psi_{n,j}$ satisfy

$$\forall 0 \leq n \leq N_j : \quad \psi_{n,j}(w_j) = P_{c_j}^n(c_j) \quad \text{and} \quad \psi_{n+1,j} = P_{c_j} \circ \psi_{n,j} \quad (1.1)$$

$$\forall l \leq n \leq N_j - q : \quad \psi_{n,j} \circ P_{c_j}^q = P_{c_j}^q \circ \psi_{n,j}. \quad (1.2)$$

$$\forall 0 \leq n \neq m \leq N_j \text{ with } n < l \text{ or } |n - m| \neq q : \quad \psi_{n,j}(V) \cap \psi_{m,j}(V) = \emptyset \quad (1.3)$$

Here the disjointness property (1.3) holds if ϵ is small enough. Note that $N_j \rightarrow \infty$ as $j \rightarrow \infty$. As the univalent maps $\psi_{n,j}$ are uniformly bounded on V , we can suppose, using the Cantor diagonal principle to extract a subsequence if necessary, that for each $n \in \mathbb{N}$

$$\psi_{n,j} \xrightarrow{j \rightarrow \infty} \psi_n$$

where $\psi_n : V \rightarrow \mathbb{C}$ is some holomorphic map, which is either univalent or constant by Hurwitz Theorem. Note that $g_{c_0} = 0$ on $\psi_n(V)$ so $\psi_n(V) \subset K_{c_0}$. By uniform convergence the properties (1.1), (1.2) and (1.3) also holds for the ψ_n (with P_{c_0}) so Properties 1. and 2. follows. By (1.1) if one ψ_n is constant they all are, so Property 3. follows. q.e.d.

Proposition 1.2 *Let θ and c_0 be as in Proposition 1.1. If $c_0 \in J_{c_0}$ then 3b. holds, $P_{c_0}^l(c_0) = P_{c_0}^{l+q}(c_0)$, $l > 0$ and the dynamic ray $R_{c_0}(\theta)$ lands on c_0 .*

Proof : Let V and ψ_n be as in the conclusion of Proposition 1.1. The image $\psi_n(V) \subset K_{c_0}$. Thus if $c_0 = \psi_0(w) \in J_{c_0} = \partial K_{c_0}$ then ψ_0 cannot be open thus all ψ_n are constant. Combining Properties 1. and 2. we have $P_{c_0}^l(c_0) = P_{c_0}^{l+q}(c_0)$. If $l = 0$ then c_0 is q -periodic, but then also the critical point 0 is q -periodic, because it is the only preimage of c_0 . This contradicts however the fact that $c_0 \in \partial M_d$. Hence $l > 0$ and c_0 is a Misiurewicz parameter with $P_{c_0}^l(c_0) = P_{c_0}^{l+q}(c_0)$.

To complete the proof we show that the ray $R_{c_0}(\theta)$ lands on c_0 . To this end let $\{c_j\}_{j \in \mathbb{N}} \subset R_{M_d}(\theta)$ be a sequence converging to c_0 as in the proof of Proposition 1.1. Define $R_{c_j}^*(\theta) = ((R_{c_j}(\theta) \cap \{z | g_{c_j}(z) \geq g_{c_j}(c_j)\}) \cup \infty)$. Then the sequence $\{R_{c_j}^*\}_{j \in \mathbb{N}}$ is a sequence of compact sets in $\overline{\mathbb{C}}$. Thus passing to a subsequence, if necessary, we can suppose the sequence converges to some compact subset $R \subset \overline{\mathbb{C}}$ containing c_0 and ∞ . Moreover $R \cap \Lambda_{c_0} = R_{c_0}(\theta)$ by continuity with respect to the parameter of the Bötcher coordinates ϕ_c . Let $z \in R \cap J_{c_0}$ be arbitrary (note that $J_{c_0} = \partial \Lambda_{c_0} = \mathbb{C} \setminus \Lambda_{c_0}$) and let $z_j \in R_{c_j}^*(\theta)$ be a sequence with $z_j \rightarrow z$ as $j \rightarrow \infty$. Arguing as in the proof of Proposition 1.1 we find a point $\hat{w} \in R_{c_0}(d^l \theta)$, an open set $\hat{V}$ and a sequence of holomorphic maps $\hat{\psi}_n : \hat{V} \rightarrow \mathbb{C}$ satisfying the properties 1., 2. and 3. of Proposition 1.1, except that c_0 is replaced by z . As above case 3b. in 3. holds, because $z \in J_{c_0}$. But then $P_{c_0}^l(z) = P_{c_0}^{l+q}(z)$. The set $\{z | P_{c_0}^l(z) = P_{c_0}^{l+q}(z)\}$ is however finite and hence $R \cap J_{c_0}$ is a singleton containing c_0 . q.e.d.

Proposition 1.3 Let θ and c_0 be as in Proposition 1.1. If c_0 belongs to a Fatou component U for P_{c_0} , then 3a. holds. Furthermore, there exists a sequence of Jordan arcs $\gamma_n : [0, 1] \rightarrow P_{c_0}^n(U)$ satisfying

1'. For all $0 \leq n$: $\gamma_n(0) = P_{c_0}^n(c_0)$ and $\gamma_{n+1} = P_{c_0} \circ \gamma_n$.

2'. For all $l \leq n$: $\gamma_n \cap \gamma_{n+q} = \gamma_n(1) = \gamma_{n+q}(0) = P_{c_0}^{n+q}(0)$.

3'. For all $0 \leq n \neq m$ with either $n < l$ or $|n - m| \neq q$: $\gamma_n \cap \gamma_m = \emptyset$.

Proof : Let V and ψ_n be as in the conclusion of Proposition 1.1. If the ψ_n were constants then by combining Property 1. and Property 2., $P_{c_0}^l(c_0) = P_{c_0}^{l+q}(c_0)$. Hence the unique critical point is preperiodic, but then it either belongs to the Julia set or is superattracting. The first possibility is excluded by hypothesis and in the second case c_0 belongs to the interior of M_d , which is impossible. Thus ψ_l is not constant so Property 3a. holds.

Let w be as in the proof of Proposition 1.1 and define an arc $\Gamma \subset V$ by

$$\Gamma = R_{c_0}(\phi) \cap \{z | g_{c_0}(w) \leq g_{c_0}(z) \leq d^q g_{c_0}(w)\}.$$

The arc Γ is naturally parametrised linearly in the potential by the interval $[0, 1]$. Define arcs $\gamma_n = \psi_n \circ \Gamma : [0, 1] \rightarrow P_{c_0}^n(U)$.

Properties 1'. and 3'. follows immediately from Properties 1. and 3a. To see also Property 2'. define

$$\widehat{V} = \{\exp(t + i2\pi\eta) | g_{c_0}(w) - \epsilon < t < d^{2q} g_{c_0}(w) + \epsilon \text{ and } |\eta - \phi| < \epsilon\},$$

$$\widehat{\Gamma} = R_{c_0}(\phi) \cap \{z | g_{c_0}(w) \leq g_{c_0}(z) \leq d^{2q} g_{c_0}(w)\}.$$

and note that the ψ_n have univalent extensions $\widehat{\psi}_n$ to $\widehat{V}$. Moreover for $n \geq l$: $\widehat{\psi}_n(\widehat{\Gamma}) = \gamma_n \cup \gamma_{n+q}$ is an arc. Thus also the interior disjointness $\gamma_n \cap \gamma_{n+q} = P_{c_0}^{n+q}(c_0)$ follows. q.e.d.

Proposition 1.4 Let θ , c_0 and U be as in Proposition 1.3. Then U is an immediate parabolic basin of exact period q and the preperiod $l = 0$.

Proof : Let $U_l = P_{c_0}^l(U)$. Then $\gamma_l \in U_l$ and U_l is q -periodic, because both $P_{c_0}^l(c_0) = \gamma_l(0)$ and $P_{c_0}^{l+q}(c_0) = \gamma_l(1) \in U_l$. But then it is either a hyperbolic, a parabolic or a rotation domain, however the first implies c_0 is in the interior of M_d and the last requires at least one critical point in the Julia set and

are hence excluded. Thus U_l is an immediate parabolic basin and so is $\bar{U}$, because the cycle of such immediate basins should contain a critical point and value. Thus U is q -periodic. Let the (exact) period of U be p . Then $\gamma_p \in U$. Let $\Phi : U \rightarrow \mathbb{C}$ be a Fatou coordinate for $P_{c_0}^p$ on U normalized by $\Phi(c_0) = 0$ and $\Phi(P_{c_0}^p(c_0)) = 1$. The map Φ has a univalent inverse branch

$$\Psi : \{z = x + iy | y > -1\} \rightarrow \Omega$$

with $\Psi(0) = c_0$. By compactness there exists $n \geq 0$ such that γ_{pn} and $\gamma_{p(n+1)}$ are contained in Ω . Let γ'_{pn} and $\gamma'_{p(n+1)}$ denote the corresponding images under Φ . The map $z \rightarrow \exp(2\pi ip/q)$ maps these two arcs onto two simple closed curves which are rotations (around the origin) of each other. Thus they intersect and so do γ'_{pn} and $\gamma'_{p(n+1)}$. Since γ_{pn} and $\gamma_{p(n+1)}$ are contained in Ω they also intersect. But then $p = q$ by Property 3'.

We prove that $l = 0$. Choose n such that $\gamma_{nq} \subset \Omega$ so that $\gamma_{(n+m)q} \subset \Omega$ for all $m \geq 0$. Then $\Phi(\gamma_{nq})$ satisfies the hypothesis of the isotopy Lemma 1.5 below, because of Properties 2' and 3'. Let $\tilde{\Gamma}$ be the corresponding isotopy of Lemma 1.5. Increasing n , if necessary, we can assume $\tilde{\Gamma}([0, 1] \times [0, 1]) \subset \Phi(\Omega)$, because of Property 1'. Define $\Gamma_n : [0, 1] \times [0, 1] \rightarrow \Omega$ as the isotopy $\Gamma_n = \Psi \circ \tilde{\Gamma}$. Define isotopies $\Gamma_k : [0, 1] \times [0, 1] \rightarrow U$, $n \geq k \geq 0$ recursively by Γ_{k-1} is the lift of Γ_k to $P_{c_0}^q$ with $\Gamma_{k-1}(1, [0, 1]) = \Gamma_k(0, [0, 1])$. An easy induction proof shows that for each $n \geq k \geq 0$ and each $0 \leq t \leq 1$: $\Gamma_k(t, 0) = \Psi(k + t)$. In particular $\Gamma_k(0, [0, 1]) = P_{c_0}^{kq}(c_0)$. But then $\gamma_0 \cap \gamma_q = P_{c_0}^q(c_0)$, which contradicts Property 3', when $l > 0$. q.e.d.

The proof of the following isotopy Lemma is left to the reader.

Lemma 1.5 Suppose $\gamma_1 : [0, 1] \rightarrow \mathbb{C}$ is an arc with $\gamma_1(0) = 0$, $\gamma_1(1) = 1$ and with $\gamma_1 \cap (\gamma_1 + n) = \emptyset$ for $|n| > 1$ and with $\gamma_1 \cap (\gamma_1 + 1) = 1$, $\gamma_1 \cap (\gamma_1 - 1) = 0$. Then there exists an isotopy of arcs $\Gamma : [0, 1] \times [0, 1] \rightarrow \mathbb{C}$ with $\Gamma(t, 1) = \gamma_1(t)$, $\Gamma(t, 0) = t := \gamma_0(t)$, $\Gamma(0, [0, 1]) = 0$, $\Gamma(1, [0, 1]) = 1$ and $\Gamma([0, 1], [0, 1]) \subset \mathbb{C} \setminus \mathbb{Z}$.

Proposition 1.6 Let θ , c_0 and U be as in Proposition 1.3. Then the dynamic ray $R_{c_0}(\theta)$ lands on the parabolic point on the boundary of U .

Proof : Let R be as in the second part of the proof of Proposition 1.2. Let α denote the parabolic point to which c_0 is iterated by $P_{c_0}^q$. Then $\alpha \in R$ by closedness and forward invariance of R . We claim that $R \cap J_{c_0} = \alpha$. To prove this let Ω be any bounded Fatou component with $R \cap \Omega \neq \emptyset$. We shall

prove that $R \cap \partial\Omega$ is a single q -periodic parabolic point, because Ω contains at most one critical point for $P_{c_0}^q$. If the degree is 2, (the critical point is simple) there is nothing to prove, because a degree two basin with locally connected boundary contains precisely one periodic orbit with period dividing that of the basin. If the degree is higher, or one does not want to rely on local connectivity, one may procede as follows: Let $z_0 \in R \cap K_{c_0}$ be arbitrary. Redoing the proof of Proposition 1.1 but with c_0 replaced by z_0 we find $w' \in R_{c_0}(\theta)$, $V' = V'(w', \epsilon)$ and holomorphic maps $\{\psi_{z_0, n} : V' \rightarrow K_{c_0}\}_{n \geq 0}$ satisfying the conclusions 1.-3. of that Proposition (with c_0 replaced by z_0). If $z_0 \in R \cap J_{c_0}$ then case 3.3b. occurs and z_0 is q -periodic. If z_0 belongs to the Fatou set then case 3.3a. occurs and in particular $\psi_{z_0, 0} : V' \rightarrow U$ is univalent with $\psi_{z_0, 0}(w') = z_0$, $\psi_{z_0, 0}(P_{c_0}^q(w')) = P_{c_0}^q(z_0)$ and

$$\psi_{z_0, 0}(\{z \in R_{c_0}(\theta) | g_{c_0}(z) \geq g_{c_0}(w')\}) \subseteq R \cap U \subseteq \psi_{z_0, 0}(R_{c_0}(\theta) \cap V').$$

Suppose there exists a bounded Fatou component Ω for which $L = R \cap \Omega$ connects two distinct q -periodic points α', β on the boundary of Ω . By the above any point $z \in L$ is connected to $P_{c_0}^q(z)$ in L through an analytic arc contained in L . Hence we can assume that α' is parabolic. Let $\phi : \Omega \rightarrow \mathbb{D}$ be a Riemann map say mapping α' to 1. Let $\hat{L} = \phi(L)$ and define $\hat{P}$ as the conjugate Blaschke product $\hat{P} = \phi \circ P_{c_0}^q \circ \phi^{-1}$. Then $\hat{L}$ connects the parabolic point 1 for $\hat{P}$ with a repelling fixed point $\hat{\beta} \neq 1$. In fact $\hat{L}$ is easily seen to contain an analytic arc $\hat{\gamma}$, which connects $\hat{\beta}$ to 1, and which is parametrized by an analytic diffeomorphism $\eta : \mathbb{R} \rightarrow \hat{\gamma}$ with $\eta(t+1) = \hat{P}(\eta(t))$.

Let $U_1, U_2 \subset \mathbb{D}$ be the two complimentary components of $\mathbb{D} \setminus \eta$. Consider say U_1 . It contains a unique connected component U'_1 of $P^{-1}(U_1)$, because the dynamics of P on η is diffeomorphically conjugate to translation by 1. The degree of the restriction $p : U'_1 \rightarrow U_1$ is at least 2 because $\overline{U_1} \cap \mathbb{S}^1$ is covered at least twice by $\overline{U'_1} \cap \mathbb{S}^1$. Thus U_1 contains a critical point. And simillarly so do U_2 . This contradicts that P has only one critical point. It follows that R contains precisely one q -periodic point. The parabolic one to which c_0 is iterated under $P_{c_0}^q$. q.e.d.

Remark 1.7 Consider the family $Q_c(z) = z^4 - 2z^3 + z^2 + z + c$, $c \in \mathbb{C}$ of quartic polynomials. For the polynomial Q_0 the real axis contains one critical point $\omega \in]-\frac{1}{3}, 0[$, whose critical value v_0 is contained in the same interval and is attracted to the parabolic fixed point 0. The two other critical points are

complex conjugate and attracted to the parabolic fixed point 1. Moreover the external ray of argument 0 equals $[1, \infty]$. For any sequence $\{c_n\}_{n \in \mathbb{N}} \subset]0, \infty$ converging to 0 the Hausdorff limit R of the segments of external rays $[v_n, \infty]$ equals $[v_0, \infty]$. In this case the segment $]0, 1[$ is like the arc L in the proof of the above theorem.

Proof of Theorem 1: The cluster set $Cl_{M_d}(\theta) = \overline{R_{M_d}(\theta)} \setminus R_{M_d}(\theta) \subseteq \partial M_d$ is a continuum and in particular connected. Let $c \in Cl_{M_d}(\theta)$. By Propositions 1.2 and 1.4, c has either parabolic or Misiurewicz dynamics. Since the period q is fixed, this implies that c varies in a finite set and is thus unique, which proves convergence. The dichotomy follows from Proposition 1.2 and Proposition 1.4. The rest of the statements 1. and 2. follow by combining Proposition 1.2, Proposition 1.4 and Proposition 1.6. q.e.d.

References

- [CG] L. Carleson and T. Gamelin. *Complex Dynamics*. Springer-Verlag, 1993.
- [DH] A. Douady and J.H. Hubbard. *Etude Dynamique des Polynômes Complexes*. Publications Mathématique d'Orsay, 1985.
- [HS] J. H. Hubbard and D Schleicher. The Spider Algorithm. In *Proceedings of Symposia in Applied Mathematics*, volume 49, 1994.
- [Mil] J. Milnor. Periodic orbits, external rays and the Mandelbrot set; an expository account. Preprint, 1995.
- [Pet] C. L. Petersen. Convergence of (pre)periodic rays in parameter spaces. Manuscript, 1997.
- [Ryd] G. Ryd. *Iterations of one parameter families of complex polynomials*. PhD thesis, Department of Mathematics, KTH Stockholm, Sweden, 1997.
- [Sch] D. Schleicher. Rational Parameter Rays of the Mandelbrot Set. November 1997, IMS Preprint 1997/13 SUNY StonyBrook.

Liste over tidligere udkomne tekster
tilsendes gerne. Henvendelse herom kan
ske til IMFUFA's sekretariat

tlf. 46 74 22 63

-
- 217/92 "Two papers on APPLICATIONS AND MODELLING
IN THE MATHEMATICS CURRICULUM"
by: Mogens Niss
- 218/92 "A Three-Square Theorem"
by: Lars Kadison
- 219/92 "RUPNOK - stationær strømning i elastiske rør"
af: Anja Boisen, Karen Birkelund, Mette Olufsen
Vejleder: Jesper Larsen
- 220/92 "Automatisk diagnosticering i digitale kredsløb"
af: Bjørn Christensen, Ole Møller Nielsen
Vejleder: Stig Andur Pedersen
- 221/92 "A BUNDLE VALUED RADON TRANSFORM, WITH
APPLICATIONS TO INVARIANT WAVE EQUATIONS"
by: Thomas P. Branson, Gestur Olafsson and
Henrik Schlichtkrull
- 222/92 On the Representations of some Infinite Dimensional
Groups and Algebras Related to Quantum Physics
by: Johnny T. Ottesen
- 223/92 THE FUNCTIONAL DETERMINANT
by: Thomas P. Branson
- 224/92 UNIVERSAL AC CONDUCTIVITY OF NON-METALLIC SOLIDS AT
LOW TEMPERATURES
by: Jeppe C. Dyre
- 225/92 "HATMODELLEN" Impedansspektroskopi i ultrarent
en-krystallinsk silicium
af: Anja Boisen, Anders Gorm Larsen, Jesper Varmer,
Johannes K. Nielsen, Kit R. Hansen, Peter Bøggild
og Thomas Hougaard
Vejleder: Petr Viscor
- 226/92 "METHODS AND MODELS FOR ESTIMATING THE GLOBAL
CIRCULATION OF SELECTED EMISSIONS FROM ENERGY
CONVERSION"
by: Bent Sørensen

- 227/92 "Computersimulering og fysik"
af: Per M. Hansen, Steffen Holm,
Peter Maibom, Mads K. Dall Petersen,
Pernille Postgaard, Thomas B. Schroeder,
Ivar P. Zeck
Vejleder: Peder Voetmann Christiansen
- 228/92 "Teknologi og historie"
Fire artikler af:
Mogens Niss, Jens Højrup, Eb Thiersen,
Hans Hedal
- 229/92 "Masser af information uden betydning"
En diskussion af informationsteorien
i Tor Norretranders' "Mærk Verden" og
en skitse til et alternativ baseret
på andenordens kybernetik og semiotik.
af: Søren Brier
- 230/92 "Vinklens tredeling - et klassisk
problem"
et matematisk projekt af
Karen Birkelund, Bjørn Christensen
Vejleder: Johnny Ottesen
- 231A/92 "Elektrondiffusion i silicium - en
matematisk model"
af: Jesper Voetmann, Karen Birkelund,
Mette Olufsen, Ole Møller Nielsen
Vejledere: Johnny Ottesen, H.B. Hansen
- 231B/92 "Elektrondiffusion i silicium - en
matematisk model" Kildetekster
af: Jesper Voetmann, Karen Birkelund,
Mette Olufsen, Ole Møller Nielsen
Vejledere: Johnny Ottesen, H.B. Hansen
- 232/92 "Undersøgelse om den simultane opdagelse
af energiens bevarelse og isærdeles om
de af Mayer, Colding, Joule og Helmholtz
udførte arbejder"
af: L. Arleth, G.I. Dybkjær, M.T. Østergård
Vejleder: Dorte Posselt
- 233/92 "The effect of age-dependent host
mortality on the dynamics of an endemic
disease and
instability in an SIR-model with age-
dependent susceptibility
by: Viggo Andreassen
- 234/92 "THE FUNCTIONAL DETERMINANT OF A FOUR-DIMENSIONAL
BOUNDARY VALUE PROBLEM"
by: Thomas P. Branson and Peter B. Gilkey
- 235/92 OVERFLADESTRUKTUR OG POREUDVIKLING AF KOKS
- Modul 3 fysik projekt -
af: Thomas Jessen
-

- 236a/93 INTRODUKTION TIL KVANTE
HALL EFFEKTEN
af: Anja Boisen, Peter Bøggild
Vejleder: Peder Voetmann Christiansen
Erland Brun Hansen
- 236b/93 STRØMSSAMMENBRUD AF KVANTE
HALL EFFEKTEN
af: Anja Boisen, Peter Bøggild
Vejleder: Peder Voetmann Christiansen
Erland Brun Hansen
- 237/93 The Wedderburn principal theorem and
Shukla cohomology
af: Lars Kadison
- 238/93 SEMIOTIK OG SYSTEMEGENSKABER (2)
Vektorbånd og tensorer
af: Peder Voetmann Christiansen
- 239/93 Valgsystemer - Modelbygning og analyse
Matematik 2. modul
af: Charlotte Gjerrild, Jane Hansen,
Maria Hermannsson, Allan Jørgensen,
Ragna Clauson-Kaas, Poul Lützen
Vejleder: Mogens Niss
- 240/93 Patologiske eksempler.
Om sære matematiske fæns betydning for
den matematiske udvikling
af: Claus Dråby, Jørn Skov Hansen, Runa
Ulsøe Johansen, Peter Meibom, Johannes
Kristoffer Nielsen
Vejleder: Mogens Niss
- 241/93 FOTOVOLTAISK STATUSNOTAT 1
af: Bent Sørensen
- 242/93 Brovedligeholdelse - bevar mig vel
Analyse af Vejdirektoratets model for
optimering af broreparationer
af: Linda Kyndlev, Kare Fundal, Kamma
Tulinius, Ivar Zeck
Vejleder: Jesper Larsen
- 243/93 TANKEEKSPERIMENTER I FYSIKKEN
Et 1.modul fysikprojekt
af: Karen Birkelund, Stine Sofia Korremann
Vejleder: Dorthe Posselt
- 244/93 RADONTRANSFORMATIONEN og dens anvendelse
i CT-scanning
Projektrapport
af: Trine Andreassen, Tine Guldager Christiansen,
Nina Skov Hansen og Christine Iversen
Vejledere: Gestur Olafsson og Jesper Larsen
- 245a+b
/93 Time-Of-Flight målinger på krystallinske
halvledere
Specialerapport
af: Linda Szkotak Jensen og Lise Odgaard Gade
Vejledere: Petr Viscor og Niels Boye Olsen
- 246/93 HVERDAGSVIDEN OG MATEMATIK
- LÆREPROCESSER I SKOLEN
af: Lena Lindenskov, Statens Humanistiske
Forskningsråd, RUC, IMFUFA
- 247/93 UNIVERSAL LOW TEMPERATURE AC CON-
DUCTIVITY OF MACROSCOPICALLY
DISORDERED NON-METALS
by: Jeppe C. Dyre
- 248/93 DIRAC OPERATORS AND MANIFOLDS WITH
BOUNDARY
by: B. Booss-Bavnbek, K.P.Wojciechowski
- 249/93 Perspectives on Teichmüller and the
Jahresbericht Addendum to Schappacher,
Scholz, et al.
by: B. Booss-Bavnbek
With comments by W.Abikoff, L.Ahlfors,
J.Cerf, P.J.Davis, W.Fuchs, F.P.Gardiner,
J.Jost, J.-P.Kahane, R.Lohan, L.Lorch,
J.Radkau and T.Söderqvist
- 250/93 EULER OG BOLZANO - MATEMATISK ANALYSE SET I ET
VIDENSKABSTEORETISK PERSPEKTIV
Projektrapport af: Anja Juul, Lone Michelser,
Tomas Højgård Jensen
Vejleder: Stig Andur Pedersen
- 251/93 Genotypic Proportions in Hybrid Zones
by: Freddy Bugge Christiansen, Viggo Andreassen
and Ebbe Thue Poulsen
- 252/93 MODELLERING AF TILFÆLDIGE FÆNOMENER
Projektrapport af: Birthe Friis, Lisbeth Helmgård,
Kristina Charlotte Jakobsen, Marina Mosbæk
Johannessen, Lotte Ludvigsen, Mette Bass Nielsen
- 253/93 Kuglepakning
Teori og model
af: Lise Arleth, Kåre Fundal, Nils Kruse
Vejleder: Mogens Niss
- 254/93 Regressionsanalyse
Materiale til et statistik kursus
af: Jørgen Larsen
- 255/93 TID & BETINGET UAFHÆNGIGHED
af: Peter Barremoës
- 256/93 Determination of the Frequency Dependent
Bulk Modulus of Liquids Using a Piezo-
electric Spherical Shell (Preprint)
by: T. Christensen and N.B.Olsen
- 257/93 Modellering af dispersion i piezoelektriske
keramikker
af: Pernille Postgaard, Jønnik Rasmussen,
Christina Specht, Mikko Østergård
Vejleder: Tage Christensen
- 258/93 Supplerende kursmateriale til
"Lineære strukturer fra algebra og analyse"
af: Mogens Brun Heefelt
- 259/93 STUDIES OF AC HOPPING CONDUCTION AT LOW
TEMPERATURES
by: Jeppe C. Dyre
- 260/93 PARTITIONED MANIFOLDS AND INVARIANTS IN
DIMENSIONS 2, 3, AND 4
by: B. Booss-Bavnbek, K.P.Wojciechowski

- 261/93 OPGAVESAMLING
Bredde-kursus i Fysik
Eksamensopgaver fra 1976-93
- 262/93 Separability and the Jones Polynomial
by: Lars Kadison
- 263/93 Supplerende kursusbemateriale til
"Lineære strukturer fra algebra
og analyse" II
af: Mogens Brun Heefelt
- 264/93 FOTOVOLTAISK STATUSNOTAT 2
af: Bent Sørensen
-
- 265/94 SPHERICAL FUNCTIONS ON ORDERED
SYMMETRIC SPACES
To Sigurdur Helgason on his
sixtyfifth birthday
by: Jacques Faraut, Joachim Hilgert
and Gestur Olafsson
- 266/94 Kommensurabilitets-oscillationer i
laterale supergitre
Fysikspeciale af: Anja Boisen,
Peter Bøggild, Karen Birkelund
Vejledere: Rafael Taboryski, Poul Erik
Lindelof, Peder Voetmann Christiansen
- 267/94 Kom til kort med matematik på
Eksperimentarium - Et forslag til en
opstilling
af: Charlotte Gjerrild, Jane Hansen
Vejleder: Bernhelm Booss-Bavnbek
- 268/94 Life is like a sewer ...
Et projekt om modellering af aorta via
en model for strømning i kloakrør
af: Anders Marcussen, Anne C. Nilsson,
Lone Michelsen, Per M. Hansen
Vejleder: Jesper Larsen
- 269/94 Dimensionsanalyse en introduktion
metaprojekt, fysik
af: Tine Guldager Christiansen,
Ken Andersen, Nikolaj Hermann,
Jannik Rasmussen
Vejleder: Jens Højgaard Jensen
- 270/94 THE IMAGE OF THE ENVELOPING ALGEBRA
AND IRREDUCIBILITY OF INDUCED REPRESENTATIONS OF EXPONENTIAL LIE GROUPS
by: Jacob Jacobsen
- 271/94 Matematikken i Fysikken.
Opdaget eller opfundet
NAT-BAS-projekt
vejleder: Jens Højgaard Jensen
- 272/94 Tradition og fornyelse
Det praktiske elevarbejde i gymnasiets
fysikundervisning, 1907-1988
af: Kristian Hoppe og Jeppe Guldager
Vejledning: Karin Beyer og Nils Hybel
- 273/94 Model for kort- og mellemdistanceløb
Verifikation af model
af: Lise Fabricius Christensen, Helle Pilemann,
Bettina Sørensen
Vejleder: Mette Olufsen
- 274/94 MODEL 10 - en matematisk model af intravenøse
anæstetikas farmakokinetik
3. modul matematik, forår 1994
af: Trine Andreasen, Bjørn Christensen, Christine
Green, Anja Skjoldborg Hansen, Lisbeth
Helmgaard
Vejledere: Viggo Andreasen & Jesper Larsen
- 275/94 Perspectives on Teichmüller and the Jahresbericht
2nd Edition
by: Bernhelm Booss-Bavnbek
- 276/94 Dispersionsmodellering
Projektrapport 1. modul
af: Gitte Andersen, Rehannah Borup, Lisbeth Friis,
Per Gregersen, Kristina Vejre
Vejleder: Bernhelm Booss-Bavnbek
- 277/94 PROJEKTARBEJDSPEÐAGOGIK - Om tre tolkninger af
problemorienteret projektarbejde
af: Claus Flensted Behrens, Frederik Voetmann
Christiansen, Jørn Skov Hansen, Thomas
Thingstrup
Vejleder: Jens Højgaard Jensen
- 278/94 The Models Underlying the Anaesthesia
Simulator Sophus
by: Mette Olufsen(Math-Tech), Finn Nielsen
(RISØ National Laboratory), Per Føge Jensen
(Herlev University Hospital), Stig Andur
Pedersen (Roskilde University)
- 279/94 Description of a method of measuring the shear
modulus of supercooled liquids and a comparison
of their thermal and mechanical response
functions.
af: Tage Christensen
- 280/94 A Course in Projective Geometry
by Lars Kadison and Matthias T. Kromann
- 281/94 Modellering af Det Cardiovasculære System med
Neural Puls kontrol
Projektrapport udarbejdet af:
Stefan Frello, Runa Ulsøe Johansen,
Michael Poul Curt Hansen, Klaus Dahl Jensen
Vejleder: Viggo Andreasen
- 282/94 Parallelle algoritmer
af: Erwin Dan Nielsen, Jan Danielsen,
Niels Bo Johansen

- 283/94 Grænser for tilfældighed
(en kaotisk talgenerator)
af: Erwin Dan Nielsen og Niels Bo Johansen
- 284/94 Det er ikke til at se det, hvis man ikke
lige ve' det!
Gymnasimatematikens begrundelsesproblem
En specialerapport af Peter Hauge Jensen
og Linda Kyndlev
Vejleder: Mogens Niss
- 285/94 Slow coevolution of a viral pathogen and
its diploid host
by: Viggo Andreassen and
Freddy B. Christiansen
- 286/94 The energy master equation: A low-temperature
approximation to Bässler's random walk model
by: Jeppe C. Dyre
- 287/94 A Statistical Mechanical Approximation for the
Calculation of Time Auto-Correlation Functions
by: Jeppe C. Dyre
- 288/95 PROGRESS IN WIND ENERGY UTILIZATION
by: Bent Sørensen
- 289/95 Universal Time-Dependence of the Mean-Square
Displacement in Extremely Rugged Energy
Landscapes with Equal Minima
by: Jeppe C. Dyre and Jacob Jacobsen
- 290/95 Modellering af uregelmæssige bølger
Et 3.modul matematik projekt
af: Anders Marcussen, Anne Charlotte Nilsson,
Lone Michelsen, Per Mørkegaard Hansen
Vejleder: Jesper Larsen
- 291/95 1st Annual Report from the project
LIFE-CYCLE ANALYSIS OF THE TOTAL DANISH
ENERGY SYSTEM
an example of using methods developed for the
OECD/IEA and the US/EU fuel cycle externality study
by: Bent Sørensen
- 292/95 Fotovoltaisk Statusnotat 3
af: Bent Sørensen
- 293/95 Geometridiskussionen - hvor blev den af?
af: Lotte Ludvigsen & Jens Frandsen
Vejleder: Anders Madsen
- 294/95 Universets udvidelse -
et metaprojekt
Af: Jesper Duelund og Birthe Friis
Vejleder: Ib Lundgaard Rasmussen
- 295/95 A Review of Mathematical Modeling of the
Controlled Cardiovascular System
By: Johnny T. Ottesen
- 296/95 RETIKULER den klassiske mekanik
af: Peder Voetmann Christiansen
- 297/95 A fluid-dynamical model of the aorta with
bifurcations
by: Mette Olufsen and Johnny Ottesen
- 298/95 Mordet på Schrödingers kat - et metaprojekt om
to fortolkninger af kvantemekanikken
af: Maria Hermannsson, Sebastian Horst,
Christina Specht
Vejledere: Jeppe Dyre og Peder Voetmann Christiansen
- 299/95 ADAM under figenbladet - et kig på en samfunds-
videnskabelig matematisk model
Et matematisk modelprojekt
af: Claus Dræby, Michael Hansen, Tomas Højgård Jensen
Vejleder: Jørgen Larsen
- 300/95 Scenarios for Greenhouse Warming Mitigation
by: Bent Sørensen
- 301/95 TOK Modellering af træers vækst under påvirkning
af ozon
af: Glenn Møller-Holst, Marina Johannessen, Birthe
Nielsen og Bettina Sørensen
Vejleder: Jesper Larsen
- 302/95 KOMPRESSORER - Analyse af en matematisk model for
aksialkompressorer
Projektrapport af: Stine Bøggild, Jakob Hilmer,
Pernille Postgaard
Vejleder: Viggo Andreassen
- 303/95 Masterlignings-modeller af Glasovergangen
Termisk-Mekanisk Relaksation
Specialerapport udarbejdet af:
Johannes K. Nielsen, Klaus Dahl Jensen
Vejledere: Jeppe C. Dyre, Jørgen Larsen
- 304a/95 STATISTIKNOTER Simple binomialfordelingsmodeller
af: Jørgen Larsen
- 304b/95 STATISTIKNOTER Simple normalfordelingsmodeller
af: Jørgen Larsen
- 304c/95 STATISTIKNOTER Simple Poissonfordelingsmodeller
af: Jørgen Larsen
- 304d/95 STATISTIKNOTER Simple multinomialfordelingsmodeller
af: Jørgen Larsen
- 304e/95 STATISTIKNOTER Mindre matematisk-statistisk opslagsværk
indeholdende bl.a. ordforklaringer, resuméer og
tabeller
af: Jørgen Larsen

- 305/95 The Maslov Index:
A Functional Analytical Definition
And The Spectral Flow Formula
By: B. Booss-Bavnbek, K. Furutani
- 306/95 Goals of mathematics teaching
Preprint of a chapter for the forthcoming International Handbook of Mathematics Education (Alan J. Bishop, ed)
By: Mogens Niss
- 307/95 Habit Formation and the Thirdness of Signs
Presented at the semiotic symposium
The Emergence of Codes and Intensions as a Basis of Sign Processes
By: Peder Voetmann Christiansen
- 308/95 Metaforer i Fysikken
af: Marianne Wilcken Bjerregaard, Frederik Voetmann Christiansen, Jørn Skov Hansen, Klaus Dahl Jensen, Ole Schmidt
Vejledere: Peder Voetmann Christiansen og Petr Viscor
- 309/95 Tiden og Tanken
En undersøgelse af begrebsverdenen Matematik udført ved hjælp af en analogi med tid
af: Anita Stark og Randi Petersen
Vejleder: Bernhelm Booss-Bavnbek
-
- 310/96 Kursusmateriale til "Lineære strukturer fra algebra og analyse" (E1)
af: Mogens Brun Heefelt
- 311/96 2nd Annual Report from the project
LIFE-CYCLE ANALYSIS OF THE TOTAL DANISH ENERGY SYSTEM
by: Hélène Connor-Lajambe, Bernd Kuemmel, Stefan Krüger Nielsen, Bent Sørensen
- 312/96 Grassmannian and Chiral Anomaly
by: B. Booss-Bavnbek, K.P. Wojciechowski
- 313/96 THE IRREDUCIBILITY OF CHANCE AND THE OPENNESS OF THE FUTURE
The Logical Function of Idealism in Peirce's Philosophy of Nature
By: Helmut Pape, University of Hannover
- 314/96 Feedback Regulation of Mammalian Cardiovascular System
By: Johnny T. Ottesen
- 315/96 "Rejsen til tidens indre" - Udarbejdelse af a + b et manuskript til en fjernsynsudsendelse + manuskript
af: Gunhild Hune og Karina Goyle
Vejledere: Peder Voetmann Christiansen og Bruno Ingemann
- 316/96 Plasmaoscillation i natriumklynger
Specialerapport af: Peter Meibom, Mikko Østergård
Vejledere: Jeppe Dyre & Jørn Borggreen
- 317/96 Poincaré og symplektiske algoritmer
af: Ulla Rasmussen
Vejleder: Anders Madsen
- 318/96 Modelling the Respiratory System
by: Tine Guldager Christiansen, Claus Dræby
Supervisors: Viggo Andreassen, Michael Danielsen
- 319/96 Externality Estimation of Greenhouse Warming Impacts
by: Bent Sørensen
- 320/96 Grassmannian and Boundary Contribution to the -Determinant
by: K.P. Wojciechowski et al.
- 321/96 Modelkompetencer - udvikling og afprøvning af et begrebsapparat
Specialerapport af: Nina Skov Hansen, Christine Iversen, Kristin Troels-Smith
Vejleder: Morten Blomhøj
- 322/96 OPGAVESAMLING
Bredde-Kursus i Fysik 1976 - 1996
- 323/96 Structure and Dynamics of Symmetric Diblock Copolymers
PhD Thesis
by: Christine Maria Papadakis
- 324/96 Non-linearity of Baroreceptor Nerves
by: Johnny T. Ottesen
- 325/96 Retorik eller realitet ?
Anvendelser af matematik i det danske Gymnasiums matematikundervisning i perioden 1903 - 88
Specialerapport af Helle Pilemann
Vejleder: Mogens Niss
- 326/96 Bevist teori
Eksemplificeret ved Gentzens bevis for konsistensen af teorien om de naturlige tal
af: Gitte Andersen, Lise Mariane Jeppesen, Klaus Frovin Jørgensen, Ivar Peter Zeck
Vejledere: Bernhelm Booss-Bavnbek og Stig Andur Pedersen
- 327/96 NON-LINEAR MODELLING OF INTEGRATED ENERGY SUPPLY AND DEMAND MATCHING SYSTEMS
by: Bent Sørensen
- 328/96 Calculating Fuel Transport Emissions
by: Bernd Kuemmel

- 329/96 The dynamics of cocirculating influenza strains conferring partial cross-immunity and
A model of influenza A drift evolution
by: Viggo Andreasen, Juan Lin and Simon Levin
- 330/96 LONG-TERM INTEGRATION OF PHOTOVOLTAICS INTO THE GLOBAL ENERGY SYSTEM
by: Bent Sørensen
- 331/96 Viskøse fingre
Specialerapport af:
Vibeke Orlén og Christina Specht
Vejledere: Jacob M. Jacobsen og Jesper Larsen
-
- 332/97 ANOMAL SWELLING AF LIPIDE DOBBELTLAG
Specialerapport af:
Stine Sofia Korremann
Vejleder: Dorthe Posselt
- 333/97 Biodiversity Matters
an extension of methods found in the literature on monetisation of biodiversity
by: Bernd Kuemmel
- 334/97 LIFE-CYCLE ANALYSIS OF THE TOTAL DANISH ENERGY SYSTEM
by: Bernd Kuemmel and Bent Sørensen
- 335/97 Dynamics of Amorphous Solids and Viscous Liquids
by: Jeppe C. Dyre
- 336/97 PROBLEM-ORIENTATED GROUP PROJECT WORK AT ROSKILDE UNIVERSITY
by: Kathrine Legge
- 337/97 Verdensbankens globale befolkningsprognose - et projekt om matematisk modellering
af: Jørn Chr. Bendtsen, Kurt Jensen, Per Pauli Petersen
Vejleder: Jørgen Larsen
- 338/97 Kvantisering af nanolederes elektriske ledningsevne
Første modul fysikprojekt
af: Søren Dam, Esben Danielsen, Martin Niss, Esben Friis Pedersen, Frederik Resen Steenstrup
Vejleder: Tage Christensen
- 339/97 Defining Discipline
by: Wolfgang Coy
- 340/97 Prime ends revisited - a geometric point of view -
by: Carsten Lunde Petersen
- 341/97 Two chapters on the teaching, learning and assessment of geometry
by Mogens Niss
- 342/97 LONG-TERM SCENARIOS FOR GLOBAL ENERGY DEMAND AND SUPPLY
A global clean fossil scenario discussion paper prepared by Bernd Kuemmel
Project leader: Bent Sørensen
- 343/97 IMPORT/EKSPORT-POLITIK SOM REDSKAB TIL OPTIMERET UDNYTTELSE AF EL PRODUCERET PÅ VE-ANLÆG
af: Peter Meibom, Torben Svendsen, Bent Sørensen
- 344/97 Puzzles and Siegel disks
by Carsten Lunde Petersen
-
- 345/98 Modeling the Arterial System with Reference to an Anesthesia Simulator
Ph.D. Thesis
by: Mette Sofie Olufsen
- 346/98 Klyngedannelse i en hulkatode-forstøvningsproces
af: Sebastian Horst
Vejledere: Jørn Borggren, NBI, Niels Boye Olsen
- 347/98 Verificering af Matematiske Modeller - en analyse af Den Danske Eulerske Model
af: Jonas Blomqvist, Tom Pedersen, Karen Timmermann, Lisbet Øhlenschläger
Vejleder: Bernhelm Booss-Bavnbek
- 348/98 Case study of the environmental permission procedure and the environmental impact assessment for power plants in Denmark
by: Stefan Krüger Nielsen
Project leader: Bent Sørensen
- 349/98 Tre rapporter fra FAGMAT - et projekt om tal og faglig matematik i arbejdsmarkedsuddannelserne
af: Lena Lindenskov og Tine Wedege
- 350/98 OPGAVERSAMLING - Bredde-Kursus i Fysik 1976 - 1998
Erstatter teksterne 3/78, 261/93 og 322/96
- 351/98 Aspects of the Nature and State of Research in Mathematics Education
by: Mogens Niss